

P-even, CP-violating Signals in Scalar-Mediated Processes



Howard E. Haber

30 August 2022

Workshop on Multi-Higgs
Models in Lisbon, Portugal



Outline

1. Brief introduction to extended Higgs models and the Higgs alignment limit
2. Sources of CP violation in extended Higgs models
 - C and P symmetries in gauge theories of spin 0 and spin 1 fields
 - Example: J^{PC} assignments of bosons in the CP-conserving 2HDM
3. P-even, CP-violating processes of extended Higgs sectors in the Higgs alignment limit
4. Observation of P-even, CP-violating scalar processes at lepton colliders
5. P-even, CP-violating signals via loop effects
6. Beyond the exact Higgs alignment limit

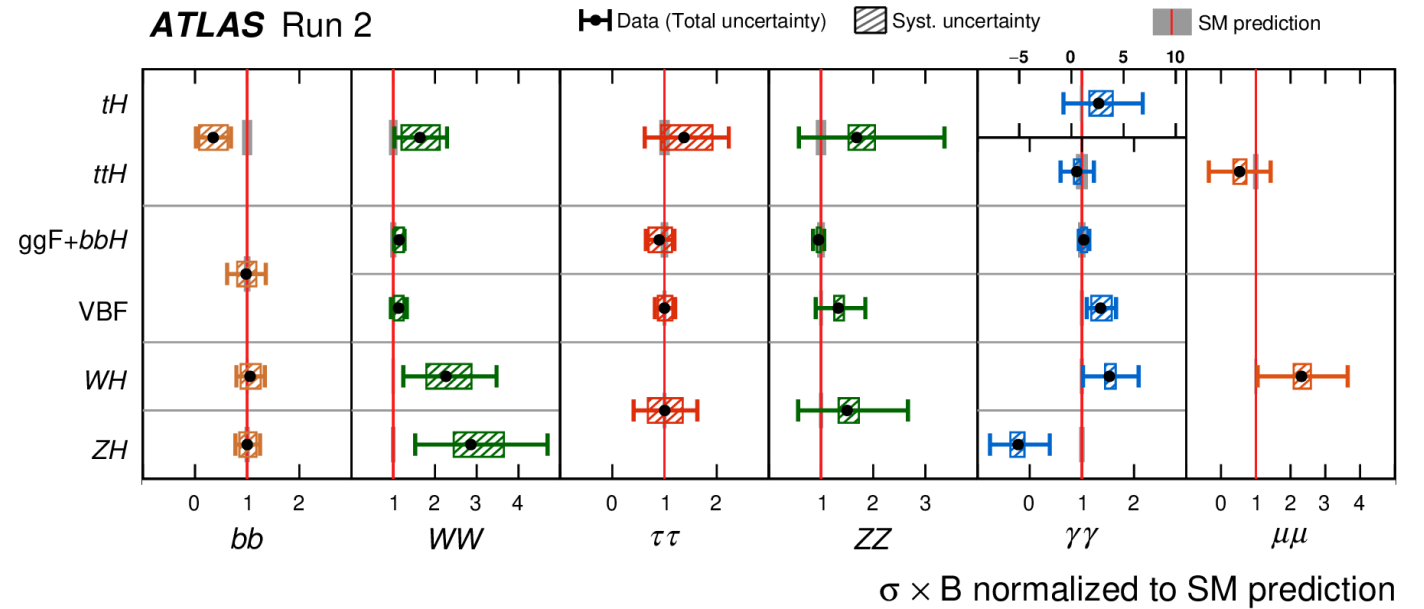
This work is based on:
H.E. Haber, V. Keus and
R. Santos, arXiv:2206.09643

Motivating extended Higgs sectors and the Higgs alignment limit

So far, there is no indication that the properties of the Higgs boson deviate from those of the Standard Model (SM).

Nevertheless, in light of the non-minimality of the SM fermions and gauge bosons, it is tempting to posit the existence of additional scalar doublets (and perhaps singlets).

For example, additional scalars can be used to produce a suitable dark matter candidate. In addition, extended Higgs sectors can significantly alter the electroweak phase transition thereby facilitating electroweak baryogenesis.



Ten years of ATLAS Higgs data yields a p -value for compatibility of the measurement and the SM prediction of 72% [taken from Nature 607, no. 7917, 52-59 (2022)].

To be compatible with a SM-like Higgs boson, the extended Higgs sector must satisfy an approximate Higgs alignment limit.

The Higgs alignment limit

Consider an extended Higgs sector with $Y = 1$, doublet scalar fields Φ_i and $SU(2) \times U(1)$ singlet scalar fields ϕ_i , with scalar field vevs, $\langle \Phi_i^0 \rangle = v_i / \sqrt{2}$ where $v^2 \equiv \sum_i |v_i|^2 \simeq (246 \text{ GeV})^2$, and $\langle \phi_j^0 \rangle = x_j$. One can then introduce new linear combinations, H_i , which define the so-called *Higgs basis*,

$$H_1 = \begin{pmatrix} H_1^+ \\ H_1^0 \end{pmatrix} = \frac{1}{v} \sum_i v_i^* \Phi_i, \quad \langle H_1^0 \rangle = v / \sqrt{2},$$

and H_2, H_3, \dots, H_n are the orthogonal linear combinations of doublet scalar fields such that $\langle H_k^0 \rangle = 0$ (for $k \neq 1$). That is H_1^0 is *aligned* in field space with the direction of the scalar field vev.

In the exact Higgs alignment limit,

$$\varphi \equiv \sqrt{2} \operatorname{Re} H_1^0 - v$$

is a scalar mass eigenstate such that the tree-level couplings of φ to itself, to gauge bosons and to fermions coincide with those of the SM Higgs boson.

More generally, φ mixes with other neutral scalar fields to form the corresponding neutral scalar mass eigenstates. Approximate Higgs alignment is realized if this mixing is suppressed. This can be achieved either in the decoupling limit or if terms in the scalar potential such as $(H_1^\dagger H_1)(H_1^\dagger H_k + \text{h.c.})$ [for $k \neq 1$] are suppressed.

Higgs alignment in the 2HDM

In the Higgs basis, the scalar potential is:

$$\begin{aligned} \mathcal{V} = & Y_1 \mathcal{H}_1^\dagger \mathcal{H}_1 + Y_2 \mathcal{H}_2^\dagger \mathcal{H}_2 + [Y_3 e^{-i\eta} \mathcal{H}_1^\dagger \mathcal{H}_2 + \text{h.c.}] \\ & + \frac{1}{2} Z_1 (\mathcal{H}_1^\dagger \mathcal{H}_1)^2 + \frac{1}{2} Z_2 (\mathcal{H}_2^\dagger \mathcal{H}_2)^2 + Z_3 (\mathcal{H}_1^\dagger \mathcal{H}_1) (\mathcal{H}_2^\dagger \mathcal{H}_2) + Z_4 (\mathcal{H}_1^\dagger \mathcal{H}_2) (\mathcal{H}_2^\dagger \mathcal{H}_1) \\ & + \left\{ \frac{1}{2} Z_5 e^{-2i\eta} (\mathcal{H}_1^\dagger \mathcal{H}_2)^2 + [Z_6 e^{-i\eta} (\mathcal{H}_1^\dagger \mathcal{H}_1) + Z_7 e^{-i\eta} (\mathcal{H}_2^\dagger \mathcal{H}_2)] \mathcal{H}_1^\dagger \mathcal{H}_2 + \text{h.c.} \right\}, \end{aligned}$$

