The Grimus-Neufeld model:
Restricting Yukawa couplings
with the neutrino sector

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Our model: the 312-νSM (i.e. the Grimus-Neufeld model)

Standard Model (SM) + one fermionic singlet + two Higgs doublets

- is not a new idea: [G-N] W. Grimus and H. Neufeld, Nucl. Phys. B **325** (1989) 18.

outline of the talk

- the Grimus-Neufeld model
  - treelevel
  - and shortly at one-loop
- the Grimus-Lavoura approximation
  - allowing the analytic prediction of neutrino masses
- determining Lagrangian parameters
  - from masses and mixings
    * in the Grimus-Lavoura approximation !
- difficulties, comparisons, plans

The bare Lagrangian of the 312-νSM

- Gauge sector $\mathcal{L}_{\mathsf{G}}$ and Fermion-Gauge sector of the SM:
  - gauge group $U(1)_{\mathsf{Y}} \otimes SU(2)_{\mathsf{L}} \otimes SU(3)_{\mathsf{color}}$
  - gauge covariant derivative $D_\mu \psi$
  - and the Lagrangian $\mathcal{L}_{\mathsf{G\text{-}F}} = \sum_\psi \bar{\psi}\, i \not{D}\, \psi$

- Gauge-Higgs sector with the gauge covariant derivative $D_\mu \phi_a$ and the Lagrangian $\mathcal{L}_{\mathsf{G\text{-}H}} = (D^\mu \phi_a)^\dagger (D_\mu \phi_a) - V(\phi_a)$

- Higgs sector: two Higgs doublets ϕ_a in the Higgs potential $V(\phi_a)$

  [H-ON] H. E. Haber and D. O'Neil, Phys. Rev. D **83** (2011) 055017 [arXiv:1011.6188 [hep-ph]]

- Fermion-Higgs sector with the Yukawa couplings (ignoring quarks)

$$\mathcal{L}_{\mathsf{F\text{-}H}} = -\overline{\ell}^0_{Lj}\, \phi_a\, Y^{\bar{a}}_{Ljk}\, e^0_{Rk} - \overline{\ell}^0_{Lj}\, \tilde{\phi}_{\bar{a}}\, \widetilde{Y}^a_{Lj} N^0 + h.c.$$

with the adjoint Higgs doublet $\tilde{\phi}_{\bar{a}} = \epsilon \phi^*_a = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} \cdot \begin{pmatrix} (\phi^+_a)^* \\ (\phi^0_a)^* \end{pmatrix} =: \begin{pmatrix} \phi^{0*}_{\bar{a}} \\ -\phi^-_{\bar{a}} \end{pmatrix}$

- Majorana sector with the Majorana singlet N^0: $D_\mu N^0 = \partial_\mu N^0$

The 312-νSM has parameters additionally to the "original" SM

- the singlet Majorana mass term M_R
- the neutrino Yukawa coupling of the first Higgs doublet

  $(Y_N^{(1)})_j \;:= \tilde{Y}^1_{Lj} = \frac{\sqrt{2}}{v}(M_D)_j \; \ldots$ the "Dirac mass" term

- the Yukawa couplings of the second Higgs doublet

  $(Y_E^{(2)})_{jk} := Y^2_{Ljk}$ to lepton doublets and charged lepton singlets ℓ_{Rj}

  $(Y_N^{(2)})_j \;:= \tilde{Y}^2_{Lj}$ to lepton doublets and neutral fermionic singlet N_R

- additional parameters in the Higgs sector see [H-ON]
  - $m^2_{H_2}$, $m^2_{H_3}$, $m^2_{H^\pm}$ masses of the additional Higgs bosons
  - θ_{12}, θ_{13} mixing angles between the neutral Higgs fields

    CP conservation forces the mixing to the pseudo-scalar A^0 to zero: $\theta_{13} = 0$

