Non-Abelian Vortices and Domain Walls in Two Higgs Doublet Models

Masafumi Kurachi
Keio University / KEK

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References

``Non-Abelian strings and domain walls in two Higgs doublet models,''
M. Eto, M. Kurachi and M. Nitta,
JHEP 08 (2018) 195, arXiv:1805.07015 [hep-ph]

``Constraints on two Higgs doublet models from domain walls,''
M. Eto, M. Kurachi and M. Nitta,
arXiv:1803.04662 [hep-ph] (to be published in Phys. Lett. B).

Outline

1. Introduction
2. Domain walls in the 2HDM
3. Non-Abelian vortices in the 2HDM
4. Summary and Prospects

1. Introduction

Introduction

Stable non-trivial field configurations (solitons) can exist when a homotopy group of the vacuum manifold of the theory is non-trivial:

(necessary condition)		(possible topological object)
$\pi_0(G/H) \neq 1$	$\longrightarrow$	Domain wall
$\pi_1(G/H) \neq 1$	$\longrightarrow$	Vortex
$\pi_2(G/H) \neq 1$	$\longrightarrow$	Monopole
$\pi_3(G/H) \neq 1$	$\longrightarrow$	Skyrmion

Such topological objects, once created during the evolution of the Universe, have an impact on cosmology

Introduction

There is no such object in the SM

Introduction

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Discovery of the remnant of
any topological object

$\Updownarrow$

Existence of the physics beyond the SM!!

Introduction

The vacuum structure and topological objects in the 2HDMs have been studied extensively

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Recent and future experimental progress
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make it more interesting at quantitative level

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make it more interesting at quantitative level

It is worth summarizing what happens in different regions of parameter space, and construct numerical solutions of various topological objects for further quantitative studies

Theoretical knowledge of topology $\Longleftrightarrow$ **Phenomenology**

2. Domain walls in the 2HDM

Two Higgs doublet model

$$\mathcal{L} = -\frac{1}{4}B_{\mu\nu}B^{\mu\nu} - \frac{1}{4}W^a_{\mu\nu}W^{a\mu\nu} + \sum_{i=1,2}\left(D_\mu\Phi_i^\dagger D^\mu\Phi_i\right) - V(\Phi_1,\Phi_2)$$

$$D_\mu\Phi_i = \partial_\mu\Phi_i - igW_\mu\Phi_i - i\frac{g'}{2}B_\mu\Phi_i$$

$$\begin{aligned} V(\Phi_1,\Phi_2) \;=\; & m_{11}^2\Phi_1^\dagger\Phi_1 \;+\; m_{22}^2\Phi_2^\dagger\Phi_2 \;-\; \left(m_{12}^2\Phi_1^\dagger\Phi_2 + \mathrm{h.c.}\right) \;+\; \frac{\beta_1}{2}\left(\Phi_1^\dagger\Phi_1\right)^2 \;+\; \frac{\beta_2}{2}\left(\Phi_2^\dagger\Phi_2\right)^2 \\ & +\; \beta_3\left(\Phi_1^\dagger\Phi_1\right)\left(\Phi_2^\dagger\Phi_2\right) \;+\; \beta_4\left(\Phi_1^\dagger\Phi_2\right)\left(\Phi_2^\dagger\Phi_1\right) \;+\; \left\{\frac{\beta_5}{2}\left(\Phi_1^\dagger\Phi_2\right)^2 + \mathrm{h.c.}\right\} \\ & +\; \left\{\left[\beta_6\Phi_1^\dagger\Phi_1 + \beta_7\Phi_2^\dagger\Phi_2\right]\left(\Phi_1^\dagger\Phi_2\right) + \mathrm{h.c.}\right\} \end{aligned}$$

Two Higgs doublet model

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Require (softly broken) Z2 symmetry

$$(\Phi_1 \to +\Phi_1,\;\; \Phi_2 \to -\Phi_2)$$

Two Higgs doublet model

$$\mathcal{L} = -\frac{1}{4}B_{\mu\nu}B^{\mu\nu} - \frac{1}{4}W^a_{\mu\nu}W^{a\mu\nu} + \sum_{i=1,2}\left(D_\mu\Phi_i^\dagger D^\mu\Phi_i\right) - V(\Phi_1,\Phi_2)$$

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$$+ \beta_3\left(\Phi_1^\dagger\Phi_1\right)\left(\Phi_2^\dagger\Phi_2\right) + \beta_4\left(\Phi_1^\dagger\Phi_2\right)\left(\Phi_2^\dagger\Phi_1\right) + \left\{\frac{\beta_5}{2}\left(\Phi_1^\dagger\Phi_2\right)^2 + \text{h.c.}\right\}$$

$$\boxed{\cancel{+ \left\{\left[\beta_6\Phi_1^\dagger\Phi_1 + \beta_7\Phi_2^\dagger\Phi_2\right]\left(\Phi_1^\dagger\Phi_2\right) + \text{h.c.}\right\}}}$$

Assume the CP invariance (for the moment)

$$(\, m_{12}^2,\ \beta_5\ :\ \text{real}\,)$$

What makes the difference? **SM** ⟷ **2HDM**

$$\mathcal{L} = -\frac{1}{4}B_{\mu\nu}B^{\mu\nu} - \frac{1}{4}W^a_{\mu\nu}W^{a\mu\nu} + \sum_{i=1,2}\left(D_\mu\Phi_i^\dagger D^\mu\Phi_i\right) - V(\Phi_1,\Phi_2)$$

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$$\begin{aligned} V(\Phi_1,\Phi_2) \;=\; & m_{11}^2\Phi_1^\dagger\Phi_1 + m_{22}^2\Phi_2^\dagger\Phi_2 - \left(m_{12}^2\Phi_1^\dagger\Phi_2 + \text{h.c.}\right) + \frac{\beta_1}{2}\left(\Phi_1^\dagger\Phi_1\right)^2 + \frac{\beta_2}{2}\left(\Phi_2^\dagger\Phi_2\right)^2 \\ & + \beta_3\left(\Phi_1^\dagger\Phi_1\right)\left(\Phi_2^\dagger\Phi_2\right) + \beta_4\left(\Phi_1^\dagger\Phi_2\right)\left(\Phi_2^\dagger\Phi_1\right) + \left\{\frac{\beta_5}{2}\left(\Phi_1^\dagger\Phi_2\right)^2 + \text{h.c.}\right\} \\ & \boxed{\cancel{+ \left\{\left[\beta_6\Phi_1^\dagger\Phi_1 + \beta_7\Phi_2^\dagger\Phi_2\right]\left(\Phi_1^\dagger\Phi_2\right) + \text{h.c.}\right\}}} \end{aligned}$$

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Relative phase of two Higgs fields: $\Phi_1 = \underline{e^{-i\alpha}} \begin{pmatrix} 0 \\ v_1 \end{pmatrix}$ $\Phi_2 = \underline{e^{+i\alpha}} \begin{pmatrix} 0 \\ v_2 \end{pmatrix}$

$$V(\Phi_1, \Phi_2) = m_{11}^2 \Phi_1^\dagger \Phi_1 + m_{22}^2 \Phi_2^\dagger \Phi_2 - \left(m_{12}^2 \Phi_1^\dagger \Phi_2 + \text{h.c.} \right) + \frac{\beta_1}{2} \left(\Phi_1^\dagger \Phi_1 \right)^2 + \frac{\beta_2}{2} \left(\Phi_2^\dagger \Phi_2 \right)^2$$
$$+ \beta_3 \left(\Phi_1^\dagger \Phi_1 \right) \left(\Phi_2^\dagger \Phi_2 \right) + \beta_4 \left(\Phi_1^\dagger \Phi_2 \right) \left(\Phi_2^\dagger \Phi_1 \right) + \left\{ \frac{\beta_5}{2} \left(\Phi_1^\dagger \Phi_2 \right)^2 + \text{h.c.} \right\}$$
$$\boxed{\cancel{+ \left\{ \left[\beta_6 \Phi_1^\dagger \Phi_1 + \beta_7 \Phi_2^\dagger \Phi_2 \right] \left(\Phi_1^\dagger \Phi_2 \right) + \text{h.c.} \right\}}}$$

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Relative phase dependent terms

Assume the CP invariance (for the moment)

$$(m_{12}^2,\ \beta_5\ :\ \mathrm{real})$$

What makes the difference? **SM** $\longleftrightarrow$ **2HDM**

Relative phase of two Higgs fields: $\Phi_1 = \underline{e^{-i\alpha}} \begin{pmatrix} 0 \\ v_1 \end{pmatrix}$ $\Phi_2 = \underline{e^{+i\alpha}} \begin{pmatrix} 0 \\ v_2 \end{pmatrix}$

$$
\begin{aligned}
V_\xi(\alpha) &= -2m_{12}^2 v_1 v_2 \cos 2\alpha + \beta_5 v_1^2 v_2^2 \cos 4\alpha \\
&= (v_1 v_2)^2 \sqrt{4\left(m_{12}^2/v_1 v_2\right)^2 + \beta_5^2} \\
&\qquad \left(-\sin\xi \cos 2\alpha + \cos\xi \cos 4\alpha\right).
\end{aligned}
$$

$$
\boxed{
\begin{aligned}
\sin\xi &= \frac{2(m_{12}^2/v_1 v_2)}{\sqrt{4(m_{12}^2/v_1 v_2)^2 + \beta_5^2}}, \\
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\end{aligned}
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V(\Phi_1, \Phi_2) &= m_{11}^2 \Phi_1^\dagger \Phi_1 + m_{22}^2 \Phi_2^\dagger \Phi_2 \boxed{- \left(m_{12}^2 \Phi_1^\dagger \Phi_2 + \text{h.c.}\right)} + \frac{\beta_1}{2}\left(\Phi_1^\dagger \Phi_1\right)^2 + \frac{\beta_2}{2}\left(\Phi_2^\dagger \Phi_2\right)^2 \\
&\quad + \beta_3 \left(\Phi_1^\dagger \Phi_1\right)\left(\Phi_2^\dagger \Phi_2\right) + \beta_4 \left(\Phi_1^\dagger \Phi_2\right)\left(\Phi_2^\dagger \Phi_1\right) \boxed{+ \left\{\frac{\beta_5}{2}\left(\Phi_1^\dagger \Phi_2\right)^2 + \text{h.c.}\right\}} \\
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Relative phase dependent terms

$|||$

$V_\xi(\alpha)$

Assume the CP invariance (for the moment)

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What makes the difference? **SM** ⟷ **2HDM**

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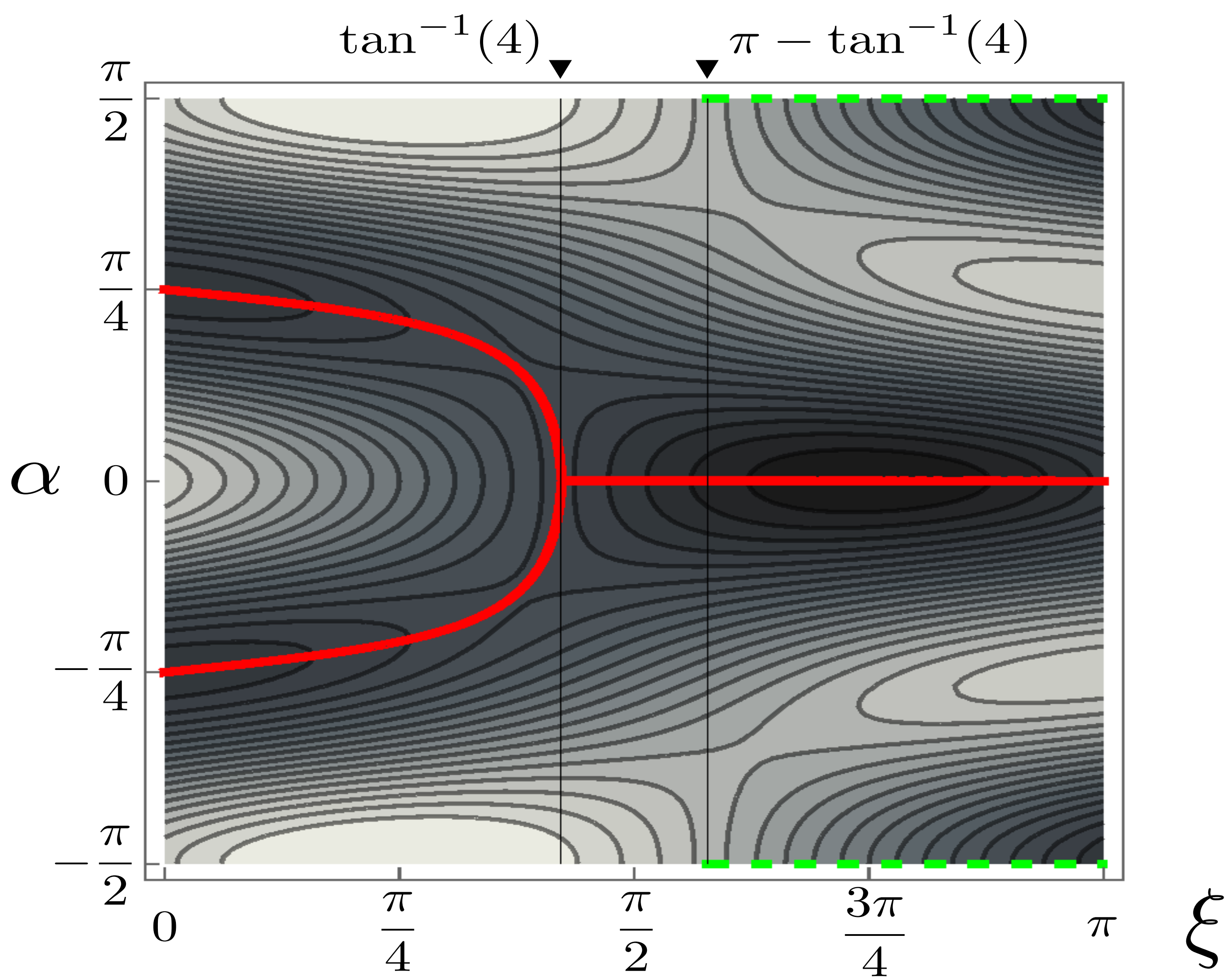
$$\begin{aligned} \sin\xi &= \frac{2(m_{12}^2/v_1 v_2)}{\sqrt{4(m_{12}^2/v_1 v_2)^2 + \beta_5^2}}, \\ \cos\xi &= \frac{\beta_5}{\sqrt{4(m_{12}^2/v_1 v_2)^2 + \beta_5^2}}. \end{aligned}$$



$0 \leq \xi \leq \pi$

m_{12}^2 can always be made positive by a field redefinition

$-\frac{\pi}{2} \leq \alpha \leq \frac{\pi}{2}$

fields with $\alpha = \delta$ and $\delta + \pi$ are physically equivalent (up to gauge transformation)

What makes the difference? SM ⟷ 2HDM

Relative phase of two Higgs fields: $\Phi_1 = \underline{e^{-i\alpha}} \begin{pmatrix} 0 \\ v_1 \end{pmatrix}$ $\Phi_2 = \underline{e^{+i\alpha}} \begin{pmatrix} 0 \\ v_2 \end{pmatrix}$

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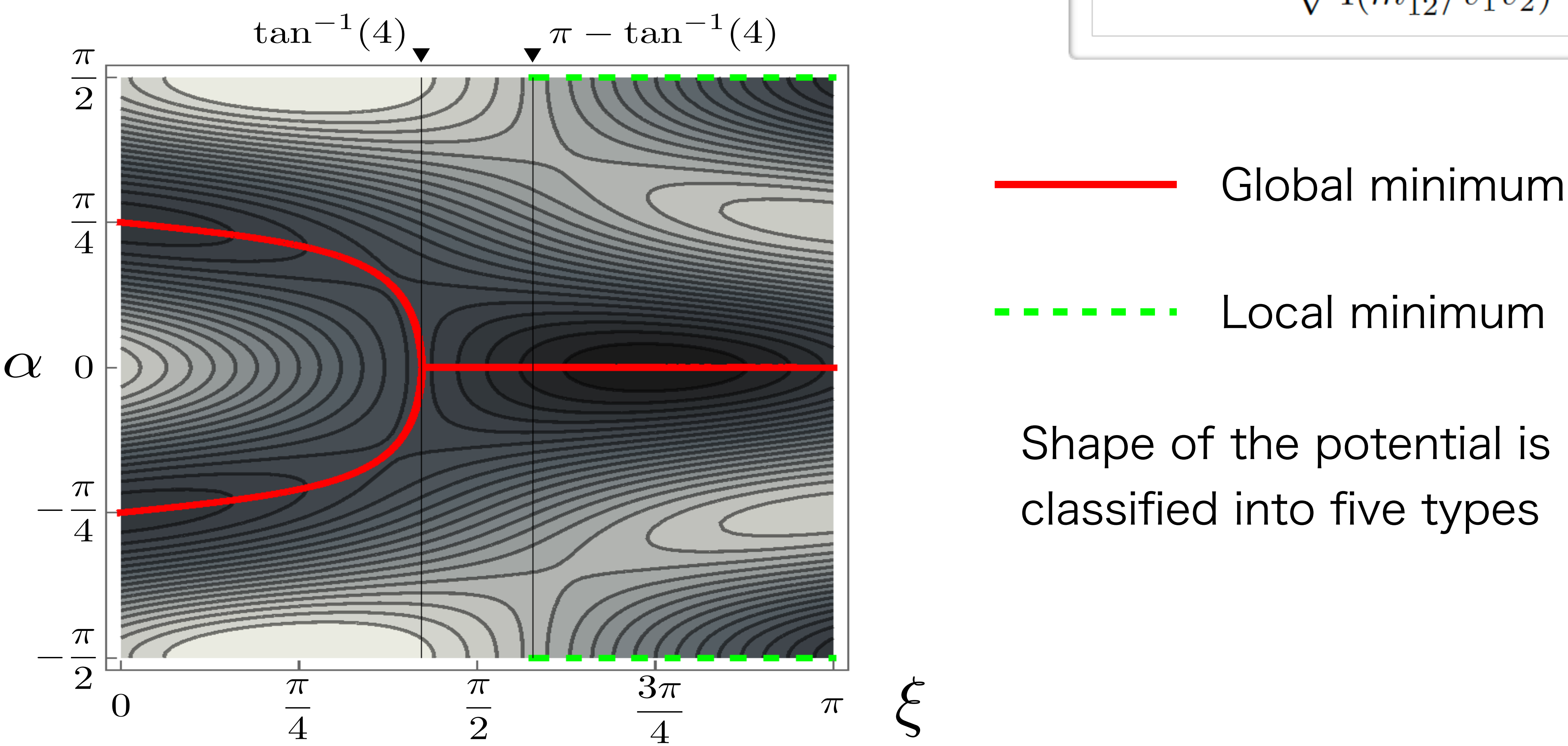

Shape of the potential is classified into five types

What makes the difference? SM ⟷ 2HDM

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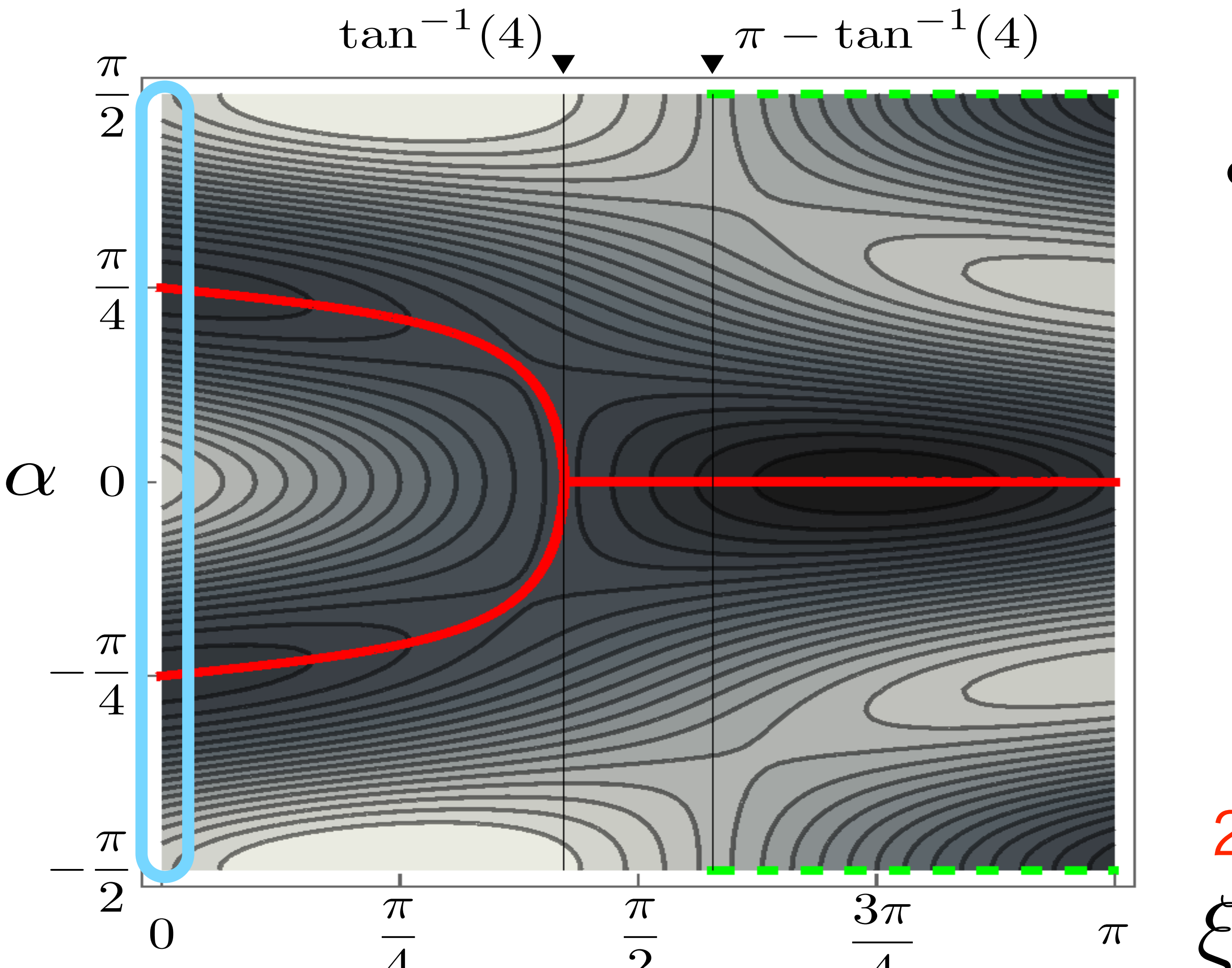
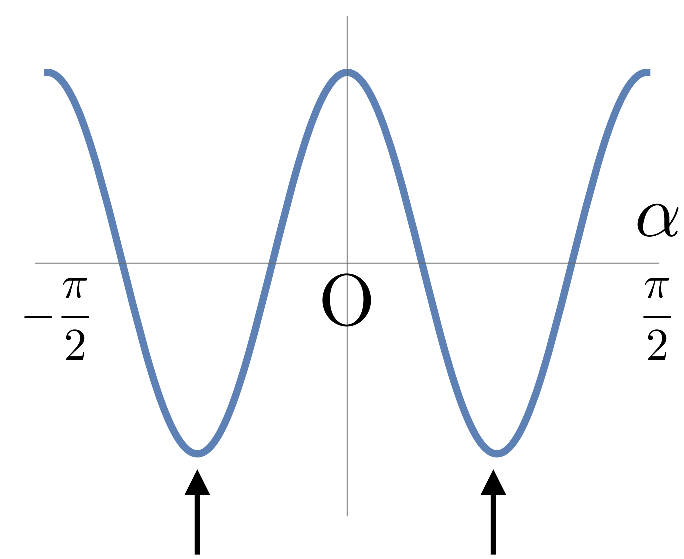


- $\mathrm{Case I} : \xi = 0$



2 minima: $\alpha = -\frac{\pi}{4}$ ⟷ $\alpha = +\frac{\pi}{4}$

CP

What makes the difference? **SM ⟷ 2HDM**

Relative phase of two Higgs fields: $\Phi_1 = \underline{e^{-i\alpha}} \begin{pmatrix} 0 \\ v_1 \end{pmatrix}$ $\quad \Phi_2 = \underline{e^{+i\alpha}} \begin{pmatrix} 0 \\ v_2 \end{pmatrix}$

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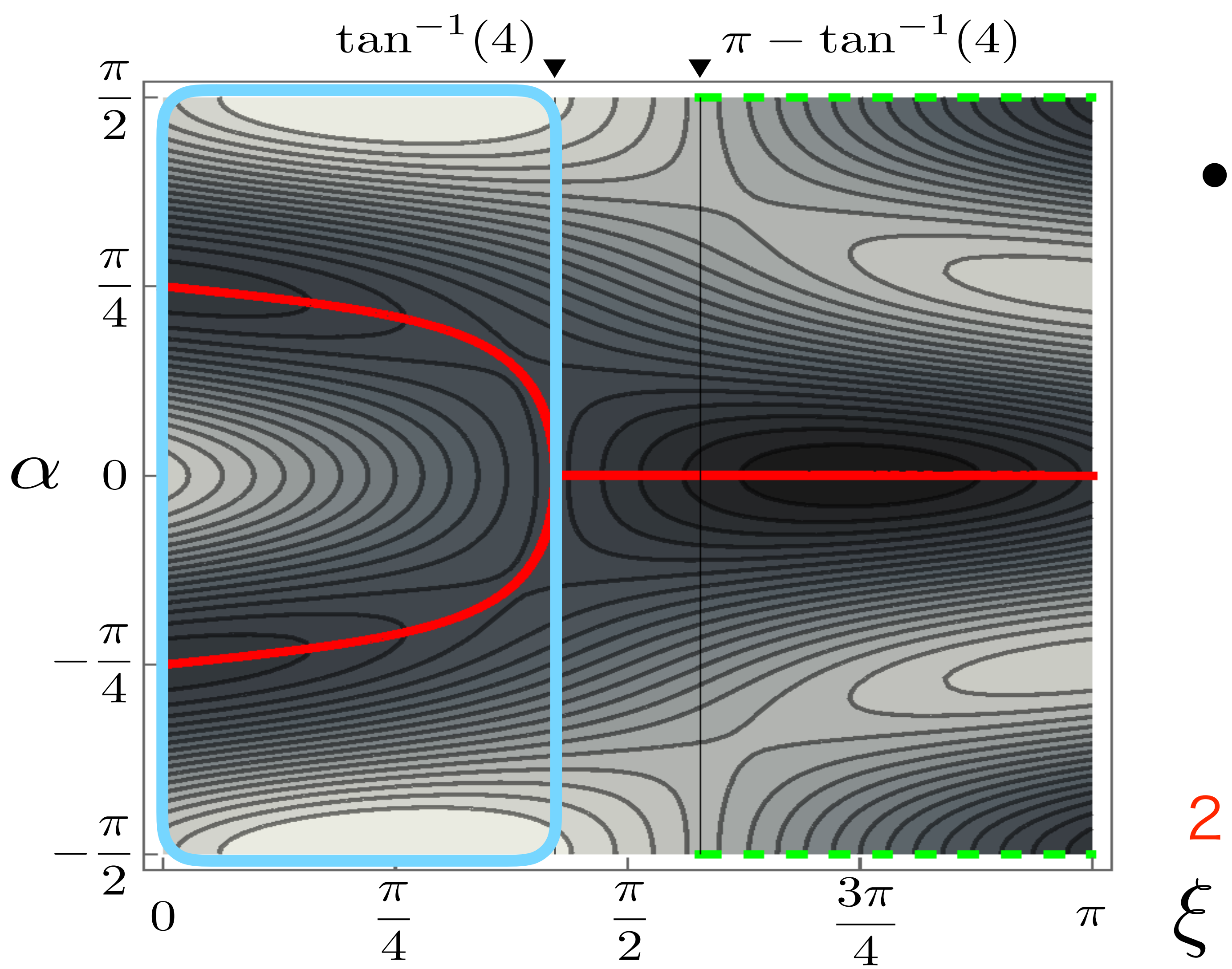
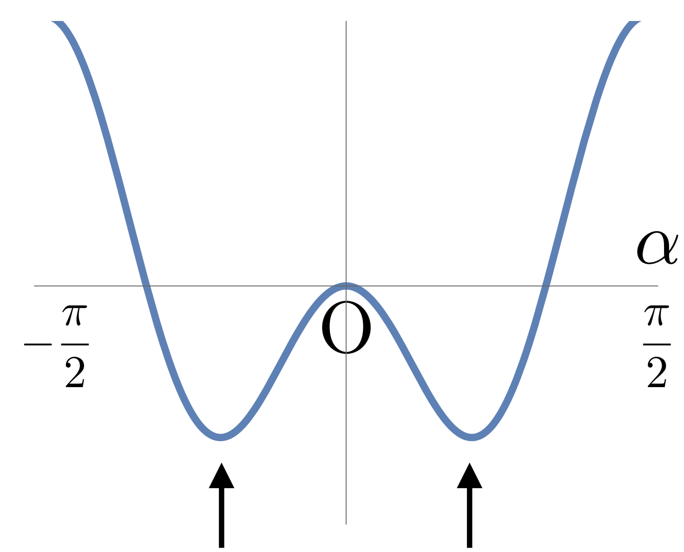


- $\mathrm{CaseII} : 0 < \xi < \tan^{-1}(4)$



2 minima: $\alpha = -\delta$ ⟷ $\alpha = +\delta$

CP

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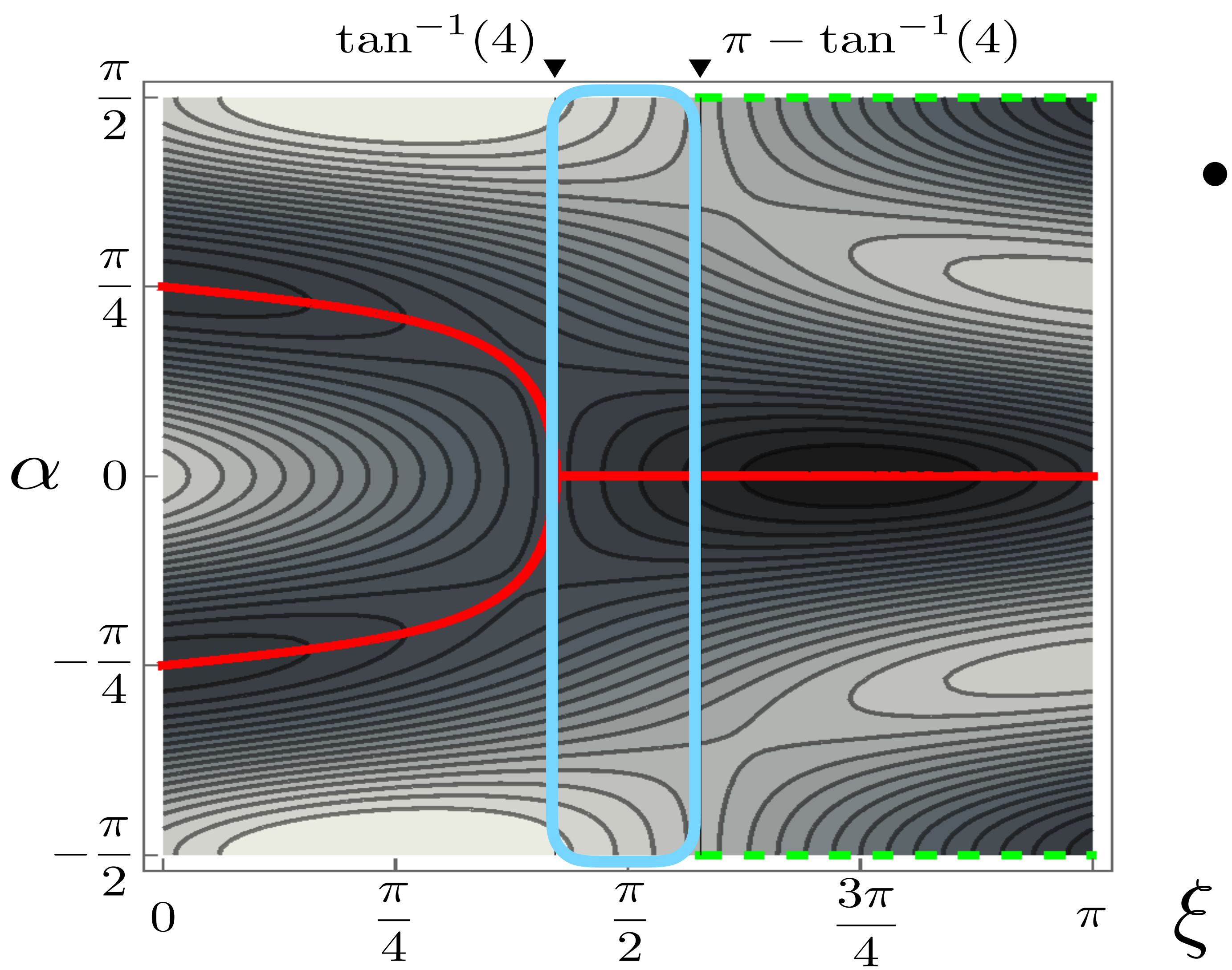
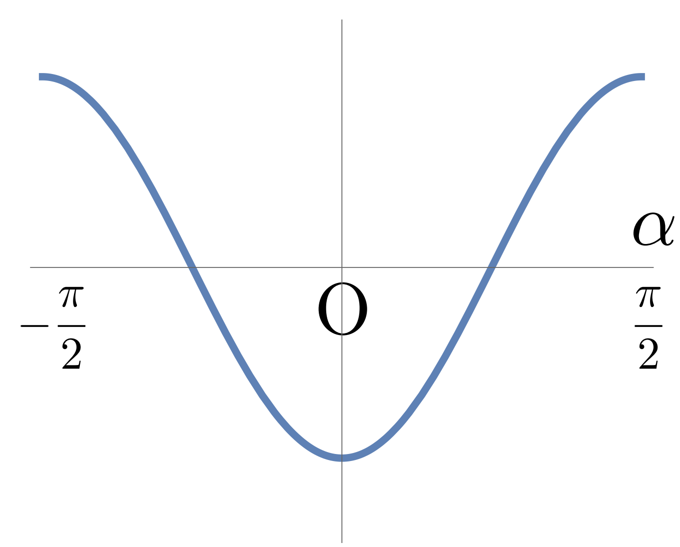


