CP-leaks in the real two-Higgs doublet model

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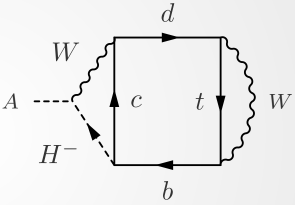


Motivation

- Mechanism of baryogenesis provides explanation for matter-anti-matter asymmetry
- Sakharovs criteria for baryogenesis:
  1. Baryon number violation
  2. C- and CP-violation
  3. Departure from the thermal equilibrium
- SM CP-violation not sufficient
- Interest in models with additional CP-violation, *e.g.* C2HDM

Extending the scalar sector of the SM

- Most general 2HDM scalar potential:

$$\begin{aligned} V_H \;=\;& m_{11}^2\Phi_1^\dagger\Phi_1 + m_{22}^2\Phi_2^\dagger\Phi_2 - \left[m_{12}^2\Phi_1^\dagger\Phi_2 + \mathrm{H.c.}\right] \\ &+ \tfrac{1}{2}\lambda_1(\Phi_1^\dagger\Phi_1)^2 + \tfrac{1}{2}\lambda_2(\Phi_2^\dagger\Phi_2)^2 + \lambda_3(\Phi_1^\dagger\Phi_1)(\Phi_2^\dagger\Phi_2) \\ &+ \lambda_4(\Phi_1^\dagger\Phi_2)(\Phi_2^\dagger\Phi_1) + \left[\tfrac{1}{2}\lambda_5(\Phi_1^\dagger\Phi_2)^2 + \mathrm{H.c.}\right] \\ &+ \left[\lambda_6(\Phi_1^\dagger\Phi_1)(\Phi_1^\dagger\Phi_2) + \lambda_7(\Phi_2^\dagger\Phi_2)(\Phi_1^\dagger\Phi_2) + \mathrm{H.c.}\right], \end{aligned}$$

  with $\{m_{11}^2, m_{22}^2, \lambda_1, \cdots, \lambda_4\} \in \mathbb{R}$ and $\{m_{12}^2, \lambda_5, \lambda_6, \lambda_7\} \in \mathbb{C}$
- Flavour changing neutral currents strongly constrained by exp.
  $\Rightarrow$ impose Z_2-symmetry ($\Phi_1 \to \Phi_1; \Phi_2 \to -\Phi_2$), provoking $m_{12}^2 = \lambda_6 = \lambda_7 = 0$, $\lambda_5 \in \mathbb{R}$ (via rephasing of Φ_2)
- Then introduce soft breaking complex $m_{12}^2 \neq 0$ for decoupling (can have heavy new scalars): C2HDM

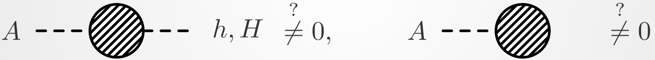
Real 2HDM

- Wider part of literature now makes the choice $\{m_{12}^2, \lambda_5\} \in \mathbb{R}$ by imposing CP-conservation in the scalar sector: **real 2HDM**
- Has CP-even physical scalars h, H and CP-odd pseudo scalar A
- Proceed to study effects of Z_2-symmetry in fermion sector and fit to experiment
- Need to accommodate CP-violating phase of CKM matrix

Main question of our work: **Are CP-conservation in the scalar and non-conservation in other sectors compatible?**

Potential inconsistency

- Strong breaking of CP via complex Yukawa couplings ($\rightarrow$ CKM-matrix)
- Note: has nothing to do with soft CP-violation via $\mathrm{Im}(m_{12}^2)$
- Loop corrections could translate this to scalar sector, *e.g.* produce scalar-pseudo scalar mixing or A-tadpole (real 2HDM has $t_A \equiv 0$)