  - Z_2, Z_3, Z_7 ... parameters of the Higgs potential,

    not fixed by tree level mass relations

312-νSM tree level for the neutral fermions

- the Yukawa coupling $(Y_N^{(1)})_j$ mixes the neutral leptons ν_j with N_R
- the mixing gives a $(3+1)\times(3+1)$ symmetric mass matrix

$$M_\nu = \begin{pmatrix} M_L & M_D^\top \\ M_D & M_R \end{pmatrix} \quad \text{with} \quad \begin{array}{rcl} M_L &=& 0_{3\times 3} \\ M_D^\top &=& (m_{De}, m_{D\mu}, m_{D\tau})^\top = \frac{v}{\sqrt{2}} Y_N^{(1)} \end{array}$$

  - M_ν has rank 2 $\Rightarrow$ only two masses are non-zero

- diagonalizing M_ν

$$U_{(\nu)}^\dagger M_\nu = \text{diag}(m_{o=\text{"zero"}}, m_{t=\text{"third"}}, m_{s=\text{"seesaw"}}, m_4) U_{(\nu)}^\top =: \hat{m} U_{(\nu)}^\top$$

with $m_o = m_t = 0$ by the unitary matrix

$$U_{(\nu)} = \begin{pmatrix} u_{eo} & u_{et} & c u_{es} & -is u_{es} \\ u_{\mu o} & u_{\mu t} & c u_{\mu s} & -is u_{\mu s} \\ u_{\tau o} & u_{\tau t} & c u_{\tau s} & -is u_{\tau s} \\ 0 & 0 & -is & c \end{pmatrix} \quad \text{where} \quad \begin{array}{rcl} c^2 &=& \frac{m_4}{m_4+m_s} \\ s^2 &=& \frac{m_s}{m_4+m_s} \end{array}$$

  - with $u_{k\alpha}$ being a unitary 3×3-matrix

312-νSM tree level for the neutral fermions

- from ${U_{(\nu)}}^\dagger M_\nu = \widehat{m} {U_{(\nu)}}^\top$ and $(Y_N^{(1)})_k = \frac{\sqrt{2}}{v}(M_D)_k$ we get

$$u_{ko}^* (Y_N^{(1)})_k = u_{kt}^* (Y_N^{(1)})_k = 0$$

- the two tree level massless "neutrinos" $\zeta_{o,t}^M$ are degenerate

- use the second Higgs coupling $(Y_N^{(2)})_k$ to distinguish them:

$$u_{ko}^* (Y_N^{(2)})_k = 0 \qquad \text{and} \qquad u_{kt}^* (Y_N^{(2)})_k =: d \neq 0$$

$\Rightarrow$ parametrize the Yukawa couplings as

$$(Y_N^{(1)})_k = \frac{\sqrt{2} m_D}{v} u_{ks} \qquad (Y_N^{(2)})_k := d\, u_{kt} + d' u_{ks}$$

with the abbreviation $m_D^2 = |(M_D)_e|^2 + |(M_D)_\mu|^2 + |(M_D)_\tau|^2$

- as a tree level identification one can take $u_{k\alpha} = (U_{\text{PMNS}})_{k\alpha}$
  - this identification chooses a basis
    for the degenerate massless "neutrinos" $\zeta_{o,t}^M$

312-νSM for the neutral fermions at one-loop level

- the model generates a loop induced mass $m_t \propto d^2$
  - * **this is the Grimus-Neufeld model**
  - and a loop correction δm_s for the seesaw mass $m_s^{[0]} \propto \frac{m_D^2}{M_R}$

determining the parameters at tree level

- we can use physical masses and couplings
  - from the Higgs sector [see Tuesdays talk of Odd Magne Ogreid]
    - * Higgs masses m_h, m_H, m_A, $m_{H^\pm}$ and Higgs-Gauge couplings
  - from the neutrino sector
    - * neutrino mixing matrix U_{PMNS}
    - * neutrino mass differences Δm^2_{atm} and Δm^2_{sol}

  ! but we have only a single mass difference at tree level, since $m_o = m_t = 0$

$\Rightarrow$ we need the one-loop level to determine parameters

Including one-loop predictions: using $\tilde{m}_i = m_{i,\text{phys}}$ and δm_i

- renormalizing the Lagrangian expressed in the mass eigenstates
  - needs a counter term $\delta^{\text{ct}} m$ for each non vanishing mass m
    * we have $m_3 > 0$ already at tree level ...

"Trick" of Grimus and Lavoura

[G-L] W. Grimus and L. Lavoura, JHEP **0011** (2000) 042 [arXiv:hep-ph/0008179].

