The general THDM in a gauge-invariant form

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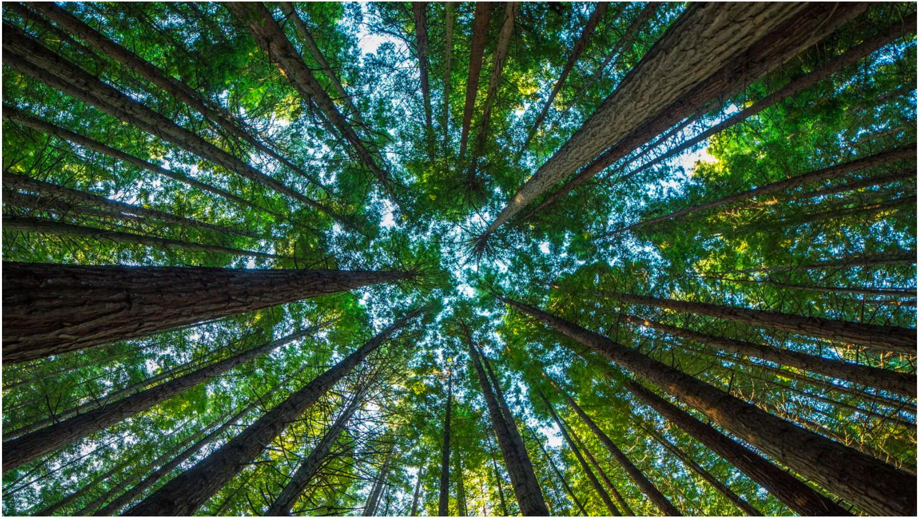
Sometimes we can't see the forest for the trees



- Gauge symmetry hidden after electroweak symmetry breaking

- Principal ideal: Keep gauge symmetry manifest.

- Let us revisit the Standard Model

- SM: One Higgs-boson doublet

$$\varphi(x) = \begin{pmatrix} \varphi^+(x) \\ \varphi^0(x) \end{pmatrix}$$

- Domain: $\varphi^+, \varphi^0 \in \mathbb{C}$
- SM potential

$$V_{SM} = -\mu^2 \varphi^\dagger \varphi + \lambda (\varphi^\dagger \varphi)^2$$

- Define real bilinear

$$K = \varphi^\dagger \varphi$$

- Domain: $K \geq 0$

$$V_{SM} = -\mu^2 K + \lambda K^2$$

- Electroweak symmtry –

$\langle K \rangle = 0$ unbroken, $\langle K \rangle > 0$ partially broken

- What do we have achieved?

  - Simple, real, potential - polynomial of degree two.

  - Gauge invariance manifest:
    Symmetries not hidden by unphysical gauge-degrees of freedom.

Bilinears in the THDM

- Two doublets, φ_1, φ_2
- Real gauge invariants

$$\varphi_1^\dagger \varphi_1, \quad \varphi_2^\dagger \varphi_2, \quad \mathrm{Re}\left(\varphi_1^\dagger \varphi_2\right), \quad \mathrm{Im}\left(\varphi_1^\dagger \varphi_2\right)$$

J. Velhinho, R. Santos and A. Barroso, Phys. Lett. B 322 (1994)

- Later real gauge invariant bilinears were introduced:

Nishi **PRD 74** (2006), MM, Manteuffel, Nachtmann, Nagel **EPJC 48** (2006)

$$K_0 = \varphi_1^\dagger \varphi_1 + \varphi_2^\dagger \varphi_2, \qquad K_1 = \varphi_1^\dagger \varphi_2 + i\varphi_2^\dagger \varphi_1,$$
$$K_2 = i\varphi_2^\dagger \varphi_1 - i\varphi_1^\dagger \varphi_2, \qquad K_3 = \varphi_1^\dagger \varphi_1 - \varphi_2^\dagger \varphi_2$$

- Bilinears derived from

$$\underline{K} = \begin{pmatrix} \varphi_1^\dagger \varphi_1 & \varphi_2^\dagger \varphi_1 \\ \varphi_1^\dagger \varphi_2 & \varphi_2^\dagger \varphi_2 \end{pmatrix} = \frac{1}{2}\left(K_0 \mathbb{1}_2 + K_i \sigma^i\right)$$

- One-to-one map $\varphi_{1/2} \leftrightarrow K_0, K_1, K_2, K_3$ (except for unphysical gauge d.o.f.)

