General lepton flavour conserving two Higgs doublets models and $g-2$ anomalies

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Outline

Based on work done in collaboration with:

Francisco J. Botella, Fernando Cornet-Gómez & Carlos Miró

☞ arXiv:2205.01115

☞ arXiv:2006.01934, PRD102 (2020)

☞ arXiv:1803.08521, PRD98 (2018)

+ work in progress

Motivation

"Anomalies" in the anomalous magnetic moments of μ and e

$$\delta a_\mu^{\text{Exp}} \equiv a_\mu^{\text{Exp}} - a_\mu^{\text{SM}} = +(2.5 \pm 0.6) \times 10^{-9}$$

Muon $g-2$ collaboration, *Phys. Rev. Lett. 126 (2021) 14*

$$\delta a_e^{\text{Exp,Cs}} \equiv a_e^{\text{Exp,Cs}} - a_e^{\text{SM}} = -(8.7 \pm 3.6) \times 10^{-13}$$

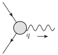
Parker et al. *Science (2018) 360:191*, α from ^{133}Cs recoil

Also

$$\delta a_e^{\text{Exp,Rb}} \equiv a_e^{\text{Exp,Rb}} - a_e^{\text{SM}} = +(4.8 \pm 3.0) \times 10^{-13}$$

Morel et al. *Nature (2020) 588:61*, α from ^{87}Rb recoil

N.B. $a_\ell = (g_\ell - 2)/2$

Vertex $\bar{\ell}\ell A^\mu$  $\gamma^\mu \to \Gamma^\mu = \gamma^\mu F_1(q^2) + i\frac{\sigma^{\mu\nu}q_\nu}{2m_\ell}F_2(q^2)$, $a_\ell = F_2(0)$

Motivation

$$\frac{\delta a_e^{\rm Exp,Cs}}{\delta a_\mu^{\rm Exp}} \simeq -\left(\frac{m_e}{m_\mu}\right)^{1.494}$$

- $\delta a_\mu^{\rm Exp}$ and $\delta a_e^{\rm Exp,Cs}$ have *opposite* signs!
- Not only the sign,
  - if NP model gives $\delta a_\ell \propto m_\ell$

$$\frac{\delta a_e}{\delta a_\mu} = \frac{m_e}{m_\mu} = \left(\frac{m_e}{m_\mu}\right)^{-0.494} \frac{\delta a_e^{\rm Exp,Cs}}{\delta a_\mu^{\rm Exp}} \simeq -13.9\,\frac{\delta a_e^{\rm Exp,Cs}}{\delta a_\mu^{\rm Exp}}$$

  - if NP model gives $\delta a_\ell \propto m_\ell^2$

$$\frac{\delta a_e}{\delta a_\mu} = \left(\frac{m_e}{m_\mu}\right)^2 = \left(\frac{m_e}{m_\mu}\right)^{0.506} \frac{\delta a_e^{\rm Exp,Cs}}{\delta a_\mu^{\rm Exp}} \simeq -\frac{1}{14.8}\,\frac{\delta a_e^{\rm Exp,Cs}}{\delta a_\mu^{\rm Exp}}$$

✎ Serious obstacle for many New Physics solutions

Motivation

- If the origin of both anomalies is beyond SM, some sort of *effective decoupling* between e and μ should be in place
- 2 Higgs Doublets Models (2HDMs) incorporate *new flavour structures* that can implement that property but
  - not in symmetry-shaped 2HDMs of types I, II, X, Y (new couplings proportional to masses)
  - not in "aligned 2HDMs" (proportionality to masses again)
  - maybe in general flavour conserving 2HDMs (gFC-2HDMs)! ✍

2HDMs

- In 2HDMs the Yukawa sector is

$$\mathscr{L}_{\mathrm{Y}} = -\bar{Q}_L^0 \left(\Phi_1 Y_{d1} + \Phi_2 Y_{d2}\right) d_R^0 - \bar{Q}_L^0 \left(\tilde{\Phi}_1 Y_{u1} + \tilde{\Phi}_2 Y_{u2}\right) u_R^0 - \bar{L}_L^0 \left(\Phi_1 Y_{\ell 1} + \Phi_2 Y_{\ell 2}\right) \ell_R^0 + \mathrm{H.c.}$$

N.B. $\tilde{\Phi}_j = i\sigma_2 \Phi_j^*$, neutrinos are massless

- Expansion around vacuum appropriate for electroweak symmetry breaking

$$\Phi_j = e^{i\theta_j} \begin{pmatrix} \varphi_j^+ \\ \frac{v_j + \rho_j + i\eta_j}{\sqrt{2}} \end{pmatrix}$$

- Higgs basis, $c_\beta \equiv \cos\beta = \frac{v_1}{v}$, $s_\beta \equiv \sin\beta = \frac{v_2}{v}$, $t_\beta \equiv \tan\beta$

$$\begin{pmatrix} H_1 \\ H_2 \end{pmatrix} = \mathcal{R}_\beta \begin{pmatrix} e^{-i\theta_1}\Phi_1 \\ e^{-i\theta_2}\Phi_2 \end{pmatrix}, \quad \text{with} \quad \mathcal{R}_\beta = \begin{pmatrix} c_\beta & s_\beta \\ -s_\beta & c_\beta \end{pmatrix}, \mathcal{R}_\beta^T = \mathcal{R}_\beta^{-1}$$

2HDMs

- Higgs basis

$$\langle H_1\rangle = \frac{v}{\sqrt{2}}\begin{pmatrix}0\\1\end{pmatrix}, \quad \langle H_2\rangle = \begin{pmatrix}0\\0\end{pmatrix}, \qquad v^2 = v_1^2 + v_2^2 = \frac{1}{\sqrt{2}G_F}$$

$$H_1 = \begin{pmatrix} G^+ \\ \frac{v+H^0+iG^0}{\sqrt{2}} \end{pmatrix}, \quad H_2 = \begin{pmatrix} \mathrm{H}^+ \\ \frac{R^0+iI^0}{\sqrt{2}} \end{pmatrix}$$

  - would-be Goldstone bosons G^0, $G^\pm$
  - physical charged scalar $H^\pm$
  - neutral scalars $\{H^0, R^0, I^0\}$, not the mass eigenstates
- Yukawa couplings again

$$\mathscr{L}_{\mathrm{Y}} = -\frac{\sqrt{2}}{v}\bar{Q}_L^0\left(H_1\mathrm{M}_d^0 + H_2\mathrm{N}_d^0\right)d_R^0 - \frac{\sqrt{2}}{v}\bar{Q}_L^0\left(\tilde{H}_1\mathrm{M}_u^0 + \tilde{H}_2\mathrm{N}_u^0\right)u_R^0 - \frac{\sqrt{2}}{v}\bar{L}_L^0\left(H_1\mathrm{M}_\ell^0 + H_2\mathrm{N}_\ell^0\right)\ell_R^0 + \mathrm{H.c.}$$

2HDMs

- Only the neutral component (↓) of H_1 has non-vanishing vev
  $\Rightarrow \mathrm{M}_f^0$ give the mass matrices, $f = u, d, \ell$
- Usual bi-unitary changes into the different fermion mass bases

$$\mathscr{L}_{\mathrm{Y}} = -\frac{\sqrt{2}}{v}\bar{Q}_L\left(H_1\mathrm{M}_d + H_2\mathrm{N}_d\right)d_R - \frac{\sqrt{2}}{v}\bar{Q}_L\left(\tilde{H}_1\mathrm{M}_u + \tilde{H}_2\mathrm{N}_u\right)u_R - \frac{\sqrt{2}}{v}\bar{L}_L\left(H_1\mathrm{M}_\ell + H_2\mathrm{N}_\ell\right)\ell_R + \mathrm{H.c.}$$

where

- M_f are the diagonal fermion mass matrices
- N_f are the new flavour structures
  (the ones that may explain the anomalies!)