- renormalize the Lagrangian expressed in interaction eigenstates
  - the counter term for the mass matrix

$$\delta^{\text{ct}} M_\nu = \begin{pmatrix} \delta^{\text{ct}} M_L & (\delta^{\text{ct}} M_D)^\top \\ \delta^{\text{ct}} M_D & \delta^{\text{ct}} M_R \end{pmatrix} \qquad \text{has} \qquad \delta^{\text{ct}} M_L = 0_{3\times 3}$$

    * since $M_L^{\text{tree}} = 0_{3\times 3}$
  - the counter term $\delta^{\text{ct}}(M_D)_k = \frac{1}{\sqrt{2}}[(\delta^{\text{ct}} v)(Y_N^{(1)})_k + v(\delta^{\text{ct}} Y_N^{(1)})_k]$
    * is fixed by the vacuum and the Higgs coupling
  - $\delta^{\text{ct}} M_R$ is "fixed" by the not measured heavy singlet ... and ignored

The Grimus-Lavoura procedure

[G-L] W. Grimus and L. Lavoura, JHEP **0011** (2000) 042 [arXiv:hep-ph/0008179].

reducing the problem to the "light" neutrinos $\zeta^{M}_{o,t,s}$:

- staying in the interaction eigenstates basis $\quad M_\nu = \begin{pmatrix} 0 & M_D^\top \\ M_D & M_R \end{pmatrix}$

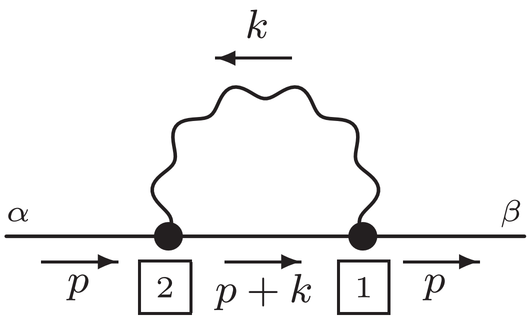
  leads to an effective 3×3-neutrino mass matrix $\mathcal{M}_\nu$

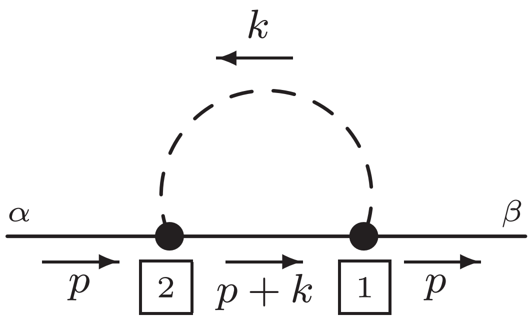
  - at tree level $\quad \mathcal{M}_\nu^{\text{tree}} = -M_D^\top \, M_R^{-1} \, M_D \;\; ,$
  - and at one-loop level $\quad \mathcal{M}_\nu^{\text{1-loop}} = \mathcal{M}_\nu^{\text{tree}} + \delta\mathcal{M}_\nu \;\; ,$

    with $\quad \delta\mathcal{M}_\nu = \delta M_L - \delta M_D^\top \, M_R^{-1} \, M_D - M_D^\top \, M_R^{-1} \, \delta M_D + M_D^\top \, M_R^{-1} \, \delta M_R \, M_R^{-1} \, M_D$

- assuming δM_R to be irrelevant (as M_R is a free unmeasurable parameter)
- assuming corrections with δM_D to be subdominant $\propto Y^2 m_{\ell^\pm} \frac{m_D}{M_R}$ or $g^2 m_{\ell^\pm} \frac{m_D}{M_R}$

$\Rightarrow$ loop corrections come from δM_L, calculated from

$\Sigma_V^{[2]}(p^2) =$ 

$\Sigma_S^{[2]}(p^2) =$ 

The Grimus-Lavoura procedure

calculating $\delta^{\text{loop}} M_L$ from the selfenergy $\Sigma^{[2]}_{(p^2)}$

- at vanishing external momentum $p^2 = \tilde{m}^2 = m^2_{\text{phys}} \sim 0$
  - for the mass term only the neutral bosons contribute
  - Z^0 and G^0 combine to a gauge invariant contribution

