



Phenomenology of a flavoured multiscalar BGL-like model with three generations of massive neutrinos

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A generic Next-to-Minimal Two Higgs Doublet Model (NTHDM) with a BGL structure

An SM extension with:

- a flavour non-universal $U(1)'$ global symmetry,
- a second Higgs Doublet Φ_2 ,
- a scalar singlet S
- three generations of right-handed neutrinos $\nu_R^{1,2,3}$, with a type-I seesaw mechanism

That follow the Branco-Grimus-Lavoura (BGL) quark textures.



$$\begin{aligned}
 -\mathcal{L}_{\text{Yukawa}} = & \overline{q_L^0} \Gamma_a \Phi^a d_R^0 + \overline{q_L^0} \Delta_a \tilde{\Phi}^a u_R^0 + \overline{\ell_L^0} \Pi_a \Phi^a e_R^0 + \overline{\ell_L^0} \Sigma_a \tilde{\Phi}^a \nu_R^0 \\
 & + \frac{1}{2} \overline{\nu_R^{c0}} (A + BS + CS^*) \nu_R^0 + \text{h.c.},
 \end{aligned}$$

$\Gamma_\alpha, \Delta_\alpha$: Yukawa matrices for the down- and up- quarks,

$\Pi_\alpha, \Sigma_\alpha$: Yukawa matrices for the charged leptons and neutrinos

B, C : Majorana-like Yukawa matrices

A : Majorana mass term

$$\Gamma_1 : \begin{pmatrix} \times & \times & \times \\ \times & \times & \times \\ 0 & 0 & 0 \end{pmatrix}, \Gamma_2 : \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ \times & \times & \times \end{pmatrix}, \Delta_1 : \begin{pmatrix} \times & \times & 0 \\ \times & \times & 0 \\ 0 & 0 & 0 \end{pmatrix}, \Delta_2 : \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & \times \end{pmatrix}$$

Note: The choice of textures implies that tree-level FCNCs will appear only in the down quark sector



BGL was introduced in: G. C. Branco, W. Grimus, and L. Lavoura, [Phys. Lett. **B380**, 119 \(1996\)](#), [arXiv:hep-ph/9601383 \[hep-ph\]](#).

Rotating the Yukawa matrices in the Higgs base:

$$(N_u)_{ij} = \left(t_\beta \delta_{ij} - \left(t_\beta + t_\beta^{-1} \right) \delta_{ij} \delta_{j3} \right) m_{u_j},$$

$$(N_d)_{ij} = \left(t_\beta \delta_{ij} - \left(t_\beta + t_\beta^{-1} \right) V_{3i}^* V_{3j} \right) m_{d_j},$$

- Only the down-quark sector has non-diagonal terms (FCNCs on the down sector)
- FCNCs suppressed by CKM matrix elements



The potential is defined as $V = V_0 + V_1$

$$V_0 = \mu_i^2 |\Phi^i|^2 + \lambda_i |\Phi^i|^4 + \lambda_3 |\Phi_1|^2 |\Phi_2|^2 + \lambda_4 |\Phi_1^\dagger \Phi_2|^2 + \mu_S^2 |S|^2 + \lambda'_1 |S|^4 \\ + \lambda'_2 |\Phi_1|^2 |S|^2 + \lambda'_3 |\Phi_2|^2 |S|^2 \quad (i = 1, 2) \text{ and}$$

$$V_1 = \mu_3^2 \Phi_2^\dagger \Phi_1 + \frac{1}{2} \mu_b^2 S^2 + a_1 \Phi_1^\dagger \Phi_2 S + a_2 \Phi_1^\dagger \Phi_2 S^\dagger + a_3 \Phi_1^\dagger \Phi_2 S^2 + a_4 \Phi_1^\dagger \Phi_2 S^{\dagger 2} + \text{h.c.} \dots$$

Given that the singlet S carries a non-trivial $U(1)'$ charge X_S , then, out of the four $a_{1,2,3,4}$ and μ_b terms, only one is allowed in the limit of an exact $U(1)'$. However, both a_1 and a_2 , as well as μ_b , can be introduced to softly break the flavour symmetry.

Also the model is gauge anomaly free^[1]

^[1]This work was inspired considering local $U(1)'$ symmetry where gauge anomalies are forbidden.



Anomaly-free solution

1. ν BGL-I Scenario

$$\Pi_1, \Sigma_1, B = \begin{pmatrix} \times & \times & 0 \\ \times & \times & 0 \\ 0 & 0 & 0 \end{pmatrix}, \quad \Pi_2, \Sigma_2 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & \times \end{pmatrix},$$

$$A = 0, \quad C = \begin{pmatrix} 0 & 0 & \times \\ 0 & 0 & \times \\ \times & \times & 0 \end{pmatrix}.$$

2. ν BGL-IIa Scenario

$$\Pi_1, \Sigma_1 = \begin{pmatrix} \times & 0 & 0 \\ 0 & \times & 0 \\ 0 & 0 & 0 \end{pmatrix},$$

$$\Pi_2 = \begin{pmatrix} 0 & 0 & 0 \\ \times & 0 & 0 \\ 0 & 0 & \times \end{pmatrix}, \quad \Sigma_2 = \begin{pmatrix} 0 & \times & 0 \\ 0 & 0 & 0 \\ 0 & 0 & \times \end{pmatrix}$$

$$A = \begin{pmatrix} \times & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \quad B = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & \times \\ 0 & \times & 0 \end{pmatrix}, \quad C = 0.$$