EW symmetry breaking

- Domain $K_0 \geq 0, \quad K_0^2 \geq K_1^2 + K_2^2 + K_3^2$.
- EW symmetry at minimum
  - Unbroken for
$$K_0 = 0 \quad \Longrightarrow \; V = 0$$
  - Charge breaking for $K_0 > 0$ and $K_0^2 > K_1^2 + K_2^2 + K_3^2$
$$\text{cond:} \qquad \partial_\mu V \equiv \frac{\partial V}{\partial K^\mu} = 0$$
  - Neutral for $K_0 > 0$ and $K_0^2 = K_1^2 + K_2^2 + K_3^2$
$$\text{cond:} \qquad \partial_\mu \big(V - u(K_0^2 - K_1^2 - K_2^2 - K_3^2) \big) = 0$$

    Lagrange mulitplier u, minimum at K_0, K_1, K_2, K_3, u.

- Minkowski-like four vector out of bilinears

$$(K^\mu) := \begin{pmatrix} K_0 \\ \boldsymbol{K} \end{pmatrix} = \begin{pmatrix} K_0 \\ K_1 \\ K_2 \\ K_3 \end{pmatrix} \quad \text{with } K_0^2 - \boldsymbol{K}^2 \geq 0$$

- Dimensionless bilinears for $K_0 > 0$

$$\boldsymbol{k} = \frac{\boldsymbol{K}}{K_0}$$

- Example: Most general tree THDM potential

$$V = \xi_0 K_0 + \boldsymbol{\xi}^{\mathrm{T}} \boldsymbol{K} + \eta_{00} {K_0}^2 + 2 K_0\, \boldsymbol{\eta}^{\mathrm{T}} \boldsymbol{K} + \boldsymbol{K}^{\mathrm{T}} E \boldsymbol{K},$$

...gauge symmetry manifest

Change of basis

- Consider the following mixing of the doublets

$$\begin{pmatrix} \varphi_1' \\ \varphi_2' \end{pmatrix} = U \begin{pmatrix} \varphi_1 \\ \varphi_2 \end{pmatrix}.$$

  with unitary 2×2 matrix U.
- The gauge invariant functions transform as

$$K_0' = K_0, \qquad \boldsymbol{K}' = R(U)\boldsymbol{K},$$

- R proper rotation

Symmetries

- Symmetries straightforward to study

  see, for instance Ferreira/MM/Nachtmann/Silva JHEP08 (2010), Bento/Boto/Silva/Trautner JHEP21 (2020)

- Example Standard CP symmetry

$$\varphi_{1/2}(x) \overset{\text{CP}}{\rightarrow} \varphi^*_{1/2}(x'), \qquad x = (t, \boldsymbol{x})^{\mathrm{T}}, \quad x' = (t, -\boldsymbol{x})^{\mathrm{T}}$$

- In terms of bilinears

  Nishi PRD74 (2006), MM/Manteuffel/Nachtmann EPJC57 (2008), Ferreira/Haber/MM/nachtmann/Silva IJMPA26 (2011)

$$K_0 \overset{\text{CP}}{\rightarrow} K_0, \qquad \begin{pmatrix} K_1 \\ K_2 \\ K_3 \end{pmatrix} \overset{\text{CP}}{\rightarrow} \begin{pmatrix} K_1 \\ -K_2 \\ K_3 \end{pmatrix}$$

The complete THDM gauge invariantly

- Up to now only potential considered.
- How to study gauge, Yukawa couplings, potential beyond tree level?
- Key: study mass matrices gauge invariantly.
- Derive couplings from mass matrices.