2HDMs

$$\mathscr{L}_{\rm Y} = -\frac{\sqrt{2}}{v}\bar{Q}_L\left(H_1 \mathrm{M}_d + H_2 \mathrm{N}_d\right) d_R - \frac{\sqrt{2}}{v}\bar{Q}_L\left(\tilde{H}_1 \mathrm{M}_u + \tilde{H}_2 \mathrm{N}_u\right) u_R - \frac{\sqrt{2}}{v}\bar{L}_L\left(H_1 \mathrm{M}_\ell + H_2 \mathrm{N}_\ell\right) \ell_R + \mathrm{H.c.}$$

- Natural Flavour Conservation:
  only one Yukawa matrix $\neq 0$ in each sector
  $\mathbb{Z}_2$ symmetry, types I, II, X, Y, with $\mathrm{N}_f = \pm t_\beta^{\mp 1}\mathrm{M}_f$

  Glashow & Weinberg, *PRD15 (1977)*

- "Aligned" 2HDM: $\mathrm{N}_f = \zeta_f \mathrm{M}_f$

  Pich & Tuzón, *PRD80 (2009)*

  RGE: unstable quark sector, stable lepton sector

  Botella, Branco, Coutinho, Rebelo & Silva-Marcos, *EPJC75 (2015)*

- General flavour conserving: diagonal N_f

  Peñuelas & Pich, *JHEP 12 (2017)*

  RGE: unstable quark sector, stable lepton sector

  Botella, Cornet-Gómez & N *PRD98 (2018)*

The I-gℓFC and II-gℓFC models

Finally

- Model I-gℓFC is defined by

$$\mathrm{N}_u = t_\beta^{-1}\mathrm{M}_u, \quad \mathrm{N}_d = t_\beta^{-1}\mathrm{M}_d, \quad \mathrm{N}_\ell = \mathrm{diag}(n_e, n_\mu, n_\tau)$$

  The couplings $\mathrm{N}_u, \mathrm{N}_d$ are the same as in 2HDMs of types I or X

- Model II-gℓFC is defined by

$$\mathrm{N}_u = t_\beta^{-1}\mathrm{M}_u, \quad \mathrm{N}_d = -t_\beta\mathrm{M}_d, \quad \mathrm{N}_\ell = \mathrm{diag}(n_e, n_\mu, n_\tau)$$

  The couplings $\mathrm{N}_u, \mathrm{N}_d$ are the same as in 2HDMs of types II or Y

- N_ℓ is diagonal, arbitrary and one loop RGE stable
- Effective decoupling among new e and μ couplings required for the $g-2$ anomalies $\leftrightarrow$ independence of n_e and n_μ

The I-gℓFC and II-gℓFC models

Completing the model

- since the quark sector is a type I or type II 2HDM, adopt a $\mathbb{Z}_2$ symmetric scalar potential

$$\mathcal{V}(\Phi_1,\Phi_2) = \mu_{11}^2\Phi_1^\dagger\Phi_1+\mu_{22}^2\Phi_2^\dagger\Phi_2+\left(\mu_{12}^2\Phi_1^\dagger\Phi_2 + \text{H.c.}\right)+\lambda_1(\Phi_1^\dagger\Phi_1)^2+\lambda_2(\Phi_2^\dagger\Phi_2)^2$$
$$+ 2\lambda_3(\Phi_1^\dagger\Phi_1)(\Phi_2^\dagger\Phi_2) + 2\lambda_4(\Phi_1^\dagger\Phi_2)(\Phi_2^\dagger\Phi_1) + \left(\lambda_5(\Phi_1^\dagger\Phi_2)^2 + \text{H.c.}\right)$$

  $\mu_{12}^2 \neq 0 \Rightarrow$ softly broken $\mathbb{Z}_2$ symmetry

- Mass matrix of the neutral scalars $\mathcal{M}_0^2$, diagonalised by a 3×3 real orthogonal matrix $\mathcal{R}$

$$\mathcal{R}^T\,\mathcal{M}_0^2\,\mathcal{R} = \text{diag}(m_{\text{h}}^2, m_{\text{H}}^2, m_{\text{A}}^2)\,,\quad \mathcal{R}^{-1} = \mathcal{R}^T$$

- Physical neutral scalars $\{\text{h},\text{H},\text{A}\}$:

$$\begin{pmatrix}\text{h}\\ \text{H}\\ \text{A}\end{pmatrix} = \mathcal{R}^T\begin{pmatrix}H^0\\ R^0\\ I^0\end{pmatrix}$$

The I-gℓFC and II-gℓFC models

- Flavour conserving Yukawa couplings of the neutral scalars

$$\mathscr{L}_N = -\sum_{S=\mathrm{h,H,A}} \sum_{f=u,d,\ell} \sum_{j=1}^{3} \frac{m_{f_j}}{v} S\, \bar{f}_j (a^S_{f_j} + i b^S_{f_j} \gamma_5) f_j$$

- Further simplifications
  1. the new Yukawa couplings are real, $\mathrm{Im}\,(n_\ell) = 0$
  2. there is no CP violation in the scalar sector,

$$\mathcal{R} = \begin{pmatrix} s_{\alpha\beta} & -c_{\alpha\beta} & 0 \\ c_{\alpha\beta} & s_{\alpha\beta} & 0 \\ 0 & 0 & 1 \end{pmatrix}, \qquad \begin{cases} s_{\alpha\beta} \equiv \sin(\alpha-\beta) \\ c_{\alpha\beta} \equiv \cos(\alpha-\beta) \end{cases}$$

$\alpha - \frac{\pi}{2}$: mixing angle in $\{\rho_j, \eta_j\} \to \{G^0, \mathrm{h}, \mathrm{H}, \mathrm{A}\}$

The I-gℓFC and II-gℓFC models

$$\mathscr{L}_N = -\sum_{S=\text{h,H,A}} \sum_{f=u,d,\ell} \sum_{j=1}^{3} \frac{m_{f_j}}{v} S\, \bar{f}_j (a^S_{f_j} + i b^S_{f_j} \gamma_5) f_j$$

Quark couplings

		a_u^S	b_u^S	a_d^S	b_d^S
I-gℓFC	h	$s_{\alpha\beta} + c_{\alpha\beta} t_\beta^{-1}$	0	$s_{\alpha\beta} + c_{\alpha\beta} t_\beta^{-1}$	0
	H	$-c_{\alpha\beta} + s_{\alpha\beta} t_\beta^{-1}$	0	$-c_{\alpha\beta} + s_{\alpha\beta} t_\beta^{-1}$	0
	A	0	$-t_\beta^{-1}$	0	$+t_\beta^{-1}$
II-gℓFC	h	$s_{\alpha\beta} + c_{\alpha\beta} t_\beta^{-1}$	0	$s_{\alpha\beta} - c_{\alpha\beta} t_\beta$	0
	H	$-c_{\alpha\beta} + s_{\alpha\beta} t_\beta^{-1}$	0	$-c_{\alpha\beta} - s_{\alpha\beta} t_\beta$	0
	A	0	$-t_\beta^{-1}$	0	$-t_\beta$

The I-gℓFC and II-gℓFC models

$$\mathscr{L}_N = -\sum_{S=\mathrm{h,H,A}} \sum_{f=u,d,\ell} \sum_{j=1}^{3} \frac{m_{f_j}}{v} S \bar{f}_j (a^S_{f_j} + i b^S_{f_j} \gamma_5) f_j$$

Lepton couplings

		a^S_ℓ	b^S_ℓ
I-gℓFC	h	$s_{\alpha\beta} + c_{\alpha\beta}\frac{\mathrm{Re}(n_\ell)}{m_\ell}$	0
	H	$-c_{\alpha\beta} + s_{\alpha\beta}\frac{\mathrm{Re}(n_\ell)}{m_\ell}$	0
	A	0	$\frac{\mathrm{Re}(n_\ell)}{m_\ell}$
II-gℓFC	h	$s_{\alpha\beta} + c_{\alpha\beta}\frac{\mathrm{Re}(n_\ell)}{m_\ell}$	0
	H	$-c_{\alpha\beta} + s_{\alpha\beta}\frac{\mathrm{Re}(n_\ell)}{m_\ell}$	0
	A	0	$\frac{\mathrm{Re}(n_\ell)}{m_\ell}$

The I-gℓFC and II-gℓFC models

- Absence of CP violation $\Leftrightarrow$ $a_f^S b_f^S = 0$
  $\Rightarrow$ absence of new contributions to electric dipole moments,
  in particular to the electron EDM
  (quite constrained $|d_e| < 1.1 \times 10^{-29}$ e·cm)

The I-gℓFC and II-gℓFC models

Yukawa couplings of the charged scalar

$$\mathscr{L}_{Ch} = -\frac{1}{\sqrt{2}v} \sum_{f=q,\ell} \sum_{j,k=1}^{3} \Big\{ \mathrm{H}^- \bar{f}_{-\frac{1}{2},j} (\alpha^f_{jk} + i\beta^f_{jk}\gamma_5) f_{\frac{1}{2},k} + \mathrm{H}^+ \bar{f}_{\frac{1}{2},k} (\alpha^{f*}_{jk} + i\beta^{f*}_{jk}\gamma_5) f_{-\frac{1}{2},j} \Big\}$$

with $q_{+\frac{1}{2},j} = u_j$; $q_{-\frac{1}{2},j} = d_j$; $\ell_{+\frac{1}{2},j} = \nu_j$; $\ell_{-\frac{1}{2},j} = \ell_j$