3. ν BGL-IIb Scenario

$$A = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & \times \\ 0 & \times & 0 \end{pmatrix}, \quad B = \begin{pmatrix} \times & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix},$$

$$C = 0.$$



Charges ν BGL-I			ν BGL-IIa	ν BGL-IIb	Charges ν BGL-I			ν BGL-IIa	ν BGL-IIb
q_L	$\begin{bmatrix} x \\ x \\ x_{tL} \end{bmatrix}$	---		---	e_R	$\begin{bmatrix} -2x - y \\ -2x - y \\ 30x - 9y \end{bmatrix}$	$\begin{bmatrix} 2x - 2y \\ -6x \\ 30x - 9y \end{bmatrix}$	$\frac{1}{3} \begin{bmatrix} 2x - 5y \\ -14x - y \\ 58x - 19y \end{bmatrix}$	
u_R	$\begin{bmatrix} y \\ y \\ x_{tR} \end{bmatrix}$	---		---	ν_R	$\begin{bmatrix} -4x + y \\ -4x + y \\ 12x - 3y \end{bmatrix}$	$\begin{bmatrix} 0 \\ -8x + 2y \\ 12x - 3y \end{bmatrix}$	$\frac{1}{3} \begin{bmatrix} -4x + y \\ -20x + 5y \\ 20x - 5y \end{bmatrix}$	
d_R	$\begin{bmatrix} 2x - y \\ 2x - y \\ 2x - y \end{bmatrix}$	---		---	Φ	$\begin{bmatrix} -x + y \\ -9x + 3y \end{bmatrix}$	$\begin{bmatrix} -x + y \\ -9x + 3y \end{bmatrix}$	$\frac{1}{3} \begin{bmatrix} 3(-x + y) \\ -19x + 7y \end{bmatrix}$	
ℓ_L	$\begin{bmatrix} -3x \\ -3x \\ 21x - 6y \end{bmatrix}$	$\begin{bmatrix} x - y \\ -7x + y \\ 21x - 6y \end{bmatrix}$	$\frac{1}{3} \begin{bmatrix} -x - 2y \\ -17x + 2y \\ 39x - 12y \end{bmatrix}$		S	$8x - 2y$	$-4x + y$	$\frac{8x - 2y}{3}$	

Table 1: Allowed charges for the various models. For model ν BGL-I and -IIa we have $x_{tL} = -7x + 2y$ and $x_{tR} = -16x + 5y$. Model ν BGL-IIb has $x_{tL} = (-13x + 4y)/3$ and $x_{tR} = (-32x + 11y)/3$. In order for the BGL textures to be preserved, we additionally require that $y \neq 4x$.



Chosen Scenario: ν BGL-I

$x = 1, y = 1/3$

	Φ_1	Φ_2	S	q_1	q_2	q_3	u_{R_1}	u_{R_2}	u_{R_3}	d_{R_1}	d_{R_2}	d_{R_3}
$U(1)_Y$	$1/2$	$1/2$	0	$1/6$	$1/6$	$1/6$	$2/3$	$2/3$	$2/3$	$-1/3$	$-1/3$	$-1/3$
$SU(2)_L$	2	2	1	2	2	2	1	1	1	1	1	1
$SU(3)_C$	1	1	1	3	3	3	3	3	3	3	3	3
$U(1)'$	$-2/3$	-8	$22/3$	1	1	$-19/3$	$1/3$	$1/3$	$-43/3$	$5/3$	$5/3$	$5/3$

	ℓ_1	ℓ_2	ℓ_3	e_{R_1}	e_{R_2}	e_{R_3}	ν_{R_1}	ν_{R_2}	ν_{R_3}
$U(1)_Y$	$-1/2$	$-1/2$	$-1/2$	-1	-1	-1	0	0	0
$SU(2)_L$	2	2	2	1	1	1	1	1	1
$SU(3)_C$	1	1	1	1	1	1	1	1	1
$U(1)'$	-3	-3	19	$-7/3$	$-7/3$	27	$-11/3$	$-11/3$	11



For the peruse of this analysis we have test our model under

- 1) STU electroweak precision observables (or oblique parameters),
- 2) Higgs observables
- 3) Most relevant Quark Flavour Violation (QFV) observables

1) STU: We use the values for the electroweak fit for the STU parameter from [41], and we use also SPheno to calculate the STU in our model.

$$\begin{aligned} S &= -0.01 \pm 0.10 \\ T &= 0.03 \pm 0.12 \\ U &= 0.02 \pm 0.11 \end{aligned}, \quad \rho_{ij} = \begin{pmatrix} 1 & 0.92 & -0.80 \\ 0.92 & 1 & -0.93 \\ -0.80 & -0.93 & 1 \end{pmatrix}$$

