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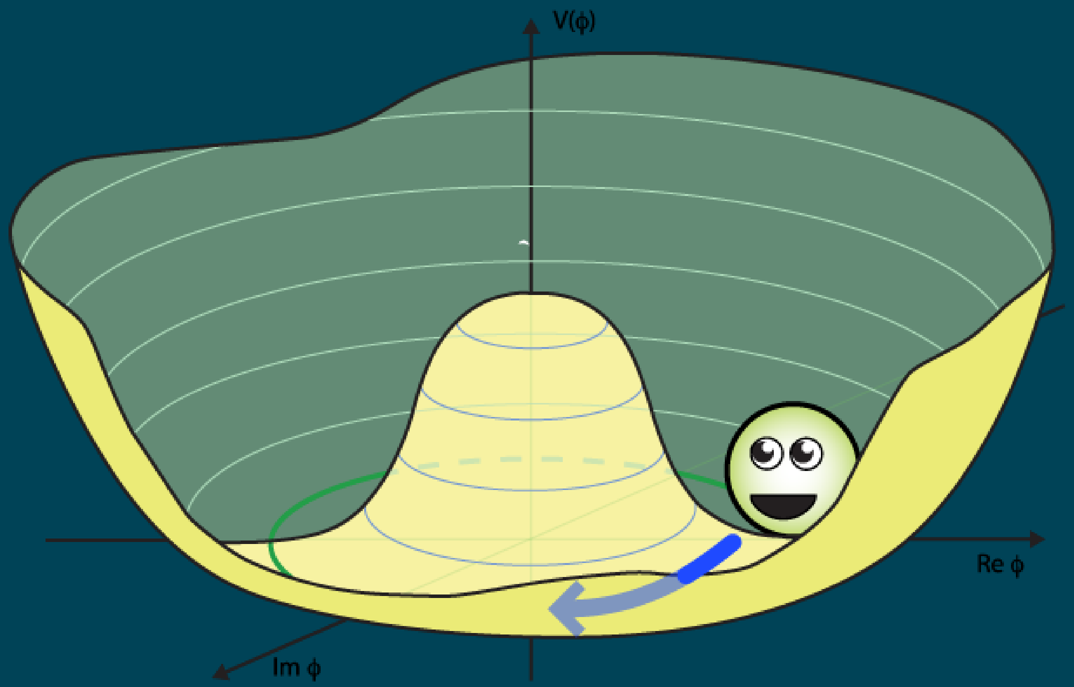
Distinguishing symmetries of the 2HDM in terms of physical observables

Talk given at workshop on Multi-Higgs models, august 2022



Odd Magne Øgreid

Based on work with Pedro Ferreira, Bohdan Grzadkowski and Per Osland.

JHEP 02 (2021) 196 and unpublished manuscript.

The zoo of 2HDM-models:

- 2HDMs are simple extension of the SM that allows for **CP violation** or provides the possibility for **Dark Matter**.
- Within the framework of 2HDM, many different models are possible.
- Imposing symmetries on the 2HDM-potential restricts the number of parameters with physical consequences.
- Six allowed symmetries (not counting custodial symmetries).
- Symmetries can be **unbroken**, **broken softly** or **broken spontaneously** (3 choices).
- Naively this yields 6 x 3 =18 different models one can consider, each with unique physical consequences. Plus, there may be more than one way to softly break a symmetry.
- What are the physical signatures of each of these models?

The 2HDM potential

$$V = V_2 + V_4$$

$$V_2 = -\frac{1}{2}\left\{m_{11}^2\Phi_1^\dagger\Phi_1 + m_{22}^2\Phi_2^\dagger\Phi_2 + \left[\underline{m_{12}^2}\Phi_1^\dagger\Phi_2 + \text{h.c.}\right]\right\}$$

$$V_4 = \frac{\lambda_1}{2}(\Phi_1^\dagger\Phi_1)^2 + \frac{\lambda_2}{2}(\Phi_2^\dagger\Phi_2)^2 + \lambda_3(\Phi_1^\dagger\Phi_1)(\Phi_2^\dagger\Phi_2) + \lambda_4(\Phi_1^\dagger\Phi_2)(\Phi_2^\dagger\Phi_1)$$
$$+\frac{1}{2}\left[\underline{\lambda_5}(\Phi_1^\dagger\Phi_2)^2 + \text{h.c.}\right] + \left\{\left[\underline{\lambda_6}(\Phi_1^\dagger\Phi_1) + \underline{\lambda_7}(\Phi_2^\dagger\Phi_2)\right](\Phi_1^\dagger\Phi_2) + \text{h.c.}\right\}$$

- 14 parameters (reducible to 11)
- 4 **complex** parameters

The freedom to choose a basis

- Initial expression of Lagrangian is defined with respect to doublets Φ_1 and Φ_2.
- We may rotate to another **basis** by the following transformation

$$\bar{\Phi}_i = U_{ij}\Phi_j$$

  where *U* is any U(2) matrix.
- Potential parameters will change under change of basis.
- Physics **must remain unchanged** if we change basis.
- Observables cannot depend on choice of basis – they should be basis-independent, i.e. **invariant** under a change of basis.
- Invariant descriptions of the physical properties of the 2HDM is important and necessary.

Higgs-Family-symmetries:
Z_2, U(1) and SO(3) symmetries of the 2HDM potential

If a basis exists so that the 2HDM potential is invariant under the transformation

$$\Phi_1 \to \Phi_1, \qquad \Phi_2 \to -\Phi_2$$

the 2HDM potential is Z_2-symmetric.

If a basis exists so that the 2HDM potential is invariant under the transformation

$$\Phi_1 \to e^{-i\theta}\Phi_1, \qquad \Phi_2 \to e^{i\theta}\Phi_2$$

the 2HDM potential is U(1)-symmetric.

If a basis exists so that the 2HDM potential is invariant under the transformation

$$\begin{pmatrix} \Phi_1 \\ \Phi_2 \end{pmatrix} \to \begin{pmatrix} e^{-i\alpha}\cos\theta & e^{-i\beta}\sin\theta \\ -e^{i\beta}\sin\theta & e^{i\alpha}\cos\theta \end{pmatrix} \begin{pmatrix} \Phi_1 \\ \Phi_2 \end{pmatrix}$$

the 2HDM potential is SO(3)-symmetric.

The challenging part is to check
if such a basis exists.

The symmetry may be hidden but can become "visible" after rotating to another basis. Expressing symmetries in terms of physical observables (basis invariants) makes this hunt for a symmetry-basis redundant.