- Expectation: lack of CP-violating counterterm $\delta\mathrm{Im}(m_{12}^2)$ in real 2HDM in order to renormalize such contributions
- Our finding: surprisingly difficult to show this inconsistency explicitly ($\Rightarrow$ rarely mentioned in the literature)
- [EPJC 81 6, 2021, arXiv:2103.05002]

Real 2HDM vs. C2HDM

- Can look at real 2HDM as limiting case of C2HDM, with $\mathrm{Im}(m_{12}^2) = 0$
  (or more specifically, with the rephasing invariant $\mathrm{Im}[\lambda_5^*(m_{12}^2)^2] = 0$)
- Corresponds to specific corner of C2HDM parameter space
- Even when tree-level parameters are set to zero, would still have the respective counterterms present
- This in contrast to the the real 2HDM as starting point with CP-conservation implied $\Rightarrow$ forbids counterterms

Some features of the real 2HDM

Potential:

$$
\begin{aligned}
V_r \quad = \quad & m_{11}^2 \Phi_1^\dagger \Phi_1 + m_{22}^2 \Phi_2^\dagger \Phi_2 - m_{12}^2 \left[\Phi_1^\dagger \Phi_2 + \Phi_2^\dagger \Phi_1\right] \\
& + \tfrac{1}{2}\lambda_1 (\Phi_1^\dagger \Phi_1)^2 + \tfrac{1}{2}\lambda_2 (\Phi_2^\dagger \Phi_2)^2 + \lambda_3 (\Phi_1^\dagger \Phi_1)(\Phi_2^\dagger \Phi_2) \\
& + \lambda_4 (\Phi_1^\dagger \Phi_2)(\Phi_2^\dagger \Phi_1) + \tfrac{1}{2}\lambda_5 \left[(\Phi_1^\dagger \Phi_2)^2 + (\Phi_2^\dagger \Phi_1)^2\right],
\end{aligned}
$$

Physical fields:

$$
\Phi_1 = \begin{pmatrix} c_\beta G^+ - s_\beta H^+ \\ \frac{1}{\sqrt{2}} \left[v c_\beta + (c_\alpha H - s_\alpha h) + i(c_\beta G^0 - s_\beta A)\right] \end{pmatrix},
$$

$$
\Phi_2 = \begin{pmatrix} s_\beta G^+ + c_\beta H^+ \\ \frac{1}{\sqrt{2}} \left[v s_\beta + (s_\alpha H + c_\alpha h) + i(s_\beta G^0 + c_\beta A)\right] \end{pmatrix}.
$$

- Inserting physical fields yields $t_A \equiv 0$, $t_{h/H} \neq 0$
- Now use vacuum conditions $t_{h/H} \stackrel{!}{=} 0$, solve for m_{11}^2, m_{22}^2 and insert back in

Some features of the real 2HDM

- This yields vanishing quadratic G^0G^0, G^+G^-, G^0A and $G^\pm H^\mp$ terms
  $\implies$ G^0 and G^+ are Goldstone bosons
- A and H^+ are already mass eigenstates
  $\implies$ No mixing of scalars and pseudo-scalars
- There are quadratic terms for HH, hh, and hH
- Use α to get rid of hH-mixing
  $\implies$ h and H are the physical neutral scalar fields
- real 2HDM parameters: $\{\alpha, \beta, m_H, m_A, m_{H^\pm}, \mathrm{Re}\left(m_{12}^2\right)\}$

Jarlskog invariant

How does CP-violation come into play?