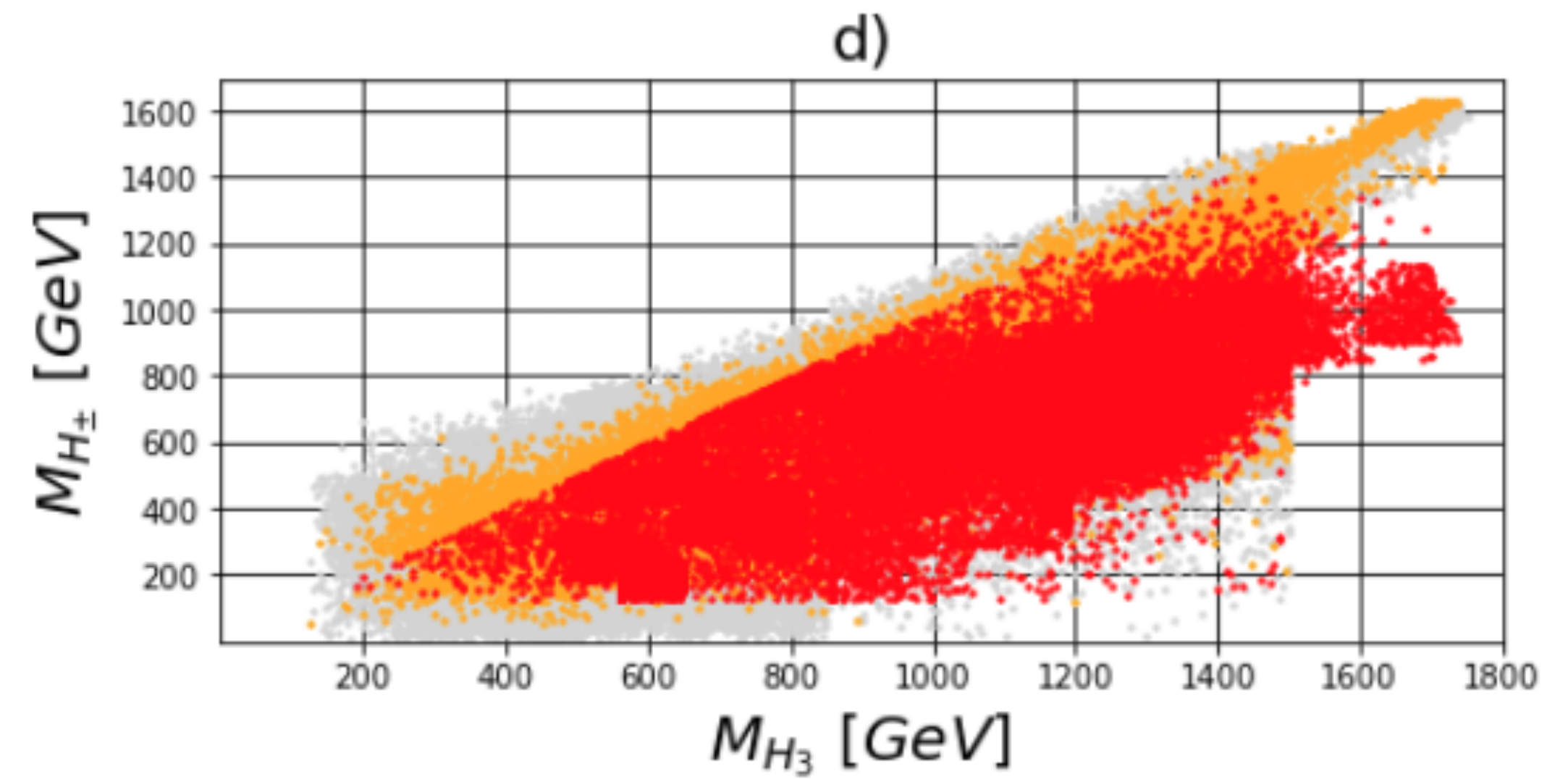
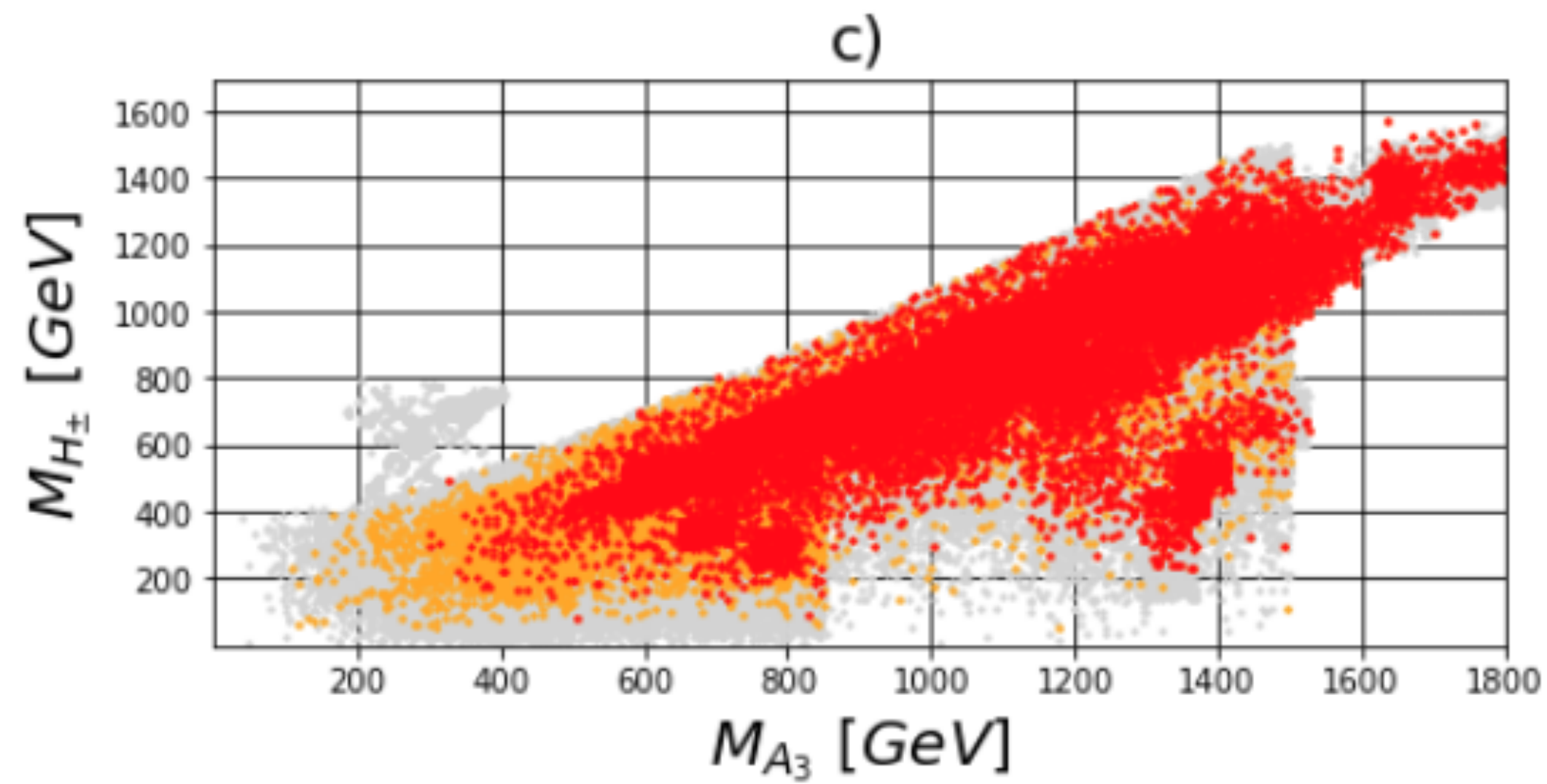