- $\mathrm{CaseIII:}\ \tan^{-1}(4) \le \xi < \pi - \tan^{-1}(4)$



1 minimum

What makes the difference? **SM** ⟷ **2HDM**

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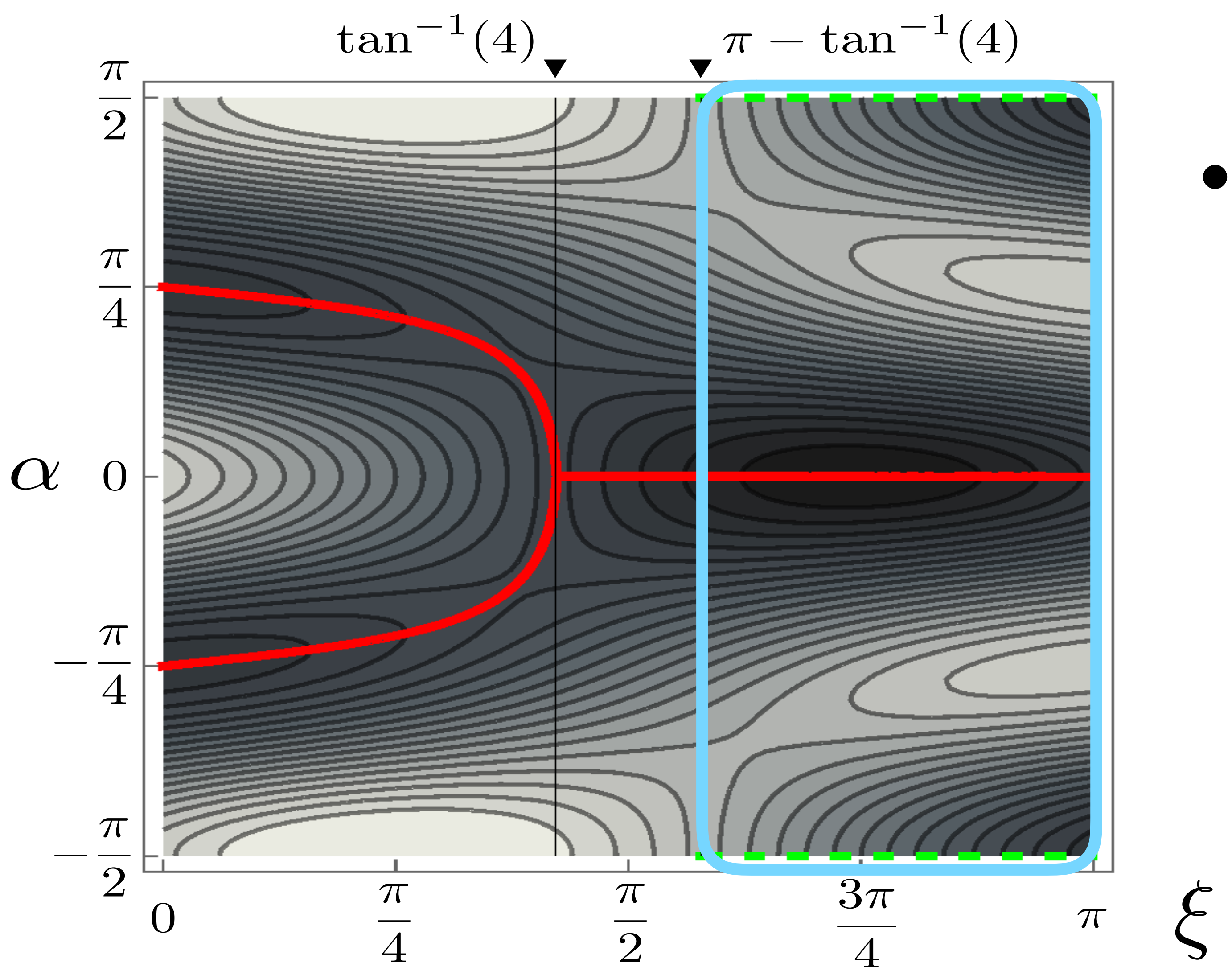
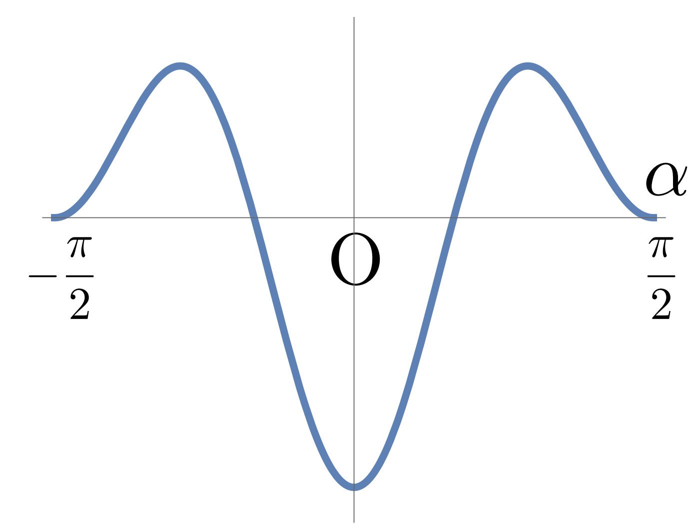
$$\sin\xi = \frac{2(m_{12}^2/v_1 v_2)}{\sqrt{4(m_{12}^2/v_1 v_2)^2 + \beta_5^2}}, \qquad \cos\xi = \frac{\beta_5}{\sqrt{4(m_{12}^2/v_1 v_2)^2 + \beta_5^2}}.$$



- CaseIV : $\pi - \tan^{-1}(4) \leq \xi < \pi$



1 global minimum

1 local minimum

What makes the difference? SM ⟷ 2HDM

Relative phase of two Higgs fields: $\Phi_1 = \underline{e^{-i\alpha}} \begin{pmatrix} 0 \\ v_1 \end{pmatrix}$ $\Phi_2 = \underline{e^{+i\alpha}} \begin{pmatrix} 0 \\ v_2 \end{pmatrix}$

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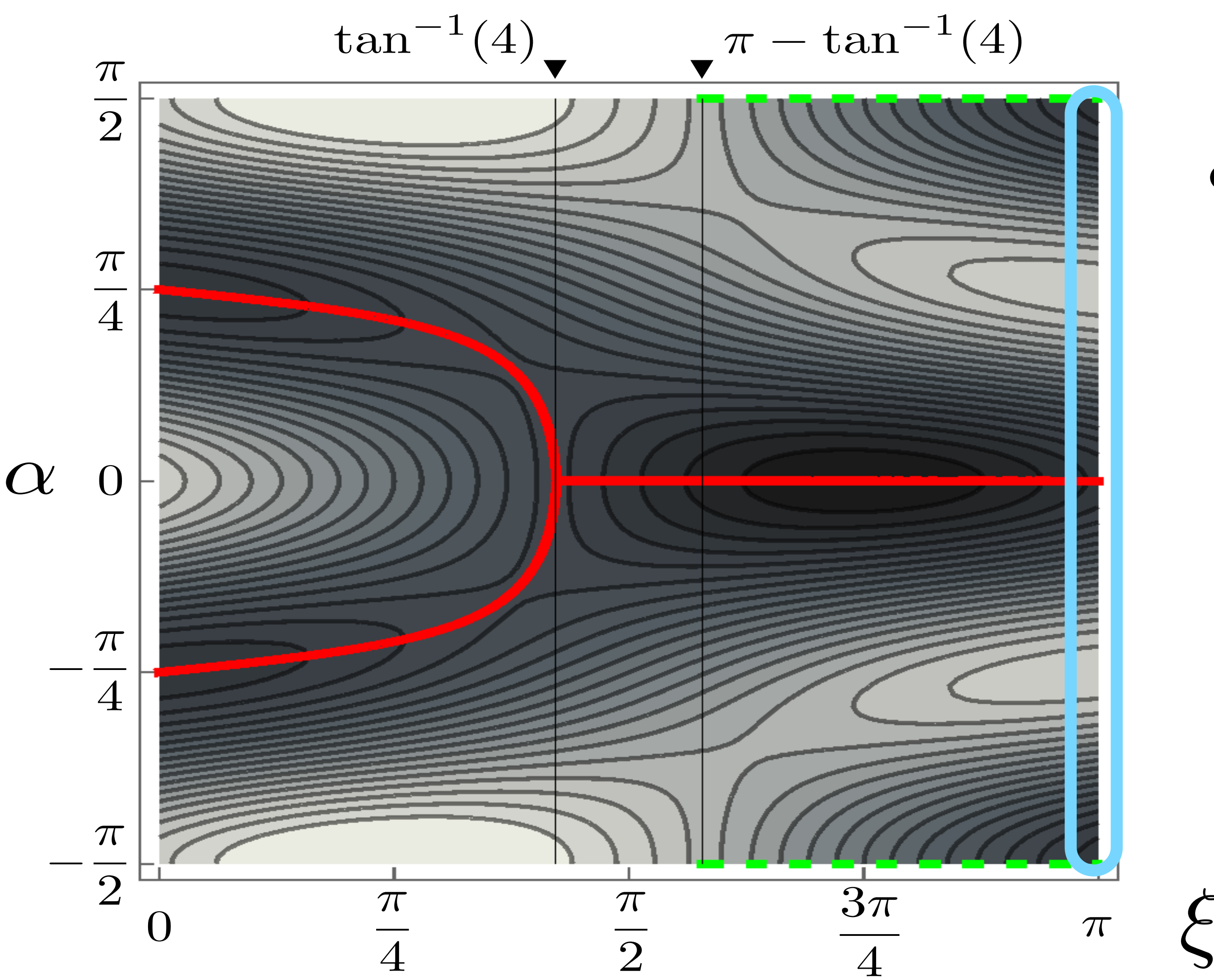
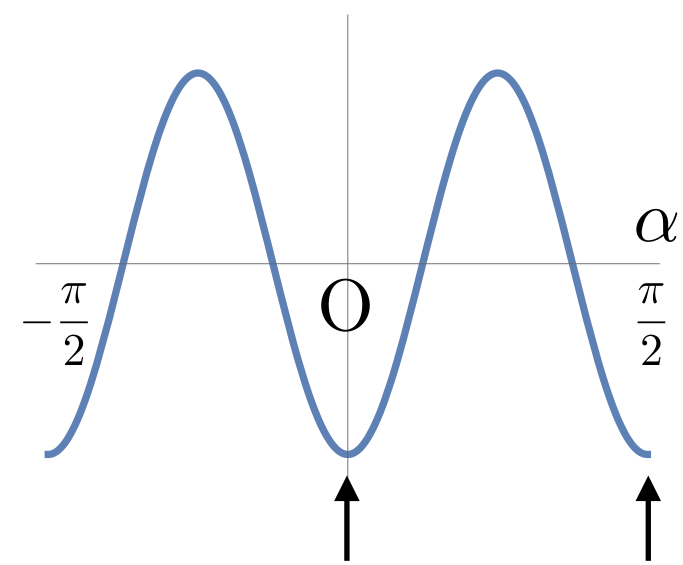


- CaseV : $\xi = \pi$



2 minima: $\alpha = 0$ ⟷ $\alpha = \pi/2$

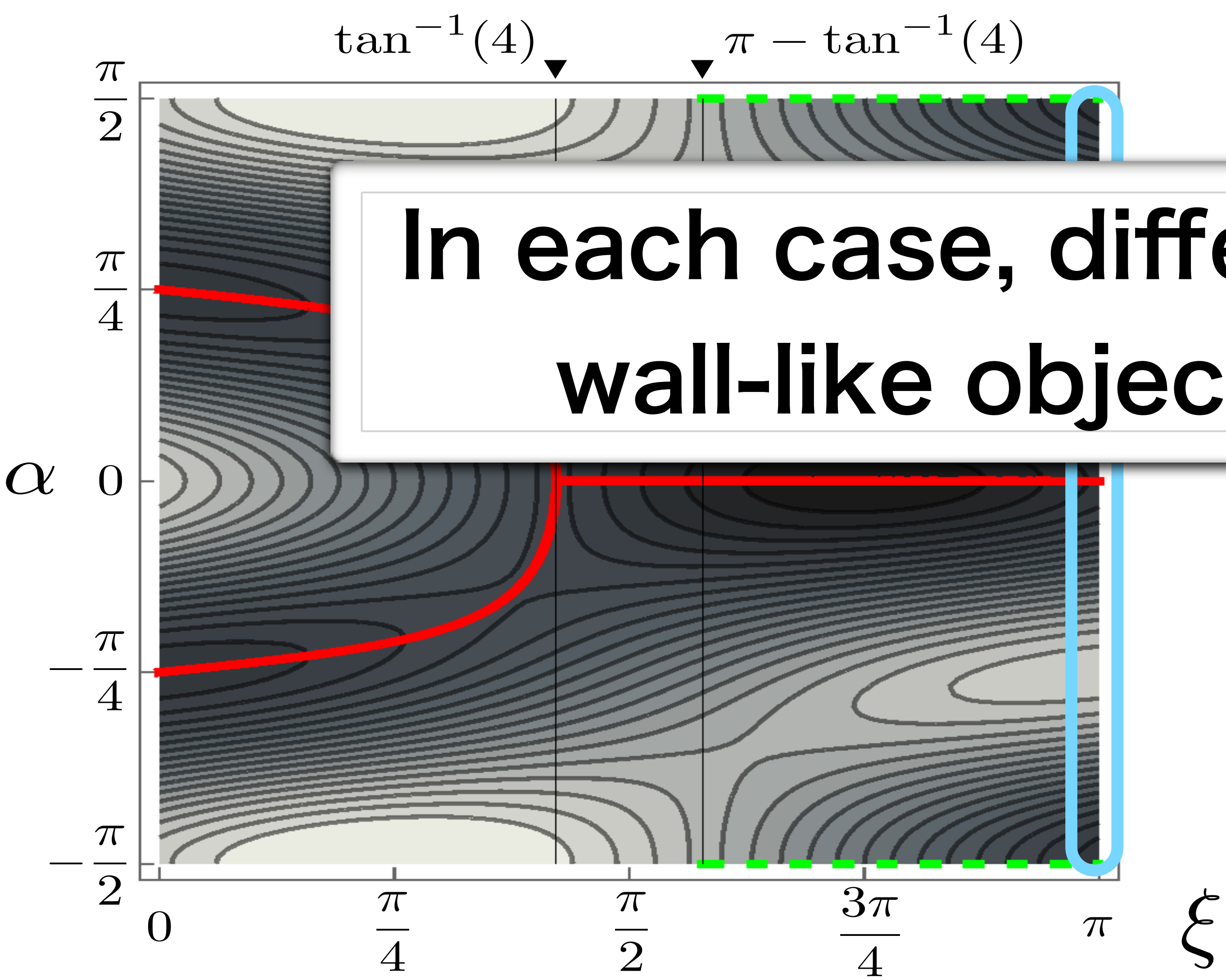
Z_2

What makes the difference? **SM ↔ 2HDM**

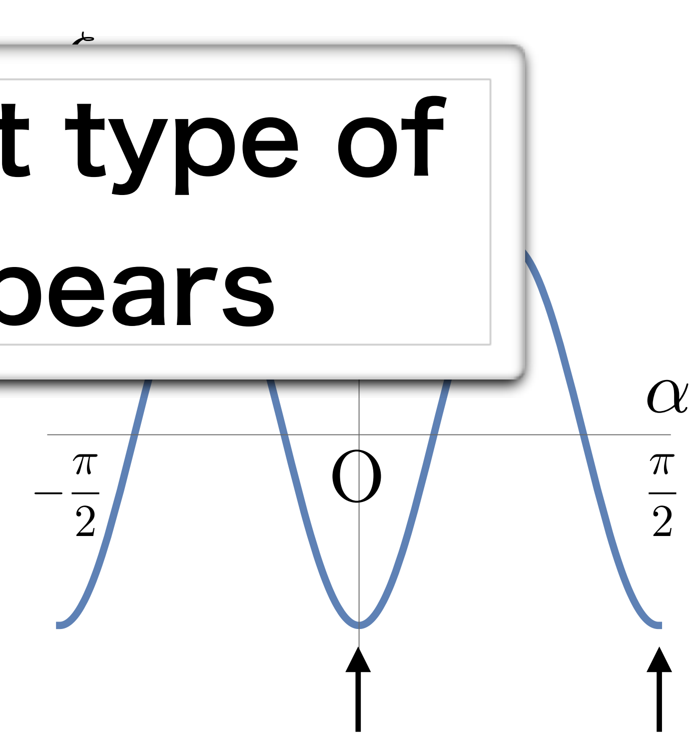
Relative phase of two Higgs fields: $\Phi_1 = \underline{e^{-i\alpha}} \begin{pmatrix} 0 \\ v_1 \end{pmatrix}$ $\Phi_2 = \underline{e^{+i\alpha}} \begin{pmatrix} 0 \\ v_2 \end{pmatrix}$

$$\begin{aligned} V_\xi(\alpha) &= -2m_{12}^2 v_1 v_2 \cos 2\alpha + \beta_5 v_1^2 v_2^2 \cos 4\alpha \\ &= (v_1 v_2)^2 \sqrt{4\left(m_{12}^2/v_1 v_2\right)^2 + \beta_5^2} \\ &\qquad \left(-\sin\xi \cos 2\alpha + \cos\xi \cos 4\alpha\right). \end{aligned}$$

$$\begin{aligned} \sin\xi &= \frac{2(m_{12}^2/v_1 v_2)}{\sqrt{4(m_{12}^2/v_1 v_2)^2 + \beta_5^2}}, \\ \cos\xi &= \frac{\beta_5}{\sqrt{4(m_{12}^2/v_1 v_2)^2 + \beta_5^2}}. \end{aligned}$$



In each case, different type of wall-like object appears

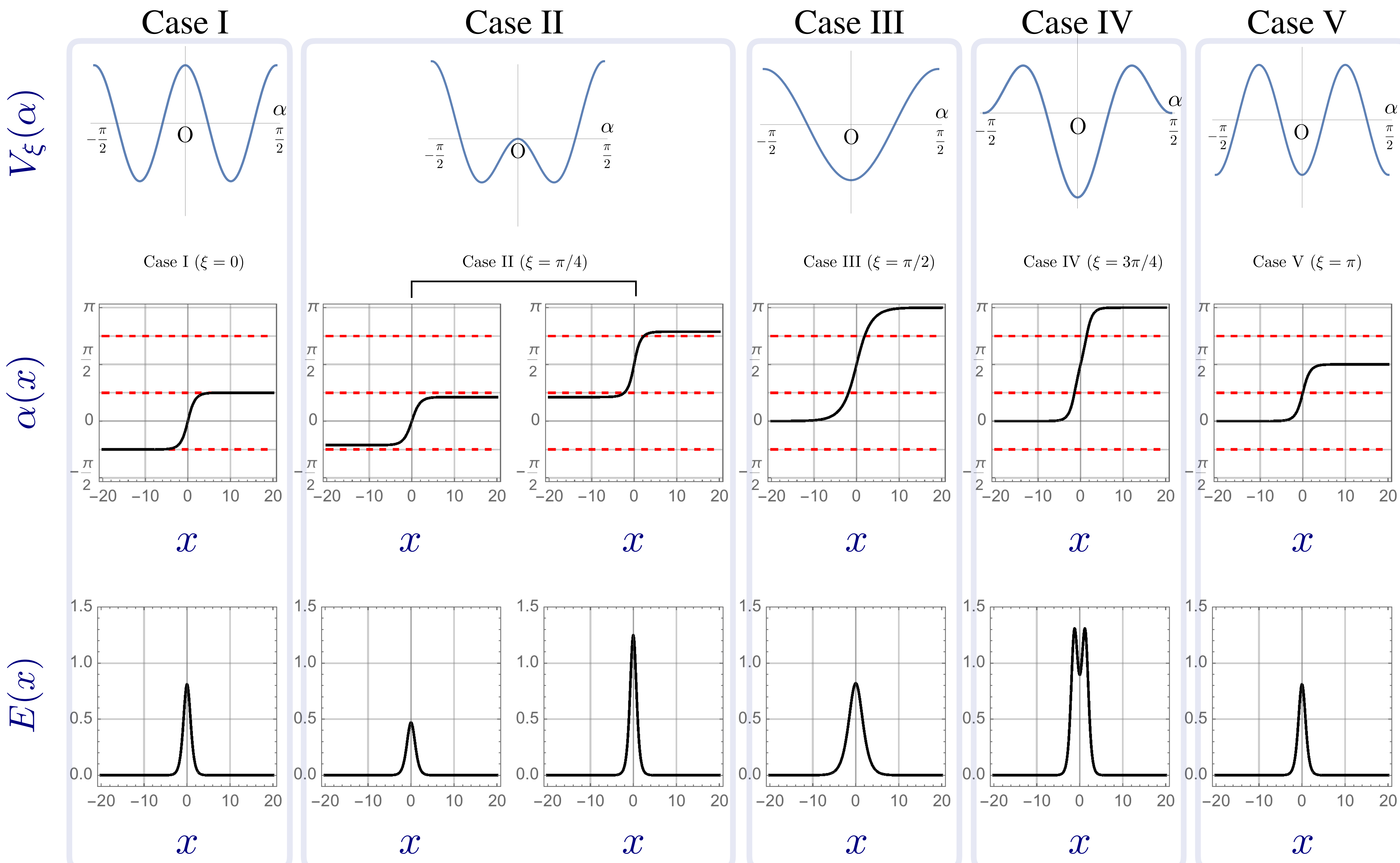


2 minima: $\alpha = 0$ ↔ $\alpha = \pi/2$

Z_2

Domain walls and membranes in 2HDMs

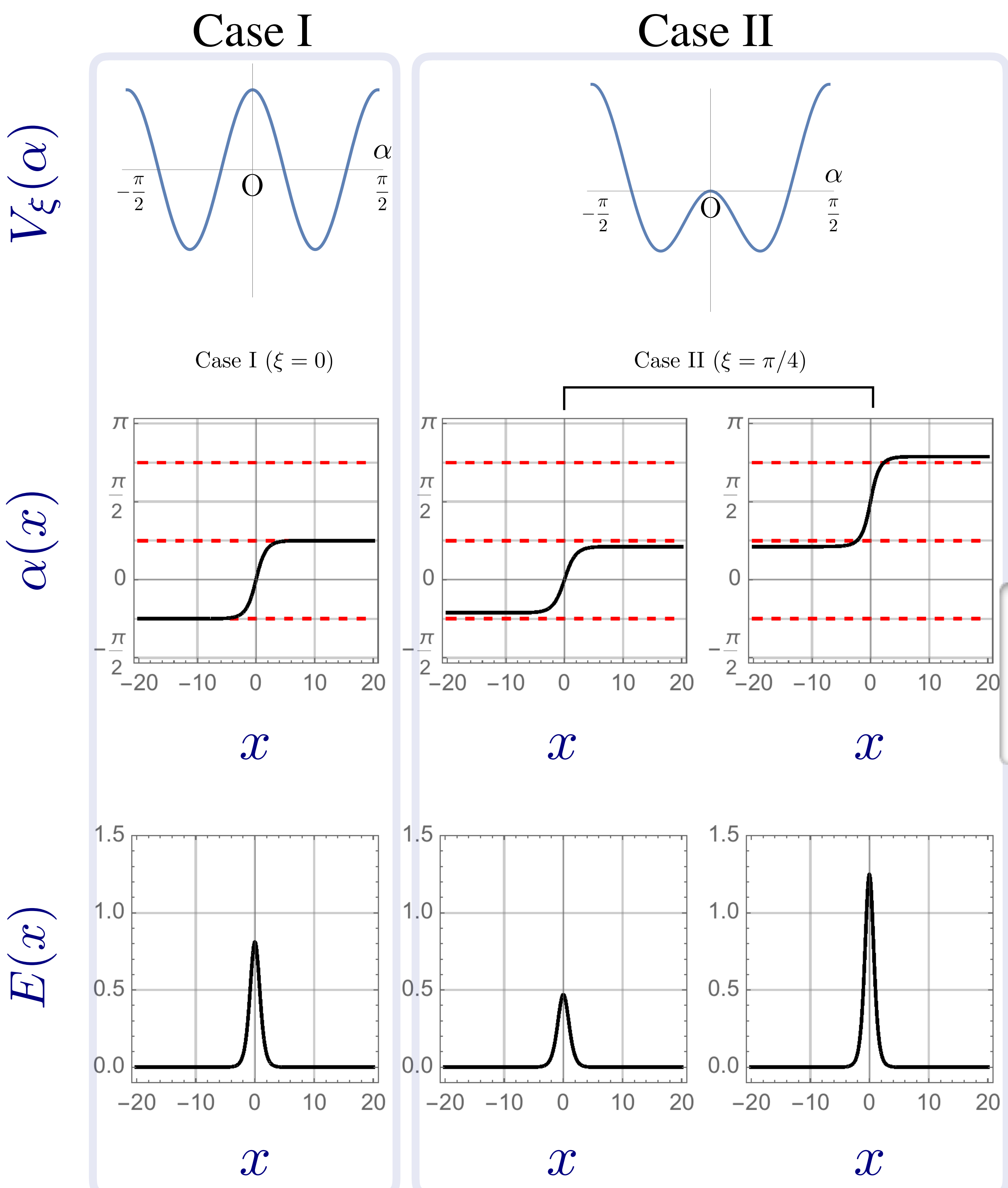
M. Eto, M. Kurachi and M. Nitta, arXiv:1803.04662 [hep-ph] (to be published in PLB)



x : coordinate vertical to the wall

Domain walls and membranes in 2HDMs

M. Eto, M. Kurachi and M. Nitta, arXiv:1803.04662 [hep-ph] (to be published in PLB)



CP domain walls

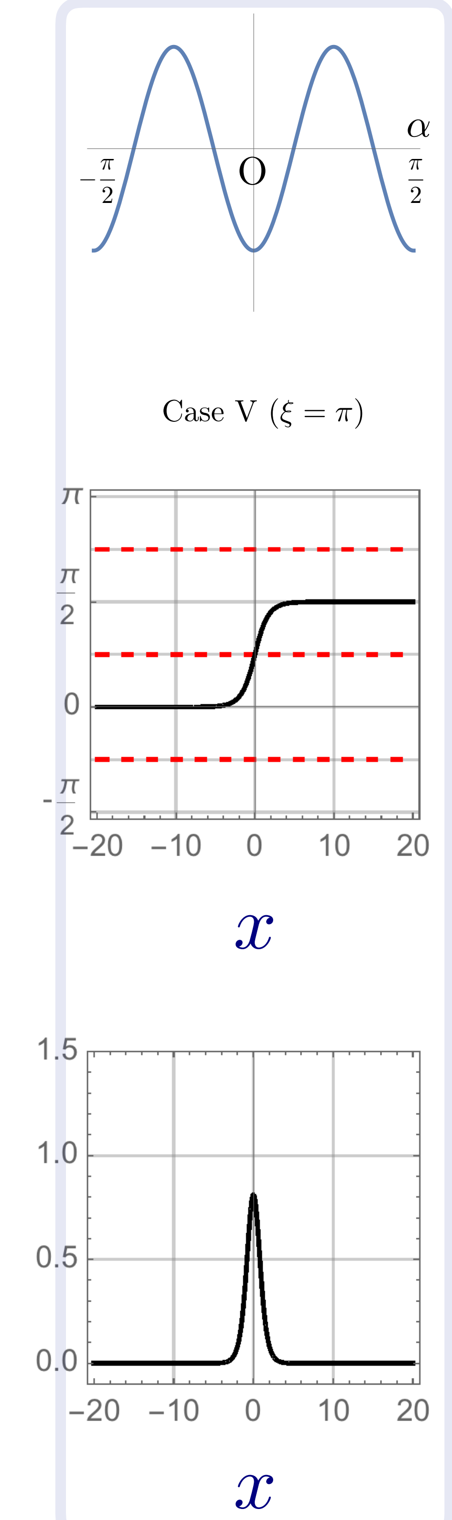
CP symmetry is broken in the vacuum, restored on the domain wall

For a comprehensive discussion, see
R. A. Battye, G. D. Brawn and A. Pilaftsis, JHEP 1108, 020 (2011)