CP-symmetries: CP1, CP2 and CP3 of the 2HDM potential

Whenever there exists a basis in which the 2HDM potential is invariant under the transformation

$$\Phi_i \to X_{ij} \Phi_j^*$$

the 2HDM is CP-symmetric, or **CP-conserving**.

X_{ij} is a U(2)-matrix.

- There are three different classes of CP-symmetries, according to the form the U(2) matrix X_{ij} can have.
- CP1: $X_{ij} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$
- CP2: $X_{ij} = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$
- CP3: $X_{ij} = \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix}$

$$0 < \theta < \pi/2$$

Each of these six symmetries imply that *there exists a basis* in which the parameters of the potential satisfy the constraints of the table below. (For some symmetries the potential can be further simplified by basis changes.)

Still, the challenging part is to check ***if such a basis exists***.

NOTE: We have not made any assumptions about the vacuum, so the symmetries may or may not be broken by the vacuum (spontaneously broken).

	V_2			V_4						
Symmetry	m_{11}^2	m_{22}^2	m_{12}^2	λ_1	λ_2	λ_3	λ_4	λ_5	λ_6	λ_7
CP1			real					real	real	real
Z_2			0						0	0
U(1)			0					0	0	0
CP2		m_{11}^2	0		λ_1					$-\lambda_6$
CP3		m_{11}^2	0		λ_1			$\lambda_1 - \lambda_3 - \lambda_4$ (real)	0	0
SO(3)		m_{11}^2	0		λ_1		$\lambda_1 - \lambda_3$	0	0	0

The 2HDM vacuum and the three ways to impose a symmetry

Most general form of charge-conserving vacuum: $\langle\Phi_1\rangle = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 \\ v_1 \end{pmatrix}, \quad \langle\Phi_2\rangle = \frac{e^{i\xi}}{\sqrt{2}}\begin{pmatrix} 0 \\ v_2 \end{pmatrix}$

	Vacuum	V_2	V_4
Unbroken symmetry	Invariant	Invariant	Invariant
Spontaneously broken symmetry	Broken	Invariant	Invariant
Softly broken symmetry	N/A	Broken	Invariant

If a basis exists… Time to get rid of this phrase!!!

The physical parameter set $\mathcal{P}$ and counting of parameters.

- Potential has initially 14 parameters
- Exploit the freedom to change basis and reduce to 11 independent parameters.
- Traditional approach:
  Work out masses and couplings expressed in terms of the initial 14 (or 11) parameters of the potential.
- Our approach:
  Work the other way around. Pick a set of 11 independent physical masses and couplings (all invariants) and express the initial 14 parameters in terms of these
- We now choose our set of 11 independent parameters to consist of:
  - Four squared masses
  - Three gauge couplings
  - Four scalar couplings

$$\mathcal{P} \equiv \{M_{H^\pm}^2, M_1^2, M_2^2, M_3^2, e_1, e_2, e_3, q_1, q_2, q_3, q\}$$

- All observables from the potential (invariants) expressible through these.
- All trilinear and quadrilinear scalar couplings expressible through these.

$$\begin{aligned} e_i &\equiv \frac{2}{g^2}\text{Coefficient}(\mathcal{L}, H_i W^- W^+) \\ q_i &\equiv \text{Coefficient}(V, H_i H^- H^+) \\ q &\equiv \text{Coefficient}(V, H^- H^- H^+ H^+). \end{aligned}$$

Satisfying: $v^2 = e_1^2 + e_2^2 + e_3^2$

Description of translation process:

Ogreid: PoS CORFU2017 (2018) 065

Remaining scalar couplings expressible in terms of $\mathcal{P}$:

Grzadkowski, Haber, Ogreid & Osland: JHEP 12 (2018) 056

CP1

Model	Invariant parts	Conditions
Softly broken CP1	V_4	$I_{6Z} = 0$
Spontaneously broken CP1	$V=V_2+V_4$	$I_{Y3Z} = I_{2Y2Z} = I_{3Y3Z} = I_{6Z} = 0$
Unbroken CP1 (CP conservation)	$V=V_2+V_4$ and vacuum	$\mathrm{Im}\, J_1 = \mathrm{Im}\, J_2 = \mathrm{Im}\, J_{30} = 0$

For definitions and translations of invariants see

Lavoura & Silva: Phys. Rev. D 50, 4619 (1994)
Gunion & Haber: Phys. Rev. D 72, 095002 (2005)
Grzadkowski, Ogreid &Osland: JHEP11 (2014) 084
Grzadkowski, Ogreid & Osland: Phys. Rev. D 94, 115002 (2016)

Softly broken CP1 - (I_{6Z} = 0) - two equivalent expressions

Expression showing that unbroken CP leads to I_{6Z} = 0.

$$I_{6Z} = c_1 \mathrm{Im} J_1 + c_2 \mathrm{Im} J_2 + c_3 \mathrm{Im} J_{30} + c_4 \mathrm{Im} J_{11} = 0$$

(Unbroken CP requires all Im J_i=0.)

Here, c_i are complicated expressions in the parameters of $\mathcal{P}$.

Expression showing explicitly that CP conservation leads to I_{6Z} = 0

$$\begin{aligned} I_{6Z} \quad = \quad & c_{21}(\Delta m_+)^2 \Delta q + c_{20}(\Delta m_+)^2 + c_{12}\Delta m_+ (\Delta q)^2 + c_{11}\Delta m_+ \Delta q \\ & + c_{10}\Delta m_+ + c_{03}(\Delta q)^3 + c_{02}(\Delta q)^2 + c_{01}\Delta q \end{aligned}$$

(Spontaneously broken CP requires

$\Delta m_+ = \Delta q = 0$)

Here, c_{ij} are complicated expressions in the parameters of $\mathcal{P}$.