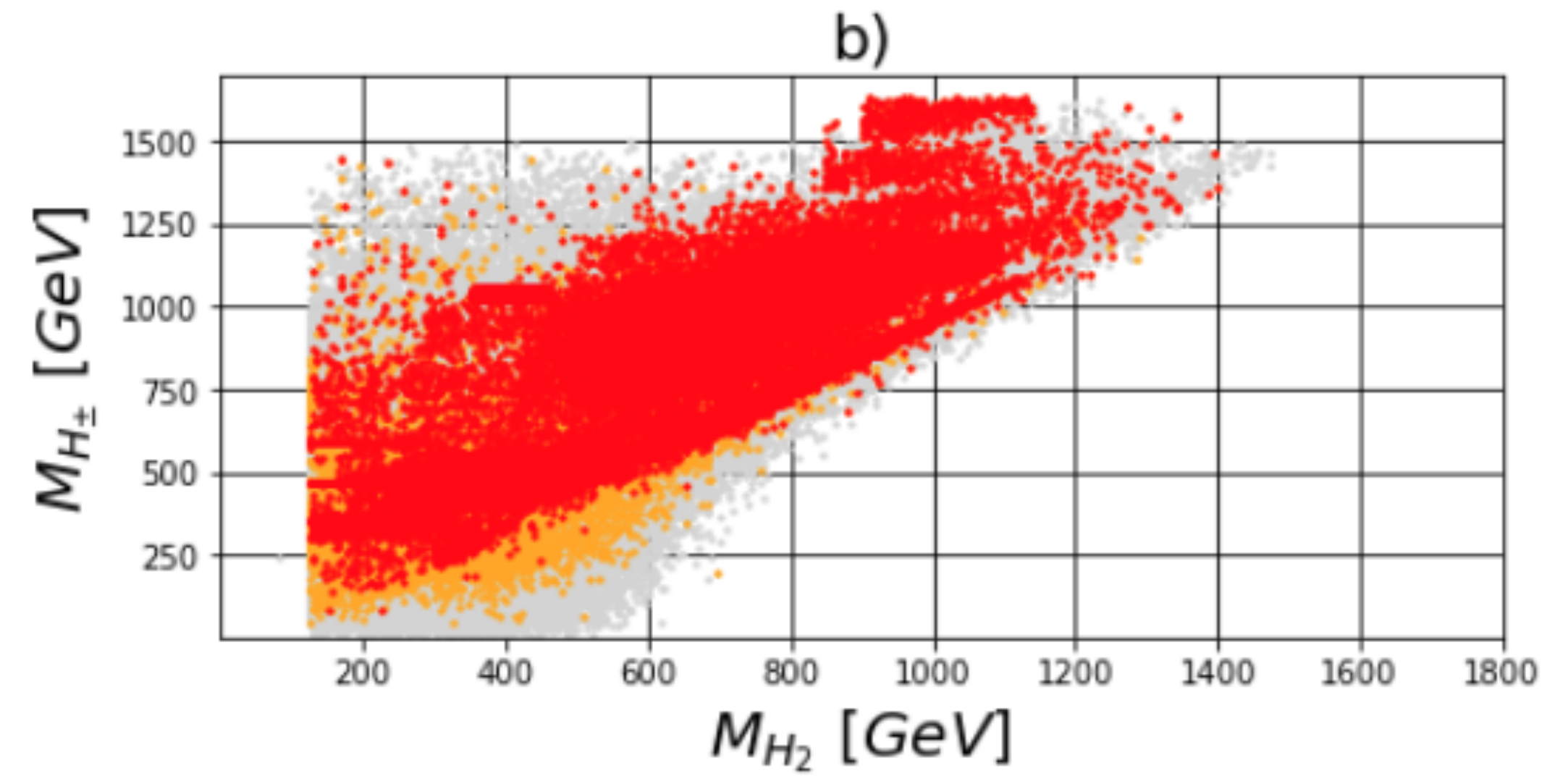
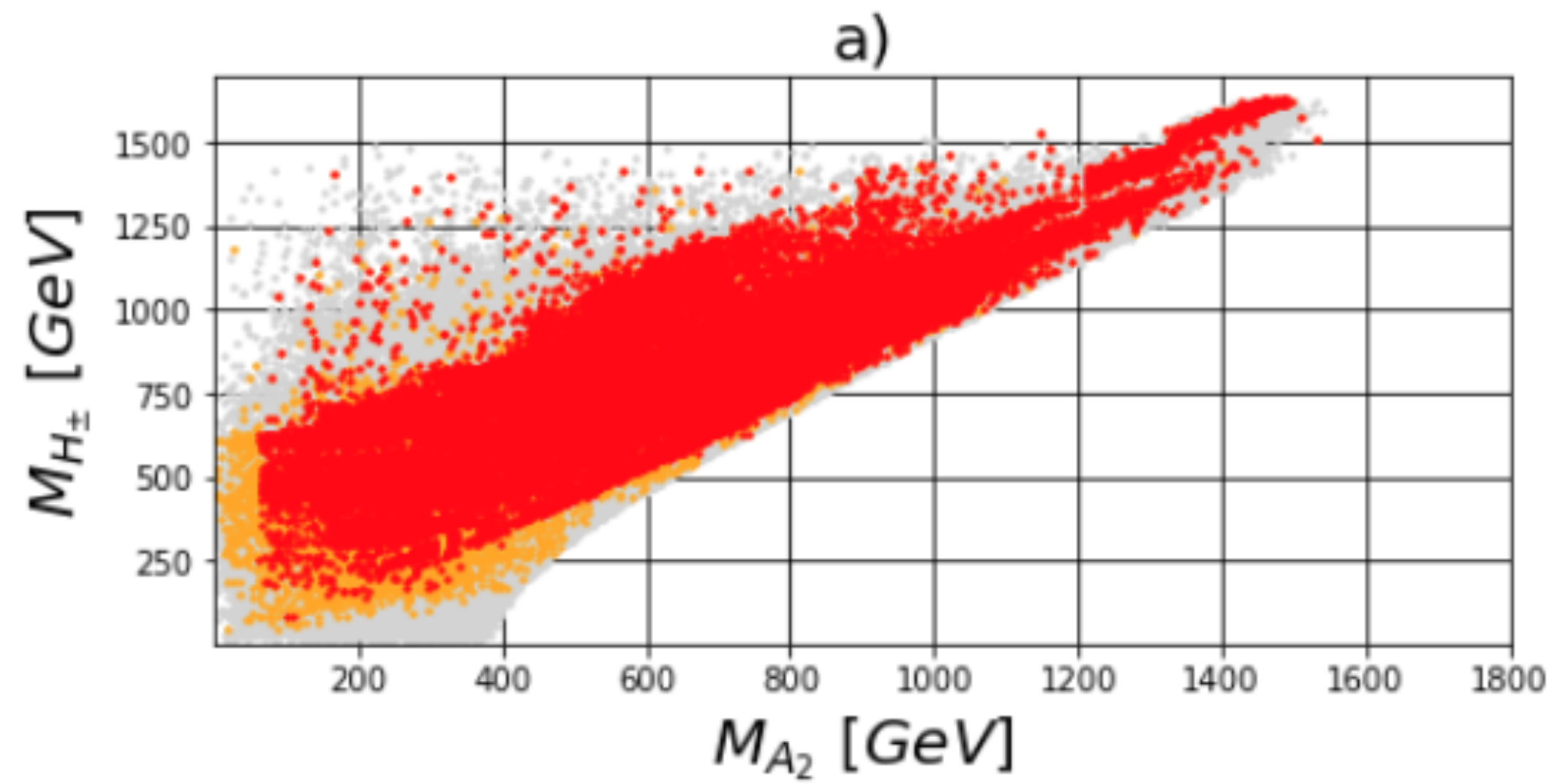
Were we require $\Delta\chi^2 < 7.815$, which is translated to 95% confidence level (CL) agreement with the electroweak fit.

