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On-Shell Renormalization of Scalar Sectors

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- Inspiration: fermions, Nielsen identities... arXiv:2107.09361
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Fermions I

$$\begin{tikzcd}
 & \delta Z_{ji} \arrow[dl, leftrightarrow] \arrow[dr, leftrightarrow] & \\
\delta m_{ji} \arrow[rr, leftrightarrow] & & \delta V_{ji}
\end{tikzcd}$$

Renormalization and mixing:

$\psi^0_j \longrightarrow Z_{jk}\,\psi_k$ flavour indices

$m^0_{ji} \longrightarrow m_i + \delta m_{ji}$

non-diagonal! $\Longrightarrow$ $\boxed{\delta V_{ji} = 0}$

Renormalization commutes with basis rotations

Fermions II

On-Shell conditions:

spinor $\not{p} u_i = m_i$

$$\boxed{\Sigma^R_{ji}(\not{p}) u_i = 0} \xRightarrow{\text{1-loop}} \left[(m_i^2 - m_j^2)\,\delta Z_{ji} \boxed{-m_j \delta m_{ji} - \delta m^\dagger_{ji} m_i} \right] u_i = -(\not{p} + m_j)\Sigma_{ji}(\not{p}) u_i$$

self-energy

Includes tadpoles!

$(m_i^2 - m_j^2)$?

- δZ^A and δm are degenerate !
- δZ^H is as usual

$$(\not{p} + m_j)\Sigma_{ji}(\not{p}) u_i \Big|^{\text{Hermitian}}_{UV} \sim \frac{1}{\epsilon_{UV}} \times \{ m_i,\ m_j,\ 2 m_i m_j,\ m_i^2 + m_j^2 \}$$

from 1-loop PV functions

associated with δm!

NO $m_i^2 - m_j^2$!

$$\dots \Big|^{A}_{UV} \sim \frac{1}{\epsilon_{UV}} \times \{ m_i^2 - m_j^2 \}$$

Fermions III

Nielsen Identity:

$$\boxed{\partial_\xi \Sigma_{ji}(\not{p}) = \left[\Lambda \Sigma + \Sigma \bar{\Lambda}\right]_{ji}}$$

→ All orders

→ Analogous for Scalars

→ $\overset{(-)}{\Lambda} = \overset{(-)}{\Lambda}(\not{p})$

→ $\overset{(-)}{\Lambda} = 0$ @ tree-level

$\Downarrow$ 1-loop

$$\partial_\xi \Sigma_{ji}(\not{p}) = \Big[\Lambda \underbrace{(\not{p} - m_i)}_{=0} + (\not{p} - m_j)\bar{\Lambda}\Big]_{ji}$$

$\Downarrow$ OS

$$(\not{p} + m_j)\,\partial_\xi \Sigma_{ji}(\not{p})\,u_i = (m_i^2 - m_j^2)\,\bar{\Lambda}\,u_i$$

Fermions IV

$$\boxed{\begin{aligned} &\delta Z^{A}_{ji}\, u_i \equiv -(\not{p}+m_j)\,\Sigma_{ji}\, u_i \text{ „}+\text{ H.c.“} \Big|_{(m_i^2-m_j^2)} \\ &\delta m = \delta m(\Sigma, \delta Z^A) \end{aligned}}$$

1-loop!

↳ Explicit computation @1-loop for SM and 2HDM fermions

- Can be shown that the same „logic“ is valid to **ALL ORDERS** in perturbation theory!

Can we apply this to Scalar Sectors?

Scalars I

Similar to fermions @1-loop

$$\Pi^R_{ji}(p^2) = \Pi_{ji}(p^2) + \delta Z^\dagger_{ji}(p^2 - m_i^2) + (p^2 - m_j^2)\,\delta Z_{ji} - \delta m^2_{ji}$$

$$\Downarrow$$

$$\partial_\xi \Pi_{ji}(p^2) = (\Lambda\Pi)_{ji} + (\Pi\Lambda^\dagger)_{ji} \xRightarrow{\text{1-loop}} \partial_\xi \Pi_{ji}(p^2) = \Lambda_{ji}\underbrace{(p^2 - m_i^2)}_{=0} + (p^2 - m_j^2)\Lambda^\dagger_{ji}$$

