Unitarity bounds for all symmetry-constrained 3HDMs

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Unitarity as a property of computability

- First championed by Lee, Quigg and Thacker, perturbative unitarity implies we trust both **perturbativity** and **unitarity**.
- It is usually done with **scattering matrices**, where we consider CM energy much larger than the masses of the particles.
- It is often a **constraint** in phenomenology.
- It is calculated with the **diagonalization** of scattering matrices.
- It is a model-dependent procedure.

An optimized method

Theorem:

Let A be a hermitian matrix. Then the following statements are equivalent:

1) The eigenvalues of A are bounded as $|\lambda_i| < c$;

2) The determinants of all the upper left k-by-k submatrices of $A + cI$ and $cI - A$ are positive;

3) The leading principal minors of $A + cI$ and $cI - A$ are positive.

A straightforward remark is that it is necessary that the diagonal elements of a scattering matrix are also bounded: $|a_{ii}| < c$

An optimized method - NHDM

A consequence of the diagonal conditions is that for any multi-Higgs doublet model we have:

$$|\lambda_{ii,ii}| < \frac{4\pi}{3}\,,$$

$$|\lambda_{ii,jj}| < 4\pi\,,$$

$$|\lambda_{ii,jj} + 2\lambda_{ij,ji}| < 4\pi\,.$$

These conditions are valid regardless of the symmetry imposed.

An optimized method - NHDM

The proposed procedure is the following:

1) Sample a very large number of random Hermitian matrices by making them check the conditions on the diagonal;

2) Loop through the random Hermitian matrices calculating the 2-by-2 determinants;

3) Check positivity of the determinants;

4) Trim the remaining Hermitian matrices;

5) Go to step 2, but now compute the 3-by-3 determinants until we reach the full n-by-n determinants.

An optimized method - advantages

New method:

- Always analytical;
- Numerically stable;
- Always polynomial;
- Faster than diagonalization.

Diagonalization:

- Sometimes purely numerical;
- Numerically unstable;
- Maybe nth roots.

The symmetries of the 3HDM

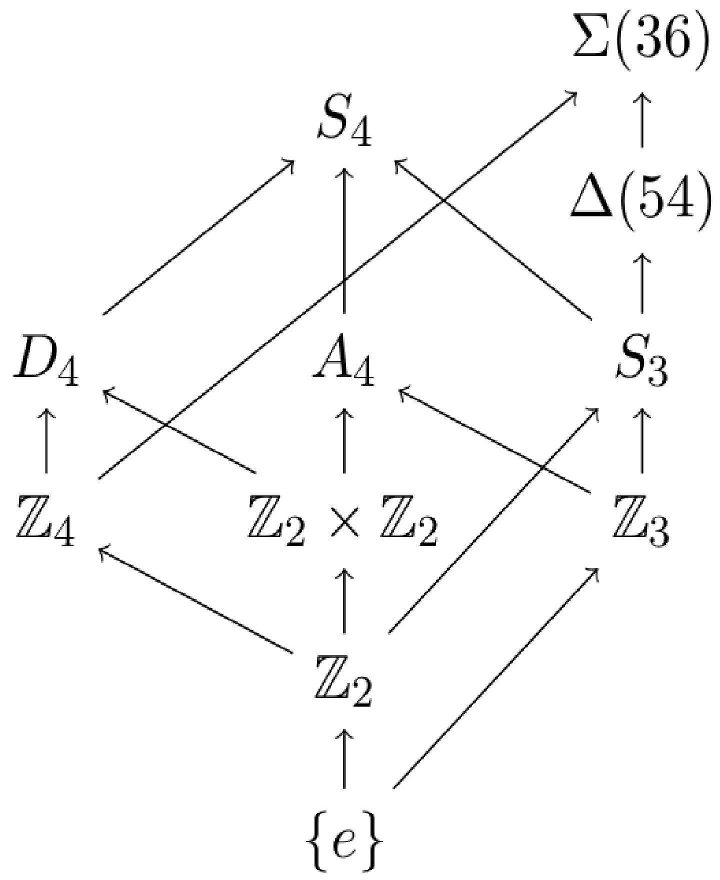

Figure 1. Tree of finite realizable groups of Higgs-family transformations in 3HDM.