Domain walls and membranes in 2HDMs

M. Eto, M. Kurachi and M. Nitta, arXiv:1803.04662 [hep-ph] (to be published in PLB)

Case V



Z_2 domain walls

CP symmetry is **not** broken in the vacuum,
broken around the domain wall

Domain walls and membranes in 2HDMs

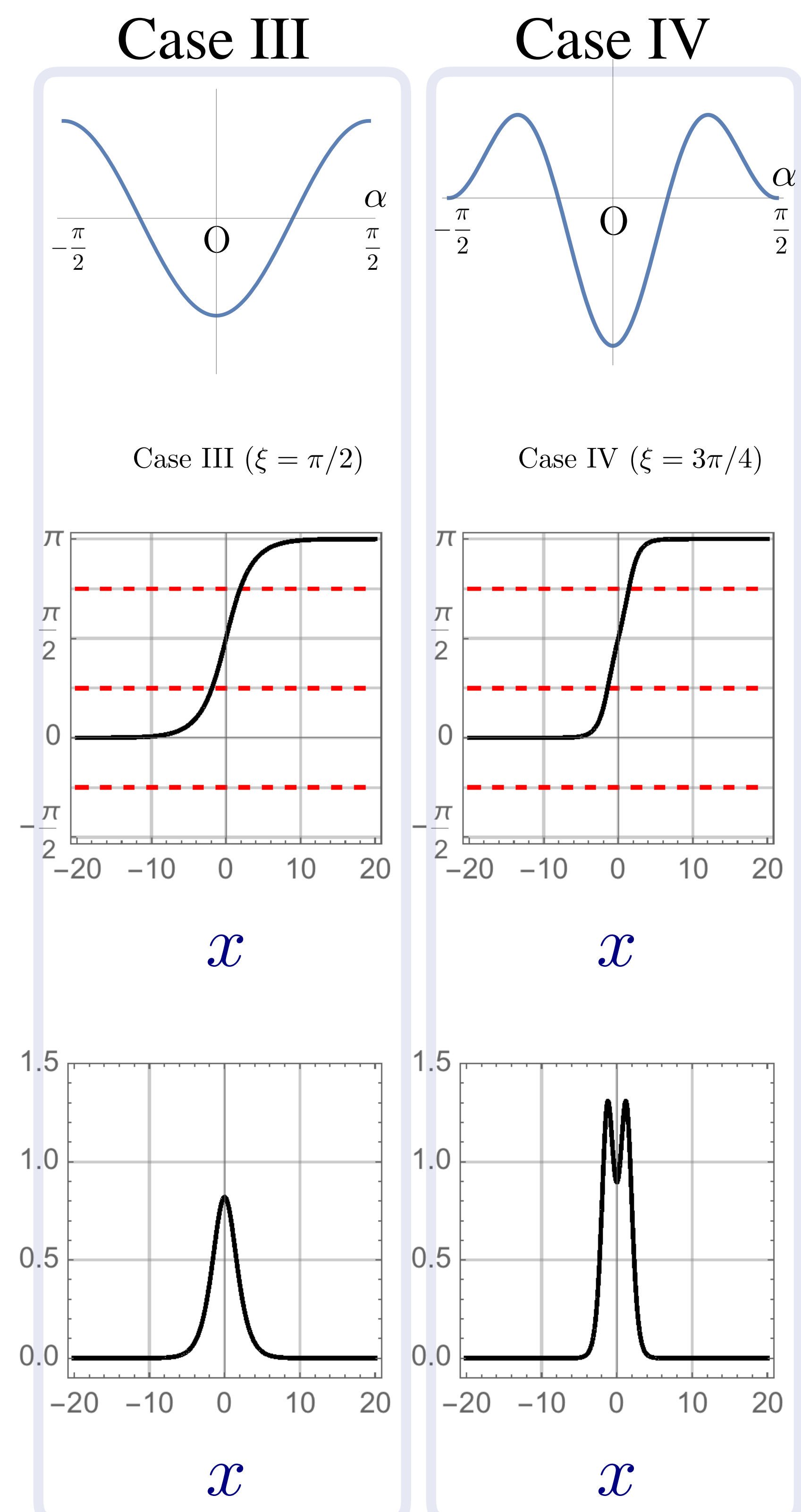
M. Eto, M. Kurachi and M. Nitta, arXiv:1803.04662 [hep-ph] (to be published in PLB)

Membranes

Not topologically stable:

fields at $x = -\infty$ and $x = \infty$ are identical

C. Bachas and T. N. Tomaras, Phys. Rev. Lett. 76, 356 (1996)



Domain walls and membranes in 2HDMs

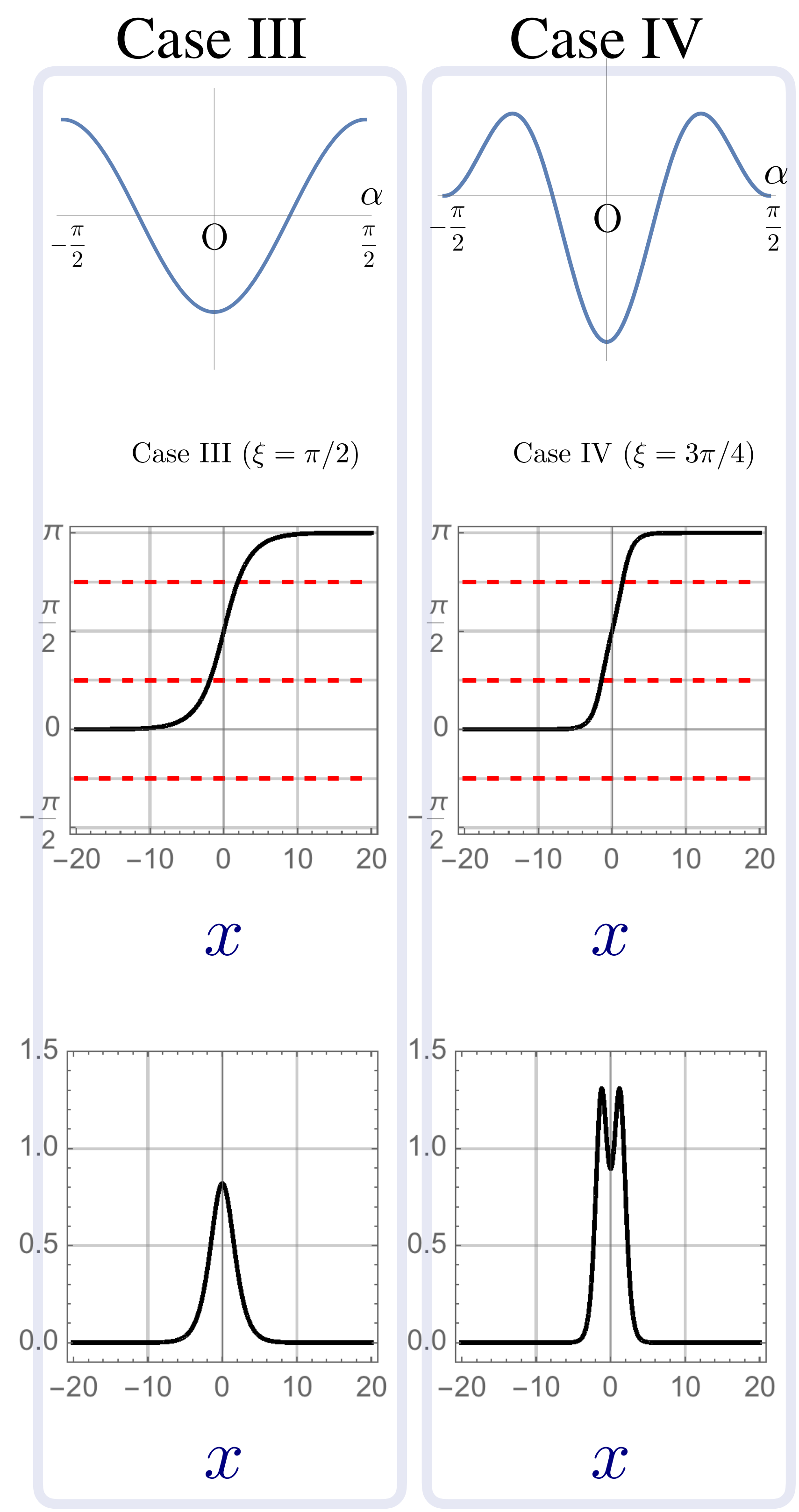
M. Eto, M. Kurachi and M. Nitta, arXiv:1803.04662 [hep-ph] (to be published in PLB)

Membranes

Not topologically stable:

fields at $x = -\infty$ and $x = \infty$ are identical

Possible existence of classically stable wall-like object, but it can quantum mechanically decay
(We will see how this happens later)



3. Non-Abelian vortices in the 2HDM

Vortices in 2HDMs

What is the vortex?

It can appear when the first homotopy group (π_1) of the vacuum manifold is nontrivial

Vortices in 2HDMs

What is the vortex?

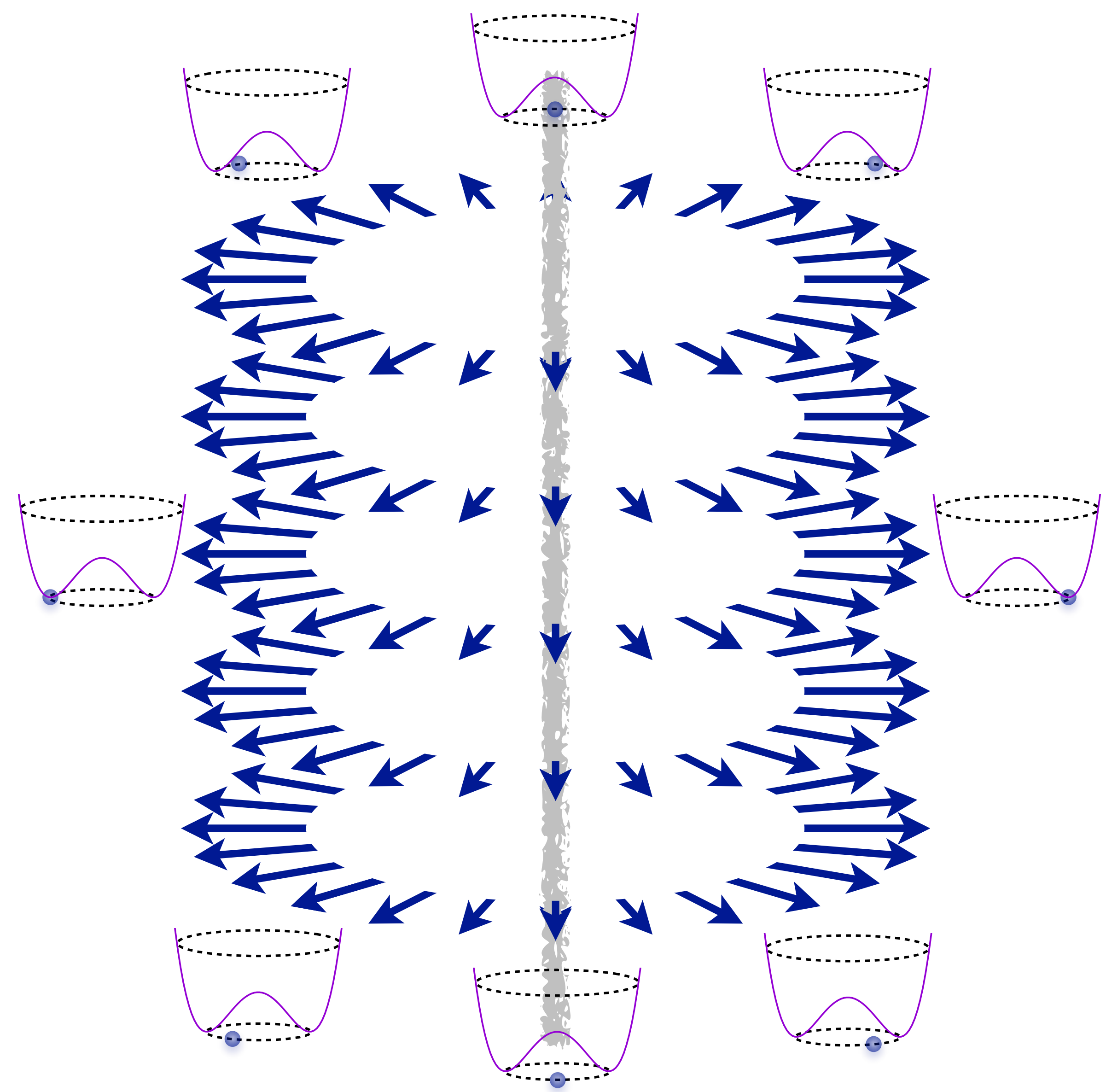
It can appear when the first homotopy group (π_1) of the vacuum manifold is nontrivial

Example: Abelian Higgs Model:

$$\mathcal{L} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + |D^\mu\phi|^2 - m^2\phi|^2 - \lambda|\phi|^4$$

When U(1) symmetry is spontaneously broken, vortex solutions appear: non-trivial winding of U(1) phase around an axis

$$\pi_1\left(U(1)\right) = \mathbb{Z}$$



Vortices in 2HDMs

What is the vortex?

It can appear when the first homotopy group (π_1) of the vacuum manifold is nontrivial

Is there any topological vortex solution in the 2HDM?

Vortices in 2HDMs

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Is there any topological vortex solution in the 2HDM?

Yes!

Vortices in 2HDMs

What is the vortex?

It can appear when the first homotopy group (π_1) of the vacuum manifold is nontrivial

Is there any topological vortex solution in the 2HDM?

Yes!

Extra U(1) degree of freedom that does not exist in the SM

$\boxed{U(1)_a \text{ transformation}}$

relative phase of Φ_1, Φ_2

$$\Phi_1 \longrightarrow e^{-i\alpha}\,\Phi_1$$

$$\Phi_2 \longrightarrow e^{+i\alpha}\,\Phi_2$$

G. R. Dvali and G. Senjanovic, Phys. Rev. Lett. 71, 2376 (1993)

$U(1)_a$ symmetry of the 2HDM Lagrangian

$$V(\Phi_1,\Phi_2) \;=\; m_{11}^2\Phi_1^\dagger\Phi_1 + m_{22}^2\Phi_2^\dagger\Phi_2 - \left(m_{12}^2\Phi_1^\dagger\Phi_2 + \mathrm{h.c.}\right) + \frac{\beta_1}{2}\left(\Phi_1^\dagger\Phi_1\right)^2 + \frac{\beta_2}{2}\left(\Phi_2^\dagger\Phi_2\right)^2$$
$$+\beta_3\left(\Phi_1^\dagger\Phi_1\right)\left(\Phi_2^\dagger\Phi_2\right) + \beta_4\left(\Phi_1^\dagger\Phi_2\right)\left(\Phi_2^\dagger\Phi_1\right) + \left\{\frac{\beta_5}{2}\left(\Phi_1^\dagger\Phi_2\right)^2 + \mathrm{h.c.}\right\}$$

$U(1)_a$ symmetry of the 2HDM Lagrangian

$$V(\Phi_1,\Phi_2) = \boxed{m_{11}^2\Phi_1^\dagger\Phi_1} + \boxed{m_{22}^2\Phi_2^\dagger\Phi_2} - \left(m_{12}^2\Phi_1^\dagger\Phi_2 + \text{h.c.}\right) + \boxed{\frac{\beta_1}{2}\left(\Phi_1^\dagger\Phi_1\right)^2} + \boxed{\frac{\beta_2}{2}\left(\Phi_2^\dagger\Phi_2\right)^2}$$
$$+ \boxed{\beta_3\left(\Phi_1^\dagger\Phi_1\right)\left(\Phi_2^\dagger\Phi_2\right)} + \boxed{\beta_4\left(\Phi_1^\dagger\Phi_2\right)\left(\Phi_2^\dagger\Phi_1\right)} + \left\{\frac{\beta_5}{2}\left(\Phi_1^\dagger\Phi_2\right)^2 + \text{h.c.}\right\}$$

terms that are invariant under $U(1)_a$ transformation

$$\Phi_1 \longrightarrow e^{-i\alpha}\,\Phi_1$$

$$\Phi_2 \longrightarrow e^{+i\alpha}\,\Phi_2$$

$U(1)_a$ symmetry of the 2HDM Lagrangian

$$
\begin{aligned}
V(\Phi_1, \Phi_2) \; = \; & m_{11}^2 \Phi_1^\dagger \Phi_1 + m_{22}^2 \Phi_2^\dagger \Phi_2 - \boxed{\left(m_{12}^2 \Phi_1^\dagger \Phi_2 + \text{h.c.} \right)} + \frac{\beta_1}{2} \left(\Phi_1^\dagger \Phi_1 \right)^2 + \frac{\beta_2}{2} \left(\Phi_2^\dagger \Phi_2 \right)^2 \\
& + \beta_3 \left(\Phi_1^\dagger \Phi_1 \right) \left(\Phi_2^\dagger \Phi_2 \right) + \beta_4 \left(\Phi_1^\dagger \Phi_2 \right) \left(\Phi_2^\dagger \Phi_1 \right) + \boxed{\left\{ \frac{\beta_5}{2} \left(\Phi_1^\dagger \Phi_2 \right)^2 + \text{h.c.} \right\}}
\end{aligned}
$$

terms that are **not** invariant under $U(1)_a$ transformation

$$\Phi_1 \longrightarrow e^{-i\alpha} \, \Phi_1$$

$$\Phi_2 \longrightarrow e^{+i\alpha} \, \Phi_2$$

$U(1)_a$ symmetry of the 2HDM Lagrangian

$$
\begin{aligned}
V(\Phi_1, \Phi_2) \;=\; & m_{11}^2 \Phi_1^\dagger \Phi_1 + m_{22}^2 \Phi_2^\dagger \Phi_2 - \boxed{\cancel{\left(m_{12}^2 \Phi_1^\dagger \Phi_2 + \text{h.c.}\right)}} + \frac{\beta_1}{2}\left(\Phi_1^\dagger \Phi_1\right)^2 + \frac{\beta_2}{2}\left(\Phi_2^\dagger \Phi_2\right)^2 \\
& + \beta_3 \left(\Phi_1^\dagger \Phi_1\right)\left(\Phi_2^\dagger \Phi_2\right) + \beta_4 \left(\Phi_1^\dagger \Phi_2\right)\left(\Phi_2^\dagger \Phi_1\right) + \boxed{\cancel{\left\{\frac{\beta_5}{2}\left(\Phi_1^\dagger \Phi_2\right)^2 + \text{h.c.}\right\}}}
\end{aligned}
$$

Let us first discuss $U(1)_a$ symmetric case: $m_{12}^2 = 0, \;\; \beta_5 = 0$

VEV of the Higgs fields breaks the symmetry **spontaneously**

$$
\Phi_1 = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 \\ v_1 \end{pmatrix}, \quad \Phi_2 = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 \\ v_2 \end{pmatrix}
$$

Vortex solution appears

$U(1)_a$ symmetry of the 2HDM Lagrangian

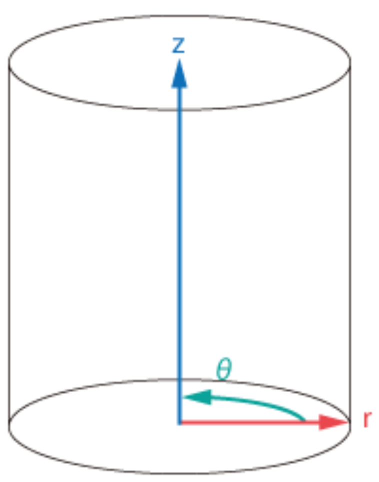
$$
\begin{aligned}
V(\Phi_1, \Phi_2) \; = \; & m_{11}^2 \Phi_1^\dagger \Phi_1 + m_{22}^2 \Phi_2^\dagger \Phi_2 - \cancel{\left(m_{12}^2 \Phi_1^\dagger \Phi_2 + \text{h.c.} \right)} + \frac{\beta_1}{2} \left(\Phi_1^\dagger \Phi_1 \right)^2 + \frac{\beta_2}{2} \left(\Phi_2^\dagger \Phi_2 \right)^2 \\
& + \beta_3 \left(\Phi_1^\dagger \Phi_1 \right) \left(\Phi_2^\dagger \Phi_2 \right) + \beta_4 \left(\Phi_1^\dagger \Phi_2 \right) \left(\Phi_2^\dagger \Phi_1 \right) + \cancel{\left\{ \frac{\beta_5}{2} \left(\Phi_1^\dagger \Phi_2 \right)^2 + \text{h.c.} \right\}}
\end{aligned}
$$

Vortex solution:

2x2 notation: $H \equiv (i\sigma_2 \Phi_1^*, \; \Phi_2)$

$$
H = \begin{pmatrix} v_1 \, f_1(r) \, e^{i\theta} & 0 \\ 0 & v_2 \, f_2(r) \end{pmatrix}
$$

$$
Z_i = -2 \frac{v_1^2}{v_1^2 + v_2^2} \frac{1}{\sqrt{g^2 + g'^2}} \, \epsilon_{ij} \frac{x^j}{r^2} \left(1 - w(r) \right)
$$



$U(1)_a$ symmetry of the 2HDM Lagrangian

$$
\begin{aligned}
V(\Phi_1, \Phi_2) \;=\; & m_{11}^2 \Phi_1^\dagger \Phi_1 + m_{22}^2 \Phi_2^\dagger \Phi_2 - \cancel{\left(m_{12}^2 \Phi_1^\dagger \Phi_2 + \text{h.c.} \right)} + \frac{\beta_1}{2} \left(\Phi_1^\dagger \Phi_1 \right)^2 + \frac{\beta_2}{2} \left(\Phi_2^\dagger \Phi_2 \right)^2 \\
& + \beta_3 \left(\Phi_1^\dagger \Phi_1 \right) \left(\Phi_2^\dagger \Phi_2 \right) + \beta_4 \left(\Phi_1^\dagger \Phi_2 \right) \left(\Phi_2^\dagger \Phi_1 \right) + \cancel{\left\{ \frac{\beta_5}{2} \left(\Phi_1^\dagger \Phi_2 \right)^2 + \text{h.c.} \right\}}
\end{aligned}
$$

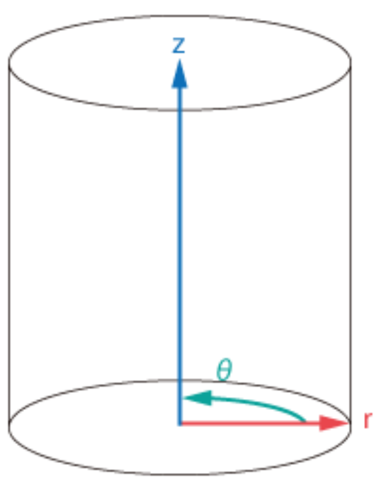
Vortex solution:

2x2 notation: $H \equiv (i\sigma_2 \Phi_1^*, \;\Phi_2)$

non-trivial winding of the phase

$$
H = \begin{pmatrix} v_1 \, f_1(r) \, e^{i\theta} & 0 \\ 0 & v_2 \, f_2(r) \end{pmatrix}
$$

$$
Z_i = -2 \frac{v_1^2}{v_1^2 + v_2^2} \frac{1}{\sqrt{g^2 + g'^2}} \epsilon_{ij} \frac{x^j}{r^2} (1 - w(r))
$$



$U(1)_a$ symmetry of the 2HDM Lagrangian

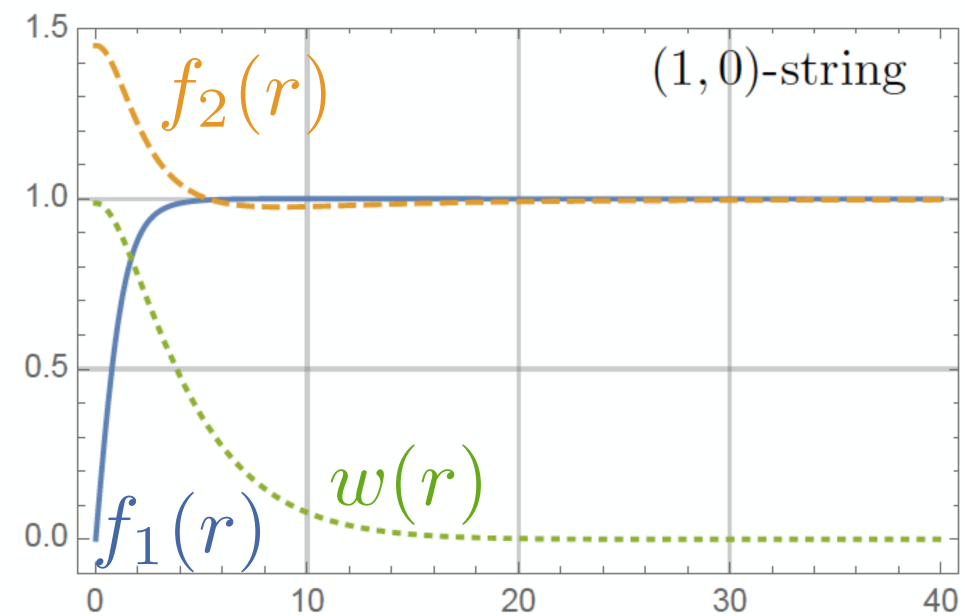
$$
\begin{aligned}
V(\Phi_1, \Phi_2) \;=\; & m_{11}^2 \Phi_1^\dagger \Phi_1 + m_{22}^2 \Phi_2^\dagger \Phi_2 - \boxed{\cancel{\left(m_{12}^2 \Phi_1^\dagger \Phi_2 + \mathrm{h.c.}\right)}} + \frac{\beta_1}{2}\left(\Phi_1^\dagger \Phi_1\right)^2 + \frac{\beta_2}{2}\left(\Phi_2^\dagger \Phi_2\right)^2 \\
& + \beta_3 \left(\Phi_1^\dagger \Phi_1\right)\left(\Phi_2^\dagger \Phi_2\right) + \beta_4 \left(\Phi_1^\dagger \Phi_2\right)\left(\Phi_2^\dagger \Phi_1\right) + \boxed{\cancel{\left\{\frac{\beta_5}{2}\left(\Phi_1^\dagger \Phi_2\right)^2 + \mathrm{h.c.}\right\}}}
\end{aligned}
$$

Vortex solution:

2x2 notation: $H \equiv \left(i\sigma_2 \Phi_1^*,\ \Phi_2\right)$

$$
H = \begin{pmatrix} v_1\, f_1(r)\, e^{i\theta} & 0 \\ 0 & v_2\, f_2(r) \end{pmatrix}
$$

$$
Z_i = -2 \frac{v_1^2}{v_1^2 + v_2^2} \frac{1}{\sqrt{g^2 + g'^2}}\, \epsilon_{ij} \frac{x^j}{r^2} \left(1 - w(r)\right)
$$



$U(1)_a$ symmetry of the 2HDM Lagrangian

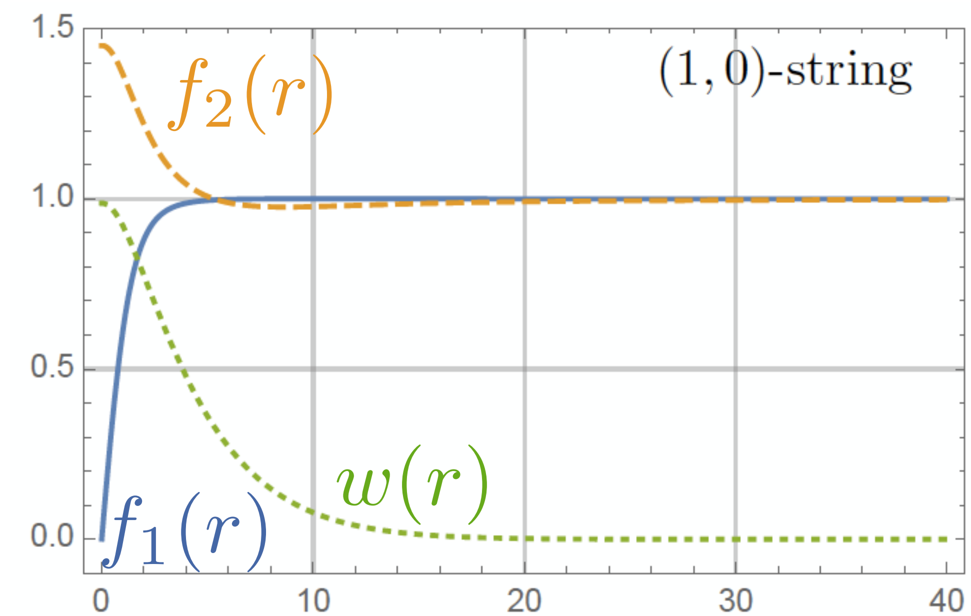
$$V(\Phi_1,\Phi_2) = m_{11}^2\Phi_1^\dagger\Phi_1 + m_{22}^2\Phi_2^\dagger\Phi_2 - \cancel{\left(m_{12}^2\Phi_1^\dagger\Phi_2 + \text{h.c.}\right)} + \frac{\beta_1}{2}\left(\Phi_1^\dagger\Phi_1\right)^2 + \frac{\beta_2}{2}\left(\Phi_2^\dagger\Phi_2\right)^2$$
$$+\beta_3\left(\Phi_1^\dagger\Phi_1\right)\left(\Phi_2^\dagger\Phi_2\right) + \beta_4\left(\Phi_1^\dagger\Phi_2\right)\left(\Phi_2^\dagger\Phi_1\right) + \cancel{\left\{\frac{\beta_5}{2}\left(\Phi_1^\dagger\Phi_2\right)^2 + \text{h.c.}\right\}}$$

Vortex solution:

2x2 notation: $H \equiv \left(i\sigma_2\Phi_1^*,\ \Phi_2\right)$

$$H = \begin{pmatrix} v_1\, f_1(r)\, e^{i\theta} & 0 \\ 0 & v_2\, f_2(r) \end{pmatrix}$$

$$Z_i = -2\frac{v_1^2}{v_1^2+v_2^2}\frac{1}{\sqrt{g^2+g'^2}}\epsilon_{ij}\frac{x^j}{r^2}\left(1-w(r)\right)$$



$$\xrightarrow{\text{Z-flux}}$$

$$\Phi_Z = -2\pi\frac{v_2^2}{v_1^2+v_2^2}\frac{1}{\sqrt{g^2+g'^2}}$$

$U(1)_a$ symmetry of the 2HDM Lagrangian

$$
\begin{aligned}
V(\Phi_1,\Phi_2) \;=\;& m_{11}^2\Phi_1^\dagger\Phi_1 + m_{22}^2\Phi_2^\dagger\Phi_2 - \boxed{\cancel{\left(m_{12}^2\Phi_1^\dagger\Phi_2 + \text{h.c.}\right)}} + \frac{\beta_1}{2}\left(\Phi_1^\dagger\Phi_1\right)^2 + \frac{\beta_2}{2}\left(\Phi_2^\dagger\Phi_2\right)^2 \\
& + \beta_3\left(\Phi_1^\dagger\Phi_1\right)\left(\Phi_2^\dagger\Phi_2\right) + \beta_4\left(\Phi_1^\dagger\Phi_2\right)\left(\Phi_2^\dagger\Phi_1\right) + \boxed{\cancel{\left\{\frac{\beta_5}{2}\left(\Phi_1^\dagger\Phi_2\right)^2 + \text{h.c.}\right\}}}
\end{aligned}
$$

Vortex solution:

2x2 notation: $H \equiv (i\sigma_2\Phi_1^*,\ \Phi_2)$

$$
H = \begin{pmatrix} v_1\, f_1(r)\, e^{i\theta} & 0 \\ 0 & v_2\, f_2(r) \end{pmatrix}
$$

$\theta\ :\ -\pi \sim \pi$

$U(1)_a$ symmetry of the 2HDM Lagrangian

$$V(\Phi_1,\Phi_2) = m_{11}^2\Phi_1^\dagger\Phi_1 + m_{22}^2\Phi_2^\dagger\Phi_2 - \boxed{\cancel{\left(m_{12}^2\Phi_1^\dagger\Phi_2 + \text{h.c.}\right)}} + \frac{\beta_1}{2}\left(\Phi_1^\dagger\Phi_1\right)^2 + \frac{\beta_2}{2}\left(\Phi_2^\dagger\Phi_2\right)^2$$
$$+\beta_3\left(\Phi_1^\dagger\Phi_1\right)\left(\Phi_2^\dagger\Phi_2\right) + \beta_4\left(\Phi_1^\dagger\Phi_2\right)\left(\Phi_2^\dagger\Phi_1\right) + \boxed{\cancel{\left\{\frac{\beta_5}{2}\left(\Phi_1^\dagger\Phi_2\right)^2 + \text{h.c.}\right\}}}$$

Vortex solution:

2x2 notation: $H \equiv (i\sigma_2\Phi_1^*,\ \Phi_2)$

$$H = \begin{pmatrix} v_1 f_1(r)\, e^{i\theta} & 0 \\ 0 & v_2 f_2(r) \end{pmatrix} = \underbrace{e^{\frac{i}{2}\theta}}_{U(1)_a\ \text{phase}:\ -\pi/2 \sim \pi/2}\ \underbrace{e^{\frac{i\sigma_3}{2}\theta}}_{\text{element of local symmetry}} \begin{pmatrix} v_1 f_1(r) & 0 \\ 0 & v_2 f_2(r) \end{pmatrix}$$