$$\Delta\chi^2 = \sum_{ij} \left(\Delta\mathcal{O}_i - \Delta\mathcal{O}_i^{(0)} \right) [(\sigma^2)^{-1}]_{ij} \left(\Delta\mathcal{O}_j - \Delta\mathcal{O}_j^{(0)} \right)$$

2) Higgs observables: For the Higgs observables we have used SPheno to calculate the values in our model and HiggsBounds/HiggsSignals for the validity of our model

[41] P. A. Zyla *et al.* (Particle Data Group), [PTEP 2020, 083C01 \(2020\)](#).

Results



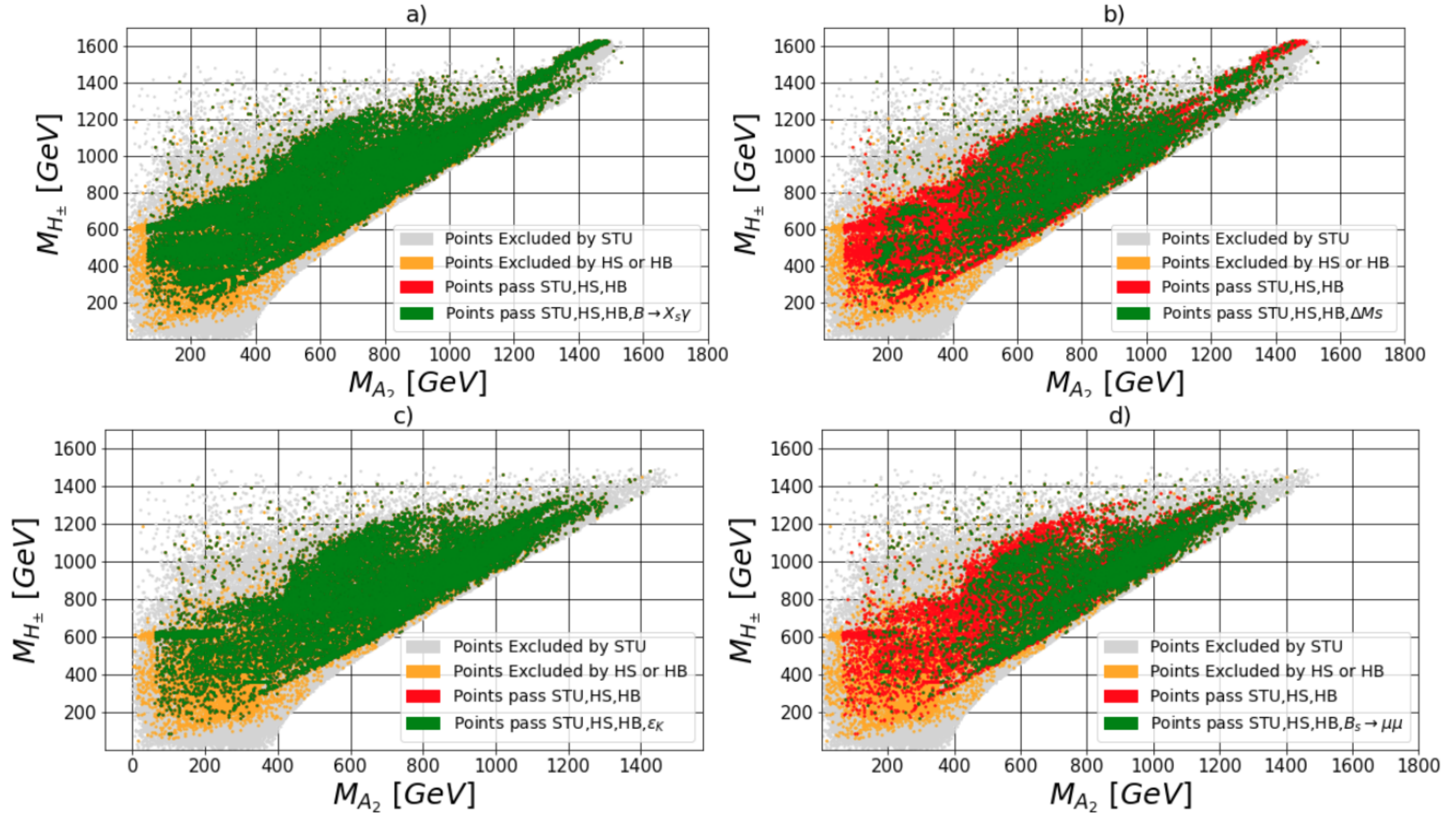
Points Excluded by STU Points Excluded by HS or HB Points pass STU,HS,HB