where $\mathcal{H}_1 \equiv H_1$ and $\mathcal{H}_2 \equiv e^{i\eta} H_2$. The phase angle η represents the freedom to rephase H_2 in the Higgs basis. At the scalar potential minimum, $Y_1 = -\frac{1}{2} Z_1 v^2$ and $Y_3 = -\frac{1}{2} Z_6 v^2$. The neutral scalar mass eigenstates are obtained by diagonalizing,

$$\mathcal{M}^2 = v^2 \begin{pmatrix} Z_1 & \text{Re}(Z_6 e^{-i\eta}) & -\text{Im}(Z_6 e^{-i\eta}) \\ \text{Re}(Z_6 e^{-i\eta}) & Y_{234} + \frac{1}{2} \text{Re}(Z_5 e^{-2i\eta}) & -\frac{1}{2} \text{Im}(Z_5 e^{-2i\eta}) \\ -\text{Im}(Z_6 e^{-i\eta}) & -\frac{1}{2} \text{Im}(Z_5 e^{-2i\eta}) & Y_{234} - \text{Re}(Z_5 e^{-2i\eta}) \end{pmatrix},$$

where $Y_{234} \equiv Y_2/v^2 + \frac{1}{2}(Z_3 + Z_4)$. $R^\top \mathcal{M}^2 R$ is diagonal, where R is a product of three rotation matrices, $R = R_{12} R_{13} R_{23}$.

k	q_{k1}	q_{k2}
1	$c_{12}c_{13}$	$-s_{12} - ic_{12}s_{13}$
2	$s_{12}c_{13}$	$c_{12} - is_{12}s_{13}$
3	s_{13}	ic_{13}

The quantities q_{kl} are functions of the neutral Higgs mixing angles θ_{12} and θ_{13} , where $c_{ij} \equiv \cos \theta_{ij}$ and $s_{ij} \equiv \sin \theta_{ij}$.

The Higgs basis field in terms of the mass-eigenstate neutral Higgs fields, h_k ($k = 1, 2, 3$):

$$\mathcal{H}_1 = \begin{pmatrix} G^+ \\ \frac{1}{\sqrt{2}} \left(v + iG + \sum_{k=1}^3 q_{k1} h_k \right) \end{pmatrix}, \quad e^{i\theta_{23}} \mathcal{H}_2 = \begin{pmatrix} H^+ \\ \frac{1}{\sqrt{2}} \sum_{k=1}^3 q_{k2} h_k \end{pmatrix}.$$

Without loss of generality, we adopt the convention where $\theta_{23} = 0$.

Exact Higgs alignment (where h_1 is identified with the observed SM-like Higgs boson) corresponds to $c_{12} = c_{13} = 1$, which implies that $Z_6 = \text{Im}(Z_5 e^{-2i\eta}) = 0$.

In the CP-conserving 2HDM, $\text{Im}(Z_5^* Z_{6,7}^2) = \text{Im}(Z_6^* Z_7) = 0$, and one can choose a real Higgs basis where all the Y_i and Z_i are real.

In this case, $c_{13} = 1$ and

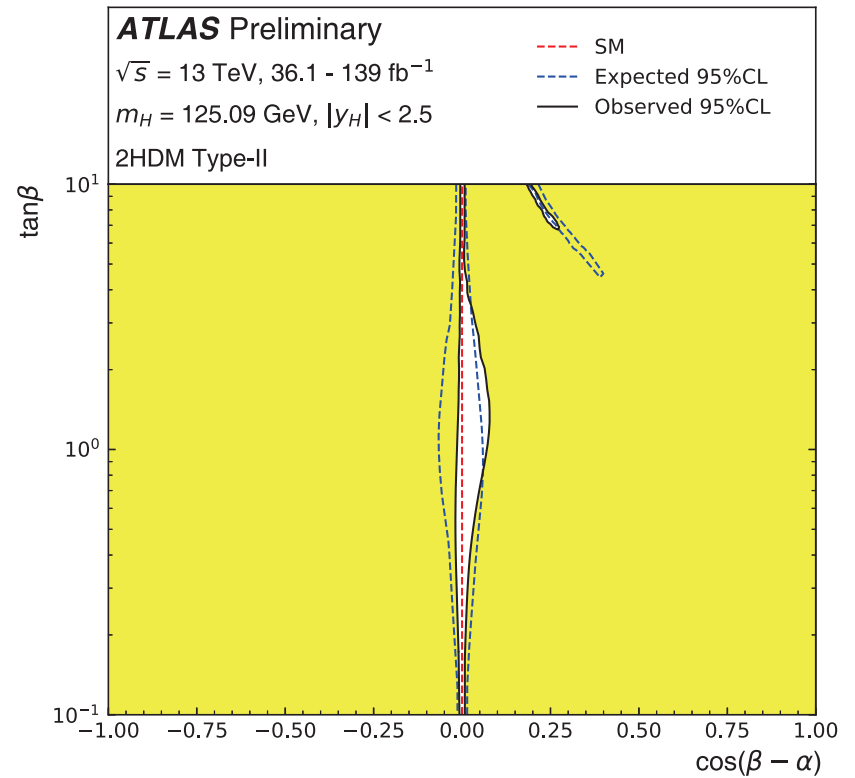
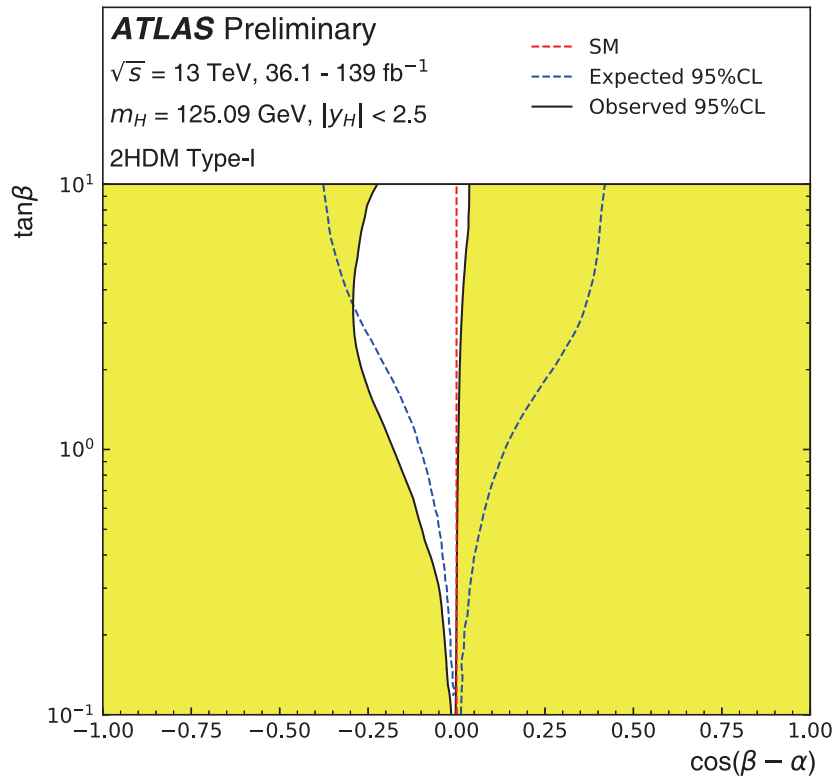
$$\varepsilon \equiv e^{i\eta} = \begin{cases} \text{sgn } Z_6, & \text{if } Z_6 \neq 0, \\ \text{sgn } Z_7, & \text{if } Z_6 = 0 \text{ and } Z_7 \neq 0. \end{cases}$$

Under the assumption that the lighter of the two neutral CP-even Higgs bosons is SM-like, we make the following identifications following the standard notation of the CP-conserving 2HDM:

$h = h_1$, $H = -\varepsilon h_2$, $A = \varepsilon h_3$ and

$$c_{12} = \sin(\beta - \alpha), \quad s_{12} = -\varepsilon \cos(\beta - \alpha).$$

Exact alignment corresponds to $\cos(\beta - \alpha) = 0$.