$\Rightarrow$ one gets an effective 1-loop improved 3×3-mass matrix $(\mathcal{M}_\nu^{\text{1-loop}})_{jk}$

  - that depends on SM parameters and on
  - the neutral Higgs masses m_h^2, m_H^2, m_A^2, and mixing angle θ_{12}
  - the heavy singlet mass $M_R \sim m_4$
  - and the Yukawa coupling parameters d, d', and m_D

- the singular values of $(\mathcal{M}_\nu^{\text{1-loop}})$ are the masses of the "light" neutrinos
  - $*$ **this is the Grimus-Lavoura approximation**

Determining the Lagrangian parameters

calculating this effective 1-loop improved 3×3-mass matrix

- we get $(\mathcal{M}_\nu^{\text{1-loop}})_{jk} = u_{jt}u_{kt}A + (u_{jt}u_{ks} + u_{js}u_{kt})B + u_{js}u_{ks}C$
  - with
$$A = d^2 f_1 \qquad B = d'df_1 + id\frac{\sqrt{2}m_D}{v}f_2$$
$$C = d'^2 f_1 + 2id'\frac{\sqrt{2}m_D}{v}f_2 + \frac{2m_D^2}{v^2}f_3$$
  - the f_i depend on the parameters of the Higgs sector
$$f_1 = s_{12}^2 L_h + c_{12}^2 L_H - L_A$$
$$f_2 = s_{12}c_{12}[L_H - L_h]$$
$$f_3 = c_{12}^2 L_h + s_{12}^2 L_H + 3L_Z - \frac{v^2}{2m_4}$$
where
$$L_\phi = \frac{1}{32\pi^2}\frac{m_\phi^2}{m_4}\ln\frac{m_4^2}{m_\phi^2}$$
$$L_Z = \frac{1}{32\pi^2}\frac{m_Z^2}{m_4}\ln\frac{m_4^2}{m_Z^2}$$
and $s_{12} = \sin\theta_{12}$, $c_{12} = \cos\theta_{12}$
  - C includes the tree level seesaw contribution: $\frac{v^2|Y_N^{(1)}|^2}{2m_4} = \frac{m_D^2}{m_4}$
- $(\mathcal{M}_\nu^{\text{1-loop}})_{jk}$ is obviously only rank 2 :
$$u_{j\alpha}^*(\mathcal{M}_\nu^{\text{1-loop}})_{jk}u_{j\beta}^* = \begin{pmatrix} 0 & 0 & 0 \\ 0 & A & B \\ 0 & B & C \end{pmatrix}_{\alpha\beta} ,$$

Diagonalizing $(\mathcal{M}_\nu^{\text{1-loop}})_{jk}$ gives the masses $\tilde{m}_t$ and $\tilde{m}_s = m_s + \delta m_s$

- using $u^*_{j\alpha}$ the mass $\tilde{m}_o = 0$ can be factored out

$\Rightarrow$ the diagonalization can be simplified to a 2×2 seesaw relation

$$R^\dagger \cdot \begin{pmatrix} A & B \\ B & C \end{pmatrix} = \begin{pmatrix} \tilde{m}_t & 0 \\ 0 & \tilde{m}_s \end{pmatrix} \cdot R^\top \quad \text{with} \quad R = e^{i\frac{\alpha}{2}} \begin{pmatrix} \bar{c} & \bar{s} \\ -\bar{s}^* & \bar{c}^* \end{pmatrix}$$

  - with $\bar{c} = e^{\frac{i}{2}(\gamma+\delta)} \cos\beta$ and $\bar{s} = e^{\frac{i}{2}(\gamma-\delta)} \sin\beta$
  - R is needed as it describes the mixing between ζ_t^M and ζ_s^M

- we can determine the masses $\tilde{m}_t$ and $\tilde{m}_s$ from A, B, and C alone :

$$\tilde{m}_t^2 + \tilde{m}_s^2 = \text{Tr}\left[\begin{pmatrix} A & B \\ B & C \end{pmatrix}^\dagger R \cdot R^\dagger \begin{pmatrix} A & B \\ B & C \end{pmatrix} \right] = |A|^2 + 2|B|^2 + |C|^2$$

$$\text{and} \quad \tilde{m}_t \, \tilde{m}_s = \det[R^\dagger] \det \begin{bmatrix} A & B \\ B & C \end{bmatrix} \det[R^*] = e^{-2i\alpha}[AC - B^2]$$