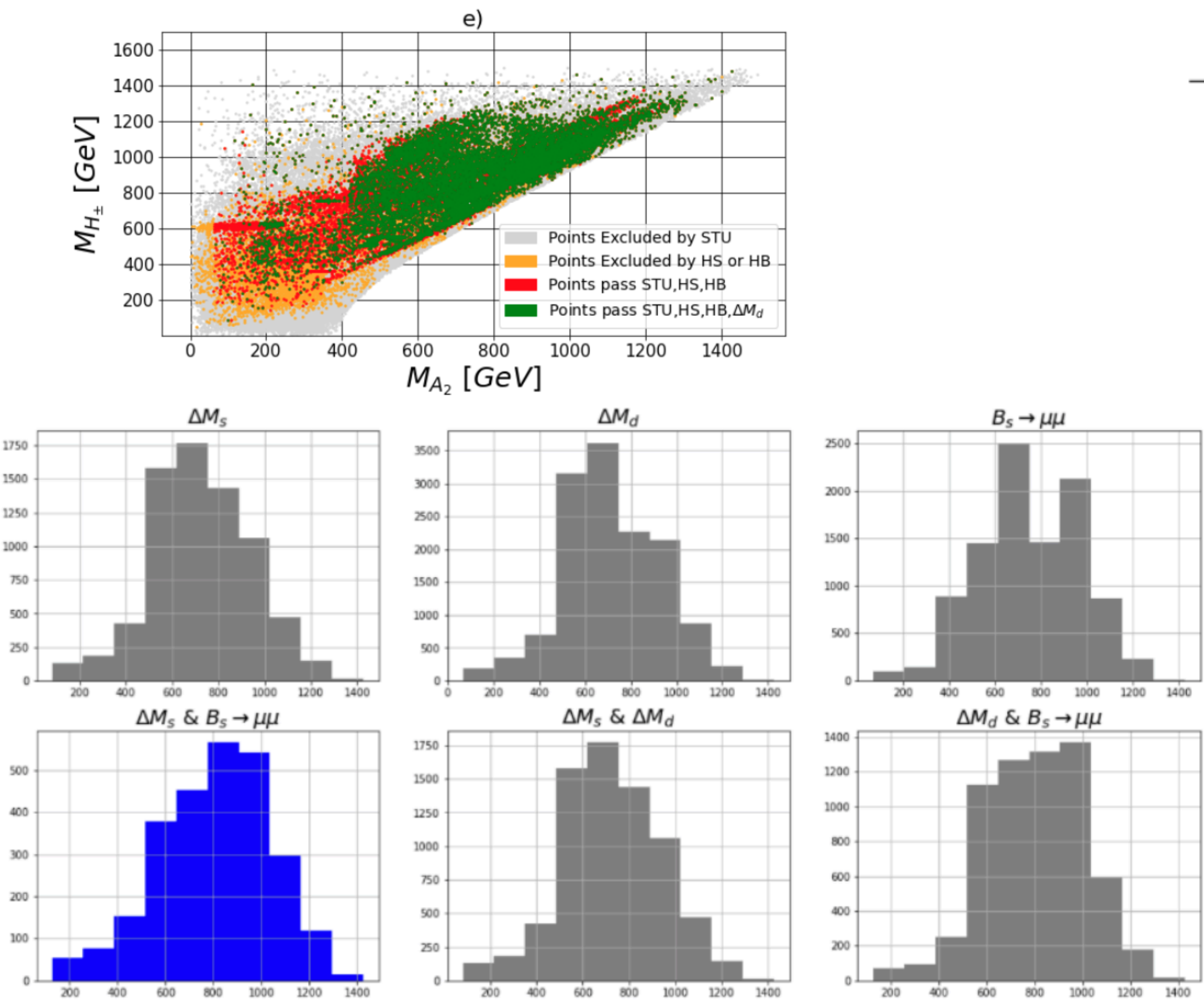


3) For the Quark Flavour Violation (QFV) observables we have only take into consideration the most relevant channels summarised in the table below.

Channel	\mathcal{O}_{SM}	σ_{SM}	\mathcal{O}_{Exp}	σ_{Exp}	σ
$\text{BR}(\text{B} \rightarrow \chi_s \gamma)$	3.29×10^{-4}	1.87×10^{-5}	3.32×10^{-4}	0.16×10^{-4}	0.075
$\text{BR}(\text{B}_s \rightarrow \mu\mu)$	3.66×10^{-9}	1.66×10^{-10}	2.80×10^{-9}	0.06×10^{-9}	0.038
ΔM_d (GeV)	3.97×10^{-13}	5.07×10^{-14}	3.33×10^{-13}	0.013×10^{-13}	0.11
ΔM_s (GeV)	1.24×10^{-11}	7.08×10^{-13}	1.17×10^{-11}	0.0014×10^{-11}	0.054
ϵ_K (GeV)	1.81×10^{-3}	2.00×10^{-4}	2.23×10^{-3}	0.011×10^{-3}	0.14