Softly broken CP1

- One constraint – reducing the number of free parameters to 10.
- I_{6Z} homogeneous polynomial of order 6 in $\{M_{H\pm}^2, M_1^2, M_2^2, M_3^2, q_1, q_2, q_3, q\}$ with $\{e_1, e_2, e_3\}$ appearing in the “coefficients” of this polynomial.
- Model name: Case SOFT-CP1

Spontaneously broken CP1 $(I_{Y3Z} = I_{2Y2Z} = I_{3Y3Z} = I_{6Z} = 0)$

- All these invariants also vanish whenever CP1 is unbroken – assuming this is not the case leads to two constraints:

$$\frac{M_{H^\pm}^2}{v^2} = \tilde{m}_+ \equiv \frac{e_1 q_1 M_2^2 M_3^2 + e_2 q_2 M_1^2 M_3^2 + e_3 q_3 M_1^2 M_2^2 - M_1^2 M_2^2 M_3^2}{2(e_1^2 M_2^2 M_3^2 + e_2^2 M_3^2 M_1^2 + e_3^2 M_1^2 M_2^2)},$$

$$q = \tilde{q} \equiv \frac{(e_2 q_3 - e_3 q_2)^2 M_1^2 + (e_3 q_1 - e_1 q_3)^2 M_2^2 + (e_1 q_2 - e_2 q_1)^2 M_3^2 + M_1^2 M_2^2 M_3^2}{2(e_1^2 M_2^2 M_3^2 + e_2^2 M_3^2 M_1^2 + e_3^2 M_1^2 M_2^2)}$$

or more compactly

$$\Delta m_+ \equiv \frac{M_{H^\pm}^2}{v^2} - \tilde{m}_+ = 0, \quad \Delta q \equiv q - \tilde{q} = 0$$

- Two constraints - model has 9 parameters
- Model name: Case D

Unbroken CP1 – CP conservation (Im J_1 = Im J_2 = Im J_{30} = 0)

- If one of the following physical configurations occur, then we have CP conservation.

$$\textbf{Case A:} \quad M_1 = M_2 = M_3.$$

$$\textbf{Case B:} \quad M_i = M_j, \quad (e_j q_i - e_i q_j) = 0.$$

$$\textbf{Case C:} \quad e_k = q_k = 0.$$

How to interpret these three different cases of unbroken CP?

Unbroken CP1 – Case C

$$\textbf{Case C:} \quad e_k = q_k = 0.$$

- Two constraints – leaving us with 9 parameters
- Couplings $H_k W^+ W^-$ and $H_k H^+ H^-$ vanish.
- This implies that couplings $H_k ZZ$ *and* $H_i H_j Z$ also vanish.
- One of the three neutral scalars, H_k does not couple to CP-even pairs. One mass eigenstate H_k is a pseudoscalar, hence CP-odd. The two remaining mass eigenstates are CP-even.

Unbroken CP1 – Case B

$$\textbf{Case B:}\quad M_1 = M_2, \qquad (e_2 q_1 - e_1 q_2) = 0$$

- Case with mass degeneracy between H_1 and H_2.
- One can then form new states

$$\begin{pmatrix} \bar{H}_1 \\ \bar{H}_2 \end{pmatrix} = \begin{pmatrix} c_\alpha & s_\alpha \\ -s_\alpha & c_\alpha \end{pmatrix} \begin{pmatrix} H_1 \\ H_2 \end{pmatrix}$$

- The new states are just as physical acceptable as the old states, i.e. they must describe the same physics.

Unbroken CP1 – Case B

$$\textbf{Case B:} \quad M_1 = M_2, \quad (e_2 q_1 - e_1 q_2) = 0$$

- Working out the couplings of the new states, we find

$$\bar{e}_1 = \bar{e}_1(\alpha), \quad \bar{e}_2 = \bar{e}_2(\alpha),$$
$$\bar{q}_1 = \bar{q}_1(\alpha), \quad \bar{q}_2 = \bar{q}_2(\alpha),$$

- The combination $(\bar{e}_2 \bar{q}_1 - \bar{e}_1 \bar{q}_2) = 0$ still holds for any value of *α.*
- Can physics depend upon an arbitrary angle *α* ? **CLEARLY NOT!**
- The mass degeneracy makes these couplings unphysical in the sense that they cannot be measured themselves. Only combinations that are independent of *α* can be measured.
- Examples: $(\bar{e}_1 \bar{q}_1 + \bar{e}_2 \bar{q}_2), \quad (\bar{e}_1^2 + \bar{e}_2^2), \quad (\bar{q}_1^2 + \bar{q}_2^2)$

Unbroken CP1 – Case B

$$\textbf{Case B:} \quad M_1 = M_2, \quad (e_2 q_1 - e_1 q_2) = 0$$

- In processes with external H_1 and H_2 one must sum corresponding squares of amplitudes (no interference).
- Consider $H^+H^- \rightarrow H_{\{1,2\}}$ (external). The sum of squared amplitudes becomes proportional to

  $(\bar{q}_1^2 + \bar{q}_2^2)$, which is physical since it is independent of *α.*

- In processes with virtual H_1 and H_2 summation should be done at the level of amplitudes.
- Consider $W^+W^- \rightarrow H_{\{1,2,3\}}$ (internal) $\rightarrow H^+H^-$. The amplitude becomes proportional to

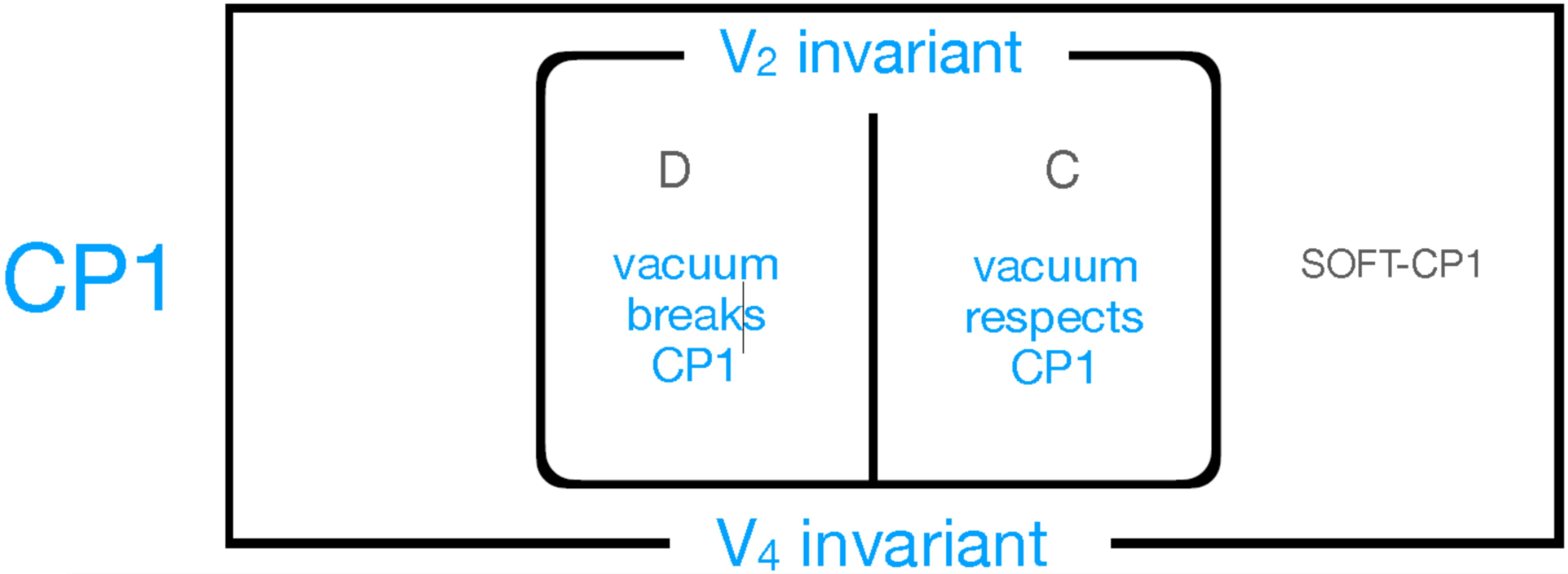
  $(\bar{e}_1 \bar{q}_1 + \bar{e}_2 \bar{q}_2 + e_3 q_3)$, which is physical since it is independent of *α.*