$\theta\ :\ -\pi \sim \pi$

element of local symmetry

$U(1)_a$ phase : $-\pi/2 \sim \pi/2$

so-called **semi-local** string

Now let us discuss effects of $U(1)_a$ breaking terms

$$V(\Phi_1,\Phi_2) \;=\; m_{11}^2\Phi_1^\dagger\Phi_1 + m_{22}^2\Phi_2^\dagger\Phi_2 - \boxed{\left(m_{12}^2\Phi_1^\dagger\Phi_2 + \text{h.c.}\right)} + \frac{\beta_1}{2}\left(\Phi_1^\dagger\Phi_1\right)^2 + \frac{\beta_2}{2}\left(\Phi_2^\dagger\Phi_2\right)^2$$
$$+\beta_3\left(\Phi_1^\dagger\Phi_1\right)\left(\Phi_2^\dagger\Phi_2\right) + \beta_4\left(\Phi_1^\dagger\Phi_2\right)\left(\Phi_2^\dagger\Phi_1\right) + \boxed{\left\{\frac{\beta_5}{2}\left(\Phi_1^\dagger\Phi_2\right)^2 + \text{h.c.}\right\}}$$

Vortex solution: $m_{12}^2 = 0,\ \beta_5 = 0$

$$H = \begin{pmatrix} v_1\, f_1(r)\, e^{i\theta} & 0 \\ 0 & v_2\, f_2(r) \end{pmatrix} = e^{\frac{i}{2}\theta} e^{\frac{i\sigma_3}{2}\theta} \begin{pmatrix} v_1 f_1(r) & 0 \\ 0 & v_2 f_2(r) \end{pmatrix}$$

Now let us discuss effects of $U(1)_a$ breaking terms

$$
\begin{aligned}
V(\Phi_1,\Phi_2) \;=\; & m_{11}^2\Phi_1^\dagger\Phi_1 + m_{22}^2\Phi_2^\dagger\Phi_2 - \boxed{\left(m_{12}^2\Phi_1^\dagger\Phi_2 + \text{h.c.}\right)} + \frac{\beta_1}{2}\left(\Phi_1^\dagger\Phi_1\right)^2 + \frac{\beta_2}{2}\left(\Phi_2^\dagger\Phi_2\right)^2 \\
& + \beta_3\left(\Phi_1^\dagger\Phi_1\right)\left(\Phi_2^\dagger\Phi_2\right) + \beta_4\left(\Phi_1^\dagger\Phi_2\right)\left(\Phi_2^\dagger\Phi_1\right) + \boxed{\left\{\frac{\beta_5}{2}\left(\Phi_1^\dagger\Phi_2\right)^2 + \text{h.c.}\right\}}
\end{aligned}
$$

Vortex solution: $m_{12}^2 \neq 0,\ \beta_5 \neq 0$

$$
H = \begin{pmatrix} v_1 f_1(r)\, e^{i\underline{\hat\theta(\theta)}} & 0 \\ 0 & v_2 f_2(r) \end{pmatrix} = e^{\frac{i}{2}\underline{\hat\theta(\theta)}} e^{\frac{i\sigma_3}{2}\underline{\hat\theta(\theta)}} \begin{pmatrix} v_1 f_1(r) & 0 \\ 0 & v_2 f_2(r) \end{pmatrix}
$$

Since the potential has non-trivial $U(1)_a$ phase dependence, vortex configurations have non-trivial θ dependence

Now let us discuss effects of $U(1)_a$ breaking terms

$$V(\Phi_1,\Phi_2) = m_{11}^2\Phi_1^\dagger\Phi_1 + m_{22}^2\Phi_2^\dagger\Phi_2 - \boxed{\left(m_{12}^2\Phi_1^\dagger\Phi_2 + \text{h.c.}\right)} + \frac{\beta_1}{2}\left(\Phi_1^\dagger\Phi_1\right)^2 + \frac{\beta_2}{2}\left(\Phi_2^\dagger\Phi_2\right)^2$$
$$+\beta_3\left(\Phi_1^\dagger\Phi_1\right)\left(\Phi_2^\dagger\Phi_2\right) + \beta_4\left(\Phi_1^\dagger\Phi_2\right)\left(\Phi_2^\dagger\Phi_1\right) + \boxed{\left\{\frac{\beta_5}{2}\left(\Phi_1^\dagger\Phi_2\right)^2 + \text{h.c.}\right\}}$$

Vortex solution: $m_{12}^2 \neq 0,\ \beta_5 \neq 0$

$$H = \begin{pmatrix} v_1\, f_1(r)\, e^{i\underline{\hat\theta(\theta)}} & 0 \\ 0 & v_2\, f_2(r) \end{pmatrix} = e^{\frac{i}{2}\underline{\hat\theta(\theta)}} e^{\frac{i\sigma_3}{2}\underline{\hat\theta(\theta)}} \begin{pmatrix} v_1 f_1(r) & 0 \\ 0 & v_2 f_2(r) \end{pmatrix}$$

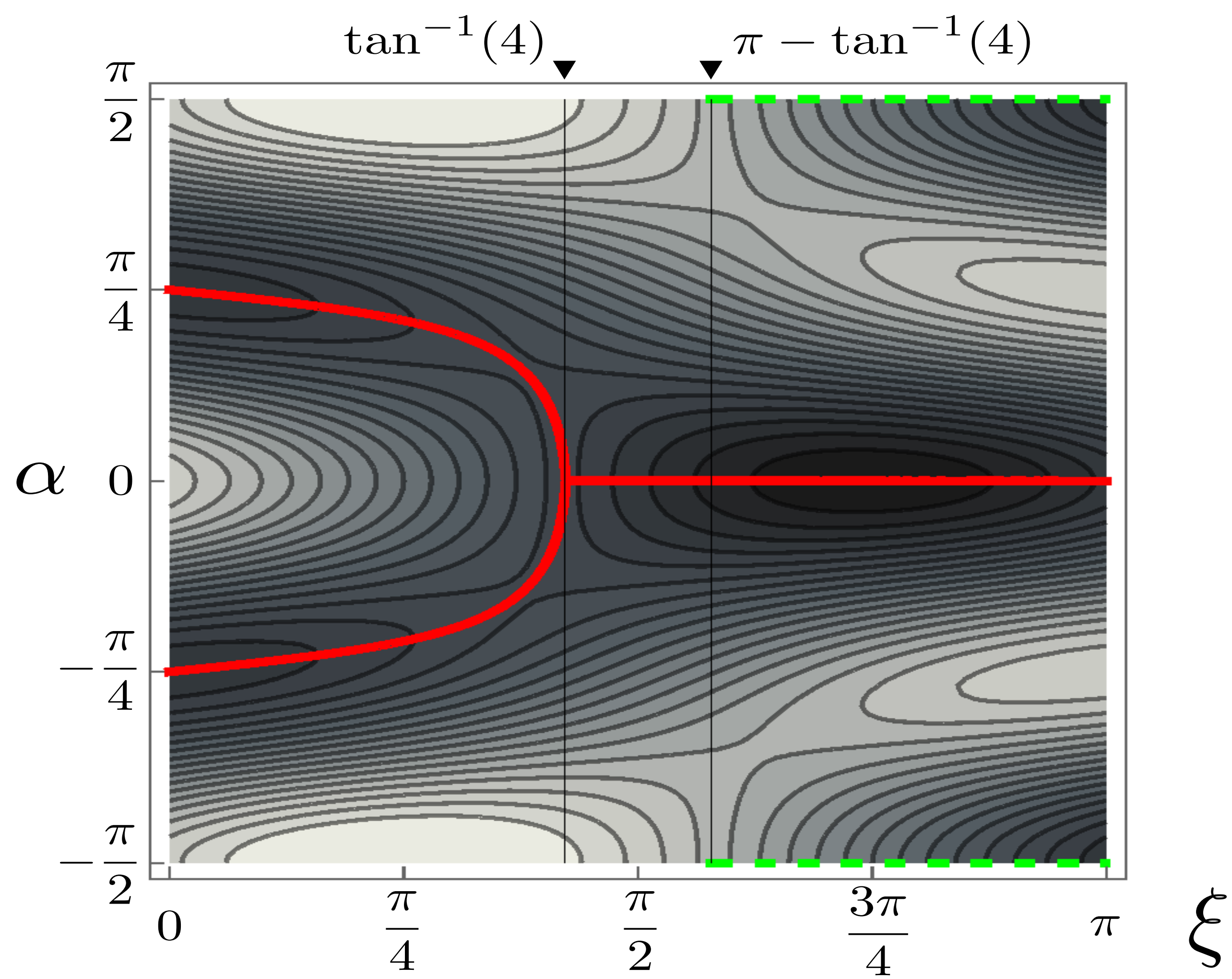
To see the potential that the vortex configuration “feels”,
let us substitute $H = e^{i\alpha}\mathrm{diag}(v_1, v_2)$ into the potential

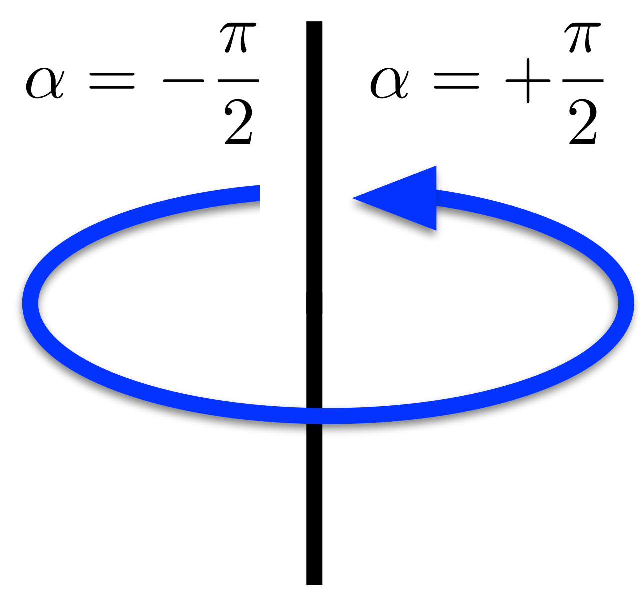
α dependent part of the potential

$$H = e^{i\alpha}\mathrm{diag}(v_1, v_2)$$

$$\begin{aligned} V_\xi(\alpha) &= -2m_{12}^2 v_1 v_2 \underline{\cos 2\alpha} + \beta_5 v_1^2 v_2^2 \underline{\cos 4\alpha} \\ &= (v_1 v_2)^2 \sqrt{4\left(m_{12}^2/v_1 v_2\right)^2 + \beta_5^2} \\ &\qquad \left(-\sin\xi \, \underline{\cos 2\alpha} + \cos\xi \, \underline{\cos 4\alpha}\right). \end{aligned}$$

$$\begin{aligned} \sin\xi &= \frac{2(m_{12}^2/v_1 v_2)}{\sqrt{4(m_{12}^2/v_1 v_2)^2 + \beta_5^2}}, \\ \cos\xi &= \frac{\beta_5}{\sqrt{4(m_{12}^2/v_1 v_2)^2 + \beta_5^2}}. \end{aligned}$$



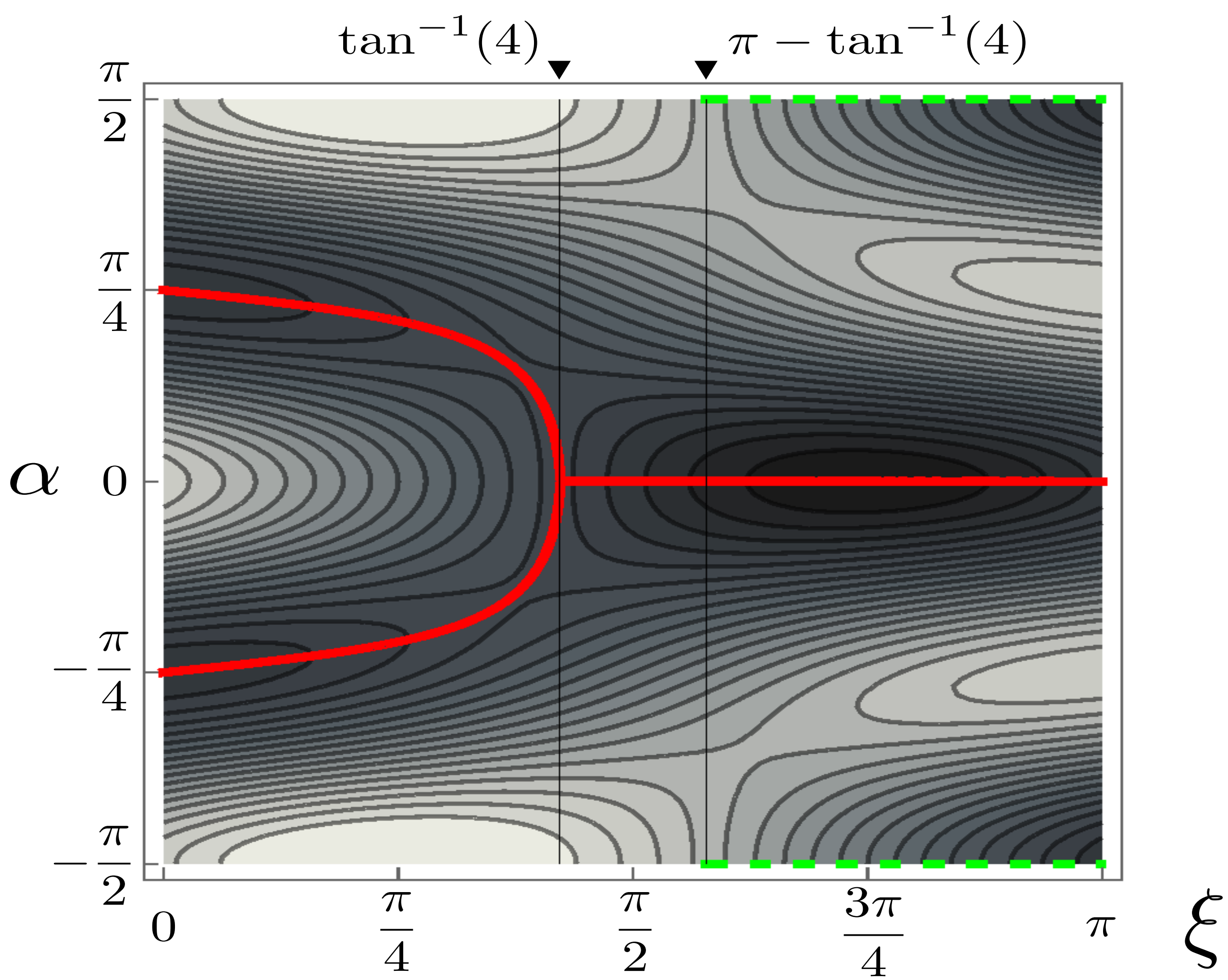


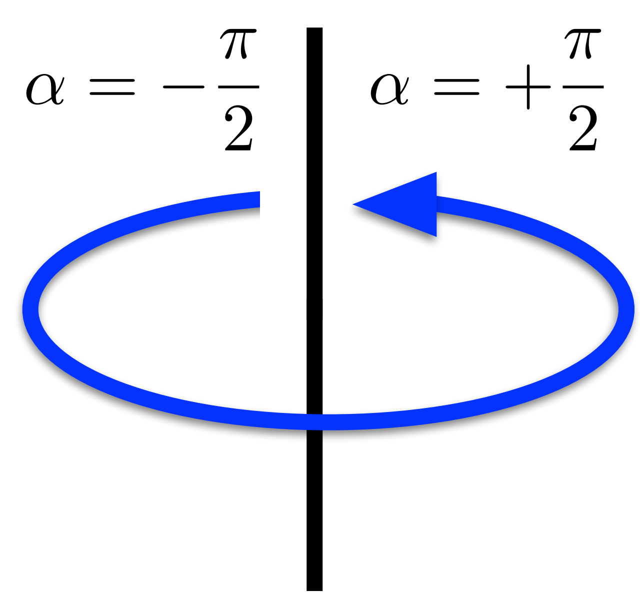
Domain wall(s) attached to the vortex appears

Domain wall - Vortex complex

M. Eto, M. Kurachi and M. Nitta,
arXiv:1803.04662 [hep-ph] (to be published in PLB)

Potential is classified into five types





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Domain wall - Vortex complex

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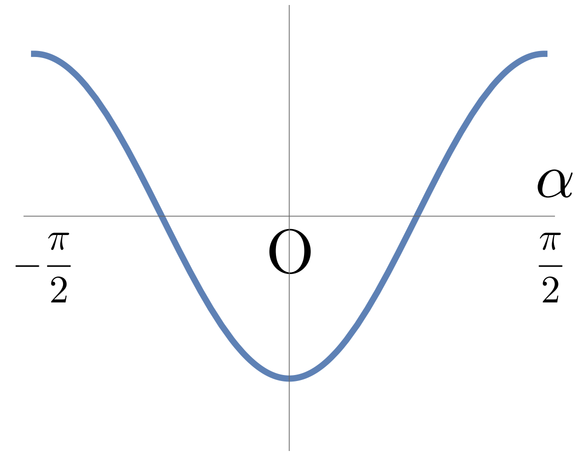
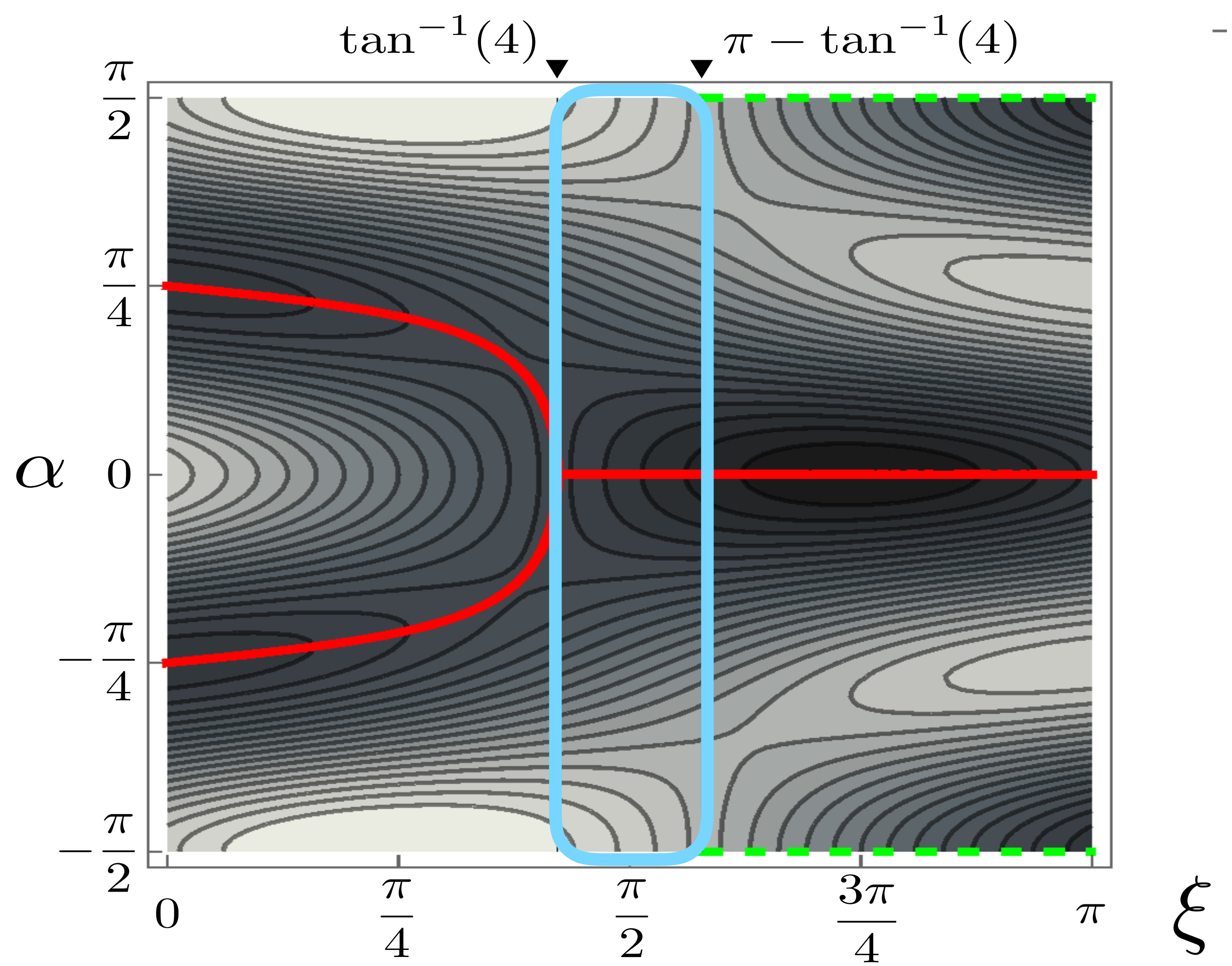
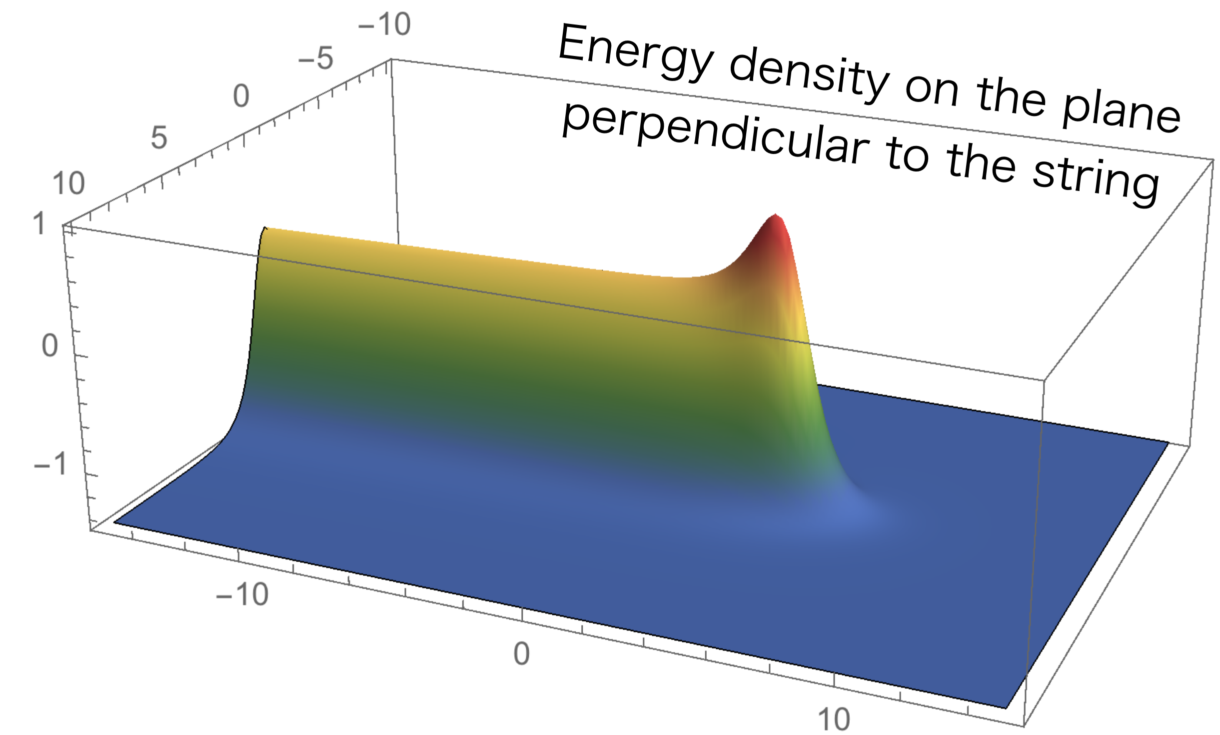
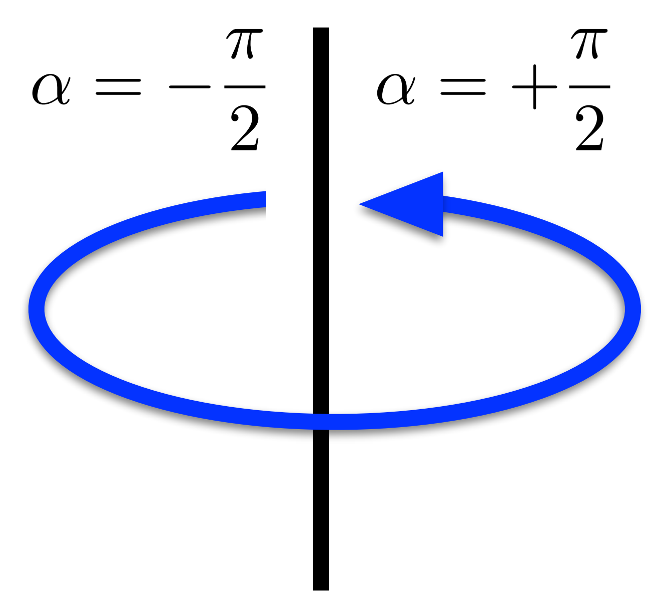




Case III ($\xi = \pi/2$)





Domain wall(s) attached to the vortex appears

Domain wall - Vortex complex

M. Eto, M. Kurachi and M. Nitta,
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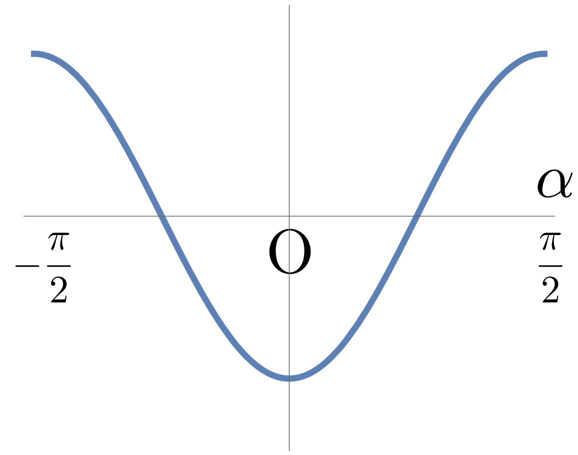
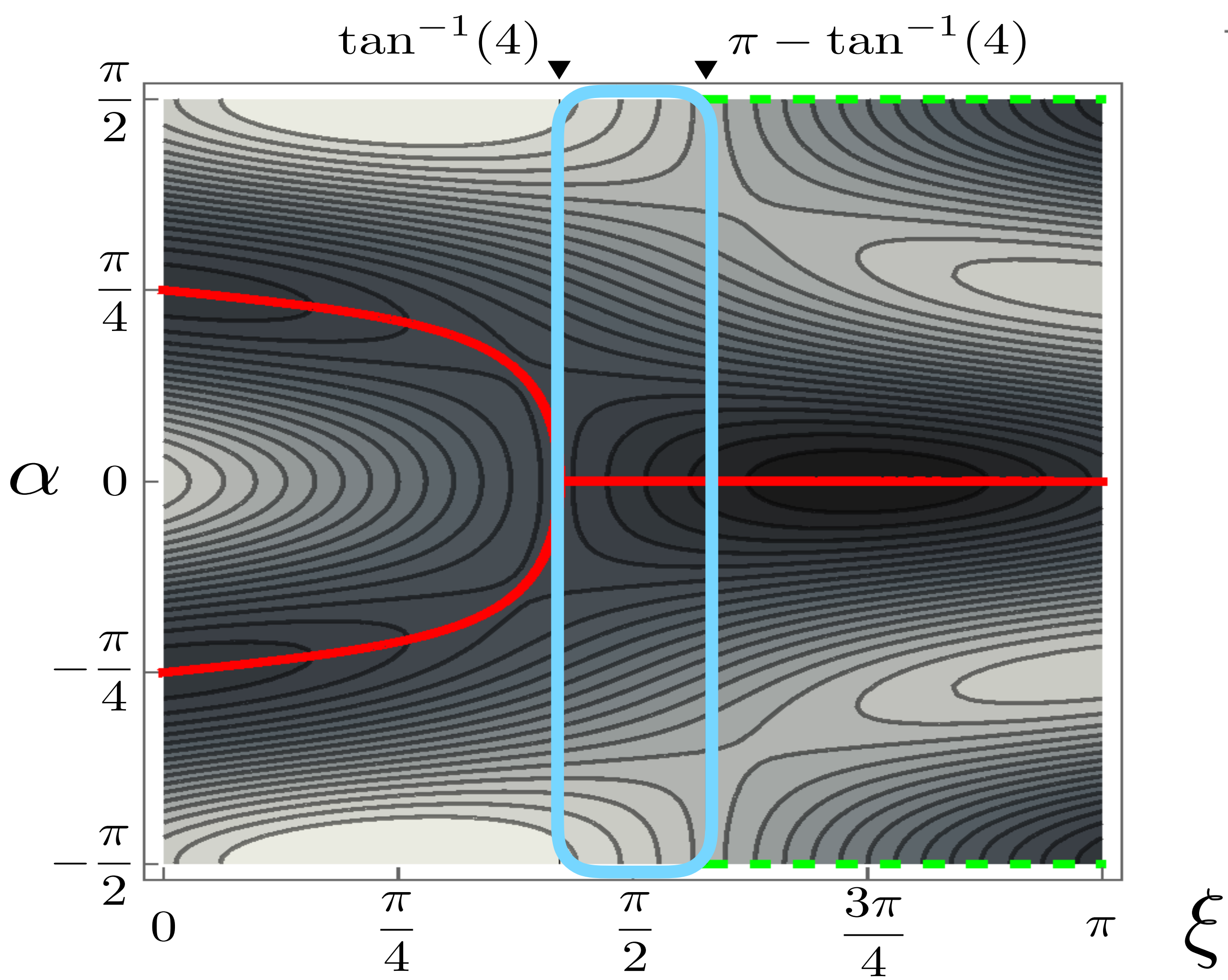
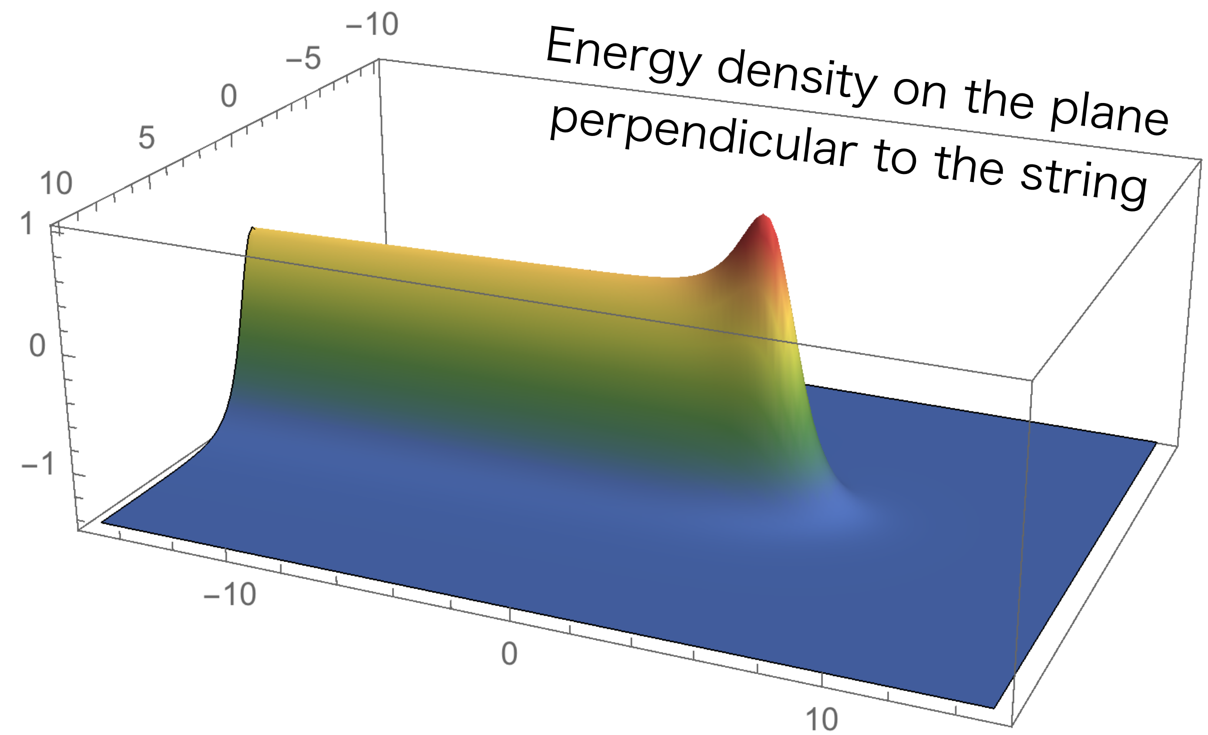
Case III ($\xi = \pi/2$)