$$\xRightarrow{\text{OS}} \partial_\xi \Pi_{ji}(m_i^2) = (m_i^2 - m_j^2)\Lambda^\dagger_{ji}$$

$$\Rightarrow \boxed{\begin{aligned} \delta Z^A_{ji} &= -\frac{1}{2}\left(\Pi_{ji}(m_i^2) + \Pi_{ji}(m_j^2)\right)\Big|_{m_i^2 - m_j^2} \\ \delta m &= \delta m(\Pi, \delta Z^A) \end{aligned}}$$

→ Extend to all orders, but....

↳ Explicit computation @1-loop in 2HDM

Scalars II

- Physical states MIX with Goldstones and longitudinal modes

- NI is in terms of $\underbrace{\Gamma^c}_{\text{Reduced}} = \underbrace{\Gamma}_{\text{1PI}} - \underbrace{\int d^4x \, \mathcal{L}_{GF}}_{= \Gamma^{GF}}$ (No effect for physical states)

$$\Rightarrow \partial_\xi \Pi_{ji}(p^2) = \left[\Lambda \Pi^c + \Pi^c \Lambda^\dagger\right]_{ji}$$

$$= \left[\Lambda(\Pi - \Gamma^{GF}) + (\Pi - \Gamma^{GF})\Lambda^\dagger + \boxed{\partial_\xi \Gamma^{GF}}\right]_{ji}$$

tree-level: $\Pi^c = (p^2 - m_G^2) + m_G^2 = p^2 \Rightarrow$ no $m_i^2 - m_j^2$!

additional disruptions

- Non-physical masses are gauge-dependent

$$\partial_\xi M_{ji}(m_G^2) = \text{—}//\text{—} + \boxed{\partial_\xi m_G^2 \, \partial_{p^2} \Pi(p^2)\Big|_{p^2 = m_G^2}}$$

Scalars III

What to do?!

- $(m_i^2 - m_G^2)\,\delta Z^A_{Gi} - \delta m_{Gi} = -\frac{1}{2}\Big(\underbrace{\Pi_{Gi}(m_i^2) + \Pi_{Gi}(m_G^2)}\Big)$ no $m_i^2 - m_G^2$!

G = „Goldstone" ≡ „Unphysical"

- Keep the same definition..?

$\Rightarrow \delta Z^A_{Gi} = 0$

$\Rightarrow \delta m_{Gi} = \frac{1}{2}\left(\Pi_{Gi}(m_i^2) + \Pi_{Gi}(m_G^2)\right)$

- ▷ $\partial_\xi \delta m_{Gi} \neq 0$, but OK

the mass is anyways gauge-dep for non-physical states

- ▷ no changes to δZ^H_{Gi}

but the gauge-dep is more complicated

Scalars IV

the real $\boxed{\text{PROBLEM!}}$

Higher orders depend on all previous orders!

exclude tree-level

(order)

$$\partial_\xi\left[(m_i^2 - m_j^2)\,\delta Z_{ji}^{(n)} - \delta m_{ji}^{(n)}\right] = -\partial_\xi\left[\Pi(m_i^2) + \boxed{\Pi^{(\not 0)}(m_i^2)\,\delta Z}\right]_{ji}^{(n)}$$

short-hand notation; includes c.t.'s up to order n

includes non-physical parts

Can
- $(m_i^2 - m_j^2)$
- $\partial_\xi \delta m_{ji} = 0$

be preserved for physical states to all orders?

Summary / Outlook

- 1-loop physical states OK
  - δZ^A as coeff. of $m_i^2 - m_j^2$; also UV finite
  - Solve for δm_{ji}
  - $\delta V_{ji} = 0$, $\partial_\xi \delta m_{ji} = 0$
  - Universal, process-independent...
- 1-loop non-physical : no $m_i^2 - m_j^2$ => $\delta Z^A \overset{?}{=} 0$?
  - δm^2 and δZ^H ensure OS and $\partial_\xi \delta m^2 \neq 0$?

! Not obvious n^{th} order extension
  - depends on all previous orders

Outlook : find the all order extension