	α^q_{ij}	β^q_{ij}
I-gℓFC	$V^*_{ji}\, t_\beta^{-1} (m_{u_j} - m_{d_i})$	$V^*_{ji}\, t_\beta^{-1} (m_{u_j} + m_{d_i})$
II-gℓFC	$V^*_{ji}\, (t_\beta^{-1} m_{u_j} + t_\beta m_{d_i})$	$V^*_{ji}\, (t_\beta^{-1} m_{u_j} - t_\beta m_{d_i})$

	α^ℓ_{ij}	β^ℓ_{ij}
I-gℓFC	$-\mathrm{Re}\,(n_{\ell_i})\,\delta_{ij}$	$\mathrm{Re}\,(n_{\ell_i})\,\delta_{ij}$
II-gℓFC	$-\mathrm{Re}\,(n_{\ell_i})\,\delta_{ij}$	$\mathrm{Re}\,(n_{\ell_i})\,\delta_{ij}$

The new contributions to δa_ℓ

- Full prediction

$$a_\ell^{\text{Th}} = a_\ell^{\text{SM}} + \delta a_\ell$$

a_ℓ^{SM}: SM contribution; δa_ℓ: corrections due to the model

- To solve the discrepancies, the aim is

$$\delta a_e \simeq \delta a_e^{\text{Exp,Cs}}, \qquad \delta a_\mu \simeq \delta a_\mu^{\text{Exp}}$$

within models I-gℓFC and II-gℓFC

- Introduce Δ_ℓ

$$\delta a_\ell = K_\ell\, \Delta_\ell, \qquad K_\ell = \frac{1}{8\pi^2}\left(\frac{m_\ell}{v}\right)^2 = \frac{1}{8\pi^2}\left(\frac{g m_\ell}{2M_W}\right)^2$$

K_ℓ are typical factors arising in one loop contributions

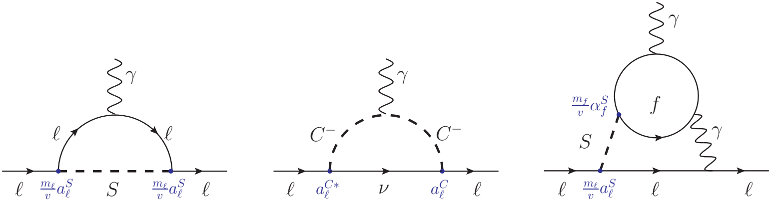
$$K_e \simeq 5.5 \times 10^{-14}, \qquad K_\mu \simeq 2.3 \times 10^{-9}$$

The new contributions to δa_ℓ

- With these values, $K_e \simeq 5.5 \times 10^{-14}$, $K_\mu \simeq 2.3 \times 10^{-9}$, we need

$$\Delta_e \simeq -16, \qquad \Delta_\mu \simeq 1$$

- Contributions at one loop and at two loops (Barr-Zee type) can be relevant



The new contributions to δa_ℓ

- To gain some insight consider the leading terms in $(m_\ell/m_\mathrm{S})^2$ of the one loop contributions in the alignment limit $s_{\alpha\beta} \to 1$

$$\Delta_\ell^{(1)} \simeq [\mathrm{Re}\,(n_\ell)]^2 \left(\frac{I_{\ell\mathrm{H}}}{m_\mathrm{H}^2} - \frac{I_{\ell\mathrm{A}} - 2/3}{m_\mathrm{A}^2} - \frac{1}{6 m_{\mathrm{H}^\pm}{}^2} \right)$$

where

$$I_{\ell\mathrm{S}} = -\frac{7}{6} - 2\ln\left(\frac{m_\ell}{m_\mathrm{S}}\right)$$

[N.B. Same in both models I-gℓFC & II-gℓFC]

- We do not consider light scalars/pseudoscalars,
  we assume $m_\mathrm{h} < m_\mathrm{H}, m_\mathrm{A}$
- Typical values of the loop function for $m_\mathrm{S} \in [0.2; 2.0]$ TeV

$$I_{e\mathrm{S}} \in [24.6; 29.2]\,, \qquad I_{\mu\mathrm{S}} \in [13.9; 18.5]$$

The new contributions to δa_ℓ

$$\Delta_\ell^{(1)} \simeq [\mathrm{Re}\,(n_\ell)]^2 \left(\frac{I_{\ell\mathrm{H}}}{m_\mathrm{H}^2} - \frac{I_{\ell\mathrm{A}} - 2/3}{m_\mathrm{A}^2} - \frac{1}{6 m_{\mathrm{H}^\pm}{}^2}\right)$$

$$I_{e\mathrm{S}} \in [24.6; 29.2]\,, \qquad I_{\mu\mathrm{S}} \in [13.9; 18.5]$$

- Dominant contributions from H and A (log enhanced), $\Delta_e^{(1)} \simeq -16$ can only come from A:

$$\Delta_e^{(1)} \simeq -[\mathrm{Re}\,(n_e)]^2 I_{e\mathrm{A}}/m_\mathrm{A}^2 \text{ requires } [\mathrm{Re}\,(n_e)]^2 \sim m_\mathrm{A}^2$$

  $\Rightarrow$ violate perturbativity requirements for Yukawa couplings or contraints from resonant dilepton searches

✎ We *do not* expect an explanation of $\delta a_e^{\mathrm{Exp,Cs}}$ in terms of one loop contributions

The new contributions to δa_ℓ

$$\Delta_\ell^{(1)} \simeq \mathrm{Re}\,(n_\ell)^2 \left(\frac{I_{\ell\mathrm{H}}}{m_\mathrm{H}^2} - \frac{I_{\ell\mathrm{A}} - 2/3}{m_\mathrm{A}^2} - \frac{1}{6 m_{\mathrm{H}^\pm}{}^2}\right)$$

$$I_{e\mathrm{S}} \in [24.6; 29.2]\,, \qquad I_{\mu\mathrm{S}} \in [13.9; 18.5]$$

- Dominant contributions from H and A (log enhanced), $\Delta_\mu^{(1)} \simeq 1$ can only come from H:

$$\Delta_\mu^{(1)} \simeq [\mathrm{Re}\,(n_\mu)]^2 I_{\mu\mathrm{H}}/m_\mathrm{H}^2 \text{ requires } [\mathrm{Re}\,(n_\mu)]^2 \sim [m_\mathrm{H}/4]^2$$

$\Rightarrow$ a not too heavy H (reasonably perturbative n_μ)
$\Rightarrow$ $m_\mathrm{A} > m_\mathrm{H}$ in order to avoid cancellations

✎ An explanation of $\delta a_\mu^{\mathrm{Exp}}$ in terms of one loop contributions *might be possible*

The new contributions to δa_ℓ

- Dominant two loop contributions: Barr-Zee diagrams
- In the same approximation (leading m_ℓ/m_{S} terms, $s_{\alpha\beta}\to 1$)

$$\Delta_\ell^{(2)} = -\left(\frac{2\alpha}{\pi}\right)\left(\frac{\mathrm{Re}\,(n_\ell)}{m_\ell}\right) F$$

F depends on

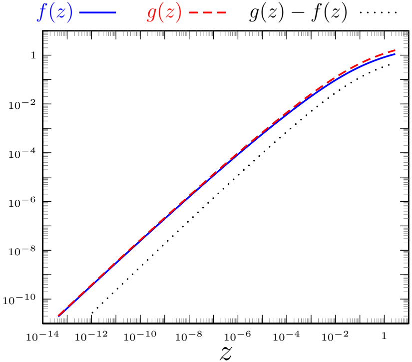
- masses of the fermions in the closed loop,
- couplings of those fermions to H and A,
- m_{H} and m_{A}

$$F_{\mathrm{I}} = \frac{\cot\beta}{3}\left[4(f_{t\mathrm{H}}+g_{t\mathrm{A}})+(f_{b\mathrm{H}}-g_{b\mathrm{A}})\right]+\frac{\mathrm{Re}\,(n_\tau)}{m_\tau}(f_{\tau\mathrm{H}}-g_{\tau\mathrm{A}})\,,$$