One domain wall is attached to the vortex

Domain wall(s) attached to the vortex appears

Domain wall - Vortex complex

M. Eto, M. Kurachi and M. Nitta,
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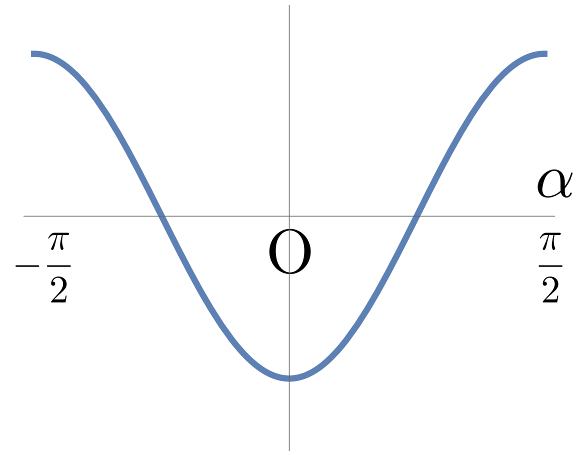
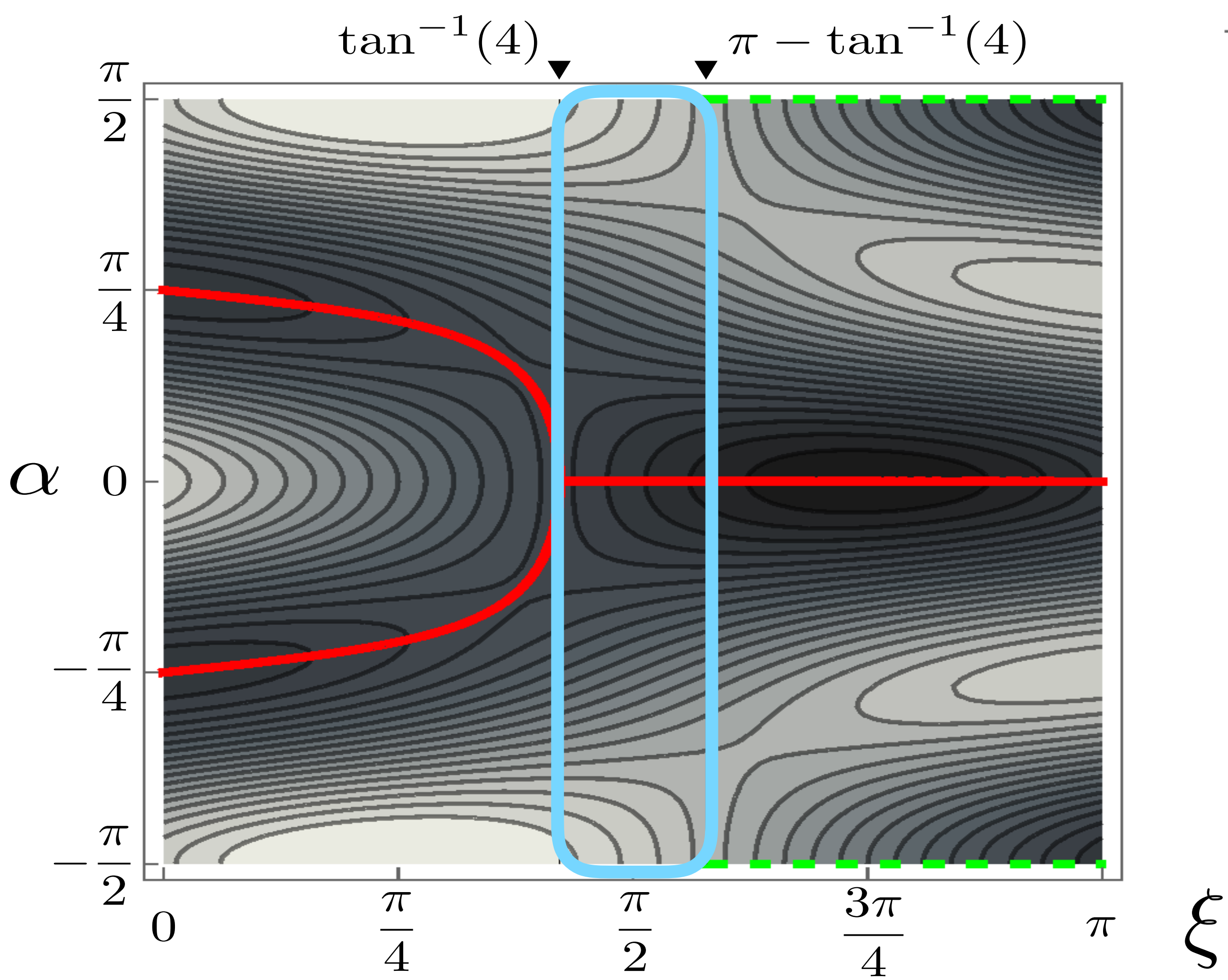
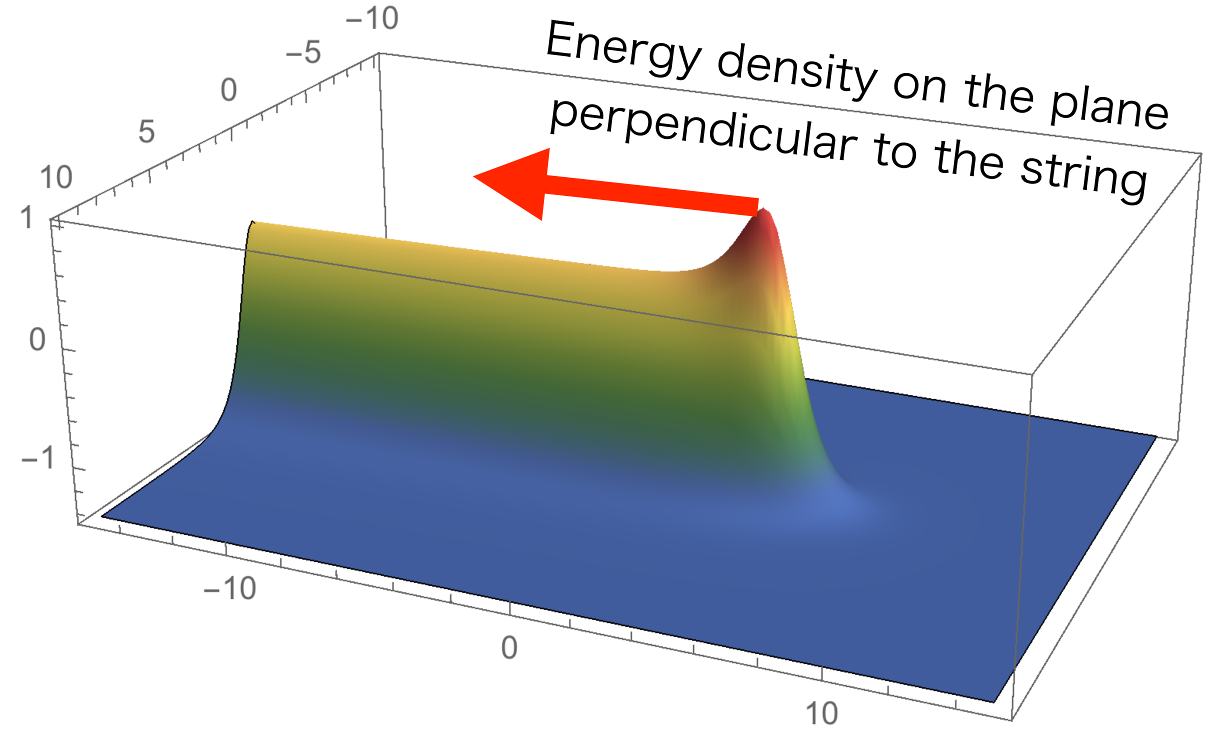




Case III ($\xi = \pi/2$)



One domain wall is attached to the vortex

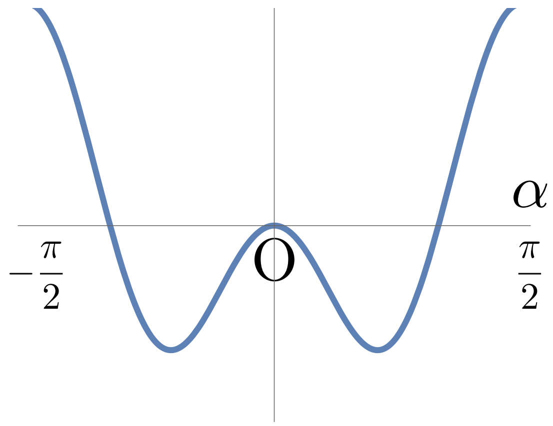
Tension of the wall drags the vortex towards the side of the wall

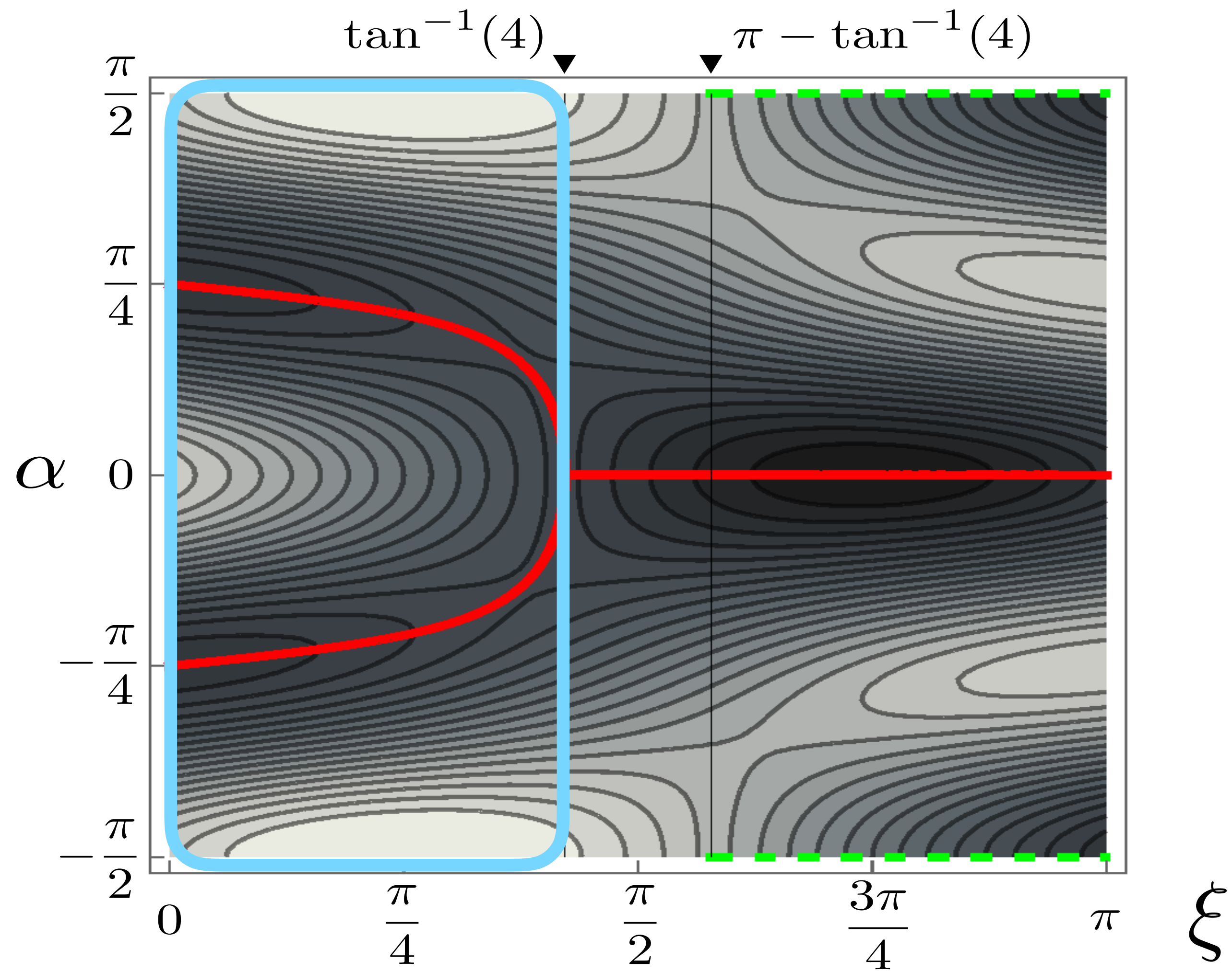
Domain wall(s) attached to the vortex appears

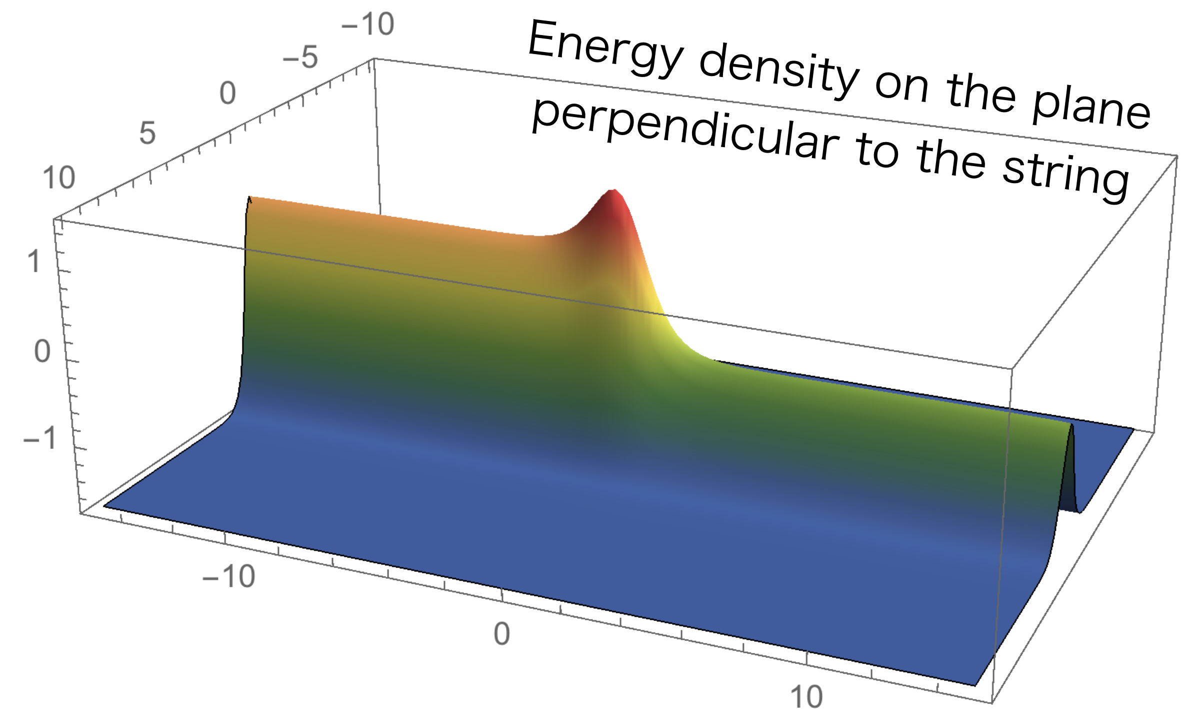
Domain wall - Vortex complex

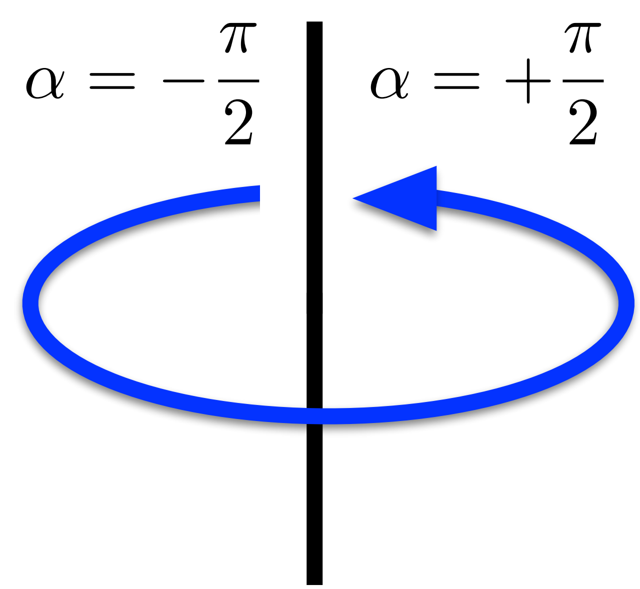
M. Eto, M. Kurachi and M. Nitta,
arXiv:1803.04662 [hep-ph] (to be published in PLB)

Case II $(\xi = \pi/4)$









Domain wall(s) attached to the vortex appears

Domain wall - Vortex complex

M. Eto, M. Kurachi and M. Nitta,
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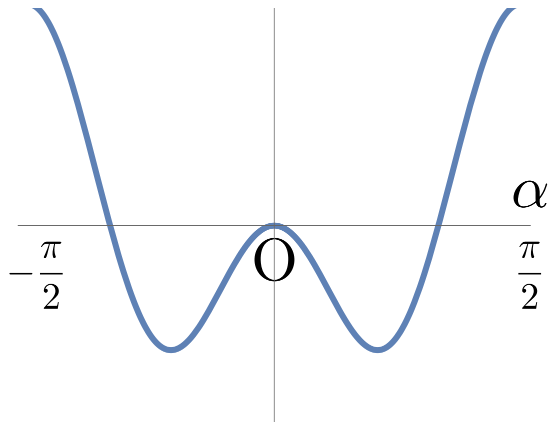
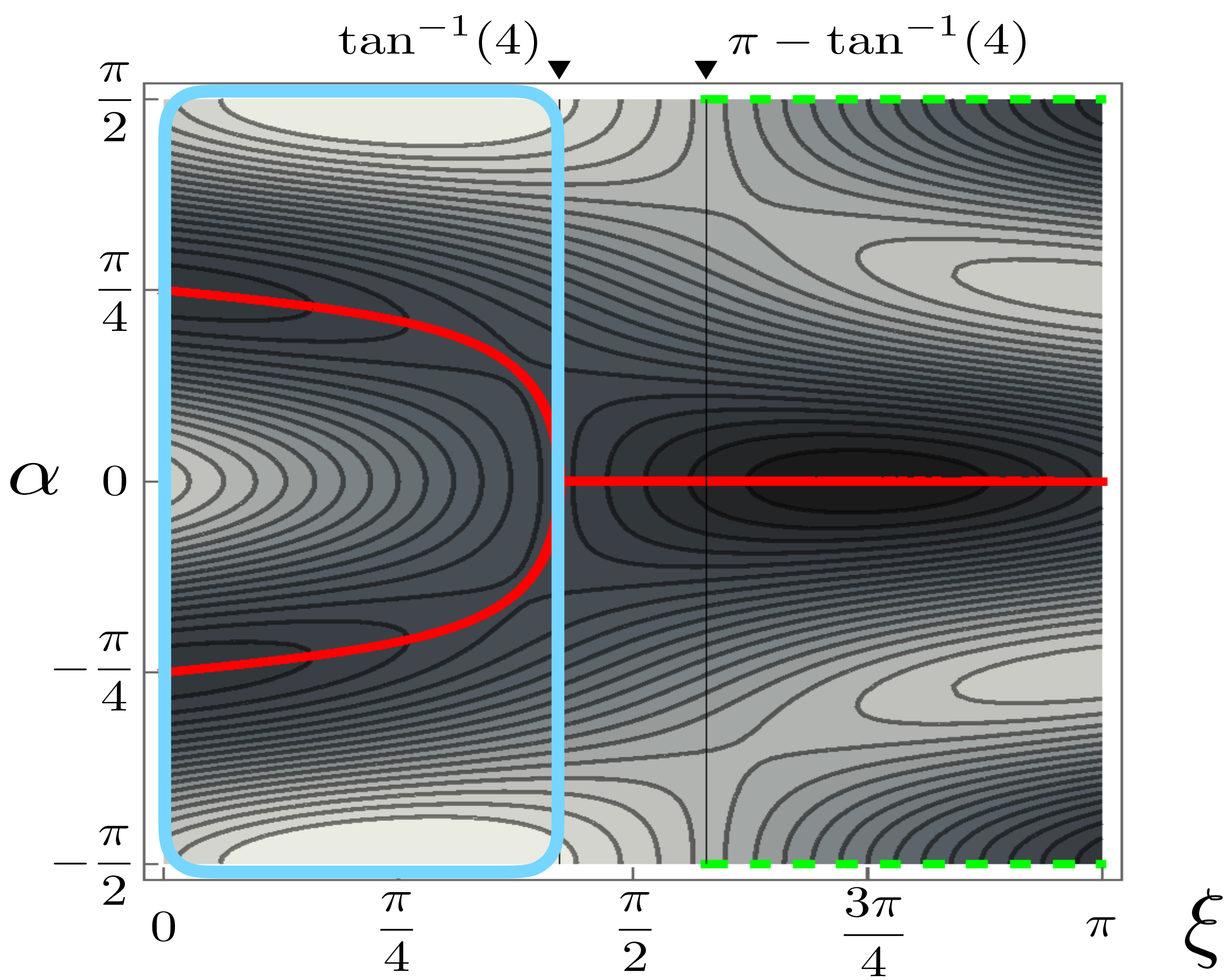
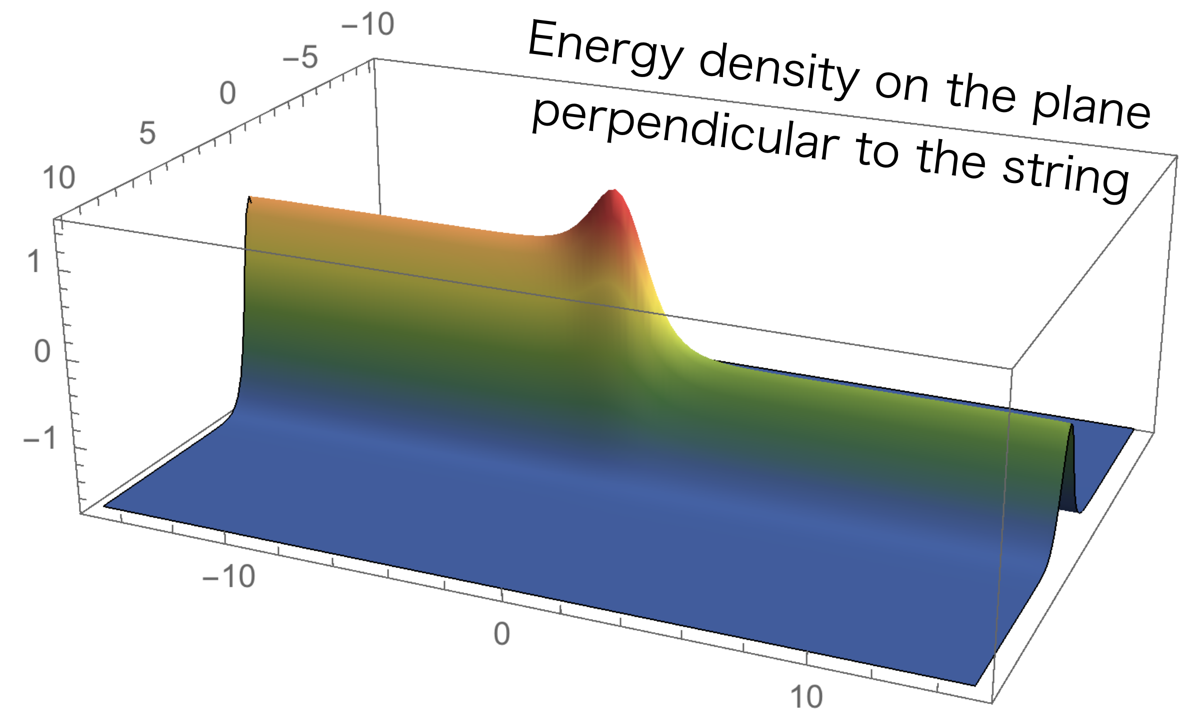




Case II $(\xi = \pi/4)$



Two domain walls are attached to the vortex

Domain wall(s) attached to the vortex appears

Domain wall - Vortex complex

M. Eto, M. Kurachi and M. Nitta,
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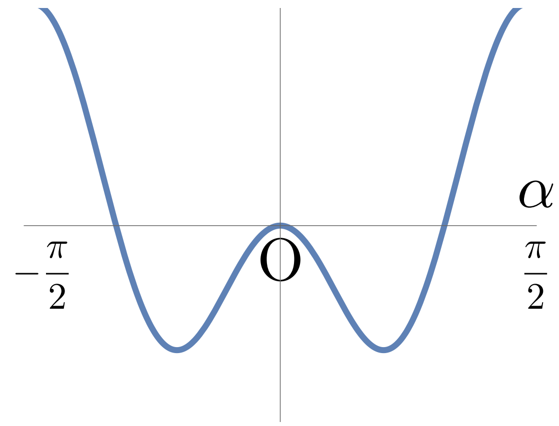
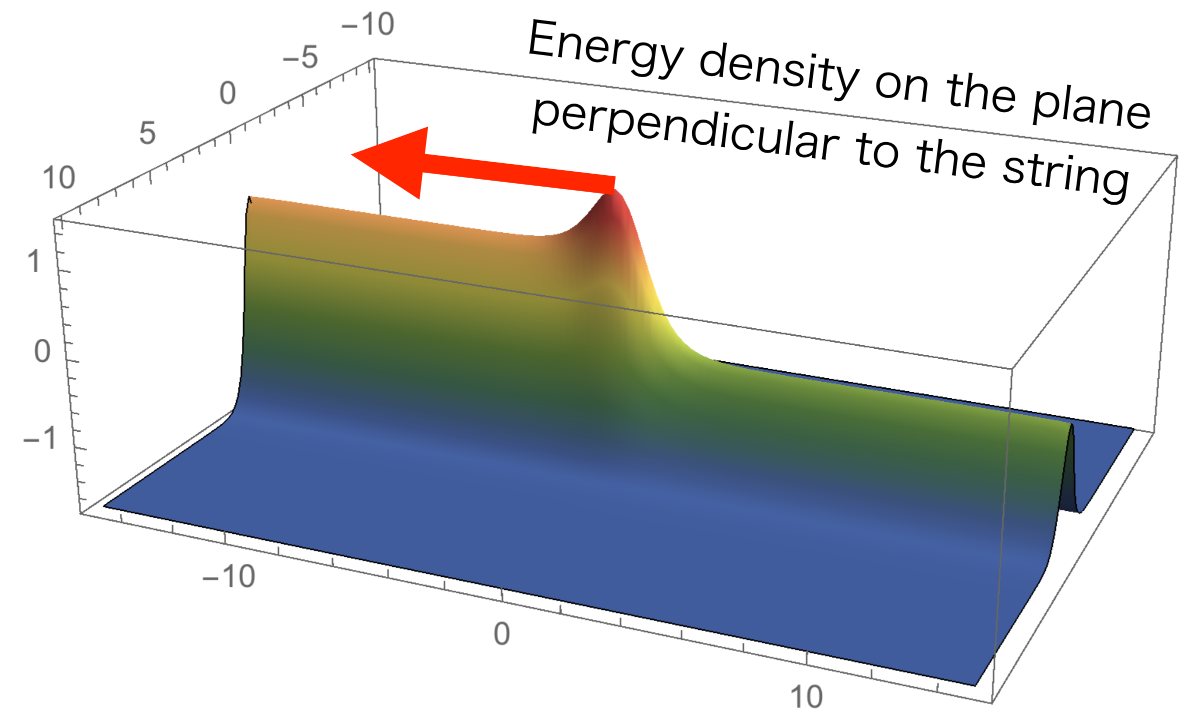
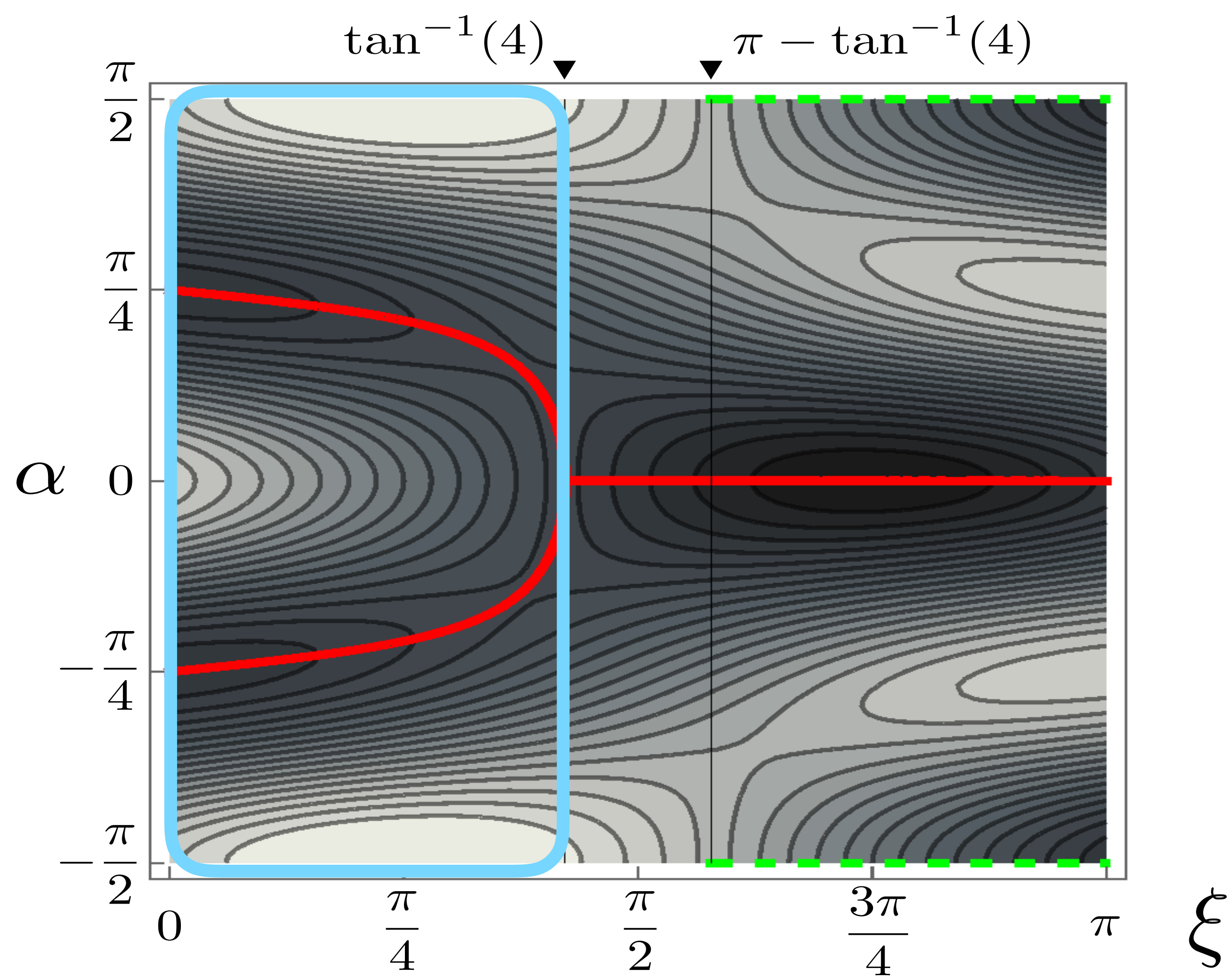
Case II ($\xi = \pi/4$)