Unbroken CP1 – Case B and Case A

$$\textbf{Case B:} \quad M_1 = M_2, \quad (e_2 q_1 - e_1 q_2) = 0$$

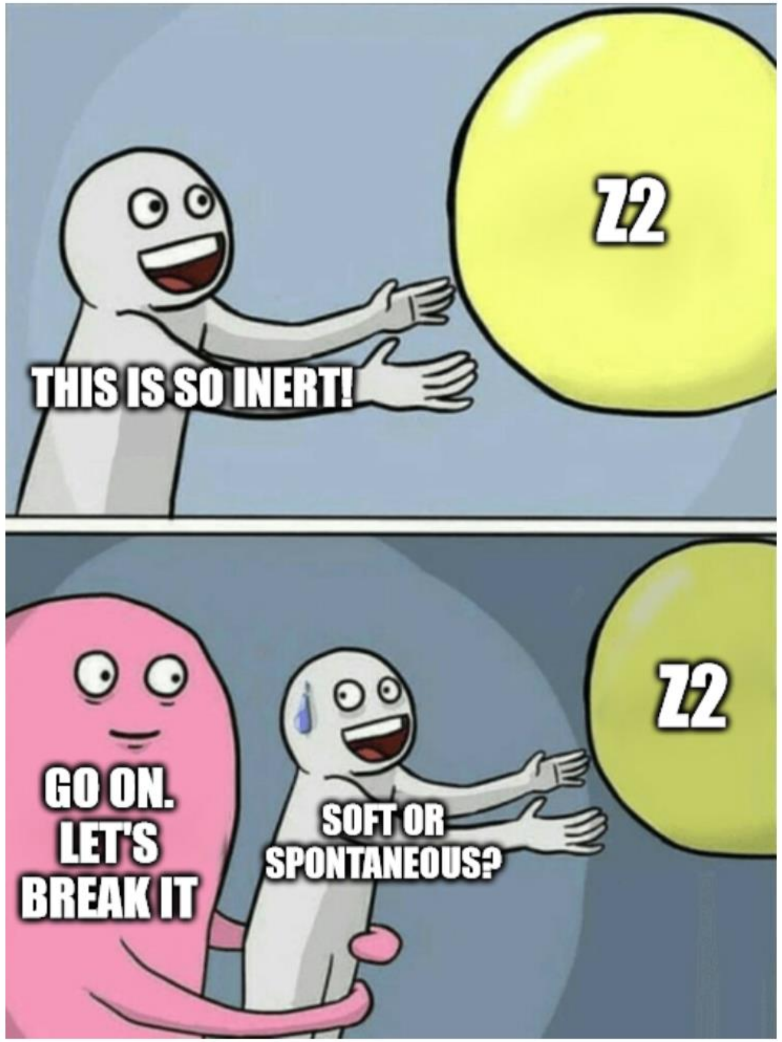
- Mass degeneracy introduces an arbitrary angle in the neutral-sector rotation matrix that makes couplings dependent upon an unphysical angle *α.*
- Picking a particular value of *α* does not make the couplings physical, but one can pick an angle such that $\bar{e}_2 = \bar{q}_2 = 0$ (or equivalently $\bar{e}_1 = \bar{q}_1 = 0$). That is similar to Case C with the mass degeneracy in addition.
- Moreover, mass degeneracy will be lost at one-loop level. It is not RG-stable. Case B must necessarily “migrate” into Case C at one-loop level.
- Similar arguments apply for $\textbf{Case A:} \quad M_1 = M_2 = M_3$
- Discarding RG-unstable cases, we are only left with $\textbf{Case C:} \quad e_k = q_k = 0$ of unbroken CP1.

CP1 - Overview



Z_2

Model	Invariant parts	Conditions
Softly broken Z_2	V_4	$[Z^{(1)}, Z^{(11)}] = 0$
Spontaneously broken Z_2 (and unbroken Z_2)	$V=V_2+V_4$ (and vacuum)	$[Z^{(1)}, Z^{(11)}] = [Z^{(1)}, Y] = [Y^{(1)}, Y] = 0$



For definitions of commutators see

Davidson & Haber: Phys. Rev. D 72, 035004 (2005)

Softly broken Z_2 when m_{12}^2 is complex (C2HDM)

$$d_{ijk} = \frac{q_1^i M_1^{2j} e_1^k + q_2^i M_2^{2j} e_2^k + q_3^i M_3^{2j} e_3^k}{v^{i+2j+k}}$$

Case SOFT-Z2-X:

$$q = d_{010} - \frac{1}{2} d_{012} - d_{101} - \frac{4\,\mathrm{Im}J_{11} + \mathrm{Im}J_2 + 2\mathrm{Im}J_{30}}{2\mathrm{Im}J_1},$$

$$\begin{aligned}
&2\mathrm{Im}J_1[2(d_{012} + d_{101} - d_{010})\mathrm{Im}J_1 + 4\,\mathrm{Im}J_{11} + \mathrm{Im}J_2 + 2\,\mathrm{Im}J_{30}]M_{H^\pm}^2 \\
&= v^2\{2(d_{010}d_{012} - d_{010}d_{101} - d_{022} + d_{200})(\mathrm{Im}J_1)^2 \\
&\qquad + [4(2d_{101} - d_{010})\mathrm{Im}J_{11} + (d_{012} - 2d_{010} + 3d_{101})\mathrm{Im}J_2 + 2(d_{101} - d_{012})\mathrm{Im}J_{30}]\mathrm{Im}J_1 \\
&\qquad + (2\,\mathrm{Im}J_{11} + \mathrm{Im}J_2)(4\,\mathrm{Im}J_{11} + \mathrm{Im}J_2 + 2\,\mathrm{Im}J_{30})\}
\end{aligned}$$

- Two constraints – leaving us with 9 free parameters
- Popular model since FCNC are constrained and CP is broken.