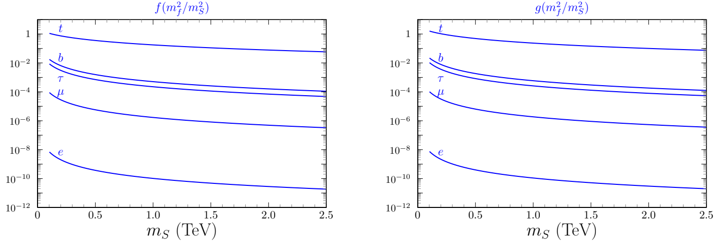
$$F_{\mathrm{II}} = \frac{\cot\beta}{3}\left[4(f_{t\mathrm{H}}+g_{t\mathrm{A}})-\tan^2\beta(f_{b\mathrm{H}}-g_{b\mathrm{A}})\right]+\frac{\mathrm{Re}\,(n_\tau)}{m_\tau}(f_{\tau\mathrm{H}}-g_{\tau\mathrm{A}})$$

The new contributions to δa_ℓ

$$f_{\text{fS}} \equiv f\left(\frac{m_{\text{f}}^2}{m_{\text{S}}^2}\right)\,, \quad g_{\text{fS}} \equiv g\left(\frac{m_{\text{f}}^2}{m_{\text{S}}^2}\right)$$



$$f_{\mathrm{fS}} \equiv f\left(\frac{m_{\mathrm{f}}^2}{m_{\mathrm{S}}^2}\right)\,, \quad g_{\mathrm{fS}} \equiv g\left(\frac{m_{\mathrm{f}}^2}{m_{\mathrm{S}}^2}\right)$$



The new contributions to δa_ℓ

- Relevant aspects
  - $f(z) \simeq g(z)$ in the range of interest
  - the largest values correspond to the heavier fermion
  - the values of f and g for the top quark contributions vary between 0.1 and 1
- Considering the dominant top quark terms, for $t_\beta \simeq 1$ and $m_{\text{H}} \simeq m_{\text{A}}$, one can realize that for $m_{\text{H}} \sim 1-2$ TeV,
  $\delta a_e^{\text{Exp,Cs}}$ can be explained with $\text{Re}\,(n_e) \sim 3-7$ GeV
- To obtain $\delta a_\mu^{\text{Exp}}$ from the same type of contribution

$$\text{Re}\,(n_\mu) = \frac{\delta a_\mu}{\delta a_e}\frac{m_e}{m_\mu}\,\text{Re}\,(n_e) \simeq -15\text{Re}\,(n_e)$$

Different signs of δa_e and δa_μ $\to$ freedom to have
opposite $\text{Re}\,(n_e)$ and $\text{Re}\,(n_\mu)$

Same assumptions $t_\beta \sim 1$, $m_{\text{A}} \sim m_{\text{H}} \sim 1-2$ TeV
$\to \text{Re}\,(n_\mu) \in -[45;105]$ GeV

Argument applies to both models I-gℓFC and II-gℓFC

The new contributions to δa_ℓ

Beyond $t_\beta \sim 1$

- $t_\beta \ll 1$ excluded in 2HDMs of types I and II by flavour constraints $\Rightarrow$ excluded in I-gℓFC and II-gℓFC as well
- What about $t_\beta \gg 1$ and δa_ℓ?
- The factor F

$$\Delta_\ell^{(2)} = -\left(\frac{2\alpha}{\pi}\right)\left(\frac{\mathrm{Re}\,(n_\ell)}{m_\ell}\right) F$$

is quite model dependent

- We consider for reference $t_\beta \sim 1$ and $m_{\mathrm{A}} \sim m_{\mathrm{H}} \sim 1 - 2$ TeV, which can reproduce the anomalies, and analyse how to maintain that prediction if, for definiteness, $t_\beta \mapsto t_\beta = 50$

$$F_{\rm I}=\frac{\cot\beta}{3}\left[4(f_{t\rm H}+g_{t\rm A})+(f_{b\rm H}-g_{b\rm A})\right]+\frac{\text{Re}\,(n_\tau)}{m_\tau}(f_{\tau\rm H}-g_{\tau\rm A})$$

- In I-gℓFC, the $\cot\beta$ suppression can be compensated with *smaller* $m_{\rm H}$, $m_{\rm A}$ and *larger* $\text{Re}\,(n_e)$: e.g. $m_{\rm A}\sim m_{\rm H}\sim 200$ GeV gives a factor of 10 with respect to $m_{\rm A}\sim m_{\rm H}\sim 1-2$ TeV, $\text{Re}\,(n_e)\mapsto 5\text{Re}\,(n_e)$ required to fully compensate the factor of 50
- That is, $\delta a_e^{\rm Exp,Cs}$ can be reproduced by the two loop contributions in the $t_\beta\gg 1$ regime with light H, A and $\text{Re}\,(n_e)\sim 15-35$ GeV
- What about δa_μ?
  $\text{Re}\,(n_\mu)\mapsto 5\text{Re}\,(n_\mu)$ gives $\text{Re}\,(n_\mu)\in -[225;505]$ GeV,
  in conflict with perturbativity requirements!
  Fortunately, for light $m_{\rm H}$, e.g. $m_{\rm H}\in[200;400]$ GeV and $|\text{Re}\,(n_\mu)|\sim m_{\rm H}/4\in[50;100]$ GeV, the one loop contributions can reproduce $\delta a_\mu^{\rm Exp}$!

The new contributions to δa_ℓ

Summarizing, *two* types of solutions

- ✎ "Solution [A]/heavy": scalars with masses in the 1–2 TeV range, $t_\beta \sim 1$, and both anomalies produced by two loop Barr-Zee contributions.
  $\mathrm{Re}\,(n_e)$ in the few GeV range, $\mathrm{Re}\,(n_\mu) \sim -15\mathrm{Re}\,(n_e)$
  Solution a priori present in *both* I-gℓFC and II-gℓFC
- ✎ "Solution [B]/light": $t_\beta \gg 1$, lighter H, $m_{\mathrm{H}} \in [200; 400]$ GeV, and a heavier A. δa_e is obtained with two loop contributions while δa_μ is one loop controlled. Contrary to solution [A], there is *no* linear relation among $\mathrm{Re}\,(n_\mu)$ and $\mathrm{Re}\,(n_e)$, and in fact both signs of $\mathrm{Re}\,(n_\mu)$ can work.
  With this simplified analysis, this second kind of solution seems to be available in the I-gℓFC model, but it is not clear if that is the case in the II-gℓFC model too. From the full numerical analysis, the answer is **no**.

Analysis

Full numerical analysis

- Markov chain MonteCarlo
- Likelihood $\mathcal{L} = e^{-\chi^2/2}$
- Usual form $\chi^2_{\mathcal{O}} = \left(\frac{\mathcal{O}_{\text{Th}} - \mathcal{O}_{\text{Exp}}}{\sigma_{\text{Exp}}}\right)^2$
  - Observable $\mathcal{O}$,
  - prediction $\mathcal{O}_{\text{Th}}$,
  - measurement $\mathcal{O}_{\text{Exp}} \pm \sigma_{\text{Exp}}$
- $+$ correlations, bounds, ...

Constraints

Shopping list

- $\delta a_\ell^{\mathrm{Exp}}$ constraints
- Scalar sector
- Fermion sector
- Higgs signal strengths
- $\mathrm{H}^\pm$ mediated contributions
  - Lepton flavour universality
  - $b \to s\gamma$, B_q^0–$\bar{B}_q^0$ mixing
- $e^+e^- \to \mu^+\mu^-, \tau^+\tau^-$ at LEP
- LHC searches
  - searches of dilepton resonances: $\sigma(pp \to S)_{[\mathrm{ggF}]} \times \mathrm{Br}\left(S \to \ell^+\ell^-\right)$, $S = \mathrm{H}, \mathrm{A}$ and $\ell = \mu, \tau$
  - searches of charged scalars: $\sigma(pp \to \mathrm{H}^\pm tb) \times \mathrm{Br}\left(\mathrm{H}^\pm \to f\right)$, $f = \tau\nu, tb$

Constraints: δa_ℓ

- The anomalies

$$\delta a_e^{\text{Exp,Cs}} = -(8.7 \pm 3.6) \times 10^{-13}\,, \quad \delta a_\mu^{\text{Exp}} = (2.5 \pm 0.6) \times 10^{-9}\,.$$

- The "natural" δa_ℓ constraint

$$\chi_0^2(\delta a_e, \delta a_\mu) = \left(\frac{\delta a_e - c_e}{\sigma_e}\right)^2 + \left(\frac{\delta a_\mu - c_\mu}{\sigma_\mu}\right)^2\,,$$

with $\delta a_\ell^{\text{Exp}} = c_\ell \pm \sigma_\ell$

- We impose a stronger requirement

$$\chi^2(\delta a_e, \delta a_\mu) = 16\chi_0^2(\delta a_e, \delta a_\mu)$$

that is $\sigma_\ell \mapsto \sigma_\ell/4$

Constraints

- Scalar sector
  - potential bounded from below
  - perturbativity and perturbative unitarity of $2 \to 2$ high energy scattering
  - electroweak precision (oblique parameters S, T)