- Need to couple scalars of real 2HDM to SM-fermions
- CP-violation then arises from complex Yukawa couplings
- In mass basis, this introduces a complex phase in V_{CKM}
- Can shift it around by quark field redefinitions
- Only rephasing invariant quantity is **Jarlskog invariant:**

$$I^{\alpha i}_{\beta j} = \operatorname{Im}\left(V_{\alpha i} V_{\beta j} V^*_{\alpha j} V^*_{\beta i}\right) = J \sum_{\gamma, k} \epsilon_{\alpha\beta\gamma}\, \epsilon_{ijk}\,,$$

- Example: $I^{11}_{22} = I^{ud}_{cs} = J$
- Diagrams to generate J need at least four V_{CKM}-factors

$\implies$ high order effect

Toy model

Build toy model with same pathology as real 2HDM, but @one-loop:

- Two neutral scalar singlets:

$$\begin{aligned} -\mathcal{L}_\Phi &= \mu_1^2 \Phi_1^* \Phi_1 + \mu_2^2 \Phi_2^* \Phi_2 + \mu^2 \Phi_1^* \Phi_2 + (\mu^2)^* \, \Phi_2^* \Phi_1 \\ &+ \lambda_1 (\Phi_1^* \Phi_1)^2 + \lambda_2 (\Phi_2^* \Phi_2)^2 + \lambda_{34} \, \Phi_1^* \Phi_1 \Phi_2^* \Phi_2 \\ &+ \lambda_5 (\Phi_1^* \Phi_2)^2 + \lambda_5{}^* (\Phi_2^* \Phi_1)^2, \end{aligned}$$

- Two charged scalar singlets:

$$\begin{aligned} -\mathcal{L}_\chi &= m_L^2 \, \chi_L \chi_L^* + m_R^2 \, \chi_R \chi_R^* + \rho_1 (\chi_L^* \chi_L)^2 \\ &+ \rho_2 (\chi_R^* \chi_R)^2 + \rho_{34} \, \chi_L^* \chi_L \chi_R^* \chi_R, \end{aligned}$$

- Yukawa-coupling equivalent:

$$\begin{aligned} -\mathcal{L}_{\Phi\chi} &= f_1 \, \Phi_1 \chi_L \chi_R^* + f_1^* \, \Phi_1^* \chi_L^* \chi_R + f_2 \, \Phi_2 \chi_L \chi_R^* \\ &+ f_2^* \, \Phi_2^* \chi_L^* \chi_R + g_1 \, \Phi_1^* \Phi_1 \chi_L^* \chi_L + g_2 \, \Phi_2^* \Phi_2 \chi_L^* \chi_L \\ &+ g_3 \, \Phi_1^* \Phi_1 \chi_R^* \chi_R + g_4 \, \Phi_2^* \Phi_2 \chi_R^* \chi_R. \end{aligned}$$

Inspired by [A. Pilaftsis; '98]

Toy model II

- Impose (softly broken) discrete symmetry D (equivalent of Z_2):

$$\Phi_1 \stackrel{D}{\rightarrow} -\Phi_1, \;\; \Phi_2 \stackrel{D}{\rightarrow} \Phi_2, \;\; \chi_L \stackrel{D}{\rightarrow} -\chi_L, \;\; \chi_R \stackrel{D}{\rightarrow} \chi_R.$$

- Conditions for CP-conserving potential:

$$\mathrm{Im}\left[\mu^2 f_1 f_2^*\right] = \mathrm{Im}\Big[\lambda_5 f_1^2 (f_2^*)^2\Big] = \mathrm{Im}\Big[\lambda_5^* (\mu^2)^2\Big] = 0.$$

- Parameterize neutral scalars as

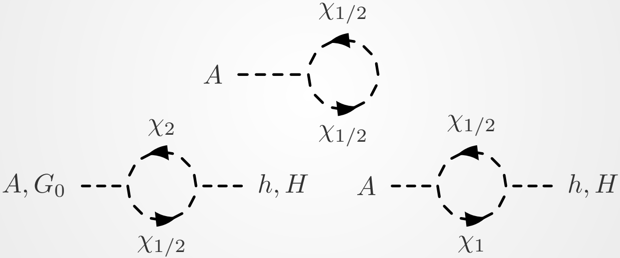
$$\begin{aligned} \Phi_1 &= \frac{1}{\sqrt{2}}\left(v_1 + c_\theta h - s_\theta H + i c_\beta G^0 - i s_\beta A\right), \\ \Phi_2 &= \frac{1}{\sqrt{2}}\left(v_2 + s_\theta h + c_\theta H + i s_\beta G^0 + i c_\beta A\right), \end{aligned}$$