- Expand the doublets

$$\varphi_1 = \begin{pmatrix} \varphi_1^+ \\ \varphi_1^0 \end{pmatrix} = \frac{1}{\sqrt{2}} \begin{pmatrix} \pi_1^1 + i\sigma_1^1 \\ \pi_1^2 + i\sigma_1^2 \end{pmatrix},$$
$$\varphi_2 = \begin{pmatrix} \varphi_2^+ \\ \varphi_2^0 \end{pmatrix} = \frac{1}{\sqrt{2}} \begin{pmatrix} \pi_2^1 + i\sigma_2^1 \\ \pi_2^2 + i\sigma_2^2 \end{pmatrix}.$$

- Real component fields

$$\phi = \left(\pi_1^1, \pi_1^2, \sigma_1^1, \sigma_1^2, \pi_2^1, \pi_2^2, \sigma_2^1, \sigma_2^2\right)^{\mathrm{T}}, \quad \text{with } \phi \in \mathbb{R}^8 .$$

- Mass matrix in terms of the 8 component fields:

$$(M_s^2)_{ij} = \frac{\partial^2}{\partial \phi_i \partial \phi_j} V, \qquad i, j \in \{1, \ldots, 8\}$$

- Mass matrix in terms of bilinears:

$$\mathcal{M} = (\mathcal{M}_{\mu\nu}) = \frac{\partial^2}{\partial K^\mu \partial K^\nu} V \equiv \partial_\mu \partial_\nu V, \qquad \mu, \nu \in \{0, \ldots, 3\}$$

- How do we get from M_s^2 to $\mathcal{M}$?

- Establish first connection between component fields and bilinears.

- Write bilinears in terms of the component fields.

$$K^{\mu} \equiv \frac{1}{2}\Delta^{\mu}_{ij}\phi^{i}\phi^{j}\,, \qquad i,j \in \{1,\ldots,8\}$$

with constants Δ^{μ}_{ij} (four 8×8 matrices).

- Connection from (gauge-dependent) components to bilinears:

$$\Gamma^{\mu}_{i} \equiv \frac{\partial K^{\mu}}{\partial \phi^{i}} = \partial_{i}K^{\mu} = \Delta^{\mu}_{ij}\phi^{j}$$

Γ is 8×4 matrix

- Now we can write the mass matrix in terms of bilinears

$$\left(M_s^2\right)_{ij} = \partial_i \partial_j V = \partial_i \left(\Gamma_j^\mu \partial_\mu V\right) = \Delta_{ij}^\mu \partial_\mu V + \Gamma_i^\mu \Gamma_j^\nu \partial_\mu \partial_\nu V \,.$$

- In matrix notation

$$M_s^2 = \Delta^\mu \partial_\mu V + \Gamma \mathcal{M} \Gamma^{\mathrm{T}}, \quad \text{with } \mathcal{M} = (\mathcal{M}_{\mu\nu})$$

- Going to a specific basis where Γ is very simple.
- Diagonalization $M_s^2 \to \bar{M}_s^2$.

- In the neutral minimum case we get

$$\bar{M}_s^2 = \begin{pmatrix} 0_{3\times 3} & & \\ & \bar{\mathcal{M}}^2_{\text{charged}} & \\ & & \bar{\mathcal{M}}^2_{\text{neutral}} \end{pmatrix} .$$

with

$$\bar{\mathcal{M}}^2_{\text{charged}} = \text{diag}\left(m^2_{H^\pm}, \quad m^2_{H^\pm} \right) = \text{diag}\left(4uK_0, \quad 4uK_0 \right)$$

neutral part, with R diagonalization matrix,

$$\bar{\mathcal{M}}^2_{\text{neutral}} = \text{diag}\left(m_1^2, m_2^2, m_3^2 \right) = R\gamma_3 \left(\mathcal{M} - 2u\widetilde{g} \right) \gamma_3^{\text{T}} R^{\text{T}}$$

with

$$\widetilde{g} = \text{diag}\left(1, -1, -1, -1\right), \qquad \gamma_3 = \sqrt{\frac{2}{K_0}} \left(\boldsymbol{K}, \quad \mathbb{1}_3 \right)$$

- What have we found?

- Mass matrix manifelsty gauge invariant.

- Potential not specified!

- In particular, the charged part reads

$$m_{H^\pm}^2 = 4uK_0 \ .$$

- Valid in any THDM at any perturbation order - even for effective models!