    ☞ one can play with M_W and the CDF value!
- Fermion sector: perturbative Yukawa couplings

$$|n_\ell| \leq n_0$$

with two different choices $n_0 = 100$ GeV or $n_0 = 250$ GeV
- Higgs signal strengths:
  - production × decay signal strengths of the usual channels
  - large lepton couplings: also include $\mathrm{h} \to \mu^+\mu^-, e^+e^-$ information

    $\Rightarrow$ Higgs alignment

Constraints

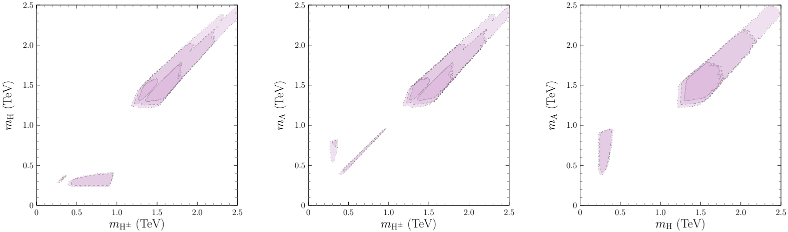
- $\mathrm{H}^{\pm}$ mediated contributions
  - Lepton flavour universality
    - purely leptonic decays $\ell_j \to \ell_k \nu\bar{\nu}$
    - decays with light pseudoscalar mesons $K, \pi \to e\nu, \mu\nu$ and $\tau \to K\nu, \pi\nu$
  - $b \to s\gamma$, B_q^0–$\bar{B}_q^0$ mixing
- $e^+e^- \to \mu^+\mu^-, \tau^+\tau^-$ at LEP

  (cross sections up to $\sqrt{s} = 208$ GeV)
- LHC searches
  - searches of dilepton resonance: $\sigma(pp \to S)_{[\mathrm{ggF}]} \times \mathrm{Br}\left(S \to \ell^+\ell^-\right)$, $S = \mathrm{H}, \mathrm{A}$ and $\ell = \mu, \tau$
  - searches of charged scalars: $\sigma(pp \to \mathrm{H}^{\pm} tb) \times \mathrm{Br}\left(\mathrm{H}^{\pm} \to f\right)$, $f = \tau\nu, tb$

Results

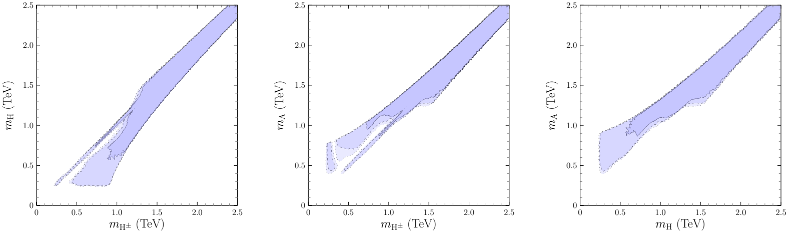
- Scalar potential with exact $\mathbb{Z}_2$
  - gives scalar masses below 1 TeV (no solution [A])
  - does not allow $t_\beta \gg 1$ (no solution [B])

  we introduce soft breaking $\mu_{12}^2 \neq 0$
- Results shown for model I-gℓFC
  - 1,2,3σ 2D-$\Delta\chi^2$ regions
- In II-gℓFC, same solution [A] regions but solution [B] regions absent, results not shown



Masses of the new scalars, $n_0 = 100$ GeV

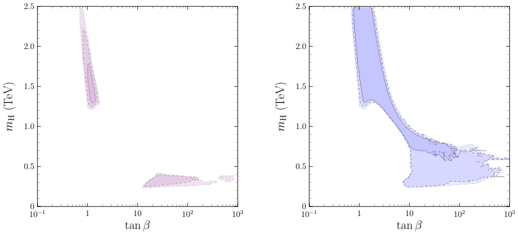
- ☞ Regions corresponding to the two solutions exist
- ☞ For small masses, $m_{\mathrm{H}} < m_{\mathrm{A}}$ and also* $m_{\mathrm{H}} < m_{\mathrm{H}^\pm}$



Masses of the new scalars, $n_0 = 250$ GeV

- Intermediate regions also exist
  (some $|\mathrm{Re}\,(n_\ell)\,| > 100$ GeV required)

Results, model I-gℓFC

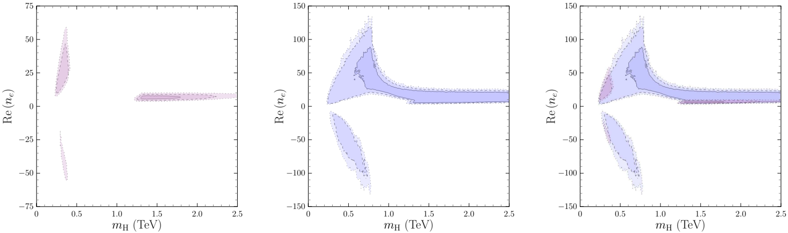


m_H vs t_β

☞ Choice of n_0 has implications for other parameters

N.B. Red/pink for $n_0 = 100$ GeV, blue for $n_0 = 250$ GeV

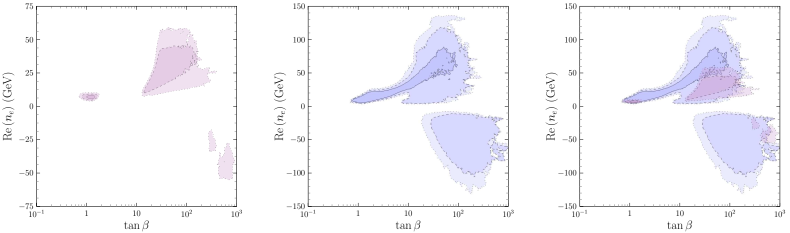
Results, model I-gℓFC



Re (n_e) vs $m_{\rm H}$

☞ Expected and unexpected regions ($\text{Re}\,(n_e) < 0$)!

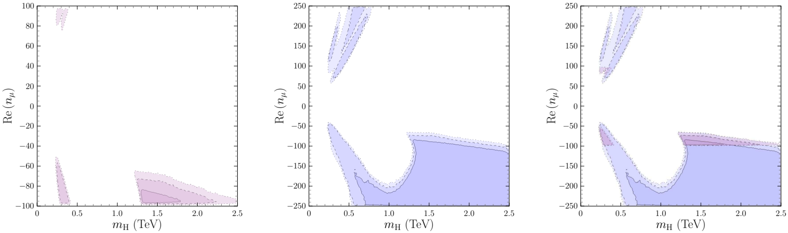
Results, model I-gℓFC



$\mathrm{Re}\,(n_e)$ vs t_β

- ☞ Expected and unexpected regions ($\mathrm{Re}\,(n_e) < 0$)!

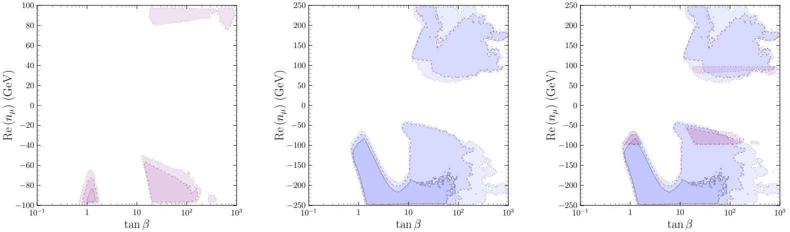
Results, model I-gℓFC



$\mathrm{Re}\,(n_\mu)$ vs m_H

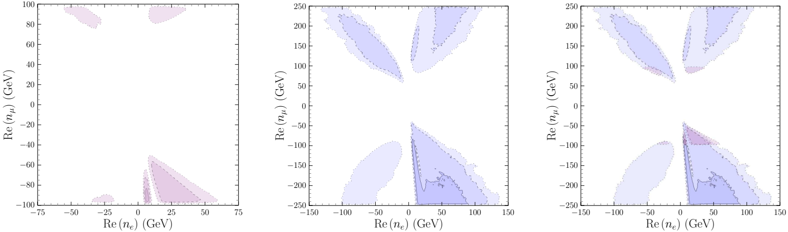
☞ Intermediate region, $m_H \sim 1$ TeV requires $\mathrm{Re}\,(n_\mu) < -150$ GeV