Regions excluded (at 95% CL) by fits to the measured rates of the productions and decay of the Higgs boson (assumed to be h of the 2HDM), highlighted in yellow. The observed best fit values for $\cos(\beta - \alpha)$ are -0.006 for the Type-I 2HDM and 0.002 for the Type-II 2HDM. Taken from The ATLAS Collaboration, ATLAS-CONF-2021-053 (December 15, 2021).

C and P symmetries in a gauge theory of spin-0 and spin-1 fields

Consider a theory of scalar fields (ϕ) and gauge fields (V).

- All kinetic energy terms (where derivatives are replaced by covariant derivatives) and mass terms of the Lagrangian separately conserve C, P and T. When scalar self-interactions are included, CP-violating interaction terms can arise.
- In the presence of (renormalizable) interactions,* P is conserved in all scattering processes.
- Gauge fields are assigned $P = -1$, since $\mathcal{L}_{\text{int}} = -j^\mu A_\mu$ conserves P.
- All scalars can be consistently assigned $P = +1$.

*excluding the topological term $\theta F\tilde{F}$, which is P-odd and C-even.

Example: Consider a gauge theory of scalar fields with CP-violating scalar self-interactions (but with no fermions). If ϕ is a mixed CP state, then the effective Lagrangian governing the $\phi \rightarrow VV$ decay, $\mathcal{L}_{\text{eff}} \sim \phi F F$, arises at one-loop due to a loop of charged scalars, which is parity conserving. Since no $\epsilon_{\mu\nu\alpha\beta}$ will appear to all orders in perturbation theory, there is no loop-induced $\phi F \tilde{F}$. Thus, CP violation must be interpreted as C violation.

A theory with CP-violating scalar self-interactions provides an example of P-even CP violation.

In contrast, CP violation arising from $\theta F \tilde{F}$ or from the Yukawa interactions of neutral scalars provide examples of P-odd CP violation, since $F \tilde{F}$, $\bar{\psi}\psi$, and $i\bar{\psi}\gamma_5\psi$ are C-even operators.

The CP-conserving 2HDM provides an instructive example. Excluding the fermions, the J^{PC} quantum numbers of the bosonic fields of the 2HDM are shown below.

bosonic field	J^{PC}	J^P
γ, Z	1^{--}	
h, H	0^{++}	
A, G	0^{+-}	
W^\pm		1^-
H^\pm, G^\pm		0^+

Perhaps you were expecting A and G to be 0^{-+} pseudoscalars. Such J^{PC} assignments arise in the context of the Yukawa interactions. But here, we are considering the bosonic sector alone.

The bosonic Lagrangian is separately C and P conserving. For example, there is no AZZ coupling due to C-invariance. If the scalar self-interactions violate CP, then C is no longer conserved.

In contrast, consider a theory of neutral scalars and fermions with (dimension-4) Yukawa interactions. In the CP-conserving 2HDM,

$$\mathcal{L}_{\text{Yuk}} = -g_h \psi \bar{\psi} h - g_H \psi \bar{\psi} H - i g_A \psi \bar{\psi} \gamma_5 \psi A,$$

which would imply that h and H are 0^{++} scalars and A is a 0^{-+} scalar. Given a theory of scalars and fermions (with renormalizable Yukawa interactions), one can consistently assign $C = +1$ to all neutral scalar fields. Thus, if ϕ is a scalar of indefinite CP, then

$$\mathcal{L}_{\text{Yuk}} = -\bar{\psi}(g_1 + i g_2 \gamma_5)\psi \phi,$$

which for $g_1 g_2 \neq 0$ violates P but conserves C. This is an example of C-even CP violation.

In extended Higgs sectors new sources of CP violation can arise:

- P-even CP violation due to a CP-violating scalar potential (explicit CP violation) or vacuum (spontaneous CP violation).
- C-even CP violation due to CP-violating neutral scalar Yukawa couplings.
- P-odd CP violation due to CP-violating charged scalar Yukawa couplings.

CP-violating Yukawa couplings can be probed via P-violating asymmetries (which arise due to the presence of an $\epsilon_{\mu\nu\alpha\beta}$).

How can we experimentally identify signals of P-even CP violation mediated by scalars?

For simplicity, proposed observables are presented in the context of the CP-violating 2HDM, with neutral scalars h_1 , h_2 and h_3 .

Proposal (Fontes, Romão, Santos and Silva: arXiv:1506.06755):[†]

- Detect the three decays, $h_3 \rightarrow h_2 Z$, $h_3 \rightarrow h_1 Z$, and $h_2 \rightarrow h_1 Z$;
- Detect the three production^{or} processes, $Z^* \rightarrow h_3 h_2$, $h_3 h_1$, $h_2 h_1$.

If CP is conserved, then one can use the J^{PC} assignments of the bosonic sector to conclude that the two final state scalars have opposite sign C.[‡] But it is not possible for h_i and h_j to have opposite sign C for all possible $i < j$.

[†]See also: A. Mendez and A. Pomarol, Phys. Lett. B 272, 313 (1991) and G. Cvetič, M. Nowakowski and A. Pilaftsis, Phys. Lett. B 301, 77 (1993).

[‡]Note that P is conserved since combining two parity-even scalars in an angular momentum $L = 1$ state (corresponding to the Z boson) yields $P = (-1)^L = -1$.

However, the proposal of Fontes et al. fails in the limit of exact Higgs alignment, where only the Zh_2h_3 coupling is nonzero (after identifying h_1 as the SM-like Higgs boson). If Higgs alignment is approximate, then the Zh_1h_j ($j \neq 1$) couplings are suppressed, making the proposed signal difficult to observe.

In the exact alignment limit, $Z_6 = \text{Im}(Z_5 e^{-2i\eta}) = 0$, and CP violation requires that $\text{Re}(Z_7 e^{-i\eta}) \text{Im}(Z_7 e^{-i\eta}) \neq 0$. Denoting h_3 as the would-be CP-odd scalar (if CP were conserved), then in the exact Higgs alignment limit, the only bosonic sources of CP violation are:

$$h_3h_3h_3, h_3h_2h_2, h_3H^+H^-, h_3h_3h_3h_1, h_3h_1h_2h_2, h_3h_1H^+H^-.$$

We propose to find similar observables in which all couplings are present in the exact alignment limit. In the 2HDM, there are four classes of processes (involving trilinear couplings) whose simultaneous observation would constitute a detection of P-even CP violation.