$\Rightarrow$ the orthonormal vectors $u^*_{j\alpha}$ do not appear in the neutrino masses:

$$\tilde{m}_{t,s} = \tilde{m}_{t,s}\left[m_h^2, m_H^2, m_A^2, \theta_{12}, v^2, \tilde{m}_4, m_D^2, d, d' = |d'| e^{i\phi'} \right]$$

taking $\tilde{m}_t$ and $\tilde{m}_s$ as measured masses

- determining d is simple:

$$\tilde{m}_t\,\tilde{m}_s = |AC - B^2| = d^2\frac{2m_D^2}{v^2}|f_1 f_3 - f_2^2| \quad\Rightarrow\quad d^2 = \frac{v^2}{2m_D^2}\frac{\tilde{m}_t\,\tilde{m}_s}{|f_1 f_3 - f_2^2|}$$

  - since the determinant equation does not depend on $|d'|$

- the trace equation, $\tilde{m}_t^2 + \tilde{m}_s^2 = |A|^2 + 2|B|^2 + |C|^2$,

  involves a fourth order polynomial in $|d'|$:

$$0 = a_4|d'|^4 + a_3|d'|^3 + a_2|d'|^2 + a_1|d'| + a_0$$

  where

$$a_4 = f_1^2 \qquad a_3 = -4\sin\phi'\frac{\sqrt{2}m_D}{v}f_1 f_2$$

$$a_2 = 2\left[d^2 f_1^2 + 2\frac{2m_D^2}{v^2}f_2^2 - (1 - 2\sin^2\phi')\frac{2m_D^2}{v^2}f_1 f_3\right]$$

$$a_1 = -4\sin\phi'\frac{\sqrt{2}m_D}{v}\left[d^2 f_1 + \frac{2m_D^2}{v^2}f_3\right]f_2$$

$$a_0 = d^4 f_1^2 + 2d^2\frac{2m_D^2}{v^2}f_2^2 + \frac{4m_D^4}{v^4}f_3^2 - \tilde{m}_t^2 - \tilde{m}_s^2$$

$\Rightarrow\ |d'| = |d'|[v^2; m_h, m_H, m_A, s_{12}; \tilde{m}_t, \tilde{m}_s, \tilde{m}_4; m_D^2; \phi']$ and $d' = |d'|e^{i\phi'}$

neutrino mass eigenstates from the Grimus-Lavoura approximation

- the "heavy" state $\tilde{\zeta}_4^M = \zeta_4^M$ with mass m_4 was "integrated out"
- the massless state $\tilde{\zeta}_o^M = \zeta_o^M$ with mass $m_o = 0$ was left untouched
- the tree level states $\zeta_{t,s}^M$ were mixed into one-loop mass eigenstates $\tilde{\zeta}_{t,s}^M$

$$\tilde{\zeta}_o^M = \zeta_o^M \quad , \quad \begin{pmatrix} \tilde{\zeta}_t^M \\ \tilde{\zeta}_s^M \end{pmatrix} = R^\top \cdot \begin{pmatrix} \zeta_t^M \\ \zeta_s^M \end{pmatrix} \quad , \quad \tilde{\zeta}_4^M = \zeta_4^M$$

$\Rightarrow$ the tree level identification with the PMNS matrix is changed:

$$\begin{aligned} u_{ko} &= (U_{\mathrm{PMNS}})_{ko} \\ u_{kt} &= (R)_{tt}(U_{\mathrm{PMNS}})_{kt} + (R)_{ts}(U_{\mathrm{PMNS}})_{ks} \qquad \text{with} \quad k = e, \mu, \tau \\ u_{ks} &= (R)_{st}(U_{\mathrm{PMNS}})_{kt} + (R)_{ss}(U_{\mathrm{PMNS}})_{ks} \end{aligned}$$