Results, model I-gℓFC



$\mathrm{Re}\,(n_\mu)$ vs t_β

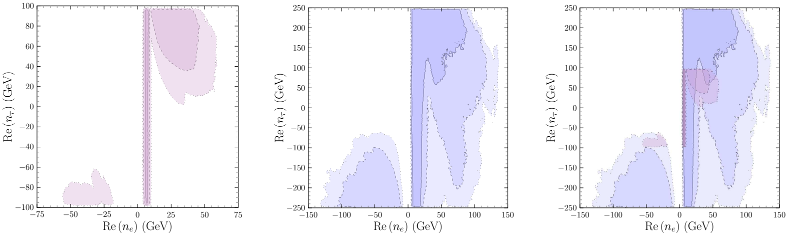
Results, model I-gℓFC



$\mathrm{Re}\,(n_\mu)$ vs $\mathrm{Re}\,(n_e)$

- Another view of unexpected regions $\mathrm{Re}\,(n_e) < 0$

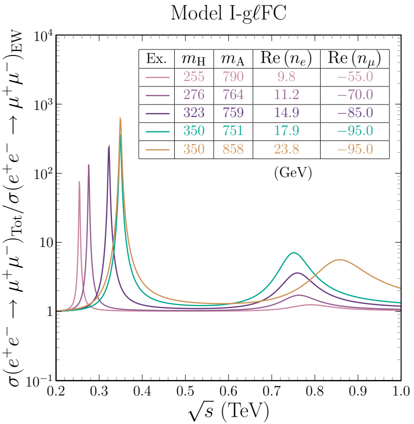
Results, model I-gℓFC



$\mathrm{Re}\,(n_\tau)$ vs $\mathrm{Re}\,(n_e)$

✎ A hint of the origin of $\mathrm{Re}\,(n_e) < 0$ regions

Results, model I-gℓFC beyond LHC



$e^+e^- \to \mu^+\mu^-$ for $\sqrt{s} \in [0.2; 1.0]$ TeV,
examples with "light" solution

Conclusions

- General 2HDMs without SFCNC in the lepton sector are a robust framework (stable under RGE)
- Lepton flavour universality is broken beyond $\propto$ mass
- Two models, I-gℓFC & II-gℓFC, to address the δa_ℓ anomalies
- Quark & scalar sector as type I, II 2HDMs, softly broken $\mathbb{Z}_2$
- Different types of solutions in agreement with constraints
  1. "Heavy", present in both models
     - new scalars have masses in the 1–2.5 TeV range,
     - $v_1 \sim v_2$,
     - both δa_ℓ from two loop Barr-Zee contributions
  2. "Light", present in I-gℓFC, not in II-gℓFC
     - new scalars have masses below 1 TeV,
     - $v_1 \ll v_2$,
     - δa_e from two loop Barr-Zee contributions, δa_μ from one loop
  3. "Intermediate"

Conclusions

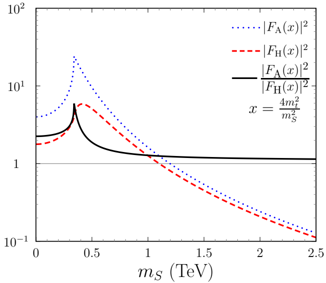
- Involved numerical analysis (subtleties & unexpected regions)
  - Gluon-gluon fusion production of scalar vs pseudoscalar
  - Role of $\mathrm{A} \to \mathrm{H}Z$
  - τ loop in Barr-Zee contributions
- Further avenues
  - Role of different $\delta a_e^{\mathrm{Exp}}$ values
  - CDF M_W through ΔS and ΔT
  - a_τ, excess in $(pp)_{\mathrm{ggF}} \to \mathrm{S} \to \tau\bar{\tau}$?

Thank you!

Backup

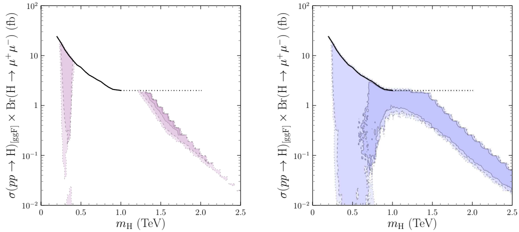
- Scalar vs pseudoscalar gluon-gluon fusion
- More results
- M_W from CDF
- Different δa_e^{Exp}
- δa_ℓ contributions in detail
- Yukawa couplings

Gluon-gluon-scalar vs gluon-gluon-pseudoscalar



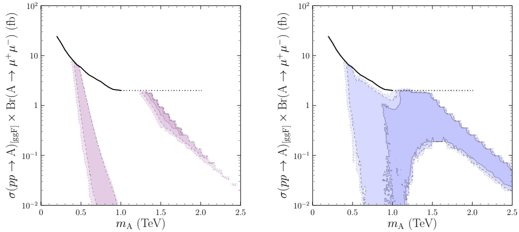
$F_{\rm H}(x)$ from $\mathrm{S}\bar{t}t$, $F_{\rm A}(x)$ from $\mathrm{S}\bar{t}\gamma_5 t$

More results, model I-gℓFC



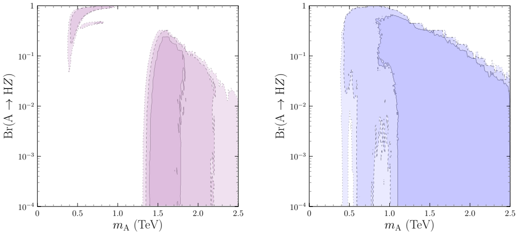
$$[pp]_{\mathrm{ggF}} \to \mathrm{H} \to \mu^+\mu^- \text{ vs. } m_\mathrm{H}$$

More results, model I-gℓFC



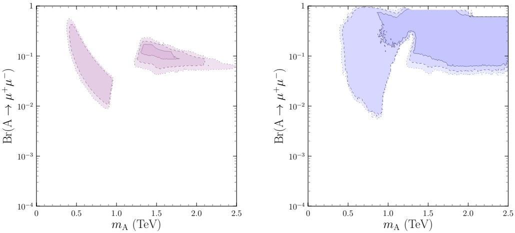
$$[pp]_{ggF} \to A \to \mu^+\mu^- \text{ vs. } m_A$$

More results, model I-gℓFC



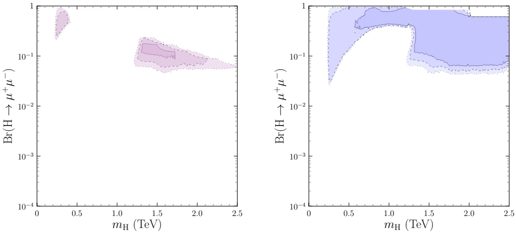
A → HZ decay

More results, model I-gℓFC



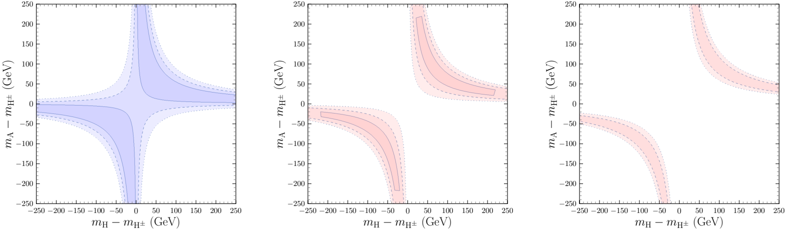
$\mathrm{A} \to \mu^+\mu^-$ decay

More results, model I-gℓFC



$\mathrm{H}\to\mu^+\mu^-$ decay

Oblique parameters and M_W from CDF



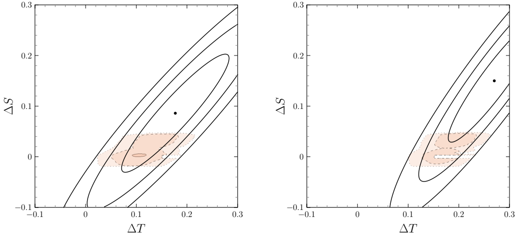
$m_A - m_{H^\pm}$ vs $m_H - m_{H^\pm}$ for $m_{H^\pm} = 1$ TeV

$\Delta S = 0.00 \pm 0.07$	$\Delta S = 0.086 \pm 0.077$	$\Delta S = 0.15 \pm 0.08$
$\Delta T = 0.05 \pm 0.06$	$\Delta T = 0.177 \pm 0.070$	$\Delta T = 0.27 \pm 0.06$
$\rho = 0.92$	$\rho = 0.89$	$\rho = 0.93$
	"Conservative"	
	2204.04204	2204.03796