1. $h_2 H^+ H^-$, $h_3 H^+ H^-$, $Z h_2 h_3$,
2. $h_2 h_k h_k$, $h_3 H^+ H^-$, $Z h_2 h_3$, (for $k = 2$ or 3),
3. $h_3 h_k h_k$, $h_2 H^+ H^-$, $Z h_2 h_3$, (for $k = 2$ or 3),
4. $h_2 h_k h_k$, $h_3 h_\ell h_\ell$, $Z h_2 h_3$, (for $k, \ell = 2$ or 3).

Note: If CP is conserved, then both $H^+ H^-$ and $h_k h_k$ in an angular momentum zero state have $C = P = +1$.

P-even, CP-violating scalar processes at lepton colliders

We shall assume that the neutral scalars (beyond the SM-like h_1) and the charged scalars of the extended Higgs sector have been discovered (at the LHC or some future collider facility).

Lepton colliders provide an ideal tool for exploring P-even, CP violation, since the relevant production processes involve tree-level purely bosonic interactions. (In contrast, the dominant production mechanism at the LHC is gluon-gluon fusion via a top quark loop.)

Since signals of P-even CP violation require the production of multiple scalars, sub-TeV lepton colliders are unlikely to provide the necessary production cross sections.

Thus, we shall focus on a number of TeV-scale lepton colliders that have been considered as the facility of choice to explore P-even CP-violating phenomena.

Accelerator	\sqrt{s} (TeV)	Integrated luminosity (ab^{-1})
CLIC	1.5	2.5
CLIC	3	5
Muon Collider	3	1
Muon Collider	10	10
Muon Collider	14	20

Accelerators used in the analysis with different CM energies proposed and the corresponding total integrated luminosity in a multiyear program (typically of order 10 years).

Coupling parameters that govern the three-scalar interactions

In the exact Higgs alignment limit, the interaction vertices that contribute to the P-even, CP-violating observables of interest are:

$$\lambda_{H^+H^-h_1} = vZ_3,$$

$$\lambda_{H^+H^-h_2} = \lambda_{h_3h_3h_2} = v \operatorname{Re}(Z_7e^{-i\eta}),$$

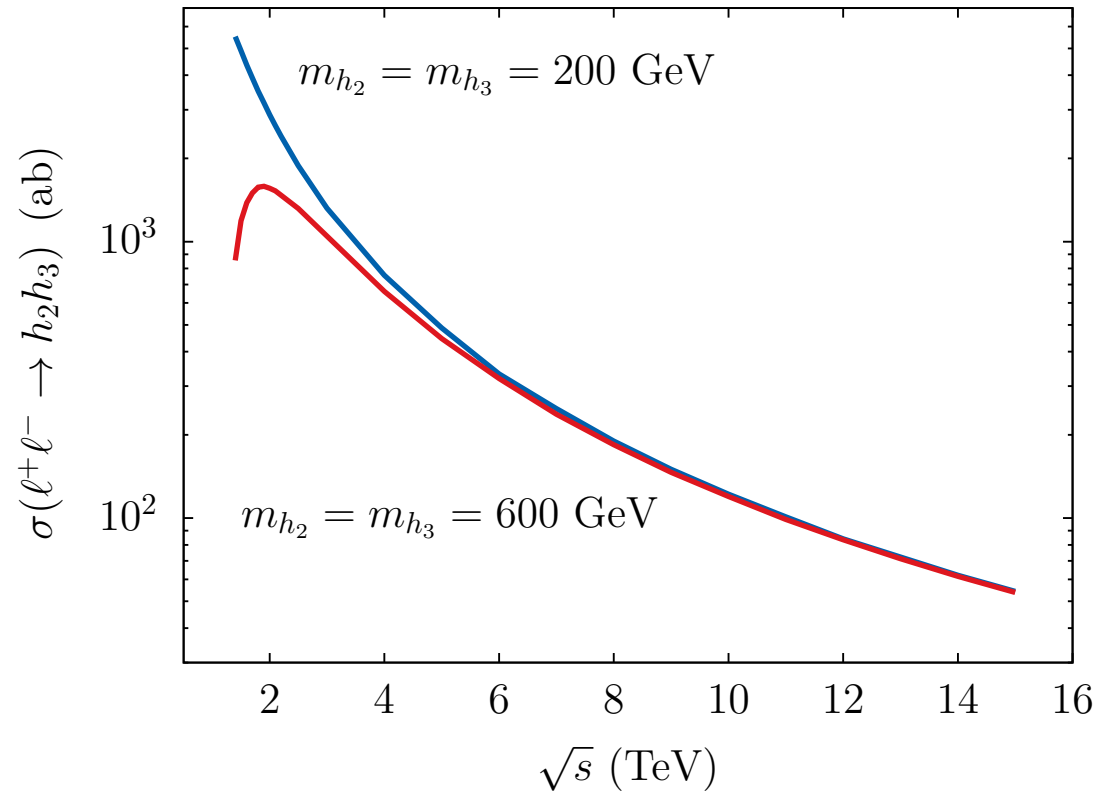
$$\lambda_{H^+H^-h_3} = \lambda_{h_2h_2h_3} = -v \operatorname{Im}(Z_7e^{-i\eta}).$$

For convenience, we shall denote:

$$\Lambda_1 \equiv Z_3, \quad \Lambda_2 \equiv \operatorname{Re}(Z_7e^{-i\eta}), \quad \Lambda_3 \equiv \operatorname{Im}(Z_7e^{-i\eta}).$$

An optimal case study: $\ell^+\ell^- \rightarrow Z^* \rightarrow h_2h_3$

$$m_{h_1} = 125 \text{ GeV}$$



The sweet spot is $\sqrt{s} \simeq 3 \text{ TeV}$, where at least 1000 events are produced at a lepton collider operating for roughly 10 years, for the scalar masses shown above.

The relevant h_2 and h_3 decay modes (if kinematically allowed):

$$h_3 \rightarrow h_2 Z, \quad h_3 \rightarrow h_2 h_2, \quad h_{2,3} \rightarrow H^\pm W^\mp, \quad h_{2,3} \rightarrow H^+ H^-, \\ h_{2,3} \rightarrow \bar{t}t, \quad h_{2,3} \rightarrow \bar{b}b \quad \text{and} \quad h_{2,3} \rightarrow \tau^+ \tau^-.$$

Because the couplings of h_2 and h_3 to other scalars can be large, $\ell^+ \ell^- \rightarrow Z^* \rightarrow h_2 h_3$ alone could signal P-even CP violation in the exact Higgs alignment limit.