Two domain walls are attached to the vortex

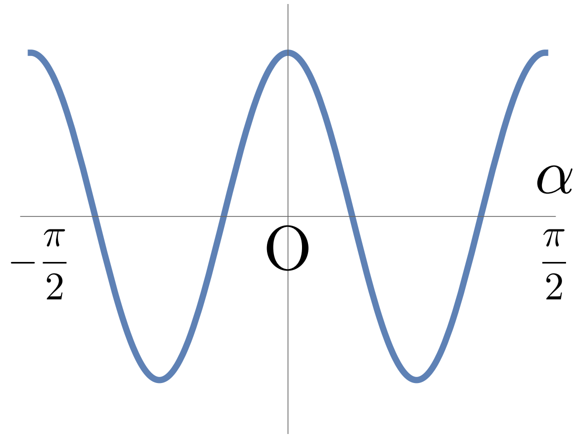
Due to the difference of the tension of two domain walls, the vortex moves toward higher-tension wall

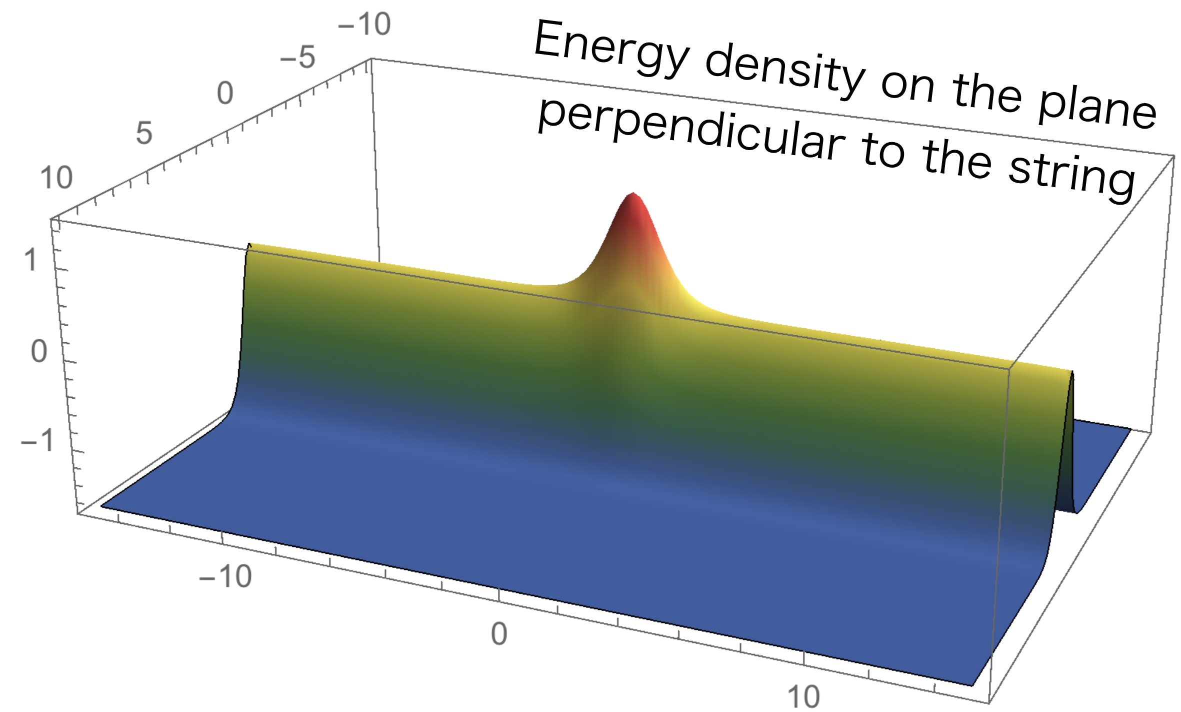
Domain wall(s) attached to the vortex appears

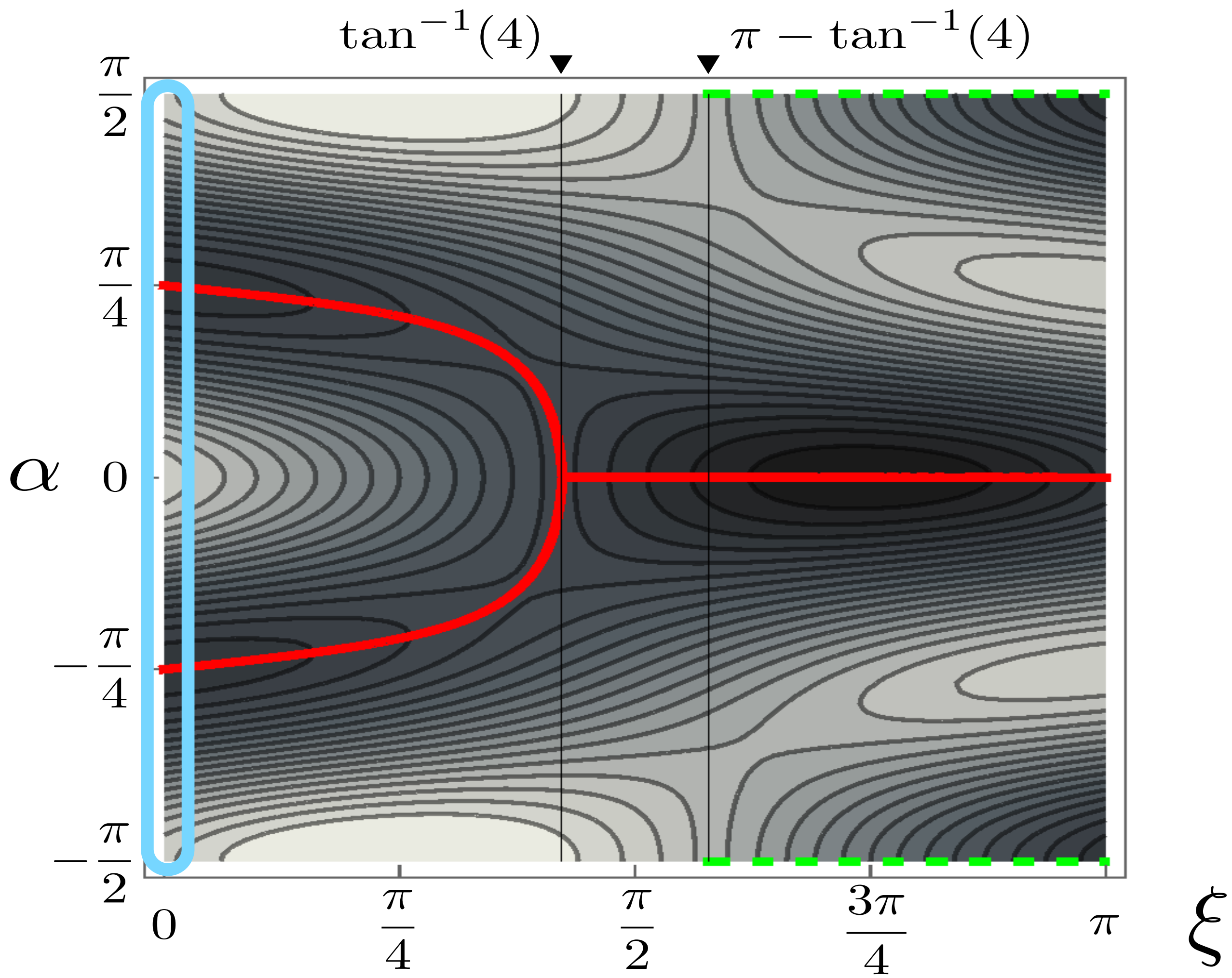
Domain wall - Vortex complex

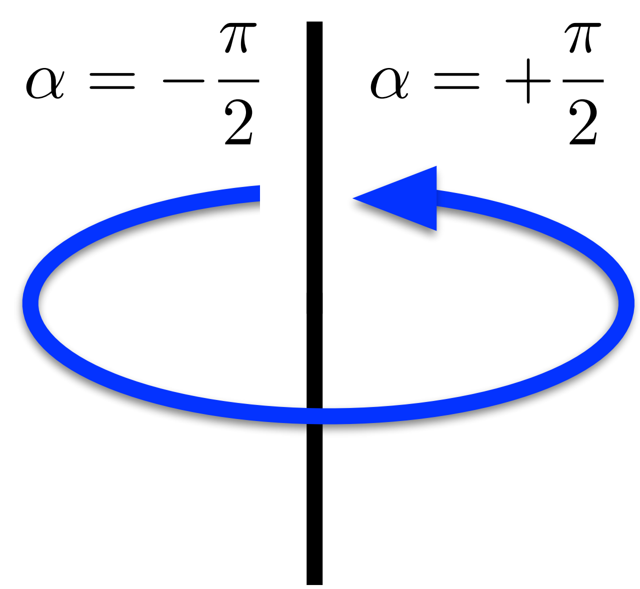
M. Eto, M. Kurachi and M. Nitta,
arXiv:1803.04662 [hep-ph] (to be published in PLB)

Case I ($\xi = 0$)









Domain wall(s) attached to the vortex appears

Domain wall - Vortex complex

M. Eto, M. Kurachi and M. Nitta,
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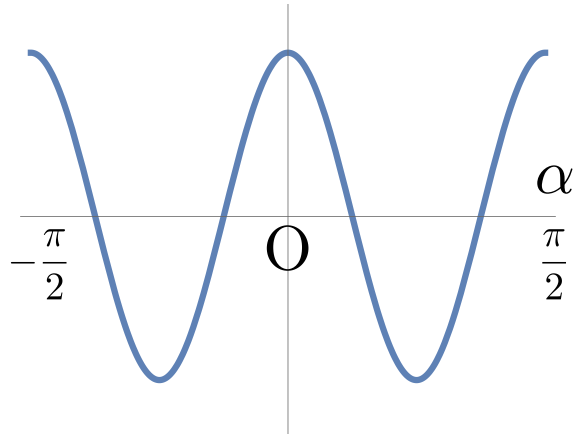
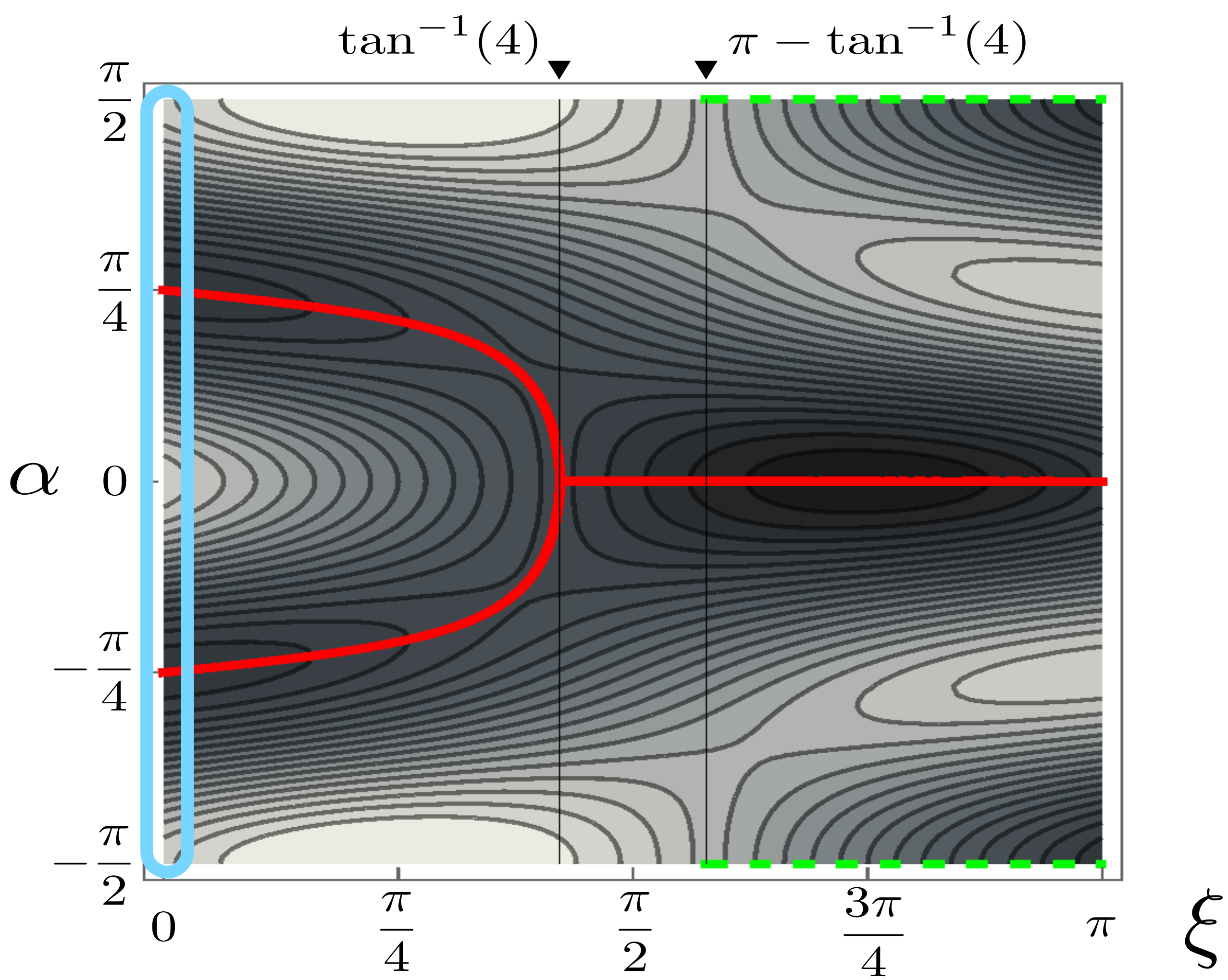
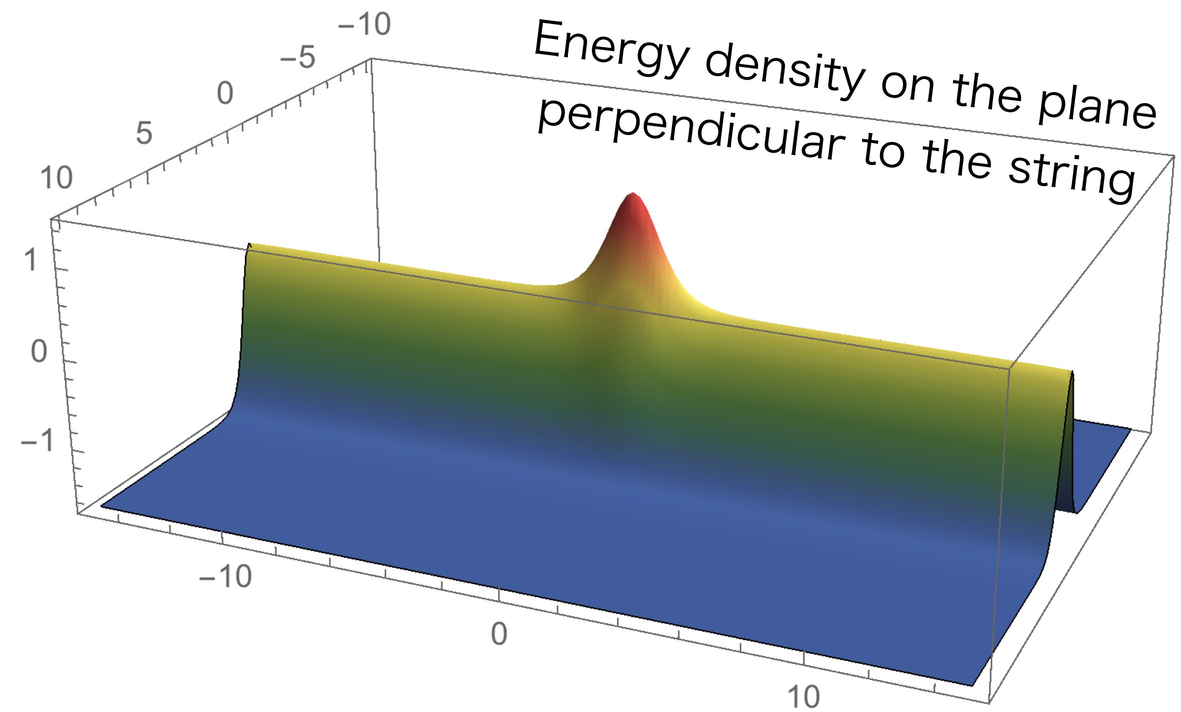




Case I ($\xi = 0$)

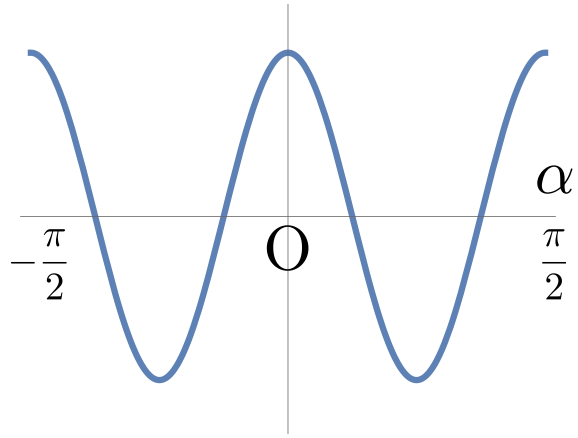


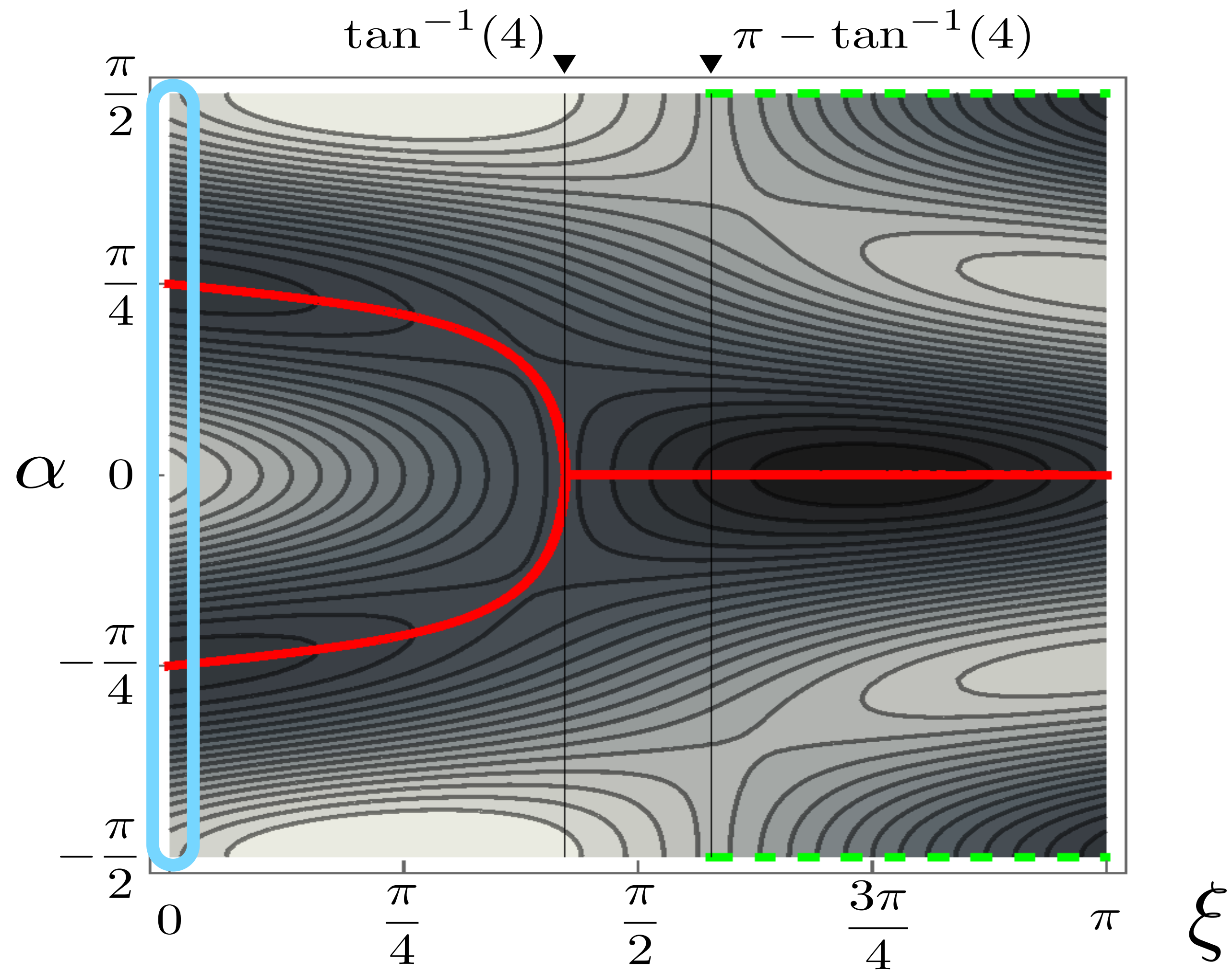
Two domain walls that have the same tension are attached to the vortex

Domain wall(s) attached to the vortex appears

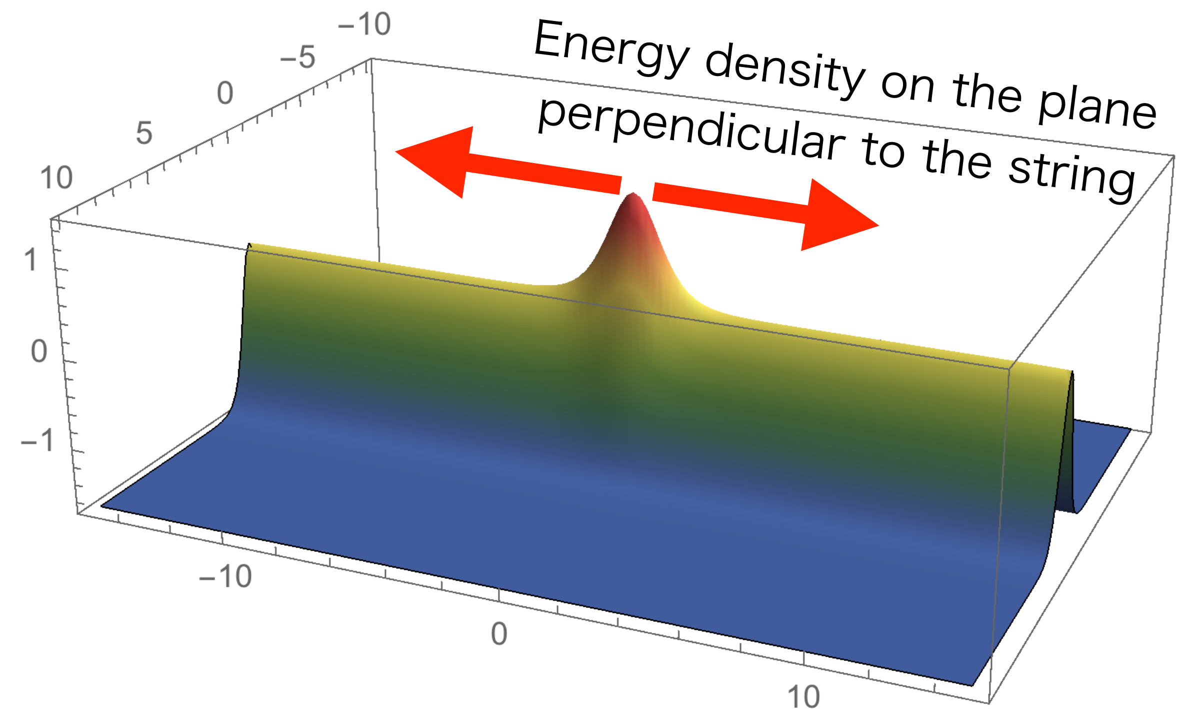
Domain wall - Vortex complex

M. Eto, M. Kurachi and M. Nitta,
arXiv:1803.04662 [hep-ph] (to be published in PLB)





Case I ($\xi = 0$)



Two domain walls that have the same tension are attached to the vortex

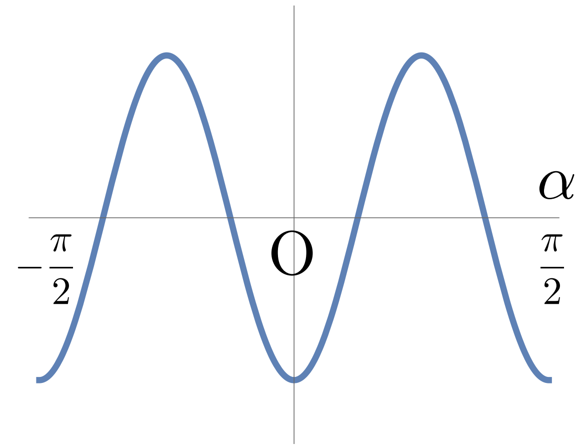
Since the tensions from both side of walls are balanced, the vortex is stable

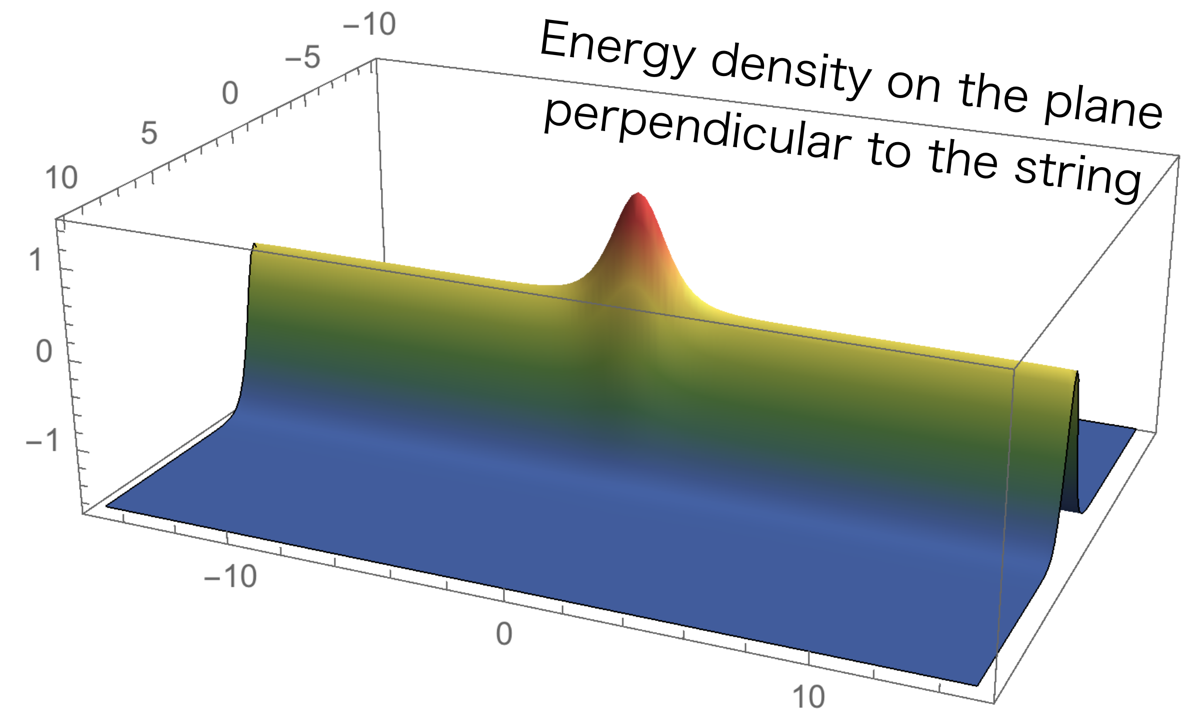
Domain wall(s) attached to the vortex appears

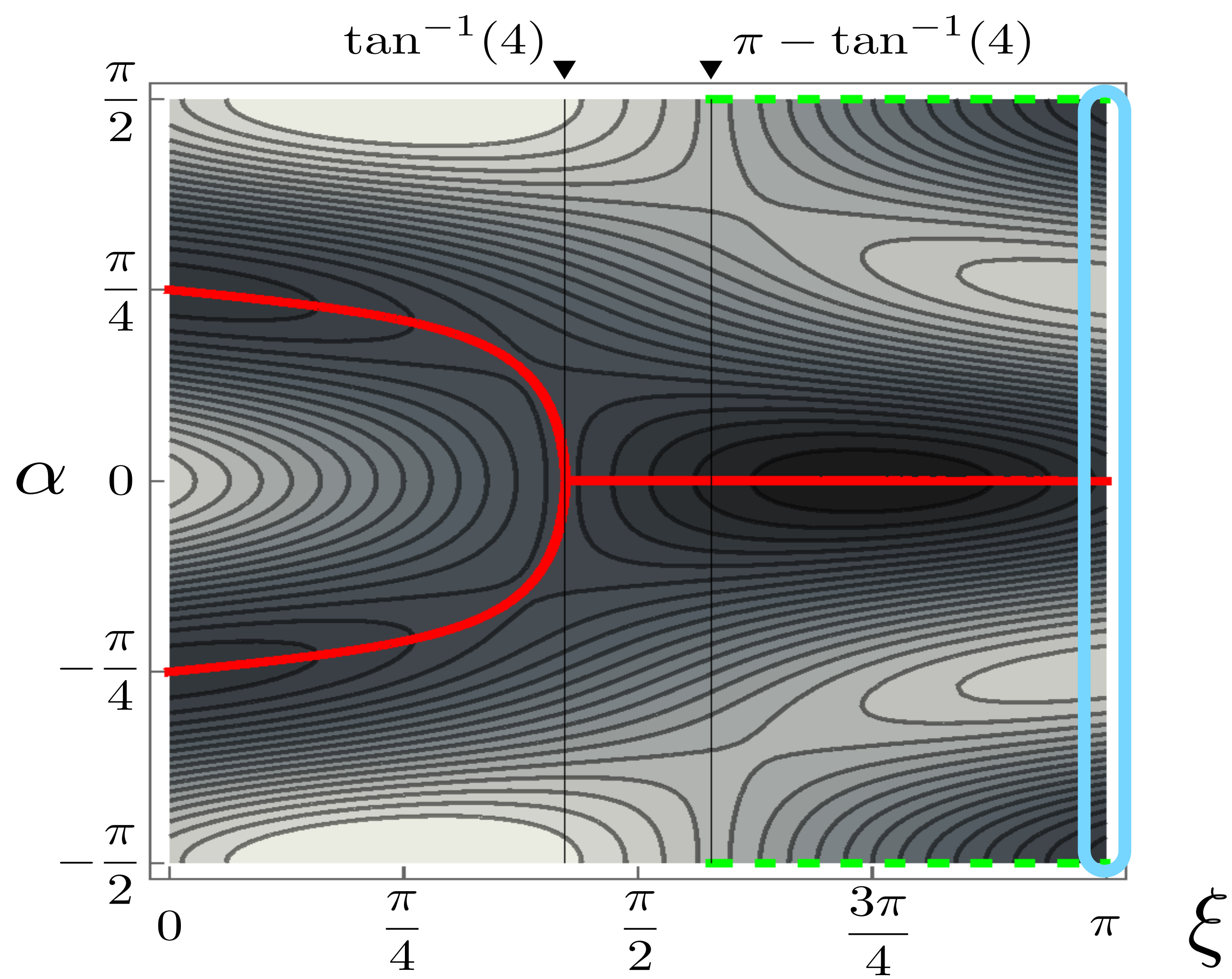
Domain wall - Vortex complex

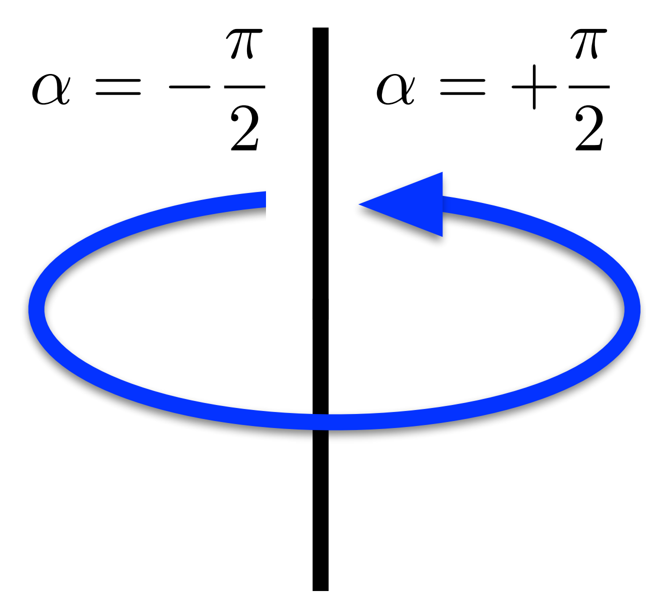
M. Eto, M. Kurachi and M. Nitta,
arXiv:1803.04662 [hep-ph] (to be published in PLB)