Oblique parameters and M_W from CDF

Bottom line: since $m_{\mathrm{H}^\pm}$ is rather irrelevant for δa_ℓ,

a simple shift in $m_{\mathrm{H}^\pm}$ may work

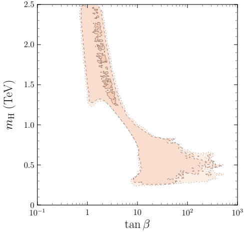


"Conservative"

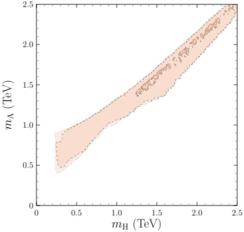
ΔT vs ΔS

Oblique parameters and M_W from CDF

"Conservative" scenario



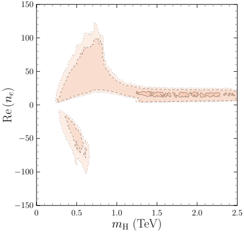
m_{H} vs t_β



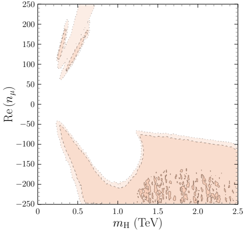
m_{A} vs m_{H}

Oblique parameters and M_W from CDF

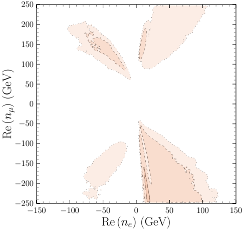
"Conservative" scenario



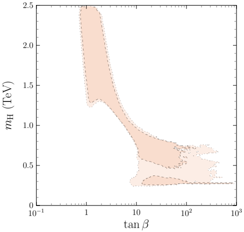
$\mathrm{Re}\,(n_e)$ vs $m_{\rm H}$



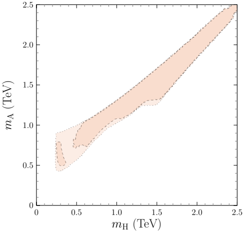
$\mathrm{Re}\,(n_\mu)$ vs $m_{\rm H}$



$\mathrm{Re}\,(n_\mu)$ vs $\mathrm{Re}\,(n_\mu)$

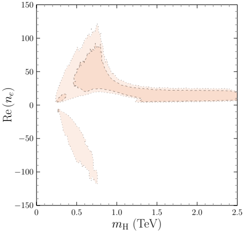


m_{H} vs t_β

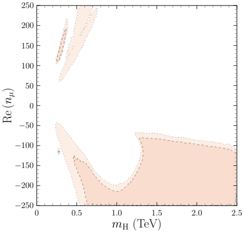


m_{A} vs m_{H}

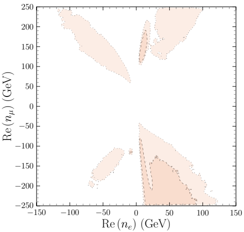
Oblique parameters and M_W from CDF



$\text{Re}\,(n_e)$ vs m_H



$\text{Re}\,(n_\mu)$ vs m_H



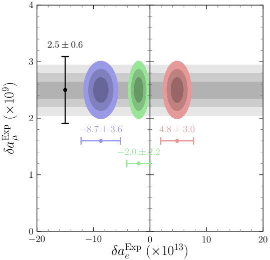
$\text{Re}\,(n_\mu)$ vs $\text{Re}\,(n_\mu)$

Different δa_e^{Exp} assumptions

Analyses with different "measurements"

$$\delta a_e^{\text{Exp,Cs}} = -(8.7 \pm 3.6) \times 10^{-13},$$
$$\delta a_e^{\text{Exp,Rb}} = (4.8 \pm 3.0) \times 10^{-13}$$
$$\delta a_e^{\text{Exp,Avg}} = -(2.0 \pm 2.2) \times 10^{-13},$$
$$\delta a_e^{\text{Exp,Bound}} = 20 \times 10^{-13}$$

N.B. Last case: $|\delta a_e| < \delta a_e^{\text{Exp,Bound}}$
In all cases $n_0 = 250$ GeV



Different $\delta a_e^{\mathrm{Exp}}$ assumptions

Simple analysis

- For a point in parameter space explaining $\delta a_e^{\mathrm{Exp,Cs}}$,
  1. consider $\mathrm{Re}\,(n_e) \mapsto \mathrm{Re}\,(n_e)\,\frac{\delta a_e^{\mathrm{Exp}}}{\delta a_e^{\mathrm{Exp,Cs}}}$, which gives $\delta a_e \simeq \delta a_e^{\mathrm{Exp}}$
  2. analyse if this new value conflicts with other observables sensitive to $\mathrm{Re}\,(n_e)$: muon decay and pseudoscalar mesons decays
- Answer
  - Short version: $\delta a_e^{\mathrm{Exp,Cs}}$ is "worst case"

    because of absolute value and sign
  - Long version in the next slides

✎ all cases can be reproduced at least with the regions arising from the previous $\mathrm{Re}\,(n_e)$ transformation

Different δa_e^{Exp} assumptions

- For muon decay

$$\left|\frac{\text{Re}\,(n_e)\,\text{Re}\,(n_\mu)}{m_{\text{H}^\pm}^2}\right| < 0.035$$

$\Rightarrow$ $|\text{Re}\,(n_e)|$ for $\delta a_e^{\text{Exp,Cs}}$ is "worse" than other cases

- For pseudoscalar meson decays, consider

$$R_{\mu e}^P = \frac{\Gamma(P^+ \to \mu^+\nu)}{\Gamma(P^+ \to \mu^+\nu)_{\text{SM}}}\frac{\Gamma(P^+ \to e^+\nu)_{\text{SM}}}{\Gamma(P^+ \to e^+\nu)}$$

Experimental values:

$$R_{\mu e}^\pi = 1 + (4.1 \pm 3.3) \times 10^{-3}\,, \qquad R_{\mu e}^K = 1 - (4.8 \pm 4.7) \times 10^{-3}$$

Model prediction:

$$R_{\mu e}^P = \frac{|1-\Delta_\mu^P|^2}{|1-\Delta_e^P|^2}\,, \qquad |1-\Delta_\ell^P|^2 = \left|1 - \frac{M_P^2}{t_\beta m_{\text{H}^\pm}^2}\frac{\text{Re}\,(n_\ell)}{m_\ell}\right|^2$$

Different δa_e^{Exp} assumptions

Experimental values:

$$R_{\mu e}^{\pi} = 1 + (4.1 \pm 3.3) \times 10^{-3}\,, \qquad R_{\mu e}^{K} = 1 - (4.8 \pm 4.7) \times 10^{-3}$$

Model prediction:

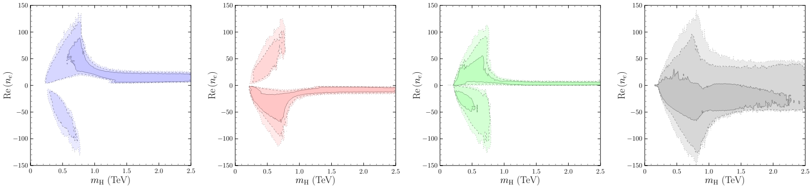
$$R_{\mu e}^{P} = \frac{|1-\Delta_\mu^P|^2}{|1-\Delta_e^P|^2}\,, \qquad |1-\Delta_\ell^P|^2 = \left|1 - \frac{M_P^2}{t_\beta m_{\mathrm{H}^\pm}^2}\frac{\mathrm{Re}\,(n_\ell)}{m_\ell}\right|^2$$

- For $\Delta_\ell^P \ll 1$,

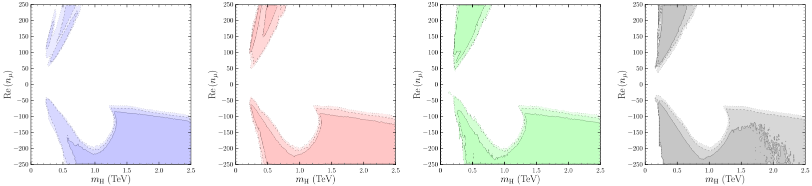
$$R_{\mu e}^{P} \simeq 1 + 2\frac{M_P^2}{t_\beta m_{\mathrm{H}^\pm}^2}\left(\frac{\mathrm{Re}\,(n_e)}{m_e} - \frac{\mathrm{Re}\,(n_\mu)}{m_\mu}\right)$$