For example, if the following two-body decays are observed,

$$h_3 \rightarrow h_2 h_2 \quad \text{and} \quad h_2 \rightarrow H^+ H^-,$$

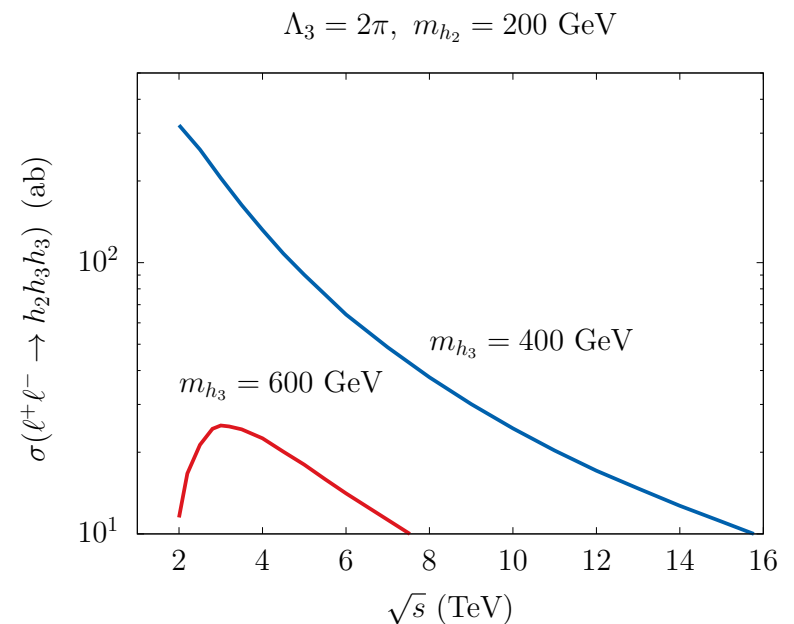
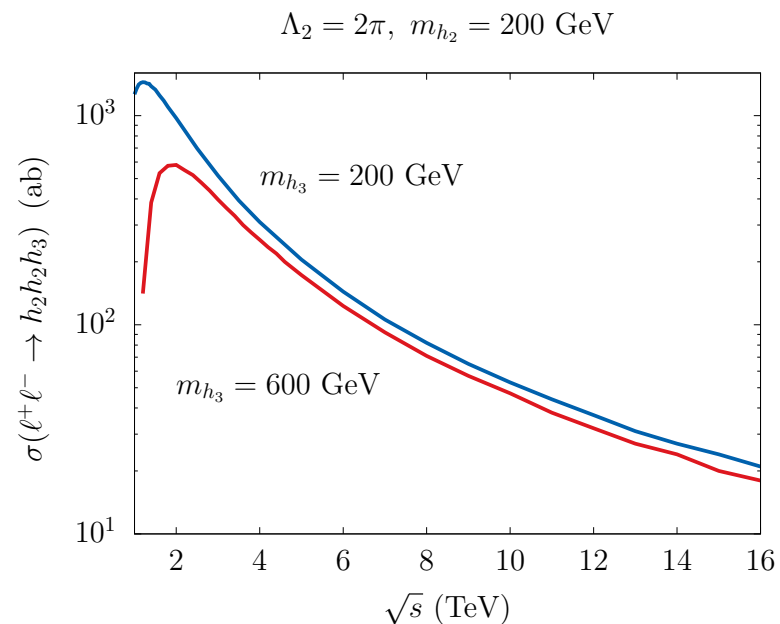
then $\ell^+ \ell^- \rightarrow Z^* \rightarrow h_2 h_3$ would provide evidence for the simultaneous observation of the $h_3 h_2 h_2$, $h_2 H^+ H^-$ and $Z h_2 h_3$ vertices, which is a signal of P-even CP violation.

If the two-body decays of h_2 and h_3 into bosonic final states are kinematically forbidden, then

- consider separately the three production processes governed by one of the sets of bosonic interactions previously mentioned.

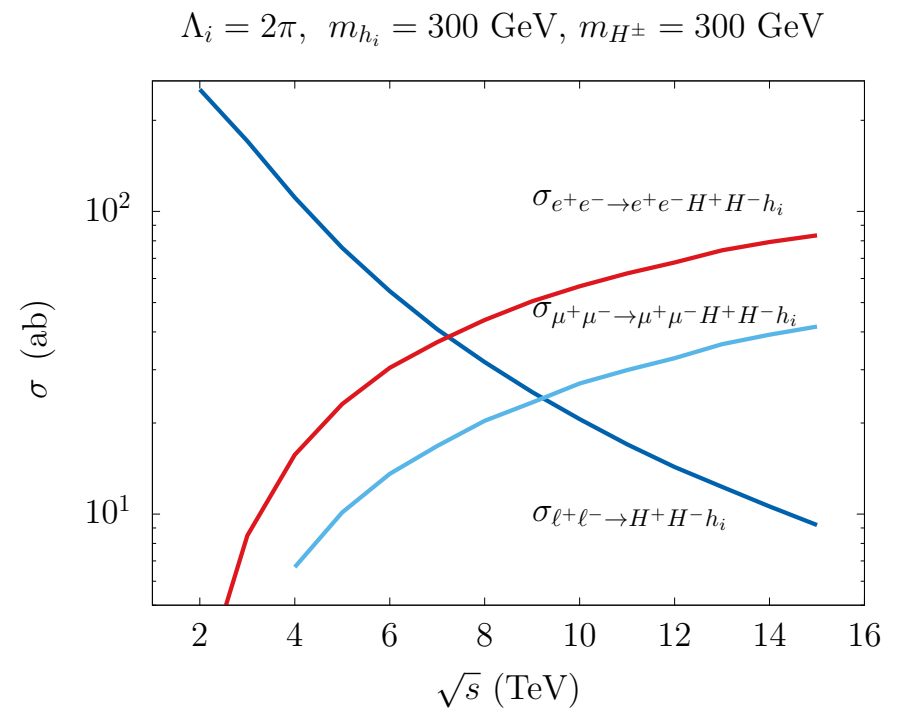
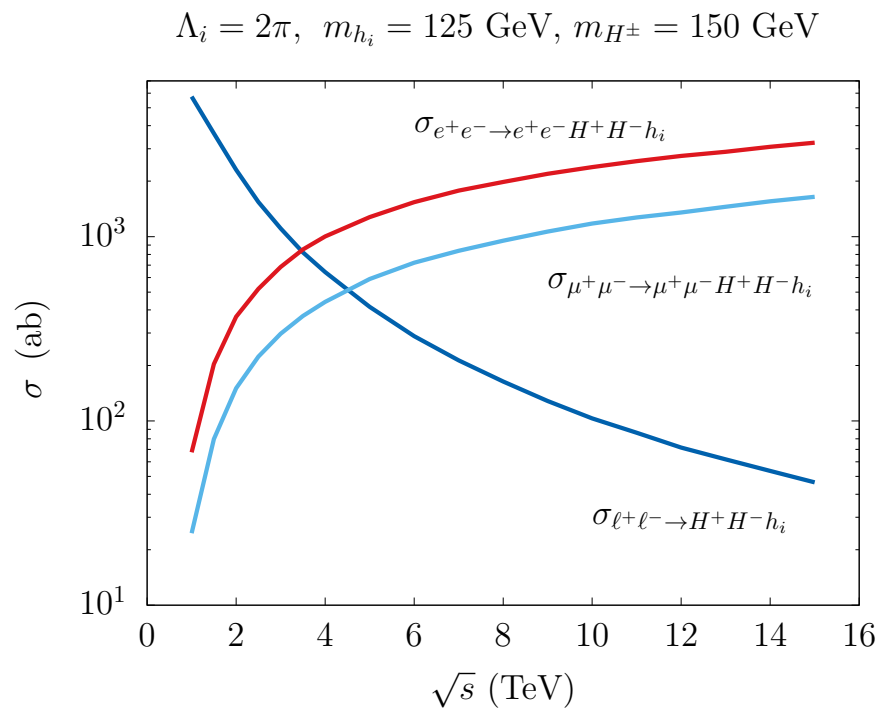
Advantage: only constrained by the collider energy.