Case V $(\xi = \pi)$





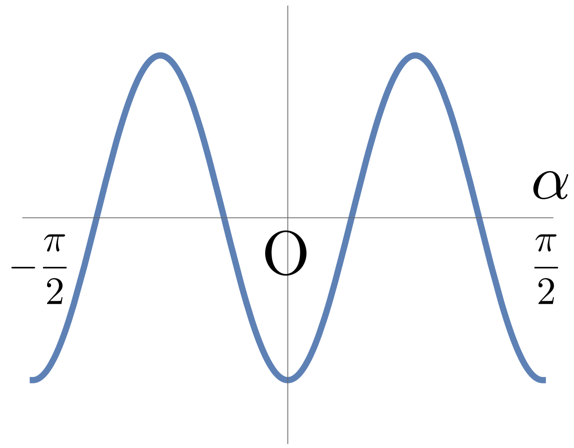


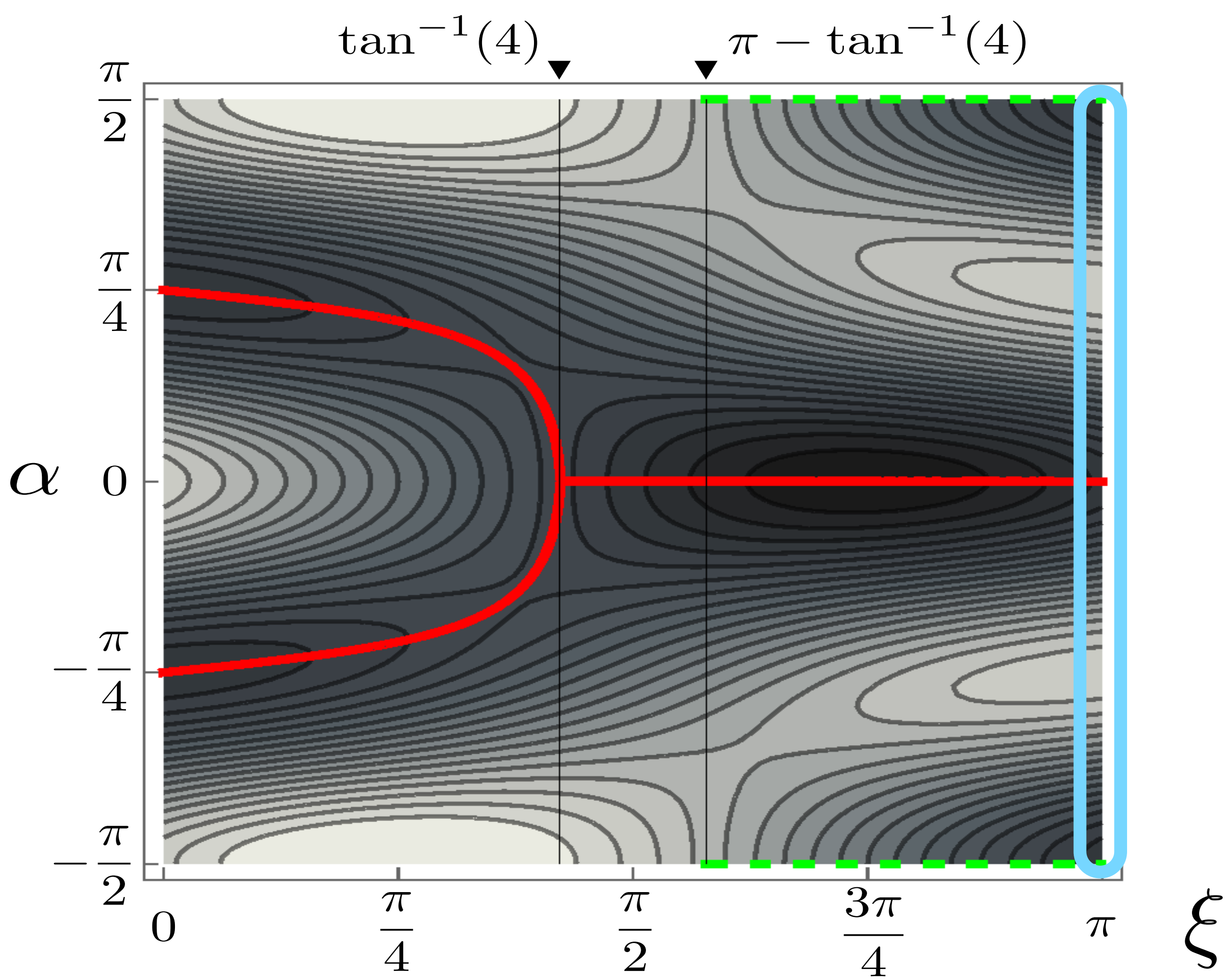


Domain wall(s) attached to the vortex appears

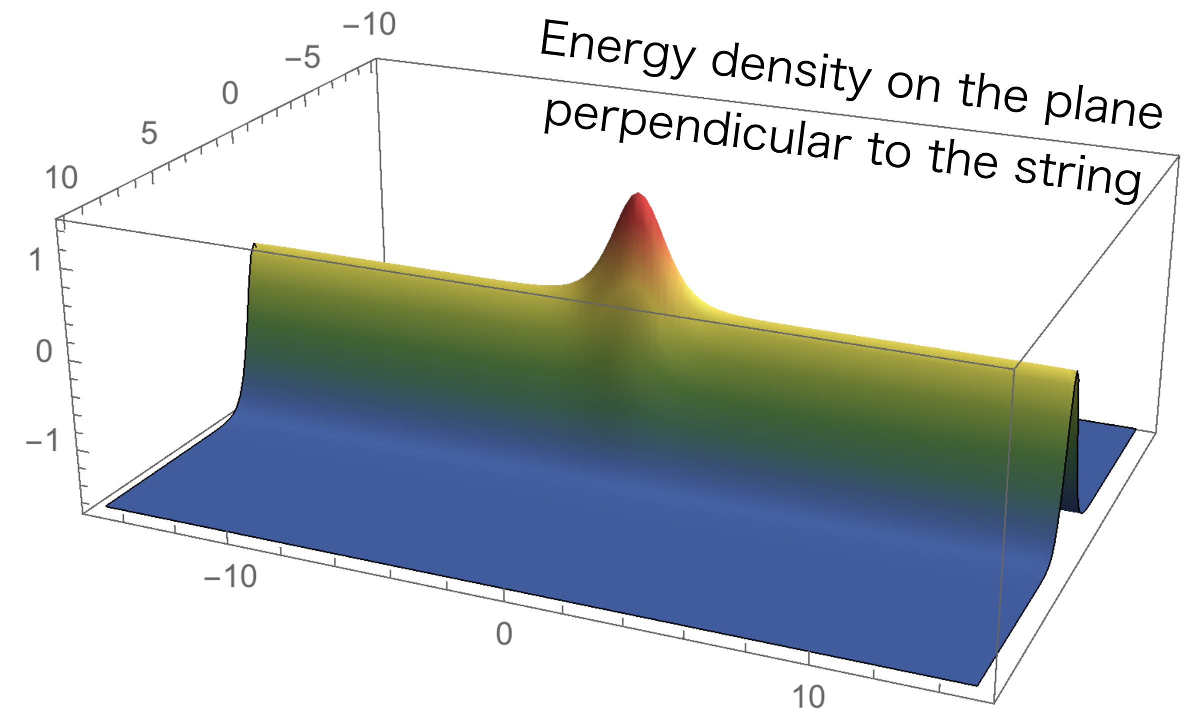
Domain wall - Vortex complex

M. Eto, M. Kurachi and M. Nitta,
arXiv:1803.04662 [hep-ph] (to be published in PLB)





Case V $(\xi = \pi)$

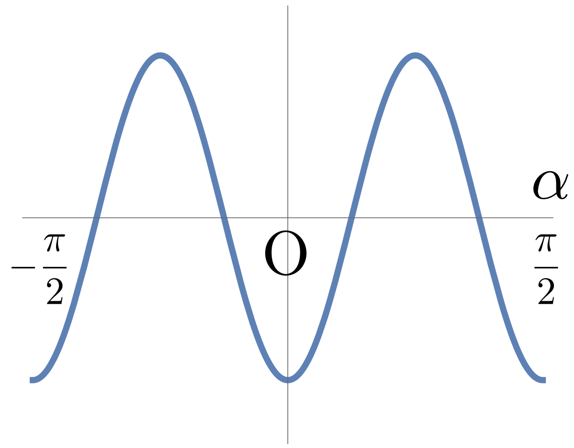


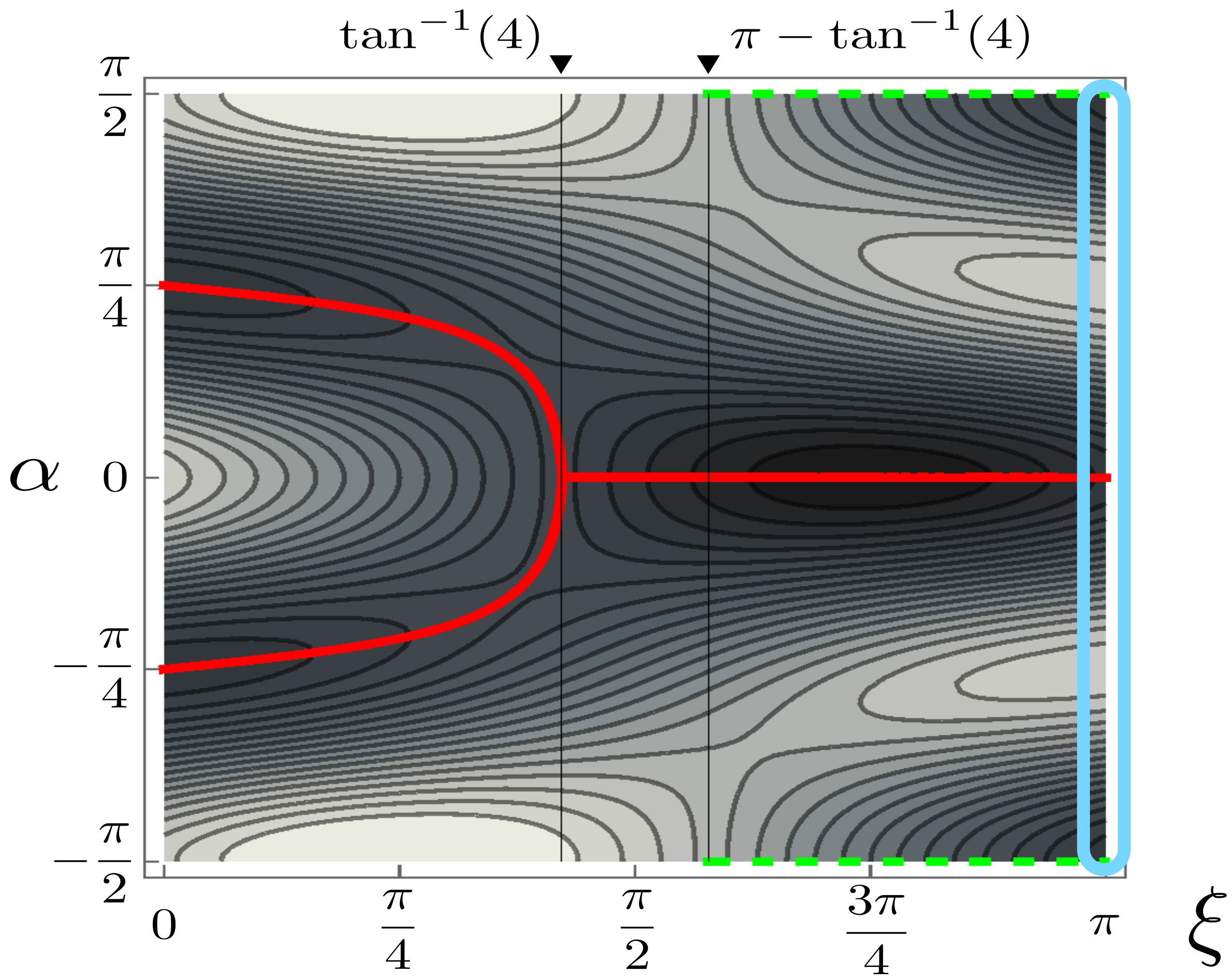
Two domain walls that have the same tension are attached to the vortex

Domain wall(s) attached to the vortex appears

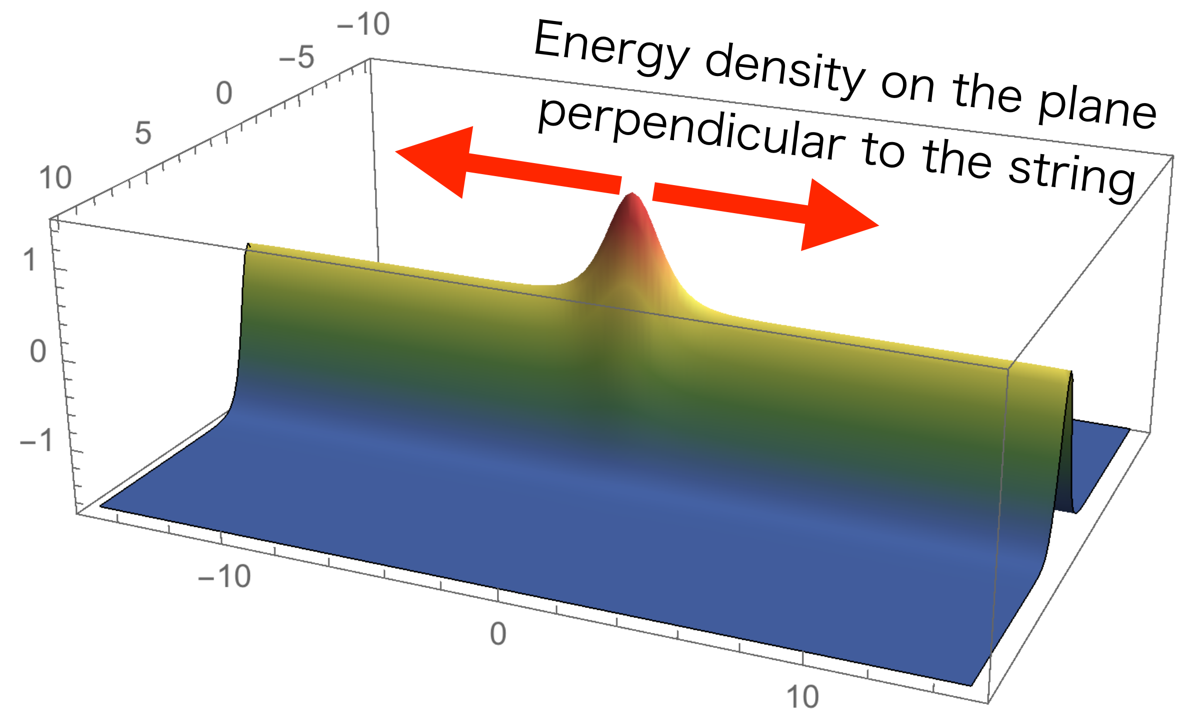
Domain wall - Vortex complex

M. Eto, M. Kurachi and M. Nitta,
arXiv:1803.04662 [hep-ph] (to be published in PLB)





Case V $(\xi = \pi)$



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Since the tensions from both side of walls are balanced, the vortex is stable

Domain wall(s) attached to the vortex appears

Domain wall - Vortex complex

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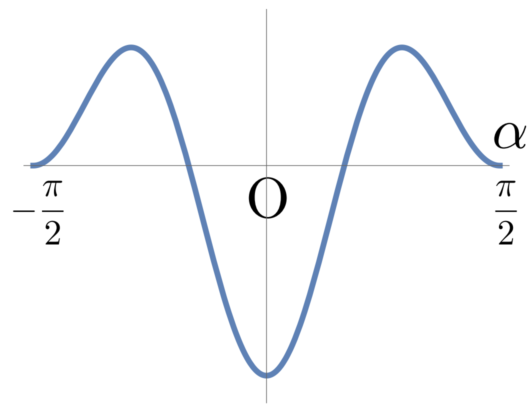
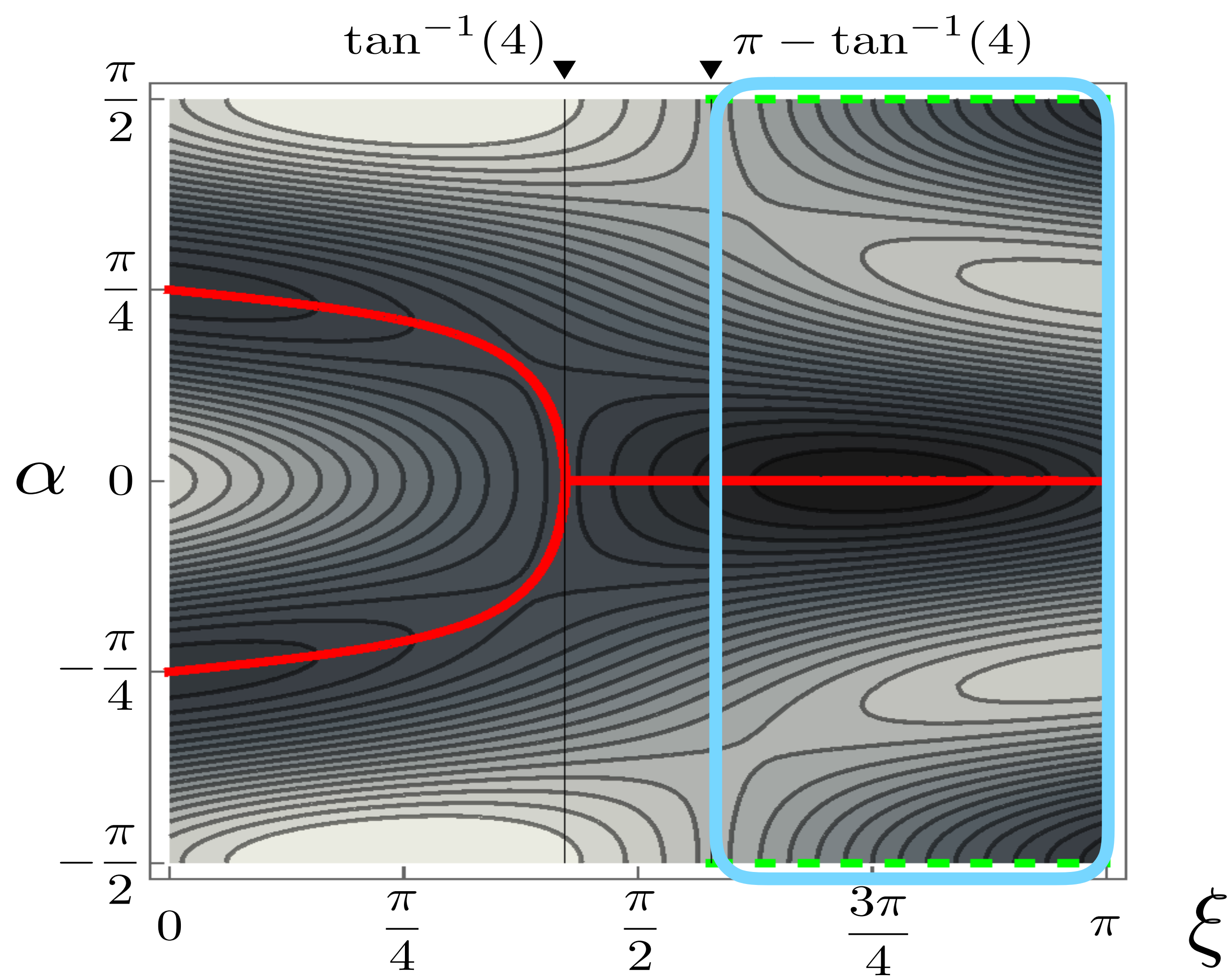
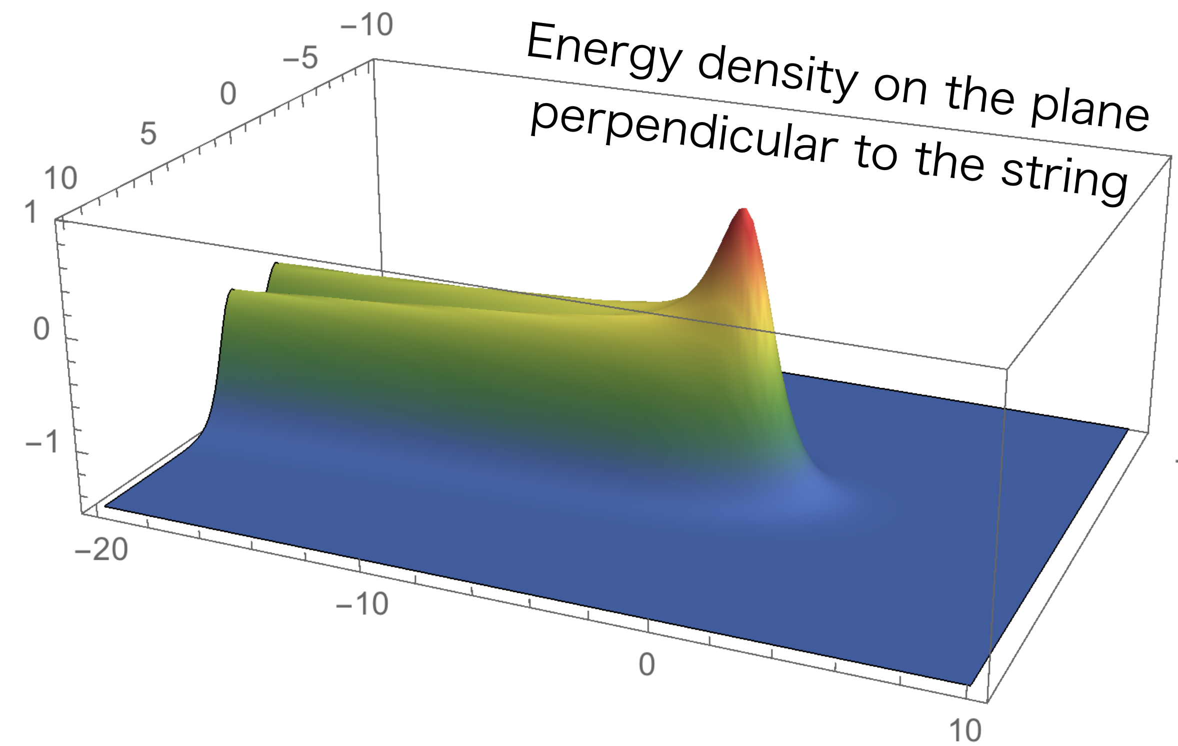
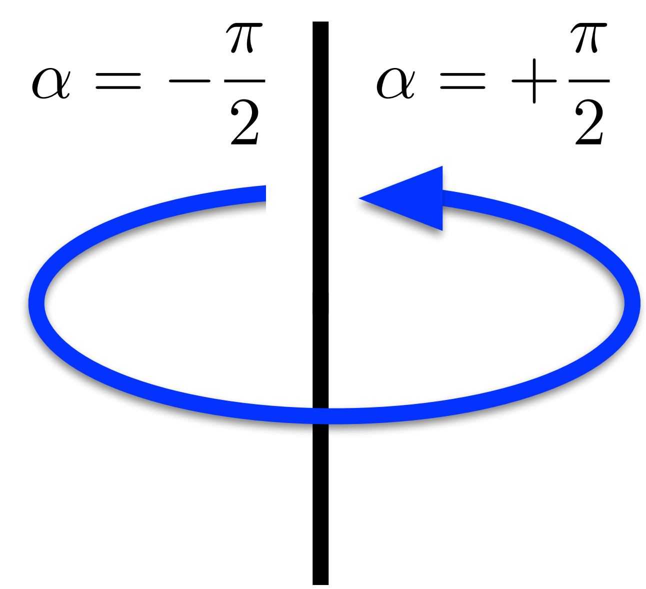




Case IV $(\xi = 3\pi/4)$



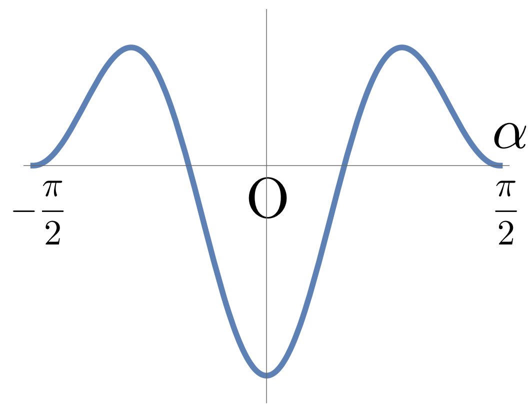


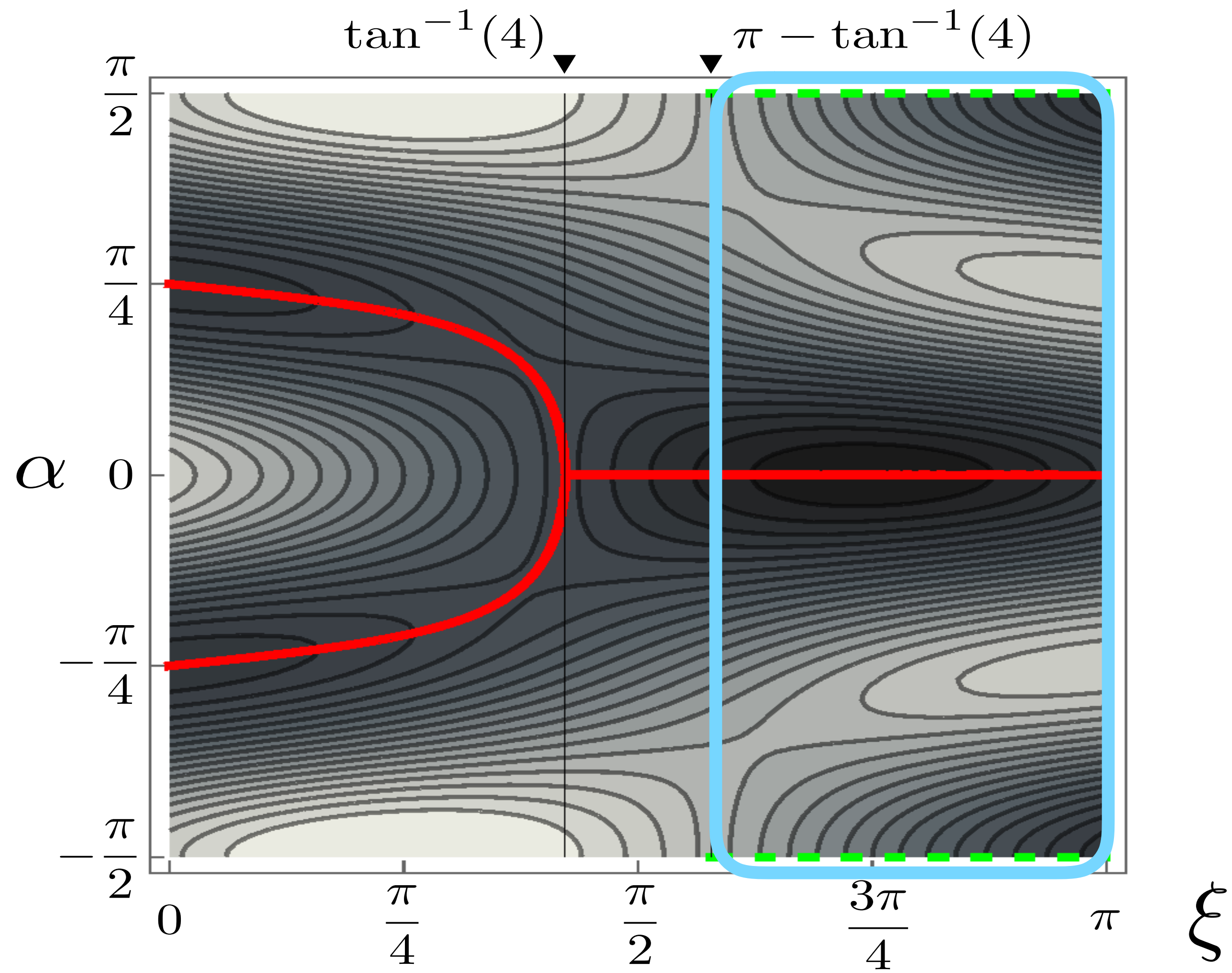
Domain wall(s) attached to the vortex appears

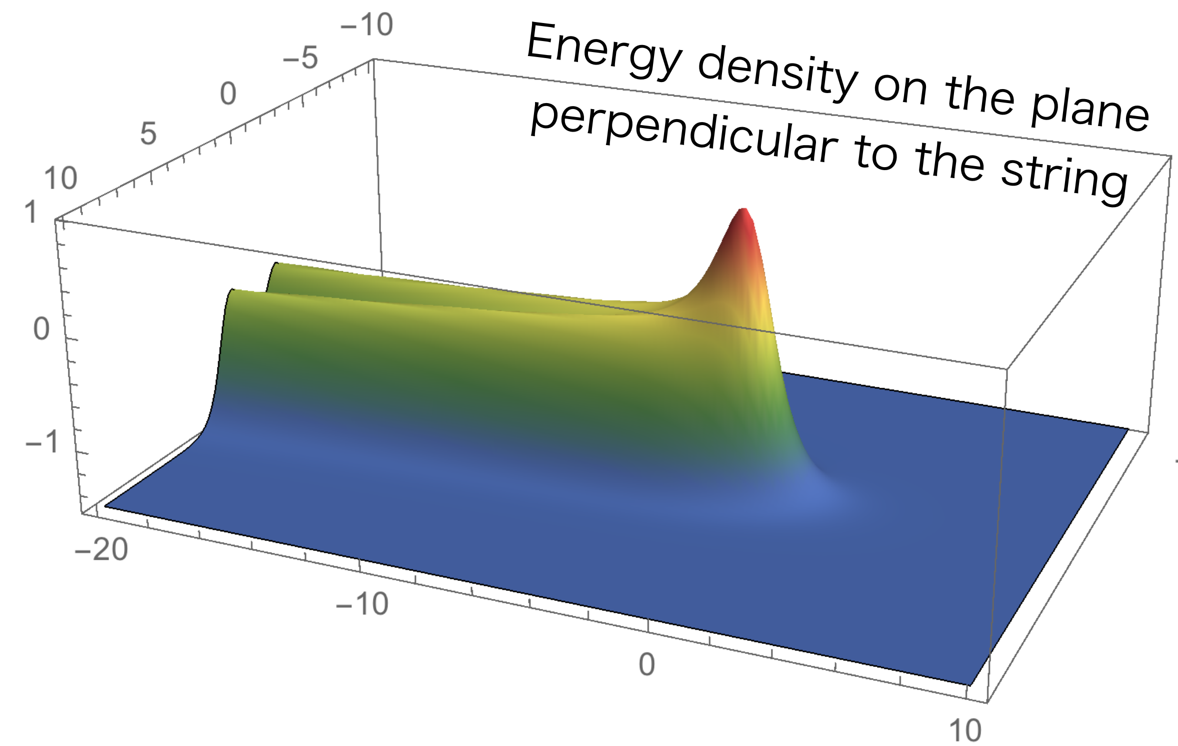
Domain wall - Vortex complex

M. Eto, M. Kurachi and M. Nitta,
arXiv:1803.04662 [hep-ph] (to be published in PLB)

Case IV $(\xi = 3\pi/4)$







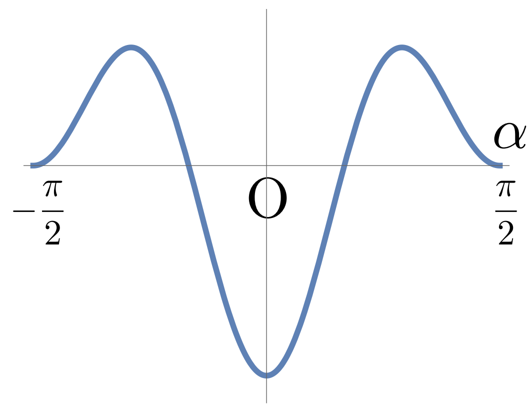
Two domain walls are attached to the vortex from one side