THDM Couplings

- Compute the couplings gauge invariantly, for instance

$$\lambda_{ijk} = \left(\partial_i M_s^2\right)^{jk} = \left(\Delta_{ij}^{\mu}\Gamma_k^{\nu} + \Delta_{ik}^{\mu}\Gamma_j^{\nu} + \Delta_{jk}^{\mu}\Gamma_i^{\nu}\right)\mathcal{M}_{\mu\nu}$$

- Example

h - $H^{\pm}$ - $H^{\pm}$

$$\lambda_{h_a H^{\pm} H^{\pm}} = \frac{1}{\sqrt{2K_0}}\left(8K_0\left(\eta_{00}k^a + \eta^a\right) - k^a m_a^2\right)$$

How to deal with Yukawa couplings?

- Yukawa couplings linear in Higgs doublets, bilinears not.
- Yukawa couplings gauge invariant - expressible in terms of bilinears!
- Trick: For neutral vaccum rank of $\underline{K}$ is one:

$$\underline{\bar{K}}^a{}_b = \frac{1}{2}\bar{K}^\mu \left(\sigma_\mu\right)^a{}_b \equiv \bar{\kappa}^a \bar{\kappa}^*_b \,.$$

- Example, Yukawa term

$$-\mathcal{L}_Y = \overline{Q}_L \big(y_u \,\widetilde{\varphi}_1 + \epsilon_u \,\widetilde{\varphi}_2\big) u_R, \quad \text{with } \widetilde{\varphi}_a = i\sigma_2 \left(\varphi_a^*\right)$$

- Neutral Higgs coupling h^a-$u_L^\dagger$-u_R,

$$\frac{1}{\sqrt{2K_0}} \left(\sigma_a\right)^b{}_c \,\bar{\kappa}^*_b \bar{\mathcal{U}}^c, \quad \text{with } \left(\mathcal{U}^c\right) = \begin{pmatrix} y_u \\ \epsilon_u \end{pmatrix}$$

Conclusion

- Mass matrices to all orders computed.
- Complete THDM formulated gauge invariantly.
  (Yukawa, scalar, gauge bosons couplings)
- Program of bilinears completed

  Sartore, MM, Schienbein, Herrmann, arXiv:2208.13719
- To do: Application to THDMs - beyond tree-level.

Thank you for your attention!

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Δ constants

$$K^\mu \equiv \frac{1}{2}\Delta^\mu_{ij}\phi^i\phi^j\,, \qquad i,j \in \{1,\ldots,8\}.$$

$$\Delta^0 = \begin{pmatrix} \mathbb{1}_2 & & & \\ & \mathbb{1}_2 & & \\ & & \mathbb{1}_2 & \\ & & & \mathbb{1}_2 \end{pmatrix}, \qquad \Delta^1 = \begin{pmatrix} & & \mathbb{1}_2 & \\ & & & \mathbb{1}_2 \\ \mathbb{1}_2 & & & \\ & \mathbb{1}_2 & & \end{pmatrix},$$

$$\Delta^2 = \begin{pmatrix} & & & \mathbb{1}_2 \\ & & -\mathbb{1}_2 & \\ & -\mathbb{1}_2 & & \\ \mathbb{1}_2 & & & \end{pmatrix}, \qquad \Delta^3 = \begin{pmatrix} \mathbb{1}_2 & & & \\ & \mathbb{1}_2 & & \\ & & -\mathbb{1}_2 & \\ & & & -\mathbb{1}_2 \end{pmatrix},$$

Bilinears linear

- $\underline{\bar{K}}$ has rank 1 for neutral vacuum:

$$\underline{\bar{K}}^{a}{}_{b} = \frac{1}{2}\bar{K}^{\mu}\left(\sigma_{\mu}\right)^{a}{}_{b} \equiv \bar{\kappa}^{a}\bar{\kappa}^{*}_{b}\,.$$

Explicitly, $\bar{\kappa}$ can be writen

$$\bar{\kappa} = \sqrt{\frac{K_0}{2}}\frac{1}{\sqrt{1+\bar{k}_3}}\begin{pmatrix} 1+\bar{k}_3 \\ \bar{k}_1 + i\bar{k}_2 \end{pmatrix} = \sqrt{\frac{K_0}{2}}\begin{pmatrix} \sqrt{1+\bar{k}_3} \\ \sqrt{1-\bar{k}_3}\,e^{i\zeta} \end{pmatrix}\,,$$

where the phase ζ was defined such that

$$\bar{k}_1 + i\bar{k}_2 = \sqrt{\bar{k}_1^2 + \bar{k}_2^2}\,e^{i\zeta} = \sqrt{1-\bar{k}_3^2}\,e^{i\zeta}\,.$$