Disadvantage: requires the observation of 3-body processes with smaller cross sections.

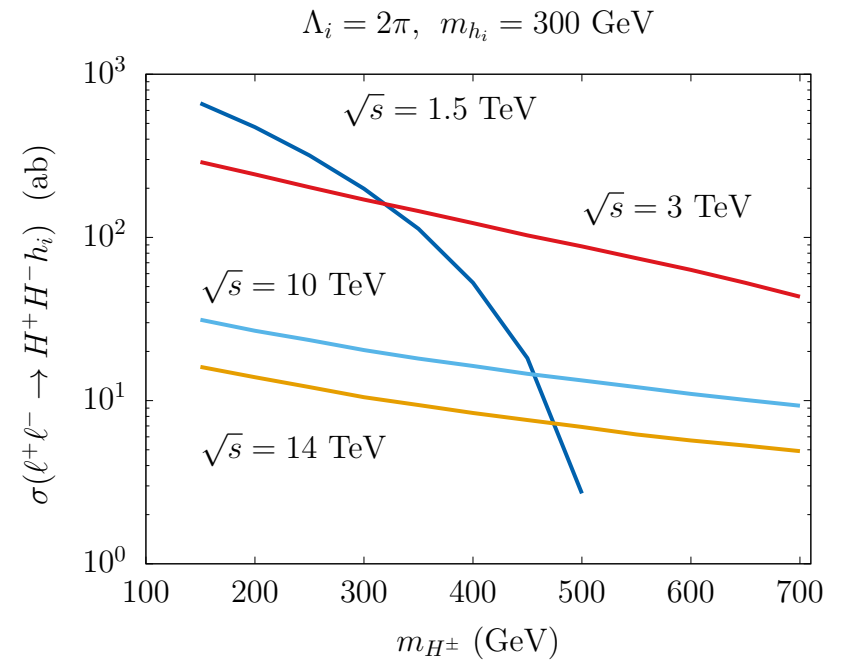
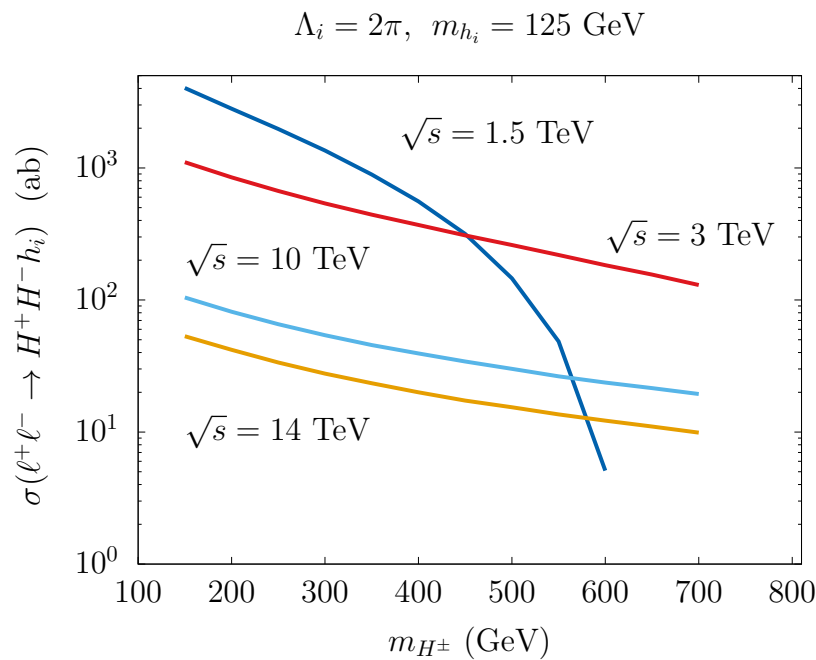


Production of charged Higgs bosons at lepton colliders

Charged Higgs bosons can be produced either via an s -channel annihilation process, $\ell^+\ell^- \rightarrow H^+H^-h_i$ or via a t -channel $\gamma\gamma$ fusion process, $\ell^+\ell^- \rightarrow \ell^+\ell^-H^+H^-h_i$.

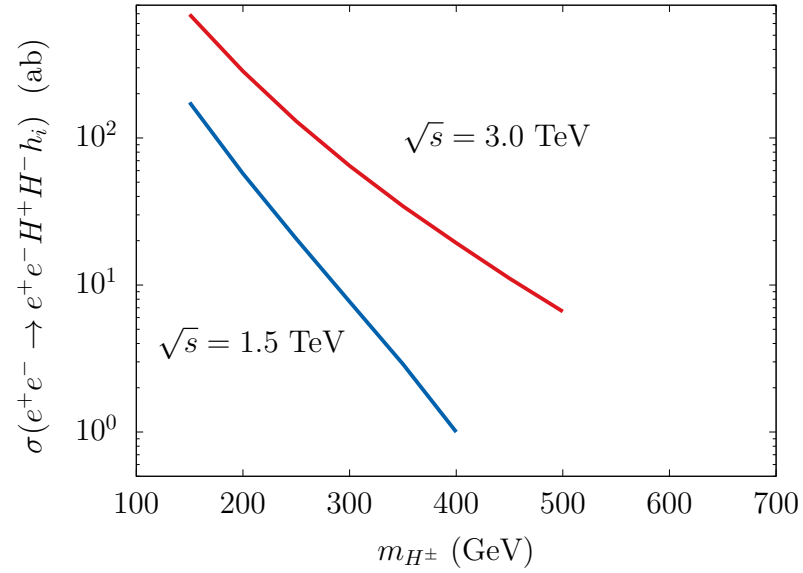


In more detail, we show below the s -channel cross section, $\sigma(\ell^+\ell^- \rightarrow H^+H^-h_i)$, as a function of the charged Higgs mass for four CM energies of $\sqrt{s} = 1.5, 3, 10$ and 14 TeV. The scalar potential parameters are chosen such that $\Lambda_i = 2\pi$.

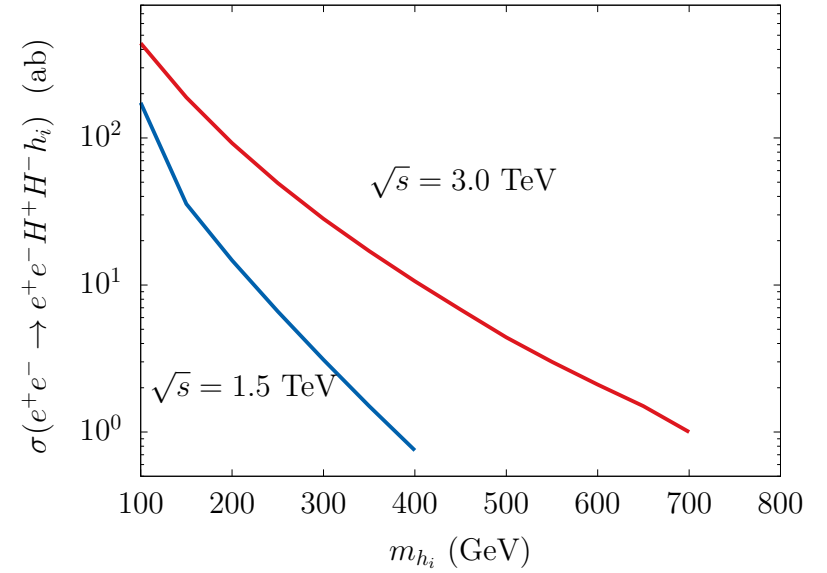


The corresponding results for the t -channel $\gamma\gamma$ fusion processes for CLIC and the muon collider, respectively, are shown below.