- Yields CP-even and -odd fields as in real 2HDM

Toy model pathology

Toy model becomes non-renormalizable

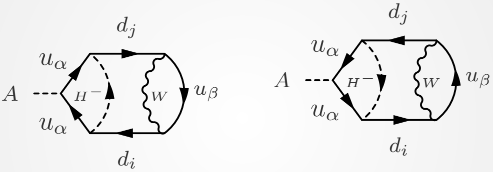
- Examples of divergent contributions @one-loop:



- When imposing $\mathrm{Im}\mu^2 = \mathrm{Im}\lambda_5 = 0$ (as in the real 2HDM) to **define** the model, one lacks the corresponding counterterms
- CP-conserving potential is not renormalizable

Jarlskog contributions to real 2HDM

- Expect same pathology in real 2HDM via the appearance of J
- Need a closed fermion loop with ≥ 4 CKM-factors $\Longrightarrow \geq 3$-loop
- Example of diagram pair where J factorizes:



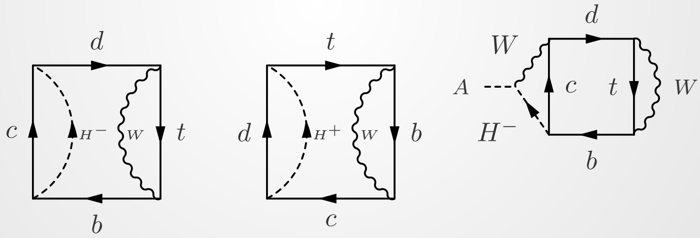
- Goal: determine leading divergence of CP-violating contributions
- Collect all diagrams containing some fixed set of quarks which could yield J, *e.g.* :

$$(iT_A)^{cd}_{tb} = \Bigg($$


$$\Bigg)^{cd}_{tb}$$

Details of calculation I

- Generate all three-loop tadpoles containing c, d, t, b (`FeynMaster`, `QGRAF` vs. `FeynArts`)
- Non-trivial check of `FeynArts` @3-loop!
- Amplitude manipulations with `FeynCalc`
- Eventually: 208 non-zero Jarlskog-diagrams in three categories:



- Caveat: we used naive dimensional regularisation (expectation: γ_5-scheme does not affect leading pole)

Loop integrals

- Use FeynCalc function `ApartFF` for partial fractioning:

$$\frac{q_1 \cdot q_2}{q_1^2[q_2^2 - m^2][(q_1 - q_2)^2 - m^2]} = \frac{1}{2q_1^2[(q_1 - q_2)^2 - m^2]} + \frac{1}{2[q_2^2 - m^2][(q_1 - q_2)^2 - m^2]} - \frac{1}{2q_1^2[q_2^2 - m^2]}$$

- Still can not get rid of “problematic” integrals of type:

$$U_5^{(1,2)}(m_1, m_2, m_3, m_4, m_5) = i\frac{e^{3\gamma_E \varepsilon}}{\pi^{3d/2}} \int \mathrm{d}^d q_1 \, \mathrm{d}^d q_2 \, \mathrm{d}^d q_3 \frac{q_1 \cdot q_2}{(q_1^2 - m_1^2)(q_2^2 - m_2^2)(q_3^2 - m_3^2)} \times \frac{1}{\left((q_1 - q_3)^2 - m_4^2\right)\left((q_2 - q_3)^2 - m_5^2\right)}.$$