Softly broken Z_2 when m_{12}^2 is real

CP1 broken spontaneously

Case SOFT-Z2-XD:

$$\frac{M_{H^\pm}^2}{v^2} = \frac{e_1q_1M_2^2M_3^2 + e_2q_2M_1^2M_3^2 + e_3q_3M_1^2M_2^2 - M_1^2M_2^2M_3^2}{2(e_1^2M_2^2M_3^2 + e_2^2M_3^2M_1^2 + e_3^2M_1^2M_2^2)},$$

$$q = \frac{(e_2q_3 - e_3q_2)^2M_1^2 + (e_3q_1 - e_1q_3)^2M_2^2 + (e_1q_2 - e_2q_1)^2M_3^2 + M_1^2M_2^2M_3^2}{2(e_1^2M_2^2M_3^2 + e_2^2M_3^2M_1^2 + e_3^2M_1^2M_2^2)},$$

$$\begin{aligned}&(2d_{010}d_{012}d_{101} + d_{010}d_{012}^2 + 2d_{010}d_{020} - 3d_{010}d_{022} - d_{010}^2d_{101} + d_{010}d_{101}^2 - d_{010}d_{200}\\ &- 2d_{012}d_{020} - d_{012}d_{022} + d_{012}d_{200} + d_{020}d_{101} - 2d_{022}d_{101} - 2d_{030} + 5d_{032} - 2d_{101}d_{111}\\ &+ d_{210})\mathrm{Im}J_1 + (4d_{010}d_{012} - 2d_{010}^2 + 2d_{020} - 4d_{022})\mathrm{Im}J_{11}\\ &+ (d_{010}d_{012} - \frac{d_{010}^2}{2} + \frac{d_{020}}{2} - d_{022})\mathrm{Im}J_2 + (2d_{010}d_{012} - d_{010}^2 + d_{020} - 2d_{022})\mathrm{Im}J_{30} = 0.\end{aligned}$$

Three constraints - leaving us with 8 free parameters.

Combination of Case SOFT-Z2-X with Case D of spontaneously broken CP1.

CP1 unbroken

Case SOFT-Z2-C:

$$e_k = q_k = 0,$$

$$\begin{aligned}&\frac{e_ie_j(M_j^2 - M_i^2) - (e_jq_i - e_iq_j)v^2}{(\mathrm{Im}J_1)^2}\\ &\times \big[2\mathrm{Im}J_1[2(d_{012} + d_{101} - d_{010})\mathrm{Im}J_1 + 4\,\mathrm{Im}J_{11} + \mathrm{Im}J_2 + 2\,\mathrm{Im}J_{30}]M_{H^\pm}^2\\ &\quad - v^2\big\{2(d_{010}d_{012} - d_{010}d_{101} - d_{022} + d_{200})(\mathrm{Im}J_1)^2\\ &\qquad + [4(2d_{101} - d_{010})\mathrm{Im}J_{11} + (d_{012} - 2d_{010} + 3d_{101})\mathrm{Im}J_2\\ &\qquad\qquad + 2(d_{101} - d_{012})\mathrm{Im}J_{30}]\mathrm{Im}J_1\\ &\qquad + (2\,\mathrm{Im}J_{11} + \mathrm{Im}J_2)(4\,\mathrm{Im}J_{11} + \mathrm{Im}J_2 + 2\,\mathrm{Im}J_{30})\big\}\big] = 0.\end{aligned}$$

Three constraints - leaving us with 8 free parameters.

Contains Case C of unbroken CP1.

Spontaneously broken Z_2 and unbroken Z_2

Z_2 broken spontaneously

Case CD:

$$e_k = q_k = 0,$$

$$M_{H^\pm}^2 = v^2 \frac{e_j q_j M_i^2 + e_i q_i M_j^2 - M_i^2 M_j^2}{e_j^2 M_i^2 + e_i^2 M_j^2},$$

$$q = \frac{(e_j q_i - e_i q_j)^2 + M_i^2 M_j^2}{2(e_j^2 M_i^2 + e_i^2 M_j^2)}.$$

Four constraints - leaving us with 7 free parameters.

Combination of Cases C and D of CP1 invariant potential.

Z_2 unbroken

Case CC:

$$e_j = q_j = e_k = q_k = 0$$

Four constraints - leaving us with 7 free parameters.

Contains two times Case C (applied to two different neutral scalars) of CP1 invariant potential.

The Inert doublet model (IDM).

Only possibility of Dark Matter in 2HDM.

U(1)

Model	Invariant parts	Conditions
Softly broken U(1)	V_4	$\Delta=0$ and $\boldsymbol{\xi} \times \mathbf{e} = \boldsymbol{\eta} \times \mathbf{e} = \mathbf{0}$ or $\Delta=\Delta_0=0$
Spontaneously broken U(1) (and unbroken U(1))	$V=V_2+V_4$ (and vacuum)	$\Delta=0$ and $\boldsymbol{\xi} \times \mathbf{e} = \mathbf{0}$ or $\Delta=\Delta_0=0$ and $\boldsymbol{\xi} \times \boldsymbol{\eta} = \mathbf{0}$

For definitions and translation of bilinear formalism quantities see

Ferreira, Haber, Nachtmann, Silva: Int.J.Mod.Phys A26 769 (2011)

Ferreira, Grzadkowski, Ogreid, Osland: JHEP 02 (2021) 196

Softly broken U(1)

Case SOFT-U1-C:

$$e_k = q_k = 0,$$

$$2(e_j^2 M_i^2 + e_i^2 M_j^2 - v^2 M_k^2) M_{H^\pm}^2$$
$$= v^2((M_j^2 - M_k^2) e_i q_i + (M_i^2 - M_k^2) e_j q_j)$$
$$+ e_j^2 (M_k^2 - M_i^2)(M_j^2 - 2M_k^2) + e_i^2 (M_k^2 - M_j^2)(M_i^2 - 2M_k^2),$$

$$2v^2(e_j^2 M_i^2 + e_i^2 M_j^2 - v^2 M_k^2) q$$
$$= v^2 (e_i q_j - e_j q_i)^2 + e_j^2 M_j^2 (M_i^2 - M_k^2) + e_i^2 M_i^2 (M_j^2 - M_k^2).$$

- Four constraints – leaving us with 7 free parameters
- CP unbroken since it contains Case C.