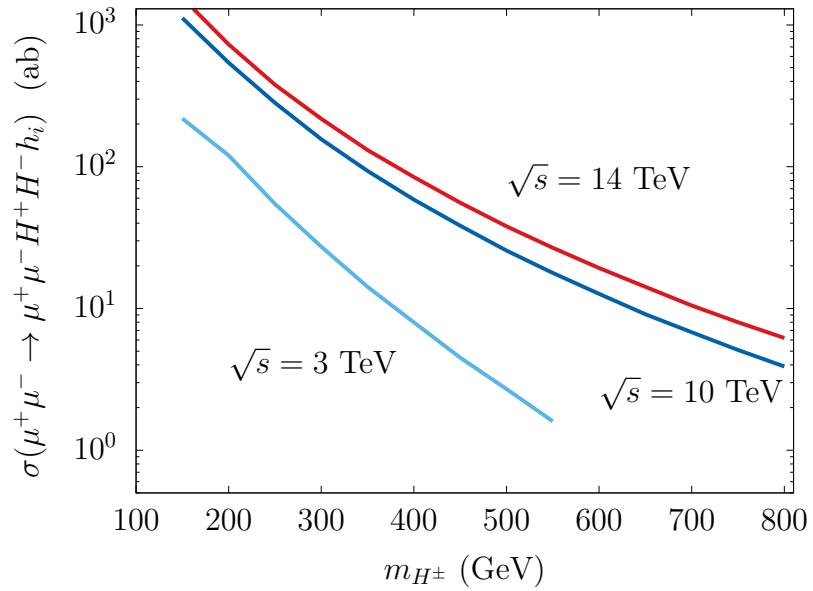
$\Lambda_i = 2\pi, m_{h_i} = 125 \text{ GeV}$



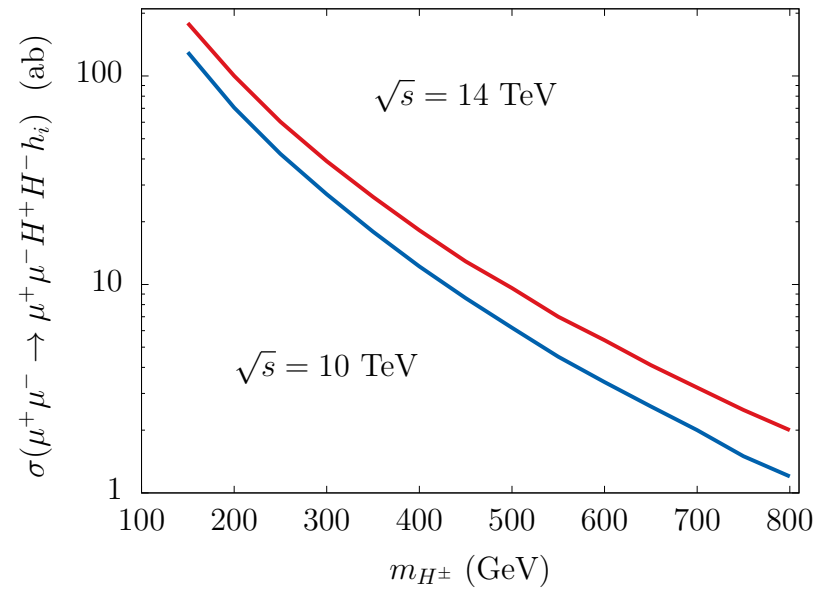
$\Lambda_i = 2\pi, m_{H^\pm} = 200 \text{ GeV}$



$\Lambda_i = 2\pi, m_{h_i} = 125 \text{ GeV}$



$\Lambda_i = 2\pi, m_{h_i} = 300 \text{ GeV}$



P-even, CP-violating signals via loop effects

One can indirectly probe the P-even, CP-violating phenomena via loop contributions to the ZZZ and ZW^+W^- form factors.[§]

$$\begin{aligned}\Gamma_V^{\alpha\beta\mu}(q, \bar{q}, P) = & f_1^V (\bar{q} - q)^\mu g^{\alpha\beta} - \frac{f_2^V}{m_W^2} (\bar{q} - q)^\mu P^\alpha P^\beta + f_3^V (P^\alpha g^{\mu\beta} - P^\beta g^{\mu\alpha}) \\ & + i f_4^V (P^\alpha g^{\mu\beta} + P^\beta g^{\mu\alpha}) + i f_5^V \epsilon^{\mu\alpha\beta\rho} (\bar{q} - q)_\rho \\ & - f_6^V \epsilon^{\mu\alpha\beta\rho} P_\rho - \frac{f_7^V}{m_W^2} (\bar{q} - q)^\mu \epsilon^{\alpha\beta\rho\sigma} P_\rho (\bar{q} - q)_\sigma.\end{aligned}$$

The form factor f_4^V is the unique form factor that is P-conserving and CP-violating. In the exact Higgs alignment limit, a nonzero scalar contribution to f_4 requires at least three neutral scalars beyond the SM-like Higgs boson.[¶]

[§]Applications to the CP-violating 2HDM can be found in Grzadkowski, OGREID and OSLAND, arXiv:1603.01388.

[¶]Note that there are two triangle diagrams with internal scalars that contribute at one loop order to the ZW^+W^- form factors, consisting of an $H^+H^-h_j$ and an $h_jh_kH^+$ loop, with corresponding ZH^+H^- and Zh_jh_k vertices, respectively. Only the latter can contribute to the P-even, CP-violating form factor f_4 .

Final state photons as a diagnostic for CP violation

1. Higgs boson decays to two photons

$h_i \rightarrow \gamma\gamma$ or $Z\gamma$, $h_j \rightarrow \gamma\gamma$ or $Z\gamma$, and the $Zh_i h_j$ vertex.

If Yukawa couplings are present, one must distinguish the effective operators $\phi F_{\mu\nu} F^{\mu\nu}$ generated by bosonic loops from $\phi \epsilon_{\mu\nu\alpha\beta} F^{\mu\nu} F^{\alpha\beta}$ (which can only be generated by fermion loops) via a measurement of the photon polarizations.

2. Higgs boson decays to multi-vector-boson final states

(i) $h_i \rightarrow \gamma\gamma$ and $h_i \rightarrow \gamma\gamma\gamma$ (for a fixed $i \neq 1$).

However, one can show that the $h_i \rightarrow \gamma\gamma\gamma$ decay amplitude vanishes at the one-loop level (independently of the presence or absence of Higgs boson couplings to fermions). Thus, the decay rate for $h_i \rightarrow \gamma\gamma\gamma$ relative to $h_i \rightarrow \gamma\gamma$ is suppressed by both a loop factor and the three-body phase space suppression.