- Concentrate on $R_{\mu e}^{K}$, neglect the $\mathrm{Re}\,(n_\mu)$ contribution:
  $\mathrm{Re}\,(n_e) > 0$ for $\delta a_e^{\text{Exp,Cs}}$ is "worse" than other cases

Different δa_e^{Exp} assumptions

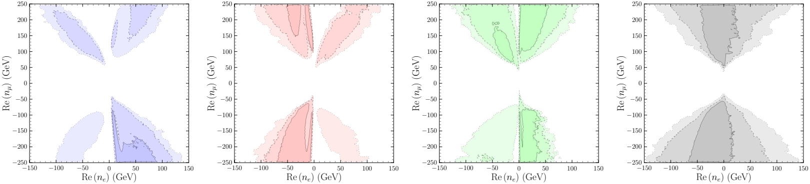


$\text{Re}\,(n_e)$ vs m_H



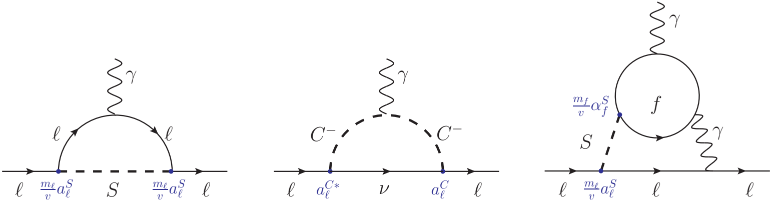
$\text{Re}\,(n_\mu)$ vs m_H

Different $\delta a_e^{\mathrm{Exp}}$ assumptions



$\mathrm{Re}\,(n_\mu)$ vs $\mathrm{Re}\,(n_e)$

Loop contributions to δa_ℓ



One loop contributions to δa_ℓ

Yukawa interactions of the form

$$\mathscr{L}_{S\ell\ell} = -\frac{m_\ell}{v} S\bar{\ell}(a_\ell^S + i b_\ell^S \gamma_5)\ell$$

give one loop contributions

$$\Delta a_\ell^{(1)} = \frac{1}{8\pi^2}\frac{m_\ell^2}{v^2}\sum_S \left\{[a_\ell^S]^2\left(2I_2(x_{\ell S}) - I_3(x_{\ell S})\right) - [b_\ell^S]^2 I_3(x_{\ell S})\right\},$$

with $x_{\ell S} \equiv m_\ell^2/m_S^2$ and

$$I_2(x) = 1 + \frac{1-2x}{2x\sqrt{1-4x}}\ln\left(\frac{1+\sqrt{1-4x}}{1-\sqrt{1-4x}}\right) + \frac{1}{2x}\ln x$$

$$I_3(x) = \frac{1}{2} + \frac{1}{x} + \frac{1-3x}{2x^2\sqrt{1-4x}}\ln\left(\frac{1+\sqrt{1-4x}}{1-\sqrt{1-4x}}\right) + \frac{1-x}{2x^2}\ln x$$

One loop contributions to δa_ℓ

For $x \ll 1$,

$$I_2(x) \simeq x\left(-\frac{3}{2} - \ln x\right) + x^2\left(-\frac{16}{3} - 4\ln x\right) + \mathcal{O}(x^3)$$
$$I_3(x) \simeq x\left(-\frac{11}{6} - \ln x\right) + x^2\left(-\frac{89}{12} - 5\ln x\right) + \mathcal{O}(x^3)$$

For $m_\ell \ll m_S$,

$$\Delta a_\ell^{(1)} = \frac{1}{8\pi^2}\frac{m_\ell^2}{m_S^2}\frac{m_\ell^2}{v^2}\left\{-[a_\ell^S]^2\left(\frac{7}{6} + \ln\left(\frac{m_\ell^2}{m_S^2}\right)\right) + [b_\ell^S]^2\left(\frac{11}{6} + \ln\left(\frac{m_\ell^2}{m_S^2}\right)\right)\right\}$$

One loop contributions to δa_ℓ

Yukawa interactions of the form

$$\mathscr{L}_{C\ell\nu} = -C^-\bar{\ell}(a_\ell^C + ib_\ell^C\gamma_5)\nu - C^+\bar{\nu}(a_\ell^{C*} + ib_\ell^{C*}\gamma_5)\ell$$

give one loop contributions

$$\Delta a_\ell^{(1)} = -\frac{1}{8\pi^2}\sum_C \left\{|a_\ell^C|^2 + |b_\ell^C|^2\right\} H(x_{\ell C})$$

where $x_{\ell C} = m_\ell^2/m_{C^\pm}^2$, and

$$H(x) = -\frac{1}{2} + \frac{1}{x} + \frac{1-x}{x^2}\ln(1-x)\,, \quad H(x) \simeq \frac{x}{6} + \frac{x^2}{12} + \mathcal{O}(x^3) \text{ for } x \ll 1$$

Two loop contributions to δa_ℓ

For quarks

$$\mathscr{L}_{S\bar{f}f} = -\frac{m_f}{v} S \bar{f} (\alpha_f^S + i\beta_f^S \gamma_5) f$$

The two loop Barr-Zee contributions to the anomalous magnetic moment of lepton ℓ

$$\Delta a_\ell^{(2)} = -\frac{\alpha^2}{4\pi^2 s_W^2} \frac{m_\ell^2}{M_W^2} \sum_f \sum_S N_c^f Q_f^2 \left\{ a_\ell^S \alpha_f^S f(z_{fS}) - b_\ell^S \beta_f^S g(z_{fS}) \right\}$$

f: fermions in the closed fermion loop

(N_c^f colour, Q_f electric charge, $z_{fS} = m_f^2/m_S^2$)

S: neutral scalar connecting the closed fermion loop with the external lepton line

Two loop contributions to δa_ℓ loop contributions to δa_ℓ

Loop functions

$$f(z) = \frac{z}{2}\int_0^1 dx\, \frac{1-2x(1-x)}{x(1-x)-z}\, \ln\left(\frac{x(1-x)}{z}\right)$$

$$g(z) = \frac{z}{2}\int_0^1 dx\, \frac{1}{x(1-x)-z}\, \ln\left(\frac{x(1-x)}{z}\right)$$

Yukawa couplings

Neutral scalars

$$\mathscr{L}_{\mathrm{S}\bar{f}f} = -\frac{\mathrm{S}}{v}\,\bar{f}\left[\mathcal{R}_{1s}\mathrm{M}_f + \mathcal{R}_{2s}\frac{\mathrm{N}_f + \mathrm{N}_f^\dagger}{2} + i\epsilon_{(f)}\mathcal{R}_{3s}\frac{\mathrm{N}_f - \mathrm{N}_f^\dagger}{2}\right] f$$
$$-\frac{\mathrm{S}}{v}\,\bar{f}\,\gamma_5\left(\mathcal{R}_{2s}\frac{\mathrm{N}_f - \mathrm{N}_f^\dagger}{2} + i\epsilon_{(f)}\mathcal{R}_{3s}\frac{\mathrm{N}_f + \mathrm{N}_f^\dagger}{2}\right) f$$

where $s = 1, 2, 3$ in correspondence with $\mathrm{S} = \mathrm{h}, \mathrm{H}, \mathrm{A}$; $f = u, d, \ell$; in terms proportional to $\mathcal{R}_{3s}$, $\epsilon_{(d)} = \epsilon_{(\ell)} = -\epsilon_{(u)} = 1$

Yukawa couplings

Charged scalars

$$\mathscr{L}_{\mathrm{H}^\pm ud} = \frac{\mathrm{H}^-}{\sqrt{2}v}\bar{d}\left[V^\dagger \mathrm{N}_u - \mathrm{N}_d^\dagger V^\dagger + \gamma_5\left(V^\dagger \mathrm{N}_u + \mathrm{N}_d^\dagger V^\dagger\right)\right] u + \frac{\mathrm{H}^+}{\sqrt{2}v}\bar{u}\left[\mathrm{N}_u^\dagger V - V\mathrm{N}_d + \gamma_5\left(\mathrm{N}_u^\dagger V + V\mathrm{N}_d\right)\right] d$$

and

$$\mathscr{L}_{\mathrm{H}^\pm \ell\nu} = -\frac{\sqrt{2}}{v}\mathrm{H}^+\,\bar{\nu}_L\,U^\dagger \mathrm{N}_\ell\,\ell_R - \frac{\sqrt{2}}{v}\mathrm{H}^-\,\bar{\ell}_R\,\mathrm{N}_\ell^\dagger U\,\nu_L$$

V and U are, respectively, the CKM and PMNS mixing matrices (massless neutrinos assumed, one can set $U \to \mathbf{1}$)