- What means o, t, s in terms of the measured neutrino mass states ?
  - depends on the scenario: normal / inverted hierarchy
  - depends on the ordering of m_t and m_s

using the measured mass differences $\Delta m^2_{\text{sol}} = \Delta m^2_{21}$ and $\Delta m^2_{\text{atm}} \approx |\Delta m^2_{31}|$
[SoNO2018] P. F. de Salas *et al.*, Phys. Lett. B **782** (2018) 633

- one neutrino is massless at one-loop
  $\Rightarrow$ the mass differences determine the masses

- for the normal hierarchy (NH) we have $\tilde{m}_o = \tilde{m}_1 = 0$ (i.e. $o = 1$)
  $\Rightarrow$ $\tilde{m}_2 = \sqrt{\Delta m^2_{\text{sol}}}$ and $\tilde{m}_3 = \sqrt{\Delta m^2_{\text{atm}}}$

- the inverted hierarchy (IH) has $\Delta m^2_{31} < 0$
  $\Rightarrow$ we have to assign $\tilde{m}_o = \tilde{m}_3 = 0$ (i.e. $o = 3$)
  - we get $\tilde{m}_1 = \sqrt{\Delta m^2_{\text{atm}}}$ and $\tilde{m}_2 = \sqrt{\Delta m^2_{\text{atm}} + \Delta m^2_{\text{sol}}}$

- the ordering of m_t and m_s influences the assignement:
  - for NH we usually expect $m_t = \tilde{m}_2 < \tilde{m}_3 = m_s$ (i.e. $t = 2$ and $s = 3$)
    * but we could also have $\overline{\text{NH}}$: $m_s = \tilde{m}_2 < \tilde{m}_3 = m_t$ (i.e. $t = 3$ and $s = 2$)
  - for IH we usually expect $m_t = \tilde{m}_1 < \tilde{m}_2 = m_s$ (i.e. $t = 1$ and $s = 2$)
    * but we could also have $\overline{\text{IH}}$: $m_s = \tilde{m}_1 < \tilde{m}_2 = m_t$ (i.e. $t = 2$ and $s = 1$)

the role of m_D^2

- m_D^2 is the size of the tree level Dirac mass term
  - and given by tree level masses: $m_D^2 = M_R m_s^{[0]} \approx m_4 m_s \neq \tilde{m}_4 \tilde{m}_s$
  - from mass measurements we have no clue about m_s or m_4
  - assuming the model has a sensible loop expansion

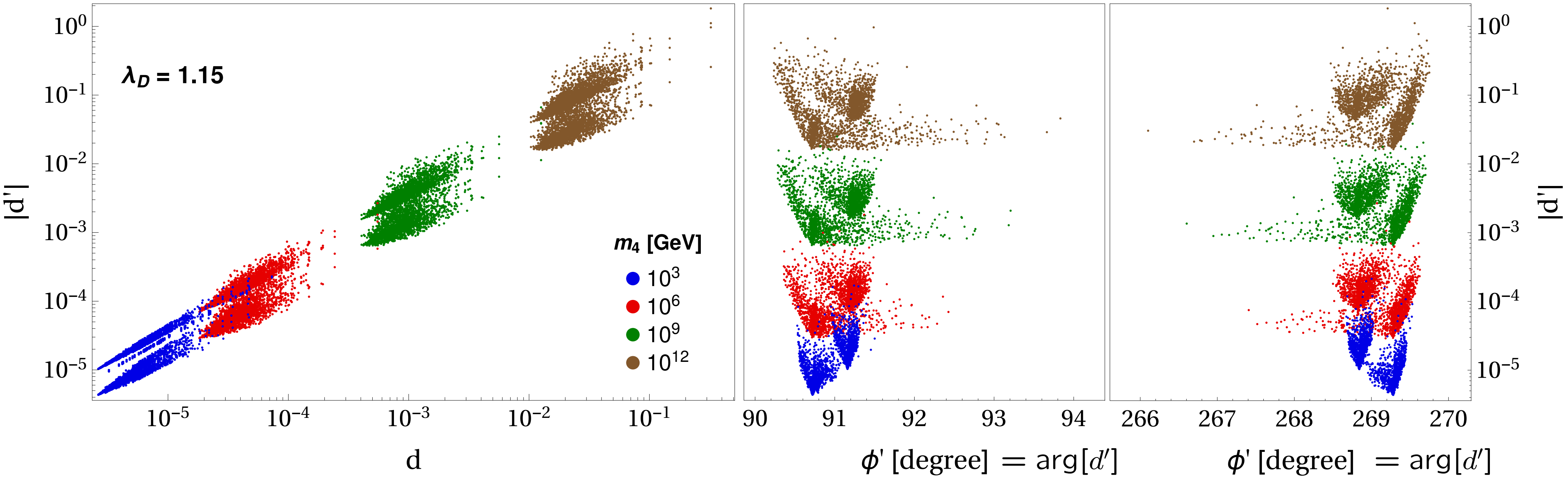
  $\Rightarrow$ the loop correction should not invalidate the tree level (i.e. $m_4 m_s = \mathcal{O}[\tilde{m}_4 \tilde{m}_s]$)