Spontaneously broken U(1) and unbroken U(1)

U(1) broken spontaneously

Case $\mathbf{C_0D}$:

$$e_k = q_k = 0, \quad M_k = 0,$$

$$M_{H^\pm}^2 = v^2 \frac{e_j q_j M_i^2 + e_i q_i M_j^2 - M_i^2 M_j^2}{2(e_j^2 M_i^2 + e_i^2 M_j^2)},$$

$$q = \frac{(e_j q_i - e_i q_j)^2 + M_i^2 M_j^2}{2(e_j^2 M_i^2 + e_i^2 M_j^2)}.$$

Five constraints - leaving us with 6 free parameters.

Combination of Cases C and D of CP1 invariant potential.

Massless pseudoscalar since continuous U(1) symmetry is spontaneously broken.

U(1) unbroken

Case BCC:

$$M_j = M_k, \quad e_j = q_j = e_k = q_k = 0.$$

Five constraints - leaving us with 6 free parameters.

Contains two times Case C (applied to two different neutral scalars) of CP1 invariant potential. Also Case B of CP1 invariant potential satisfied.

The Inert doublet model (IDM) with mass degeneracy.

CP2

Model	Invariant parts	Conditions
Softly broken CP2	V_4	$\eta = 0$
Spontaneously broken CP2 (CP2 cannot be unbroken)	$V=V_2+V_4$	$\xi = \eta = 0$

For definitions and translation of bilinear formalism quantities see

Ferreira, Haber, Nachtmann, Silva: Int.J.Mod.Phys A26 769 (2011)

Ferreira, Grzadkowski, Ogreid, Osland: JHEP 02 (2021) 196

Softly broken CP2

Change to basis where $\lambda_6 = \lambda_7 = \text{Im}\,\lambda_5 = 0$

$$V(\Phi_1,\Phi_2) = -\frac{1}{2}\left\{m_{11}^2\Phi_1^\dagger\Phi_1 + m_{22}^2\Phi_2^\dagger\Phi_2 + \left[m_{12}^2\Phi_1^\dagger\Phi_2 + \text{h.c.}\right]\right\}$$
$$+\frac{\lambda_1}{2}\left\{(\Phi_1^\dagger\Phi_1)^2 + (\Phi_2^\dagger\Phi_2)^2\right\} + \lambda_3(\Phi_1^\dagger\Phi_1)(\Phi_2^\dagger\Phi_2)$$
$$+\lambda_4(\Phi_1^\dagger\Phi_2)(\Phi_2^\dagger\Phi_1) + \frac{\text{Re}\lambda_5}{2}\left[(\Phi_1^\dagger\Phi_2)^2 + (\Phi_2^\dagger\Phi_1)^2\right]$$

Seemingly five different ways to softly break CP2:

Option I:	$m_{11}^2 \neq m_{22}^2$,	m_{12}^2 complex,	(8 model parameters)
Option II:	$m_{11}^2 \neq m_{22}^2$,	m_{12}^2 real,	(7 model parameters)
Option III:	$m_{11}^2 \neq m_{22}^2$,	$m_{12}^2 = 0$,	(6 model parameters)
Option IV:	$m_{11}^2 = m_{22}^2$,	m_{12}^2 complex,	(7 model parameters)
Option V:	$m_{11}^2 = m_{22}^2$,	m_{12}^2 real,	(6 model parameters)

Option II and Option IV are identical modulo a change of basis.

Option III and Option V are identical modulo a change of basis.

Softly broken CP2

Change to basis where $\lambda_6 = \lambda_7 = \mathrm{Im}\,\lambda_5 = 0$

$$\begin{aligned} V(\Phi_1,\Phi_2) = &-\frac{1}{2}\left\{m_{11}^2\Phi_1^\dagger\Phi_1 + m_{22}^2\Phi_2^\dagger\Phi_2 + \left[m_{12}^2\Phi_1^\dagger\Phi_2 + \mathrm{h.c.}\right]\right\} \\ &+ \frac{\lambda_1}{2}\left\{(\Phi_1^\dagger\Phi_1)^2 + (\Phi_2^\dagger\Phi_2)^2\right\} + \lambda_3(\Phi_1^\dagger\Phi_1)(\Phi_2^\dagger\Phi_2) \\ &+ \lambda_4(\Phi_1^\dagger\Phi_2)(\Phi_2^\dagger\Phi_1) + \frac{\mathrm{Re}\lambda_5}{2}\left[(\Phi_1^\dagger\Phi_2)^2 + (\Phi_2^\dagger\Phi_1)^2\right] \end{aligned}$$

Seemingly five different ways to softly break CP2:

$$\begin{array}{llll} \text{Option I:} & m_{11}^2 \neq m_{22}^2, & m_{12}^2 \text{ complex,} & (8 \text{ model parameters}) \\ \text{Option II:} & m_{11}^2 \neq m_{22}^2, & m_{12}^2 \text{ real,} & (7 \text{ model parameters}) \\ \text{Option III:} & m_{11}^2 \neq m_{22}^2, & m_{12}^2 = 0, & (6 \text{ model parameters}) \\ \cancel{\text{Option IV:}} & \cancel{m_{11}^2 = m_{22}^2,} & \cancel{m_{12}^2 \text{ complex,}} & \cancel{(7 \text{ model parameters})} \\ \cancel{\text{Option V:}} & \cancel{m_{11}^2 = m_{22}^2,} & \cancel{m_{12}^2 \text{ real,}} & \cancel{(6 \text{ model parameters})} \end{array}$$

Option II and Option IV are identical modulo a change of basis.

Option III and Option V are identical modulo a change of basis.

All different physics models contained in Options I, II and III.

Softly broken CP2

Option I: $m_{11}^2 \neq m_{22}^2, \quad m_{12}^2$ complex

leads to

Case SOFT-CP2:

$$v^2(e_1q_2 - e_2q_1) + e_1e_2(M_2^2 - M_1^2) = 0,$$
$$v^2(e_1q_3 - e_3q_1) + e_1e_3(M_3^2 - M_1^2) = 0,$$
$$v^2(e_2q_3 - e_3q_2) + e_2e_3(M_3^2 - M_2^2) = 0,$$
$$2v^4q = e_1^2M_1^2 + e_2^2M_2^2 + e_3^2M_3^2.$$

Most general way to softly break CP2. CP is broken explicitly by the soft terms.

Softly broken CP2

Option III: $m_{11}^2 \neq m_{22}^2, \quad m_{12}^2 = 0$

Depending on the form of the vacuum this leads to either…

Case SOFT-CP2-CC:

$$e_j = q_j = e_k = q_k = 0, \quad q = \frac{M_i^2}{2v^2}.$$

Z_2 is unbroken since it contains Case CC.