- In terms of bilinears, for instance for the charge conserving EW breaking case:

$$\widehat{M}_s^2 \stackrel{CC}{=} 2u \begin{pmatrix} A_{55} & 0_{5\times 3} \\ 0_{3\times 5} & B_{33} \end{pmatrix} + \begin{pmatrix} 0_{5\times 5} & 0_{5\times 3} \\ 0_{3\times 5} & \gamma_3 \mathcal{M} \gamma_3^{\mathrm{T}} \end{pmatrix} = \begin{pmatrix} \widehat{\mathcal{M}}^2_{\mathrm{CC}} & 0_{5\times 3} \\ 0_{3\times 5} & \widehat{\mathcal{M}}^2_{\mathrm{neutral}} \end{pmatrix} ,$$

- SM: simple potential with one Higgs doublet

$$V_{SM} = -\mu^2 \varphi^\dagger \varphi + \lambda (\varphi^\dagger \varphi)^2$$

- Electroweak (spontaneous) - symmetry breaking for $\mu^2 > 0$, stability: $\lambda > 0$.

- Vacuum expectation value

$$\langle \varphi \rangle = \begin{pmatrix} 0 \\ v \end{pmatrix}, \quad v = \sqrt{\mu^2/(\lambda)}$$

Example: two doublets

- Two Higgs doublets $\varphi_1 = \begin{pmatrix} \varphi_1^+ \\ \varphi_1^0 \end{pmatrix}, \quad \varphi_2 = \begin{pmatrix} \varphi_2^+ \\ \varphi_2^0 \end{pmatrix}$
- Matrix of doublets

$$\phi = \begin{pmatrix} \varphi_1^{\mathrm{T}} \\ \varphi_2^{\mathrm{T}} \end{pmatrix} = \begin{pmatrix} \varphi_1^+ \, \varphi_1^0 \\ \varphi_2^+ \, \varphi_2^0 \end{pmatrix}$$

- Building matrix of invariants

$$\underline{K} = \phi\phi^\dagger = \begin{pmatrix} \varphi_1^\dagger\varphi_1 & \varphi_2^\dagger\varphi_1 \\ \varphi_1^\dagger\varphi_2 & \varphi_2^\dagger\varphi_2 \end{pmatrix}$$

- Basis: Pauli matrices and unit matrix $\sigma_0 = \mathbb{1}_2$

$$\underline{K} = \frac{1}{2} K_\alpha \sigma_\alpha, \qquad \alpha = 0, 1, \ldots, 3$$

- $K_\alpha = \mathrm{tr}\,(\underline{K}\sigma_\alpha)$, $(\alpha = 0, \ldots, 3)$ translates to

$$\varphi_1^\dagger\varphi_1 = (K_0 + K_3)/2, \quad \varphi_1^\dagger\varphi_2 = (K_1 + iK_2)/2,$$
$$\varphi_2^\dagger\varphi_2 = (K_0 - K_3)/2, \quad \varphi_2^\dagger\varphi_1 = (K_1 - iK_2)/2$$

Electroweak symmetry breaking

- EW symmetry breaking given by global minimum

$$\langle \phi \rangle = \left\langle \begin{pmatrix} \varphi_1^+ & \varphi_1^0 \\ \vdots & \\ \varphi_n^+ & \varphi_n^0 \end{pmatrix} \right\rangle = \begin{pmatrix} v_1^+ & v_1^0 \\ \vdots & \\ v_n^+ & v_n^0 \end{pmatrix}, \quad \langle \underline{K} \rangle = \langle \phi \rangle \langle \phi \rangle^\dagger$$

- Unbroken $SU(2)_L \times U(1)_Y$ corresponds to $\langle \underline{K} \rangle$ of rank 0, $\underline{K} = 0, \qquad \varphi_i = 0.$

- Fully broken EW symmetry corresponds to $\langle \underline{K} \rangle$ of rank 2

- $SU(2)_L \times U(1)_Y \ \rightarrow U(1)_{em}$ corresponds to $\langle \underline{K} \rangle$ of rank 1