- we set the scale of m_D^2 with the measured masses
  - and parametrize the difference by a multiplicative parameter

$$m_D^2 = m_4 m_s := \lambda_D^2 \tilde{m}_4 \tilde{m}_s = \lambda_D^2 \tilde{m}_4 \begin{cases} \sqrt{\Delta m^2_{\text{atm}}} & \ldots \quad \text{NH} \\ \sqrt{\Delta m^2_{\text{sol}}} & \ldots \quad \overline{\text{NH}} \\ \sqrt{\Delta m^2_{\text{atm}} + \Delta m^2_{\text{sol}}} & \ldots \quad \text{IH} \\ \sqrt{\Delta m^2_{\text{atm}}} & \ldots \quad \overline{\text{IH}} \end{cases}$$

  - a conservative range is $\frac{1}{2} \leq \lambda_D \leq 2$

d and $|d'|$ are fully determined:



- scatterplot of the values d and $|d'|$
  - shown values of ϕ' allow physical solutions for the fourth order equation
  - the Higgs masses m_H and m_A vary between 0.2 and 1.2 TeV
  - the mixing angle θ_{12} gives a stable, perturbative, and unitary Higgs potential

  taking the data points ⟨https://doi.org/10.18279/MIDAS.2HDMpar.61451⟩
  from the bachelor thesis of A. Kunčinas:
  ⟨http://talpykla.elaba.lt/elaba-fedora/objects/elaba:23352542/datastreams/MAIN/content⟩

Determining the rotation matrix R

- the 2×2 seesaw relation gives

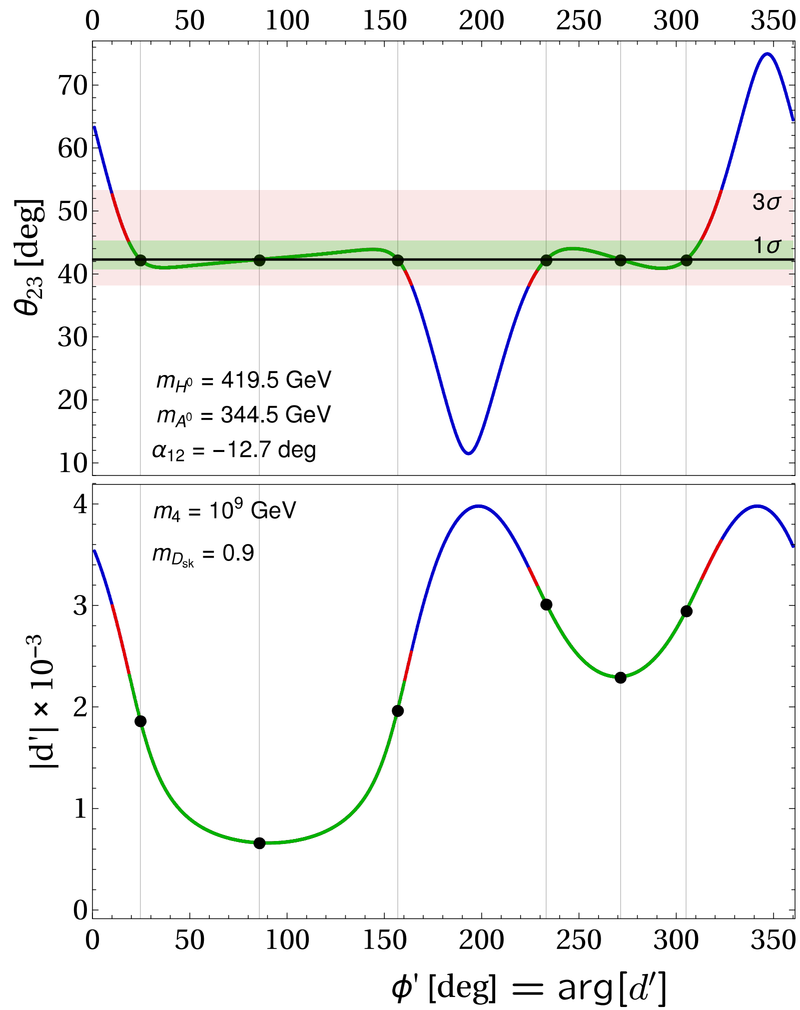
$$\tan^2\beta = \frac{|A|^2 + |B|^2 - \tilde{m}_t^2}{\tilde{m}_s^2 - |A|^2 - |B|^2}$$