(ii) $h_i \rightarrow \gamma\gamma$ or $Z\gamma$ and $h_i \rightarrow Z\gamma\gamma$ or $h_i \rightarrow ZZ\gamma$ ($i \neq 1$)

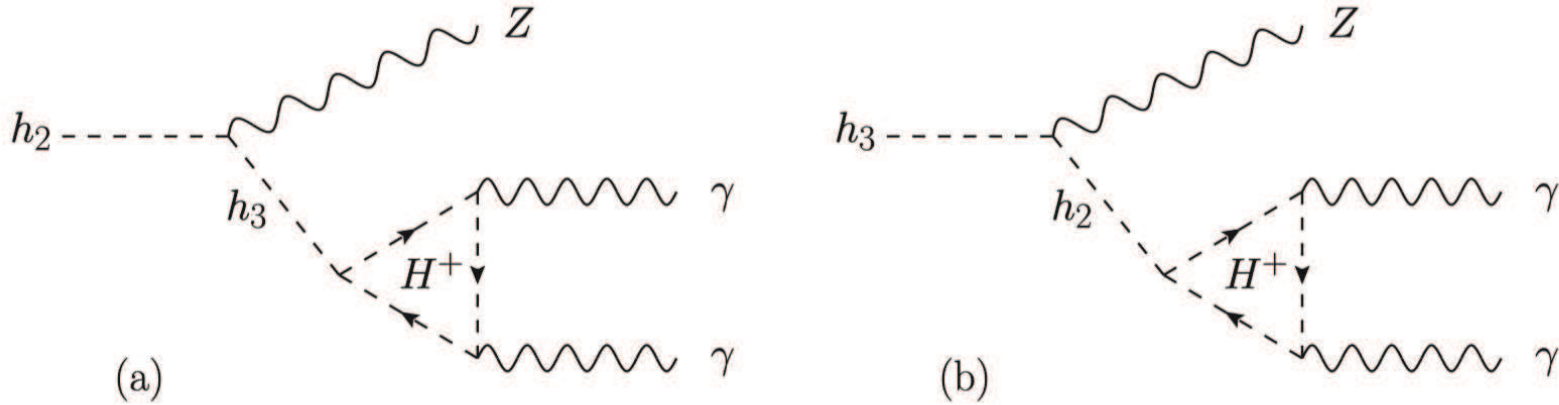


Figure 8: Sample one-loop diagrams contributing to the decay process $h_{2,3} \rightarrow Z\gamma\gamma$.

In the absence of fermion loops, the observation of $h_i \rightarrow \gamma\gamma$ and $h_i \rightarrow Z\gamma\gamma$ would violate C and constitute a signal of P-even CP violation. When fermion loops contribute, one must isolate the contribution of the $\phi F_{\mu\nu} F^{\mu\nu}$ effective operator associated with the triangle subdiagram above.

Nevertheless, the three-body phase space suppression (along with the loop-suppressed branching ratio) render this proposal impractical in most extended Higgs models.

Extensions: beyond the 2HDM and/or beyond the Higgs alignment limit

- If extended Higgs sectors beyond the 2HDM are considered, then additional channels can be employed in the exact Higgs alignment limit. For example, the original proposal of Fontes et al. may now be viable by observing:

$$h_i h_j Z, \quad h_i h_k Z \quad \text{and} \quad h_j h_k Z, \quad (\text{for } i \neq j \neq k \text{ and } i, j, k, \neq 1).$$

- If the alignment limit is only approximately realized, then additional observables can be employed. For example, in the 2HDM, if we allow for precisely one Higgs alignment suppressed vertex then one can consider:^{||}

1. $h_i H^+ H^-$, $Z h_1 h_i$, (for $i \neq 1$),
2. $h_i h_j h_j$, $Z h_1 h_i$, (for $i, j \neq 1$),
3. $h_1 h_i h_j$, $Z h_i h_j$, (for $i, j \neq 1$ and $i \neq j$),

which have the benefit of requiring the observation of only two vertices.

^{||}By observation, one can treat the SM-like h_1 as a CP-even scalar.

vertex	self-coupling
$H^+ H^- h_1$	$v [Z_3 - s_{12} \text{Re}(Z_7 e^{-i\eta}) + s_{13} \text{Im}(Z_7 e^{-i\eta})]$
$H^+ H^- h_2$	$v [\text{Re}(Z_7 e^{-i\eta}) + s_{12} Z_3]$
$H^+ H^- h_3$	$-v [\text{Im}(Z_7 e^{-i\eta}) - s_{13} Z_3]$
$h_1 h_1 h_1$	$3v Z_1$
$h_2 h_2 h_2$	$3v [\text{Re}(Z_7 e^{-i\eta}) + s_{12} (Z_{34} + \text{Re}(Z_5 e^{-2i\eta}))]$
$h_3 h_3 h_3$	$-3v [\text{Im}(Z_7 e^{-i\eta}) - s_{13} (Z_{34} - \text{Re}(Z_5 e^{-2i\eta}))]$
$h_1 h_2 h_2$	$v [Z_{34} + \text{Re}(Z_5 e^{-2i\eta}) - 3s_{12} \text{Re}(Z_7 e^{-i\eta}) + s_{13} \text{Im}(Z_7 e^{-i\eta})]$
$h_1 h_3 h_3$	$v [Z_{34} - \text{Re}(Z_5 e^{-2i\eta}) - s_{12} \text{Re}(Z_7 e^{-i\eta}) + 3s_{13} \text{Im}(Z_7 e^{-i\eta})]$
$h_2 h_1 h_1$	$v [s_{12} (3Z_1 - 2Z_{34} - 2 \text{Re}(Z_5 e^{-2i\eta})) + 3 \text{Re}(Z_6 e^{-i\eta})]$
$h_2 h_3 h_3$	$v [\text{Re}(Z_7 e^{-i\eta}) + s_{12} (Z_{34} - \text{Re}(Z_5 e^{-2i\eta}))]$
$h_3 h_1 h_1$	$v [s_{13} (3Z_1 - 2Z_{34} + 2 \text{Re}(Z_5 e^{-2i\eta})) - 3 \text{Im}(Z_6 e^{-i\eta})]$
$h_3 h_2 h_2$	$-v [\text{Im}(Z_7 e^{-i\eta}) - s_{13} (Z_{34} + \text{Re}(Z_5 e^{-2i\eta}))]$
$h_1 h_2 h_3$	$-v [s_{13} \text{Re}(Z_7 e^{-i\eta}) - s_{12} \text{Im}(Z_7 e^{-i\eta})]$

Cubic self-couplings of the physical Higgs scalars of the 2HDM in the approximate Higgs alignment limit without decoupling (where $|Z_6| \ll 1$). Charged fields point *into* the vertex. The first order corrections to the exact Higgs alignment limit, which are linear in s_{12} , s_{13} and $Z_6 e^{-i\eta}$, are exhibited.

Conclusions

- P-even CP violation can arise extended Higgs sectors.
- The physics of P-even CP violation is distinct from CP-violating phenomena that originate from the Yukawa sector.
- If new sources of CP violation are present in an extended Higgs sector, then an important task of future Higgs studies will be to determine their origins.
- To discover and probe P-even CP violation of an extended Higgs sector, our initial studies suggest that a multi-TeV lepton collider will be required.**

** Although not discussed in this talk, a multi-TeV $\gamma\gamma$ collider (which could be obtained using an e^+e^- collider by converting the electrons to photons via Compton backscattering of laser light) can also be employed to identify P-even CP-violating signals.