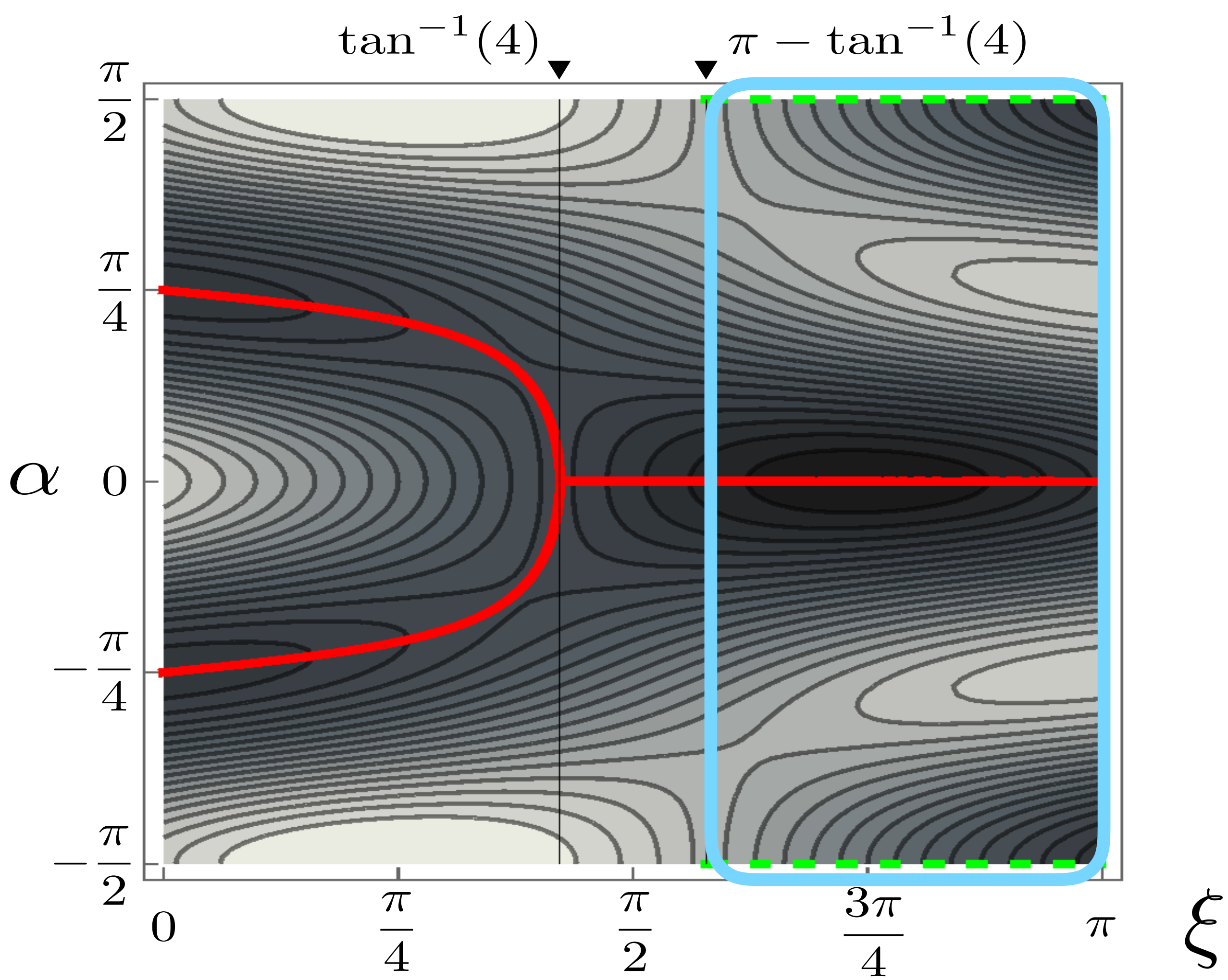
Domain wall(s) attached to the vortex appears

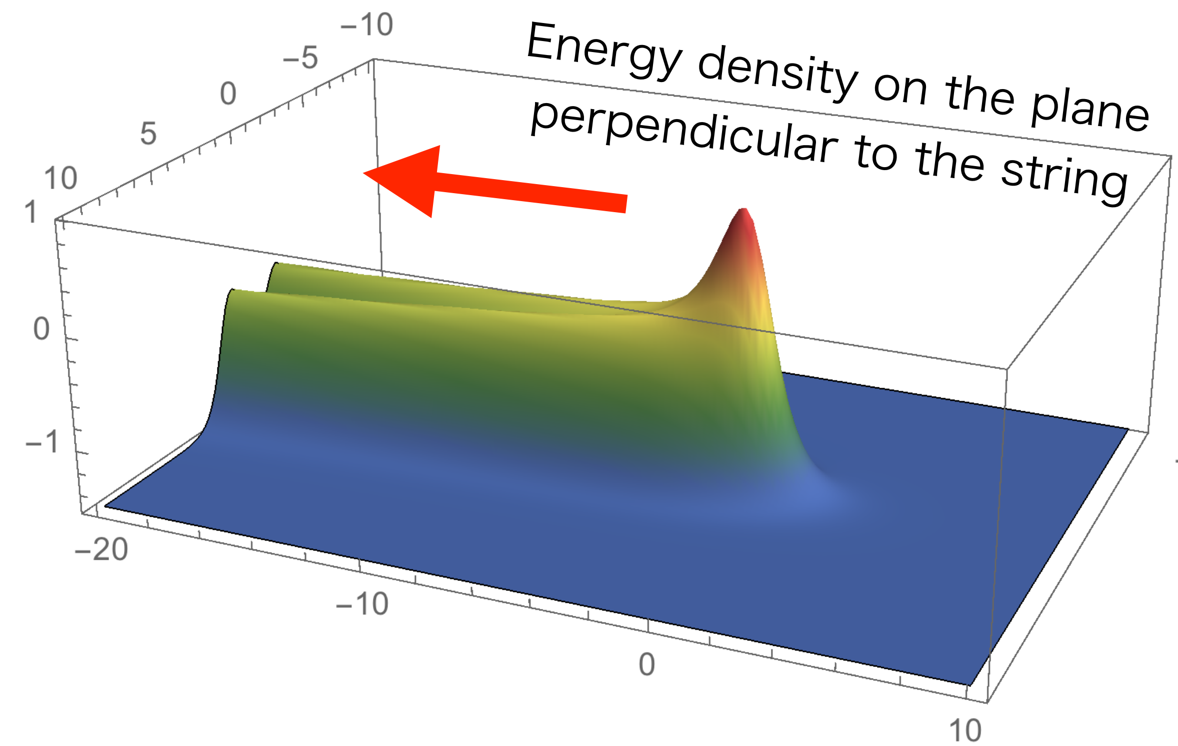
Domain wall - Vortex complex

M. Eto, M. Kurachi and M. Nitta,
arXiv:1803.04662 [hep-ph] (to be published in PLB)

Case IV $(\xi = 3\pi/4)$







Two domain walls are attached to the vortex from one side

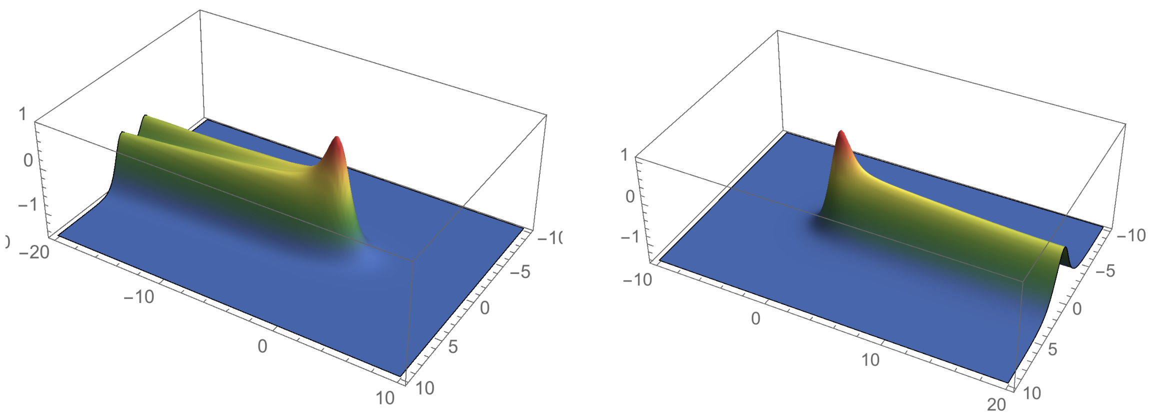
Tension of walls drags the vortex towards the side of walls

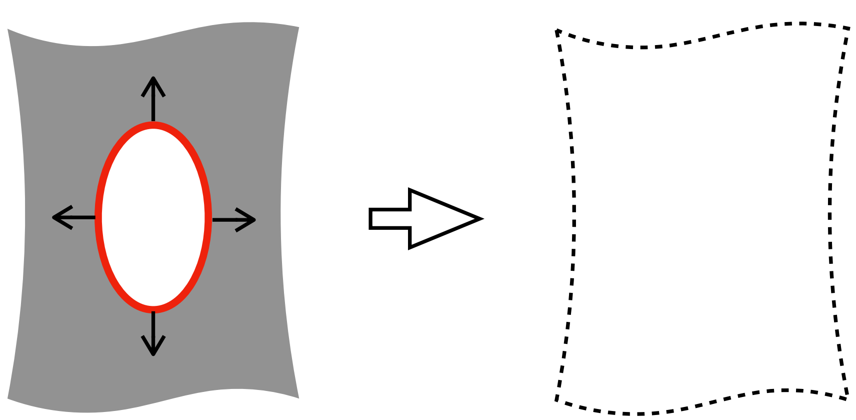
Domain wall(s) attached to the vortex appears

Stability of the domain wall / membrane

$$\tan^{-1}(4) \leq \xi < \pi$$

(Cases III, and IV)



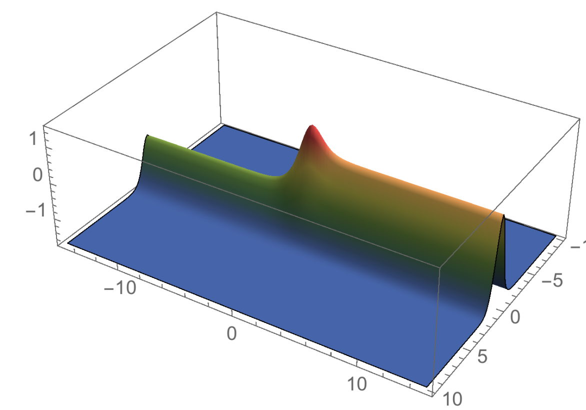


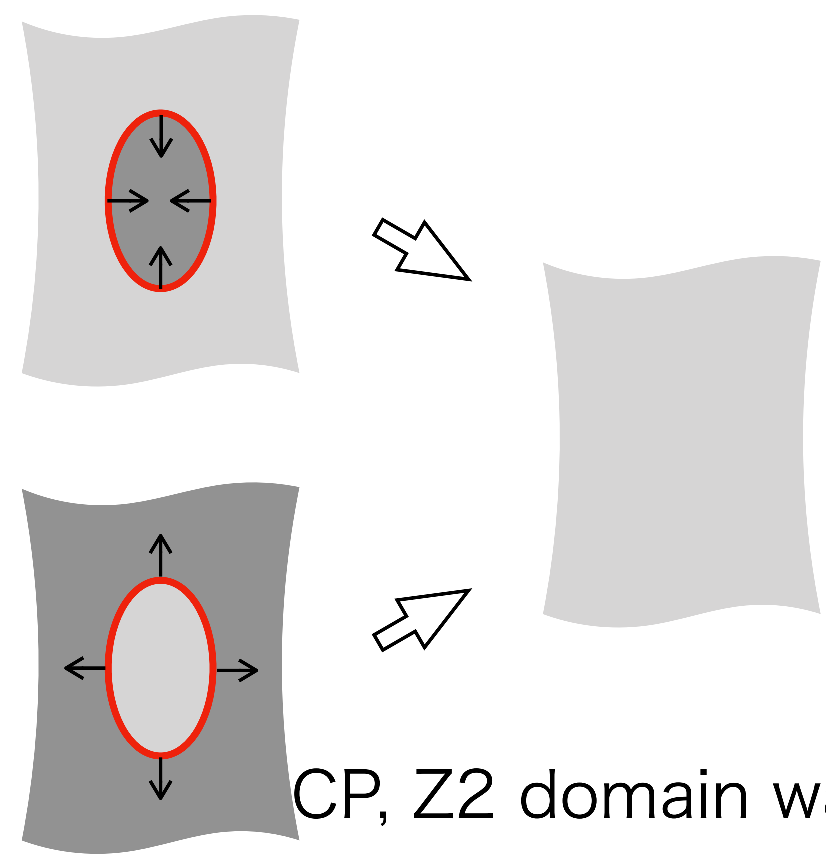
membrane disappears by creating a hole bounded by a vortex

(Cosmologically safe)

$$0 \leq \xi < \tan^{-1}(4) \quad \& \quad \xi = \pi$$

(Cases I, II, and V)





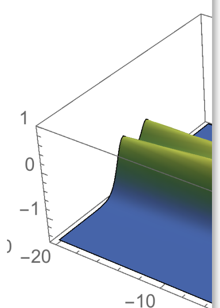
CP, Z2 domain walls remain

(Cosmologically ruled out)

Stability of the domain wall / membrane

$\tan^{-1}(4) \leq \xi < \pi$

(Cases III, and IV)



mem

a hole bounded by a vortex

(Cosmologically safe)

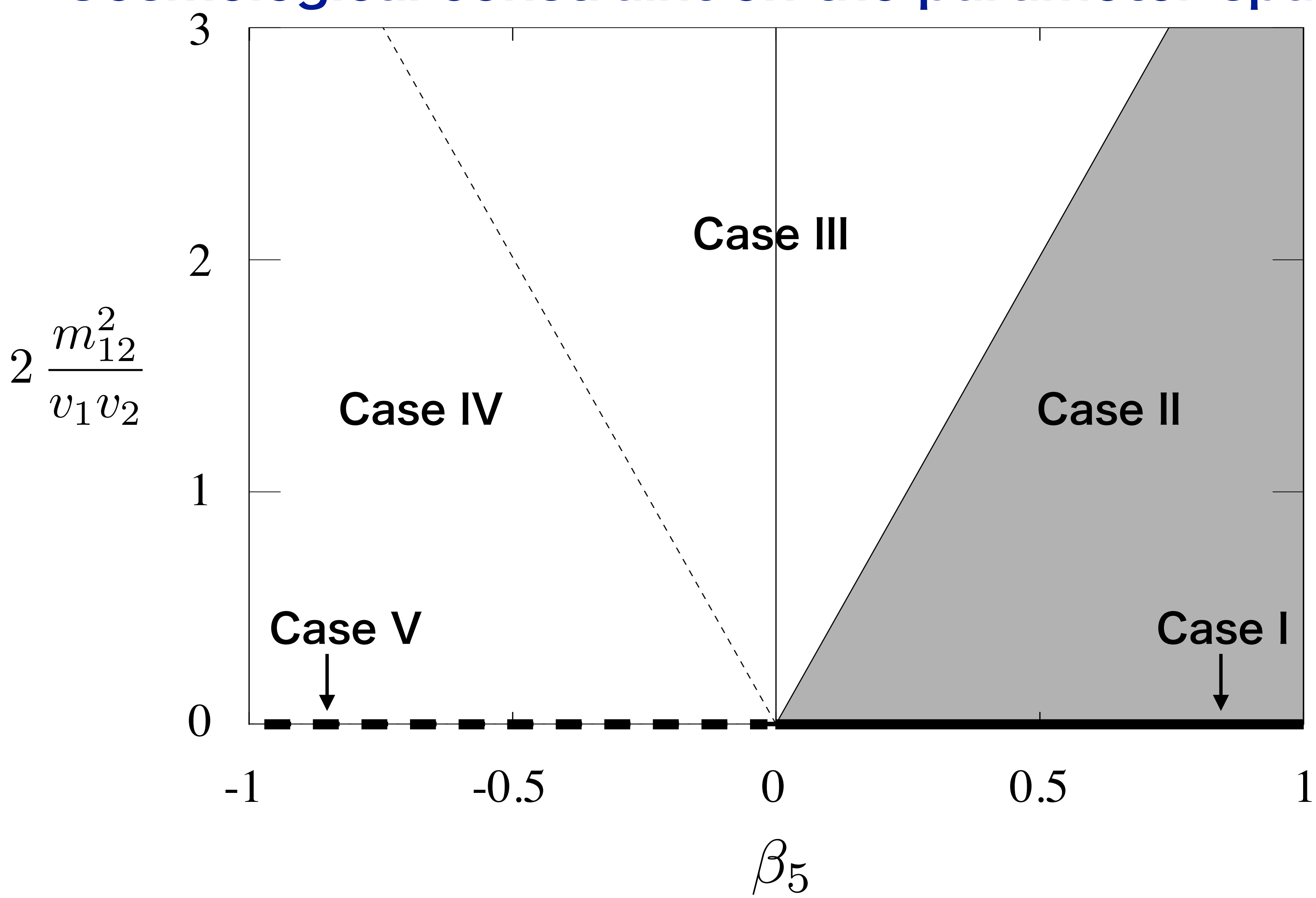
$0 \leq \xi < \tan^{-1}(4)$ **&** $\xi = \pi$

(Cases I, II, and V)

ain walls remain

(Cosmologically ruled out)

Cosmological constraint on the parameter space



Summary and Prospect

- Relative phase plays important role for the vacuum structure in the 2HDM
- Depending on the parameter choice, various kinds of domain walls and membranes are created
- Vortex attached by domain wall(s) can be also created, which plays the role to let the membranes decay
- constraints on the parameter space was placed from cosmological requirement
- Numerical solutions of various objects were obtained

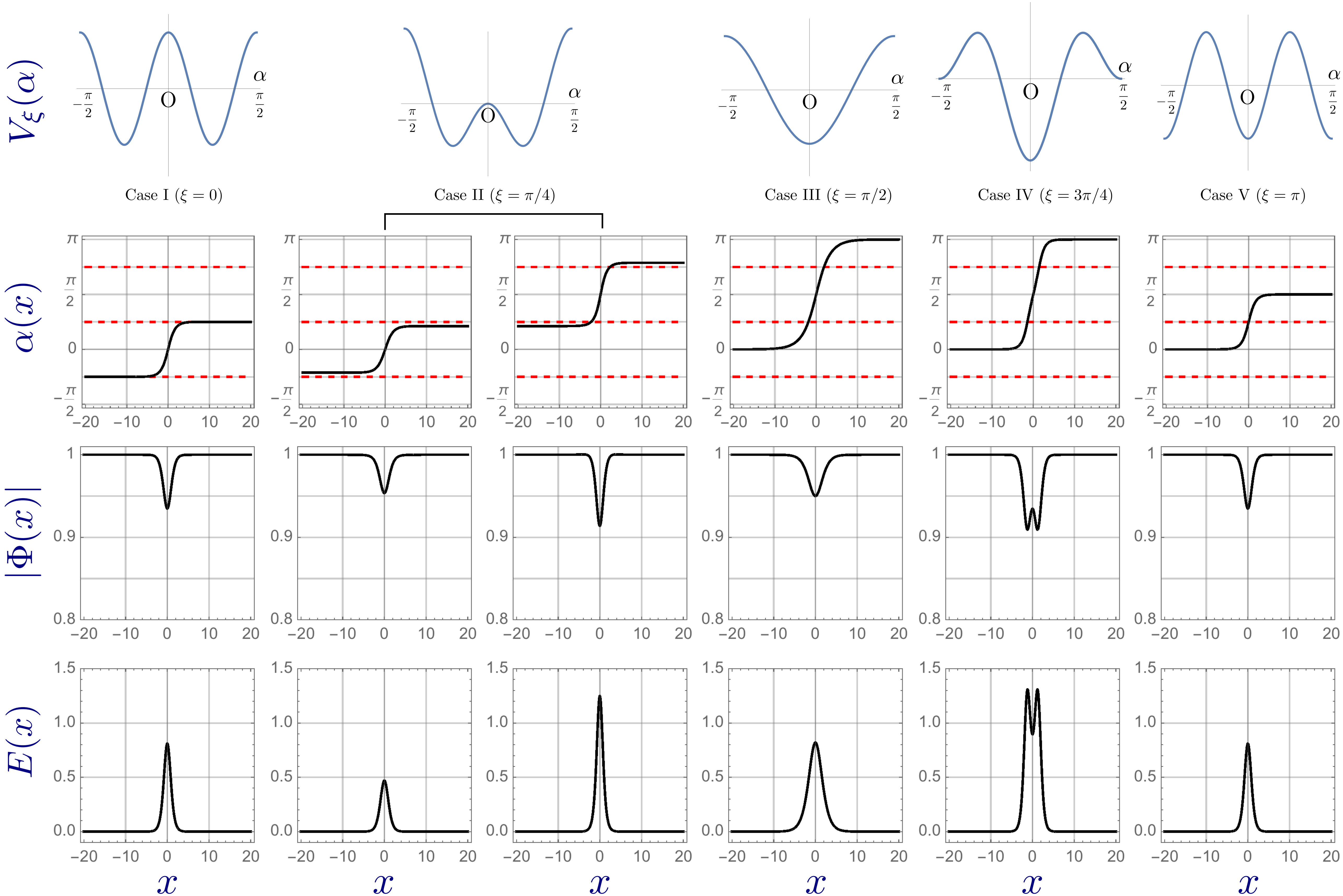
Future directions:

- Effect of the explicit CP violation
- Gravitational waves
- Baryogenesis (new source of CP violation)

Backups

Domain walls and membranes in 2HDMs

M. Eto, M. Kurachi and M. Nitta, arXiv:1803.04662 [hep-ph]



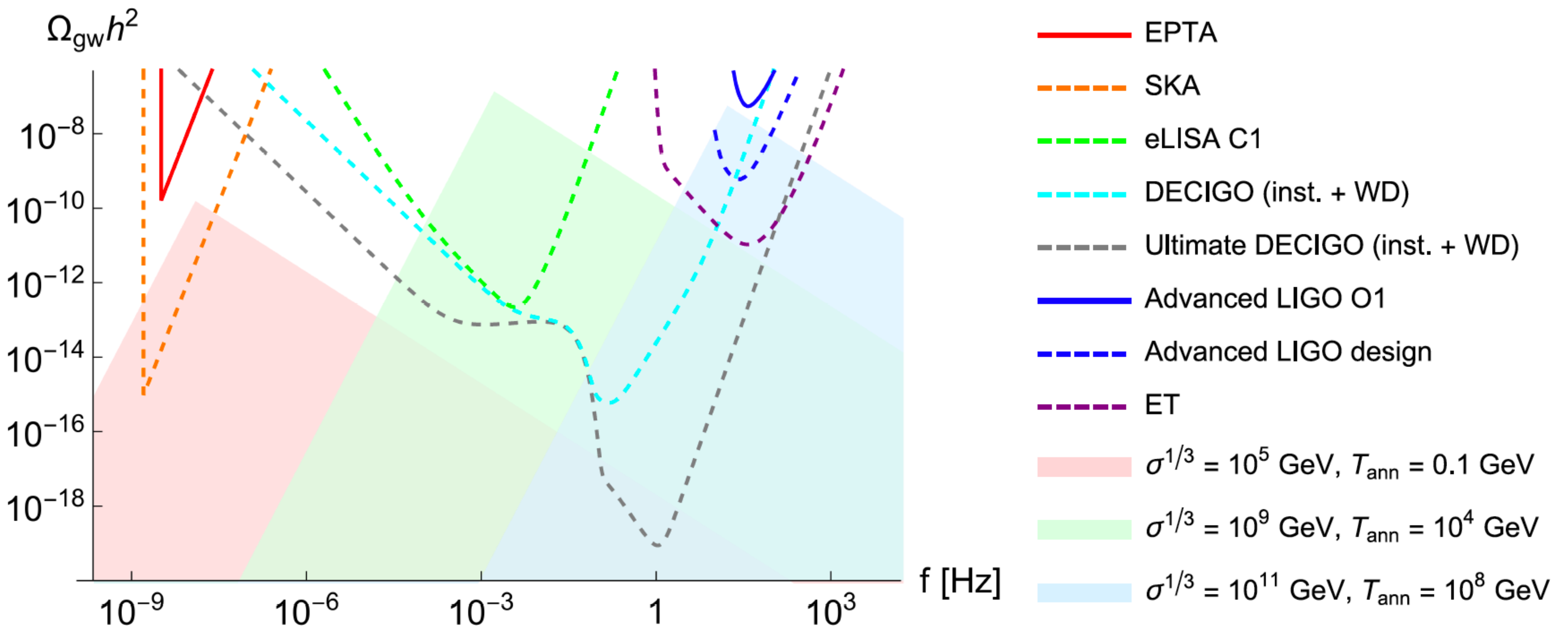
Gravitational Wave?

Remnant of creations and annihilations of domain walls

$$\Omega_{\rm gw}h^2(t_0)_{\rm peak} = 7.2\times 10^{-18}\,\tilde{\epsilon}_{\rm gw}\mathcal{A}^2\left(\frac{g_{*s}(T_{\rm ann})}{10}\right)^{-4/3}\underline{\left(\frac{\sigma}{1\,{\rm TeV}^3}\right)^2}\,\underline{\left(\frac{T_{\rm ann}}{10^{-2}\,{\rm GeV}}\right)^{-4}}. \quad (3.9)$$

$$f_{\rm peak} \simeq \left(\frac{R(t_{\rm ann})}{R(t_0)}\right)H(t_{\rm ann})$$

$$= 1.1\times 10^{-9}\,{\rm Hz}\left(\frac{g_*(T_{\rm ann})}{10}\right)^{1/2}\left(\frac{g_{*s}(T_{\rm ann})}{10}\right)^{-1/3}\underline{\left(\frac{T_{\rm ann}}{10^{-2}\,{\rm GeV}}\right)}. \quad (3.10)$$



K. Saikawa, arXiv:1703.02576 [hep-ph]

Effect of the explicit CP violation

If m_{12}^2 and/or β_5 are complex,

$$
\begin{aligned}
V_\xi(\alpha) &= -2 m_{12}^2 v_1 v_2 \cos 2\alpha + \beta_5 v_1^2 v_2^2 \cos 4\alpha \\
&= (v_1 v_2)^2 \sqrt{4\left(m_{12}^2 / v_1 v_2\right)^2 + \beta_5^2} \\
&\qquad \left(-\sin\xi \cos 2\alpha + \cos\xi \cos 4\alpha\right).
\end{aligned}
$$

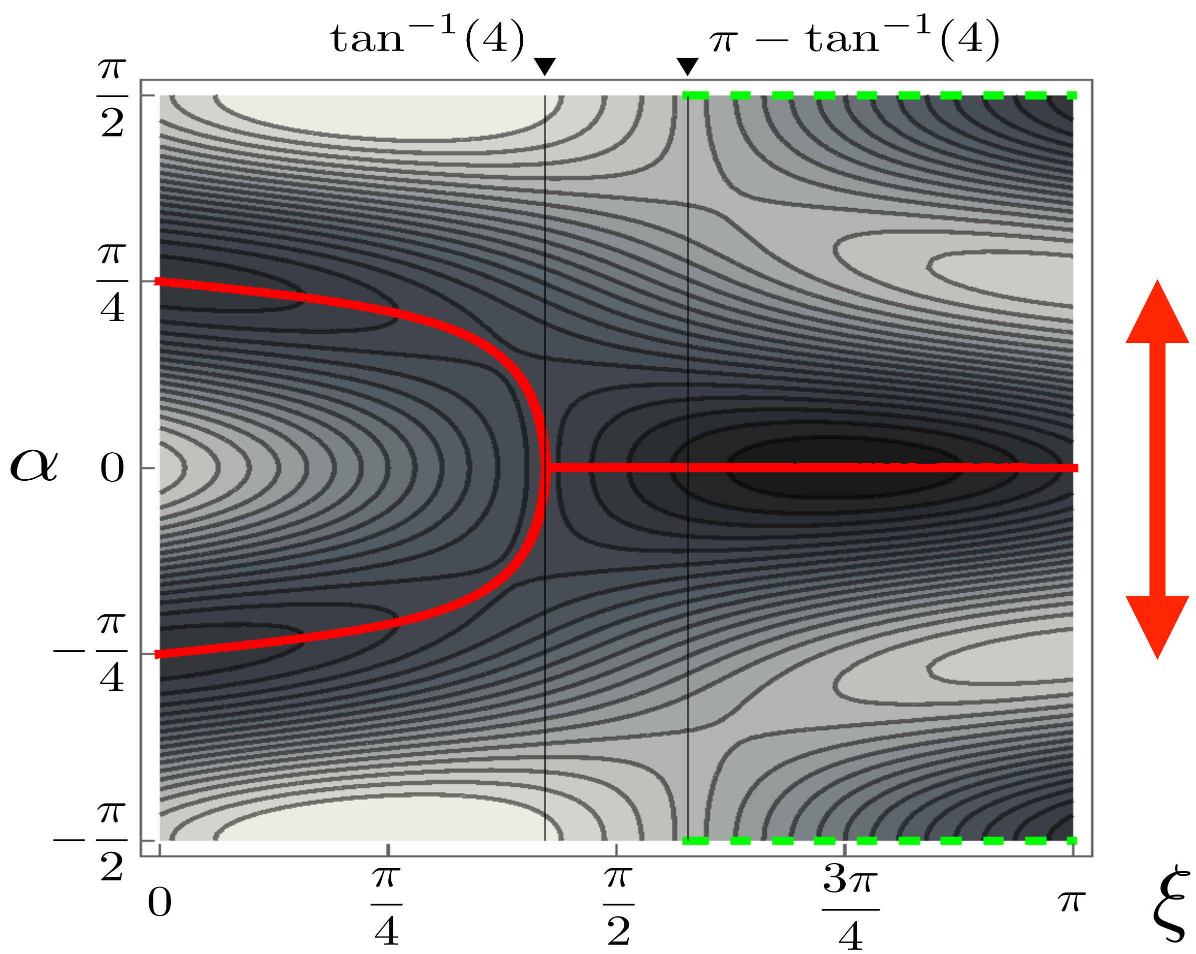
$$
-\sin\xi \cos(2\alpha + \underline{\delta_1}) + \cos\xi \cos(4\alpha + \underline{\delta_2})
$$

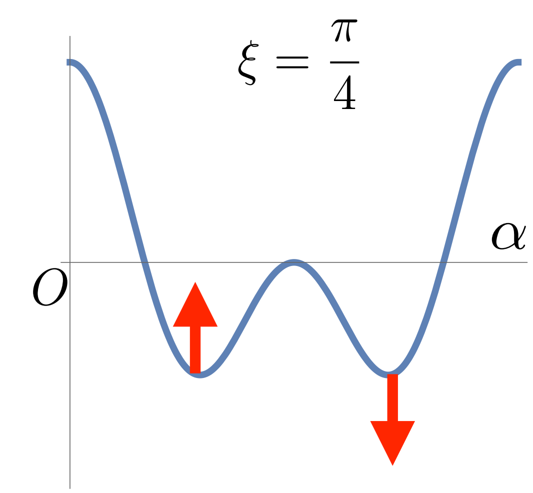
where $m_{12}^2 = e^{i\delta_1} |m_{12}^2|$, $\beta_5 = e^{i\delta_2} |\beta_5|$

Effect of the explicit CP violation

If m_{12}^2 and/or β_5 are complex,

the reflection symmetry will be lost





Domain walls become unstable

Phenomenological constraint on the magnitude of the explicit CP violation (EDM, Higgs decay···)

see, e.g.,

V. Keus, S. F. King, S. Moretti and K. Yagyu, JHEP 1604, 048 (2016)

Lifetime of the domain wall