… or it leads to

Case SOFT-CP2-CD:

$$v^2(e_i q_j - e_j q_i) + e_i e_j (M_j^2 - M_i^2) = 0,$$

$$e_k = q_k = 0, \quad q = \frac{e_i^2 M_i^2 + e_j^2 M_j^2}{2v^4},$$

$$M_{H\pm}^2 = \frac{v^2(e_i q_i M_j^2 + e_j q_j M_i^2 - M_i^2 M_j^2)}{2(e_i^2 M_j^2 + e_j^2 M_i^2)}.$$

Z_2 is spontaneously broken since it contains Case CD.

Softly broken CP2

Option II: $m_{11}^2 \neq m_{22}^2, \quad m_{12}^2$ real

Depending on the form of the vacuum this leads to either…

Case SOFT-CP2-C:

$$v^2(e_i q_j - e_j q_i) + e_i e_j (M_j^2 - M_i^2) = 0,$$

$$e_k = q_k = 0, \quad q = \frac{e_i^2 M_i^2 + e_j^2 M_j^2}{2v^4}.$$

CP1 is unbroken since it contains Case C.

… or it leads to

Case SOFT-CP2-D:

$$v^2(e_1 q_2 - e_2 q_1) + e_1 e_2 (M_2^2 - M_1^2) = 0,$$
$$v^2(e_1 q_3 - e_3 q_1) + e_1 e_3 (M_3^2 - M_1^2) = 0,$$
$$v^2(e_2 q_3 - e_3 q_2) + e_2 e_3 (M_3^2 - M_2^2) = 0,$$
$$M_{H^\pm}^2 = \frac{v^2(e_1 q_1 M_2^2 M_3^2 + e_2 q_2 M_3^2 M_1^2 + e_3 q_3 M_1^2 M_2^2 - M_1^2 M_2^2 M_3^2)}{2(e_1^2 M_2^2 M_3^2 + e_2^2 M_3^2 M_1^2 + e_3^2 M_1^2 M_2^2)},$$
$$q = \frac{e_1^2 M_1^2 + e_2^2 M_2^2 + e_3^2 M_3^2}{2v^4}.$$

CP1 is spontaneously broken since it contains Case D (but not Case C).

Spontaneously broken CP2

CP2 broken spontaneously

Case CCD:

$$e_j = q_j = e_k = q_k = 0,$$

$$2M_{H^\pm}^2 = e_i q_i - M_i^2, \quad 2v^2 q = M_i^2.$$

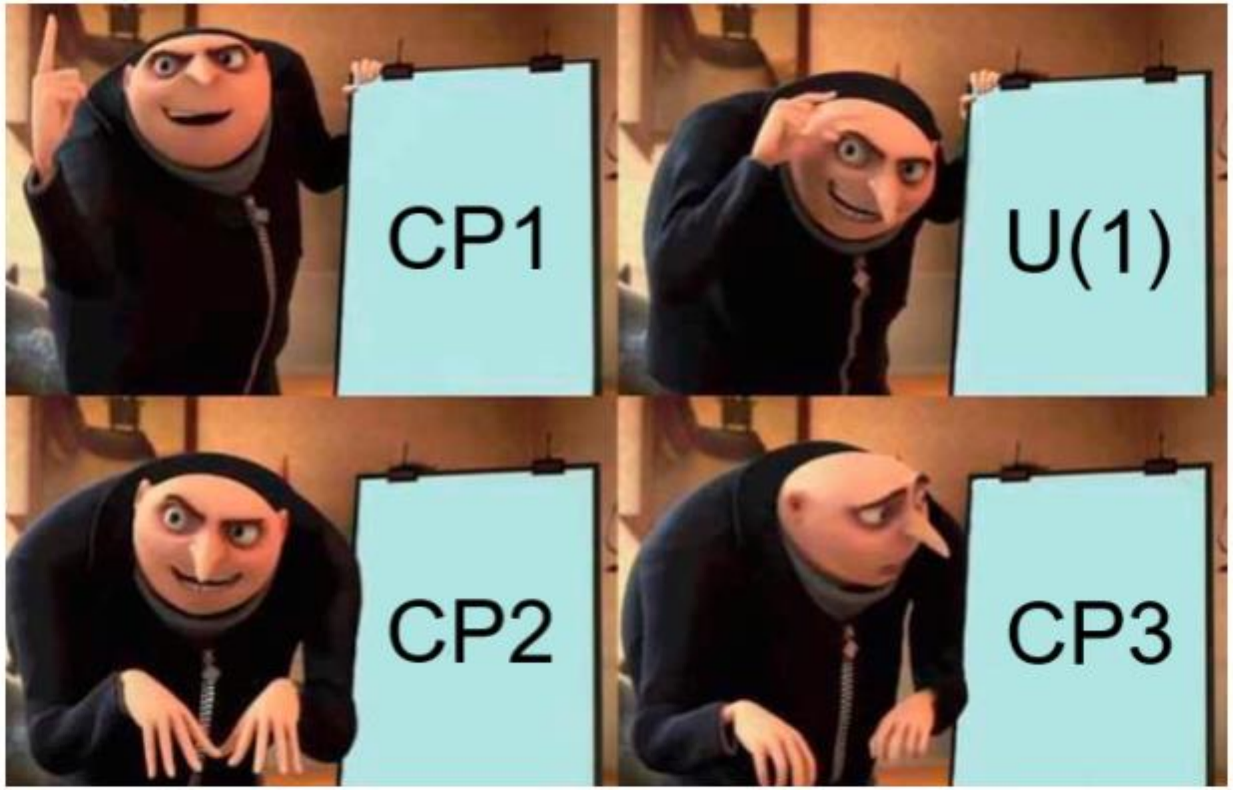
Six constraints – leaving us with 5 free parameters.

Case results from combining three CP1 symmetries (C, C and D).

CP2 can not be unbroken.