Integration by parts reduction

- Can use integration by parts (IBP) method to determine "problematic" integral
- Simplest example ($a > 1$):

$$F(a) = \int \mathrm{d}k^d \frac{1}{(k^2 - m^2)^a}$$

- Use the fact that:

$$0 = \int \mathrm{d}k^d \frac{\partial}{\partial k_\mu} \frac{k_\mu}{(k^2 - m^2)^a} = \int \mathrm{d}k^d \left[\frac{d}{(k^2 - m^2)^a} - 2a \frac{k^2 - m^2 + m^2}{(k^2 - m^2)^{a+1}} \right]$$

- Leads to recursion relation

$$F(a) = \frac{d - 2(a+1)}{2(a+1)m^2} F(a-1)$$

- `FIRE` uses this method to break down loop integrals into set of master integrals (here: $F(1)$)
- Can decompose $U_5^{(1,2)}(m_1, m_2, m_3, m_4, m_5)$ into scalar integrals

Numerical result

- Decompose all relevant amplitudes
- Numerical evaluation using `FIESTA`
- Yields non-vanishing leading pole for $(T_A)^{cd}_{tb}$

$-i\Big($ 

$\Big)^{cd}_{tb} = \frac{2392.6(\mathrm{GeV})^3}{\varepsilon^3} \times J + \mathcal{O}(\varepsilon^{-2})$

- Non-trivial test of `FIESTA` @3-loop with up to five different propagator masses

End of the story?

Analytic result

- Can use analytic results for leading ε-poles with the same decomposition as for numerical evaluation
- Eventually: remarkably simple result for full tadpole

$$-i\Big($$


$$\Big)^{\alpha i}_{\beta j} = \frac{1}{\varepsilon^3} \frac{g^5}{8 m_W^3 s_\beta c_\beta} M^{\alpha i}_{\beta j}\, I^{\alpha i}_{\beta j} + \mathcal{O}(\varepsilon^{-2}),$$

with $M^{\alpha i}_{\beta j} = (m^2_{u_\alpha} - m^2_{u_\beta})(m^2_{d_i} - m^2_{d_j})(m^2_{u_\alpha} - m^2_{d_i} + m^2_{u_\beta} - m^2_{d_j})$

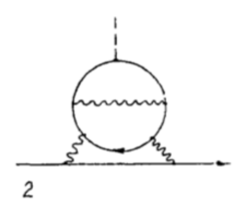
- **But**: contribution vanishes when summing over all families!

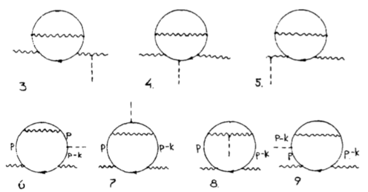
$$\sum_{\alpha<\beta} \sum_{i<j} (T_A)^{\alpha i}_{\beta j} = \mathcal{O}(\varepsilon^{-2})$$

- Note: does **not** vanish due to anti-symmetry in $\alpha \leftrightarrow \beta$ or $i \leftrightarrow j$
  $\Longrightarrow$ Some unknown (family-) symmetry at play?

Similarities to electric dipole moment (EDM)

- SM electron- and W-EDM vanish up to 3- and 2-loop respectively





- [Khriplovich, Pospelov; '91]: *"We cannot get rid of the feeling that this simple result (...) should have a simple transparent explanation. Unfortunately, we have not been able to find it."*

Conclusions

Q: Does the CKM-induced CP-violation leak into the CP-conserving sector of the real 2HDM?

A: **Probably yes, but only @ $> 3 - loop$, in sub-leading divergencies or finite contributions**

- Toy model exhibits expected pathology of real 2HDM
- Non-trivial checks:
  - `FeynArts` works @ 3-loop (tadpoles)
  - `Fiesta` works @3-loop with up to five different mass scales
- Individual CP-violating 3-loop contributions are non-vanishing
- Leading divergence vanishes when summing over all families
- **How to solve the riddle of "accidental" cancellations of leading poles?**

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- **How to solve the riddle of "accidental" cancellations of leading poles?**

Thank you!

Backup slides