$$e^{2i\alpha} = \frac{\tilde{m}_t\,\tilde{m}_s}{AC - B^2}$$

$$e^{i\delta} = -\frac{(A^*B + B^*C)\tan\beta}{|A|^2 + |B|^2 - \tilde{m}_t^2} = \ldots$$

$$e^{i\gamma} = \frac{e^{i\alpha}}{\tilde{m}_s}(Ce^{-i\delta} - B\tan\beta) = \ldots$$

. . . indicate different possible analytic expressions



- R then determines the vectors $u_{k\alpha}$:

$$u_{ko} = (U_{\text{PMNS}})_{ko}$$

$$u_{kt} = (R)_{tt}(U_{\text{PMNS}})_{kt} + (R)_{ts}(U_{\text{PMNS}})_{ks}$$

$$u_{ks} = (R)_{st}(U_{\text{PMNS}})_{kt} + (R)_{ss}(U_{\text{PMNS}})_{ks} \qquad \text{with} \quad k = e, \mu, \tau$$

- which fully determines the Yukawa couplings:

$$(Y_N^{(1)})_k = \frac{\sqrt{2}m_D}{v}u_{ks} \qquad (Y_N^{(2)})_k = d\,u_{kt} + d' u_{ks}$$

Difficulties

- for a value of ϕ', the fourth order equation for $|d'|$ can have
  - no solution: this value of ϕ' does not give a physical point
  - one solution: this value of ϕ' does give a physical point
    - $*$ if the resulting mixing matrix is compatible with the measured U_{PMNS}
  - more than one solution:
    each value of d' can give a distinct physical point
    - $*$ if the resulting mixing matrix is compatible with the measured U_{PMNS}

$\Rightarrow$ one still has to check the obtained parameter point

- in a similar way as in a Monte Carlo method
  - $*$ reproduction of the input neutrino masses: $\tilde{m}_1 = 0$, $\tilde{m}_2$, $\tilde{m}_3$, $\tilde{m}_4 \approx m_4 \approx M_R$
    checks the numerics

$+$ the "success rate" with the Grimus-Lavoura approximation for valid parameter points is much higher than random points

- $*$ the approximation "ignores" the neutrino mixing matrix

Comparison with SPheno and FlexibleSUSY

[Drau] ⟨http://talpykla.elaba.lt/elaba-fedora/objects/elaba:29420143/datastreams/MAIN/content⟩

- S. Draukšas implemented the GN model in SPheno and FlexibleSUSY
  - the code generation worked
  - SPheno could not reproduce the small neutrino masses
  - FlexibleSUSY gave qualitatively an expected spectrum
    - time was (too) short
    - implementing the parameter selection from the GL-approximation was difficult

Plans

- We want a full renormalization of the GN model
  - see talk of Vytautas Dūdėnas on Tuesday
  - giving us an estimate of the size of the GL approximation
- We want to extend the comparison with SPheno and FlexibleSUSY
- We want to make predictions with the determined Yukawa couplings

Thank you

for discussion

and comments

and of course for the workshop! ☺