The symmetries of the 3HDM

Discrete symmetries in the 3HDM	
Unitary	$\mathbb{Z}_2, \mathbb{Z}_3, \mathbb{Z}_2 \times \mathbb{Z}_2, \mathbb{Z}_4, S_3, D_4, A_4, S_4, \Delta(54), \Sigma(36)$
Anti-unitary (GCP)	$\mathbb{Z}_2^{(\mathrm{CP})}, \mathbb{Z}_2 \times \mathbb{Z}_2^{(\mathrm{CP})}, \mathbb{Z}_2 \times \mathbb{Z}_2 \times \mathbb{Z}_2^{(\mathrm{CP})}, CP4, \mathbb{Z}_3 \rtimes \mathbb{Z}_2^{(\mathrm{CP})},$ $S_3 \times \mathbb{Z}_2^{(\mathrm{CP})}, \Delta(54) \rtimes \mathbb{Z}_2^{(\mathrm{CP})}$

Table 1. Full list of discrete symmetries in the 3HDM, where $\mathbb{Z}_2^{(\mathrm{CP})}$ stands for the usual CP.

Continuous symmetries in the 3HDM	
Abelian	$\mathrm{U}(1)_1, \mathrm{U}(1)_2, \mathrm{U}(1)_2 \times \mathbb{Z}_2, \mathrm{U}(1) \times \mathrm{U}(1)$
Non-abelian	$\mathrm{U}(2), \mathrm{O}(2), \mathrm{SU}(3), \mathrm{SO}(3)$

Table 2. List of continuous symmetries in the 3HDM.

The 3HDM with S_3 symmetry

$$
\begin{aligned}
V_{S_3} =& r_1 \left[(\phi_1^\dagger \phi_1)^2 + (\phi_2^\dagger \phi_2)^2 \right] + r_3 |\phi_3|^4 + 2 r_4 (\phi_1^\dagger \phi_1)(\phi_2^\dagger \phi_2) \\
& + 2 r_5 (\phi_1^\dagger \phi_1 + \phi_2^\dagger \phi_2)(\phi_3^\dagger \phi_3) + 2 r_7 |\phi_1^\dagger \phi_2|^2 + 2 r_8 \left[|\phi_1^\dagger \phi_3|^2 + |\phi_2^\dagger \phi_3|^2 \right] \\
& + \left[2 c_{11} (\phi_1^\dagger \phi_3)(\phi_2^\dagger \phi_3) + 2 c_{12} \left[(\phi_1^\dagger \phi_2)(\phi_3^\dagger \phi_2) + (\phi_2^\dagger \phi_1)(\phi_3^\dagger \phi_1) \right] + h.c. \right]
\end{aligned}
$$

The 3HDM with S_3 symmetry

One of the many scattering matrices we compute is: $M_0^+ \supset A = 2 \begin{pmatrix} r_4 & c_{12} & c_{12}^* \\ c_{12}^* & r_5 & c_{11} \\ c_{12} & c_{11}^* & r_5 \end{pmatrix}$

And thus, we compute the conditions (and similarly for $8\pi I$ - A):

$$D_1(A + 8\pi I) > 0 \Rightarrow r_4 > -4\pi\,,$$

$$D_2(A + 8\pi I) > 0 \Rightarrow (r_4 + 4\pi)\,(r_5 + 4\pi) - |c_{12}|^2 > 0\,,$$

$$\begin{aligned} D_3(A + 8\pi I) > 0 \Rightarrow\, & 2\Re\left[c_{11}c_{12}^2\right] + (r_1 + 4\pi)\,(r_5 + 4\pi)^2 \\ & - (r_4 + 4\pi)\,|c_{11}|^2 - 2\,(r_5 + 4\pi)\,|c_{12}|^2 > 0 \\ & \Leftrightarrow \det\,(A + 8\pi I) > 0\,. \end{aligned}$$

Conclusion

- We computed perturbative unitarity for all symmetry-constrained 3HDMs;
- We presented a faster and more stable method than diagonalization;
- Thus, we can probe more points in the parameter space;
- We can now define perturbative unitarity analytically for any given model.