Change of basis

- Consider the following unitary mixing of the doublets

$$\begin{pmatrix} \varphi_1(x)^{\mathrm{T}} \\ \vdots \\ \varphi_n(x)^{\mathrm{T}} \end{pmatrix} \to U \begin{pmatrix} \varphi_1(x)^{\mathrm{T}} \\ \vdots \\ \varphi_n(x)^{\mathrm{T}} \end{pmatrix}$$

- Bilinears transform as

$$\phi \to U\phi, \qquad \underline{K} = \phi\phi^\dagger \to U\underline{K}U^\dagger$$

hence,

$$\begin{aligned} K_0 &= \mathrm{tr}\,(\underline{K}\lambda_0) \to \mathrm{tr}\,\left(U\underline{K}U^\dagger\lambda_0\right) = \mathrm{tr}\,\left(U\underline{K}\lambda_0 U^\dagger\right) = \mathrm{tr}\,(\underline{K}\lambda_0) = K_0, \\ K_a &= \mathrm{tr}\,(\underline{K}\lambda_a) \to \mathrm{tr}\,\left(U\underline{K}U^\dagger\lambda_a\right) = \mathrm{tr}\,\Big(\underbrace{U^\dagger\lambda_a U}_{\equiv R_{ab}\lambda_b}\,\underline{K}\Big) = R_{ab}K_b \end{aligned}$$

- Change of basis correspond to proper rotations $R = (R_{ab}) \in SO(n^2-1)$.

- Under a change of basis

$$K_0 \to K_0, \quad \boldsymbol{K} \to R\boldsymbol{K}$$

The potential

$$V = \xi_0 K_0 + \boldsymbol{\xi}^{\mathrm{T}} \boldsymbol{K} + \eta_{00} {K_0}^2 + 2 K_0\, \boldsymbol{\eta}^{\mathrm{T}} \boldsymbol{K} + \boldsymbol{K}^{\mathrm{T}} E \boldsymbol{K}$$

changes to

$$V' = \xi_0 K_0 + \boldsymbol{\xi}^{\mathrm{T}} R\boldsymbol{K} + \eta_{00} {K_0}^2 + 2 K_0\, \boldsymbol{\eta}^{\mathrm{T}} R\boldsymbol{K} + \boldsymbol{K}^{\mathrm{T}} R^{\mathrm{T}} E R\boldsymbol{K}$$

- Potential invariant, $V = V'$, iff

$$\boldsymbol{\xi} = R\, \boldsymbol{\xi}, \quad \boldsymbol{\eta} = R\, \boldsymbol{\eta}, \quad E = R\, E\, R^{\mathrm{T}}.$$

Symmetries

- Symmetry desirable to restrict nHDM.

- Symmetries easily formulated in terms of bilinears.

$$V = \xi_0 K_0 + \boldsymbol{\xi}^{\mathrm{T}} \boldsymbol{K} + \eta_{00} {K_0}^2 + 2 K_0\, \boldsymbol{\eta}^{\mathrm{T}} \boldsymbol{K} + \boldsymbol{K}^{\mathrm{T}} E \boldsymbol{K}$$

- Transformation $K_0 \to K_0$, $\boldsymbol{K} \to \bar{R}\boldsymbol{K}$, $\bar{R} \in O(n^2-1)$ is symmetry of potential iff

$$\boldsymbol{\xi} = \bar{R}\, \boldsymbol{\xi}, \qquad \boldsymbol{\eta} = \bar{R}\, \boldsymbol{\eta}, \qquad E = \bar{R}\, E\, \bar{R}^{\mathrm{T}}$$

- $\bar{R} \in O(n^2-1)$, keeping kinetic terms invariant.

I. F. Ginzburg, M. Krawczyk, PRD 72 (2005)
I. P. Ivanov and C. C. Nishi, PRD 82 (2010)
MM, O. Nachtmann, JHEP 11 151 (2011)
P. M. Ferreira, H. E. Haber, MM, O. Nachtmann, J. P. Silva, Int.J.Mod.Phys. A26 (2011)
A. Aranda, C. Bonilla, J. L. Diaz-Cruz, PLB 717 (2012)