CP3

Model	Invariant parts	Conditions
Softly broken CP3	V_4	Δ=0 and **η** = **0**
Spontaneously broken CP3 (and unbroken CP3)	$V=V_2+V_4$ (and vacuum)	Δ=0 and **ξ** = **η** = **0**



For definitions and translation of bilinear formalism quantities see

Ferreira, Haber, Nachtmann, Silva: Int.J.Mod.Phys A26 769 (2011)

Ferreira, Grzadkowski, Ogreid, Osland: JHEP 02 (2021) 196

Softly broken CP3

Change to basis where $m_{11}^2 = m_{22}^2$

$$V(\Phi_1,\Phi_2) = -\frac{1}{2}\left\{m_{11}^2\left(\Phi_1^\dagger\Phi_1+\Phi_2^\dagger\Phi_2\right)+\left[m_{12}^2\Phi_1^\dagger\Phi_2+\text{h.c.}\right]\right\}$$
$$+\frac{\lambda_1}{2}\left\{(\Phi_1^\dagger\Phi_1)^2+(\Phi_2^\dagger\Phi_2)^2\right\}+\lambda_3(\Phi_1^\dagger\Phi_1)(\Phi_2^\dagger\Phi_2)$$
$$+\lambda_4(\Phi_1^\dagger\Phi_2)(\Phi_2^\dagger\Phi_1)+\frac{\lambda_1-\lambda_3-\lambda_4}{2}\left[(\Phi_1^\dagger\Phi_2)^2+(\Phi_2^\dagger\Phi_1)^2\right]$$

Three different ways to softly break CP3:

Option I:	m_{12}^2 complex,	(6 model parameters)
Option II:	m_{12}^2 imaginary,	(5 model parameters)
Option III:	m_{12}^2 real,	(5 model parameters)

Option I: m_{12}^2 complex

leads to

Case SOFT-CP3-C:

$$e_k = q_k = 0, \quad v^2(e_iq_j - e_jq_i) + e_ie_j(M_j^2 - M_i^2) = 0,$$
$$2v^4q = e_i^2M_i^2 + e_j^2M_j^2,$$
$$(e_j^2M_i^2 + e_i^2M_j^2 - v^2M_k^2)$$
$$\times\left[2v^2M_{H^\pm}^2 + e_i^2M_i^2 + e_j^2M_j^2 - v^2(2M_k^2 + e_iq_i + e_jq_j)\right]$$
$$= 2e_i^2e_j^2(M_j^2 - M_i^2)^2,$$

The most general way to softly break CP3.
CP1 is unbroken since it contains Case C.

Softly broken CP3

Option II: m_{12}^2 imaginary

Depending on the form of the vacuum this leads to

Case SOFT-CP3-BCC:

$$e_j = q_j = e_k = q_k = 0,$$

$$M_j = M_k, \quad 2v^2 q = M_i^2.$$

U(2) is unbroken since it contains Case BCC.

… or it leads to

Case SOFT-CP3-C$_0$D:

$$e_k = q_k = 0, \quad M_k = 0,$$

$$v^2(e_i q_j - e_j q_i) + e_i e_j (M_j^2 - M_i^2) = 0,$$

$$2v^4 q = e_i^2 M_i^2 + e_j^2 M_j^2,$$

$$2(e_j^2 M_i^2 + e_i^2 M_j^2) M_{H^\pm}^2 = v^2 (e_j q_j M_i^2 + e_i q_i M_j^2 - M_i^2 M_j^2).$$

U(2) is spontaneously broken since it contains Case C$_0$D.

Softly broken CP3

Option III: m_{12}^2 real

Depending on the form of the vacuum this leads to

Case SOFT-CP3-CC:

$$e_j = q_j = e_k = q_k = 0, \quad 2v^2 q = M_i^2,$$

$$M_{H^\pm}^2 = \frac{1}{2}(2M_k^2 - M_i^2 + e_i q_i).$$

Z_2 is unbroken since it contains Case CC.

… or it leads to

Case SOFT-CP3-CD:

$$e_k = q_k = 0, \quad v^2(e_i q_j - e_j q_i) + e_i e_j (M_j^2 - M_i^2) = 0,$$
$$2v^4 q = e_i^2 M_i^2 + e_j^2 M_j^2,$$
$$2(e_j^2 M_i^2 + e_i^2 M_j^2) M_{H^\pm}^2 = v^2 (e_j q_j M_i^2 + e_i q_i M_j^2 - M_i^2 M_j^2),$$
$$e_i^2 M_j^2 (M_j^2 - M_k^2) + e_j^2 M_i^2 (M_i^2 - M_k^2) = 0.$$

Z_2 is spontaneously broken since it contains Case CD.

Spontaneously broken CP3 and unbroken CP3

CP3 broken spontaneously

Case $\mathbf{C_0CD}$:

$$e_j = q_j = e_k = q_k = 0, \quad M_j = 0,$$
$$2M_{H^\pm}^2 = e_i q_i - M_i^2, \quad 2v^2 q = M_i^2.$$

Six constraints - leaving us with 5 free parameters.

Combination of Cases C, C and D of CP1 invariant potential.

Massless pseudoscalar since continuous CP3 symmetry is spontaneously broken.

CP3 unbroken

Case BCCD:

$$e_j = q_j = e_k = q_k = 0, \quad M_j = M_k,$$
$$2M_{H^\pm}^2 = e_i q_i - M_i^2, \quad 2v^2 q = M_i^2.$$

Six constraints - leaving us with 5 free parameters.

Combination of Cases B, C, C and D of CP1 invariant potential.

SO(3)

Model	Invariant parts	Conditions
Softly broken SO(3)	V_4	$\Delta=\Delta_0=0$ and $\boldsymbol{\eta} = \mathbf{0}$
Spontaneously broken SO(3) (SO(3) cannot be unbroken)	$V=V_2+V_4$	$\Delta=\Delta_0=0$ and $\boldsymbol{\xi} = \boldsymbol{\eta} = \mathbf{0}$

For definitions and translation of bilinear formalism quantities see

Ferreira, Haber, Nachtmann, Silva: Int.J.Mod.Phys A26 769 (2011)

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Softly broken SO(3) and spontaneously broken SO(3)

SO(3) broken softly

Case SOFT-SO3-BCC:

$$M_j = M_k, \quad e_j = q_j = e_k = q_k = 0,$$
$$2M_{H^\pm}^2 = e_i q_i - M_i^2 + 2M_j^2, \quad 2v^2 q = M_i^2.$$

Seven constraints - leaving us with 4 free parameters.

Combination of Cases B, C and C of CP1 invariant potential.

Mass degeneracy present.

SO(3) broken spontaneously

Case $\mathbf{B_0C_0C_0D}$:

$$M_j = M_k = 0, \quad e_j = q_j = e_k = q_k = 0,$$
$$2M_{H^\pm}^2 = e_i q_i - M_i^2, \quad 2v^2 q = M_i^2.$$

Eight constraints - leaving us with 3 free parameters.

Combination of Cases B, C, C and D of CP1 invariant potential.

Massless pseudoscalars since continuous SO(3) symmetry is spontaneously broken.

Thank you for nor falling aZZZZZZleep!

- Imposing symmetries on the 2HDM potential and/or vacuum has physical implications. Has been presented for all possible combinations.
- Different physics depending on if the symmetry is softly broken, spontaneously broken or unbroken.
- If (and when) we discover the 2HDM particle zoo and measure all 11 masses/couplings we can identify if any symmetries are present and in which part of the potential/vacuum.