Results

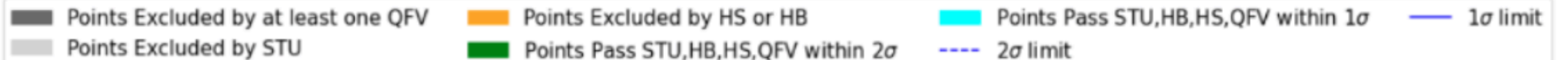
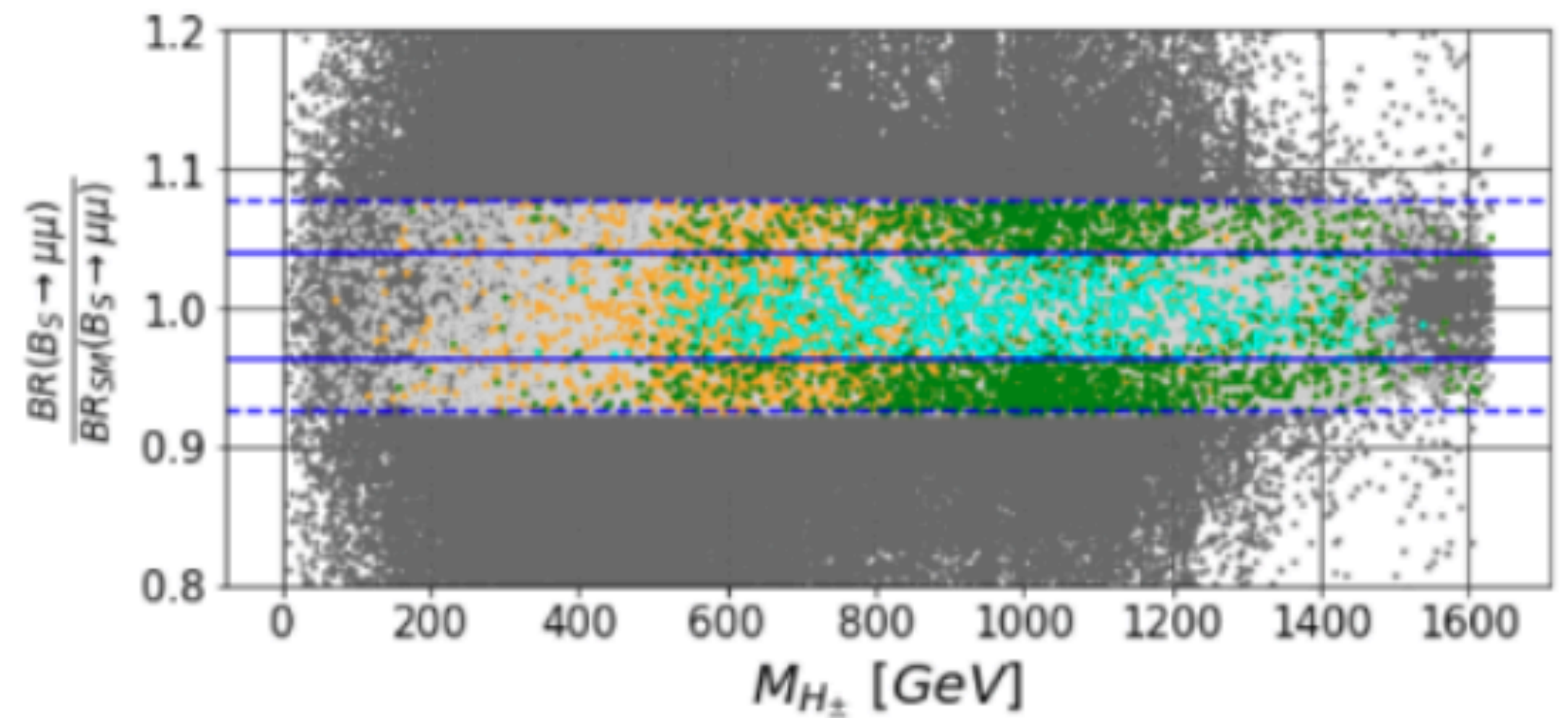
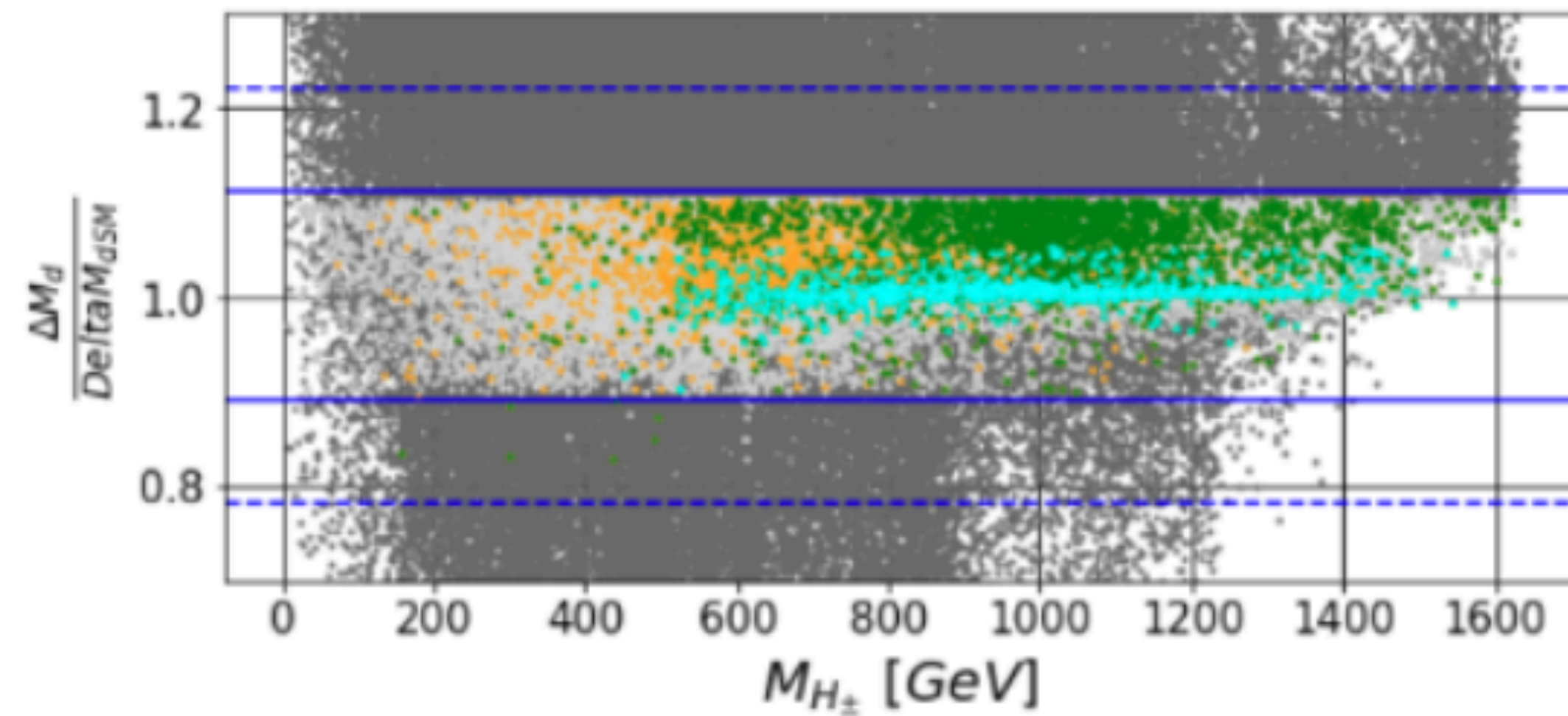
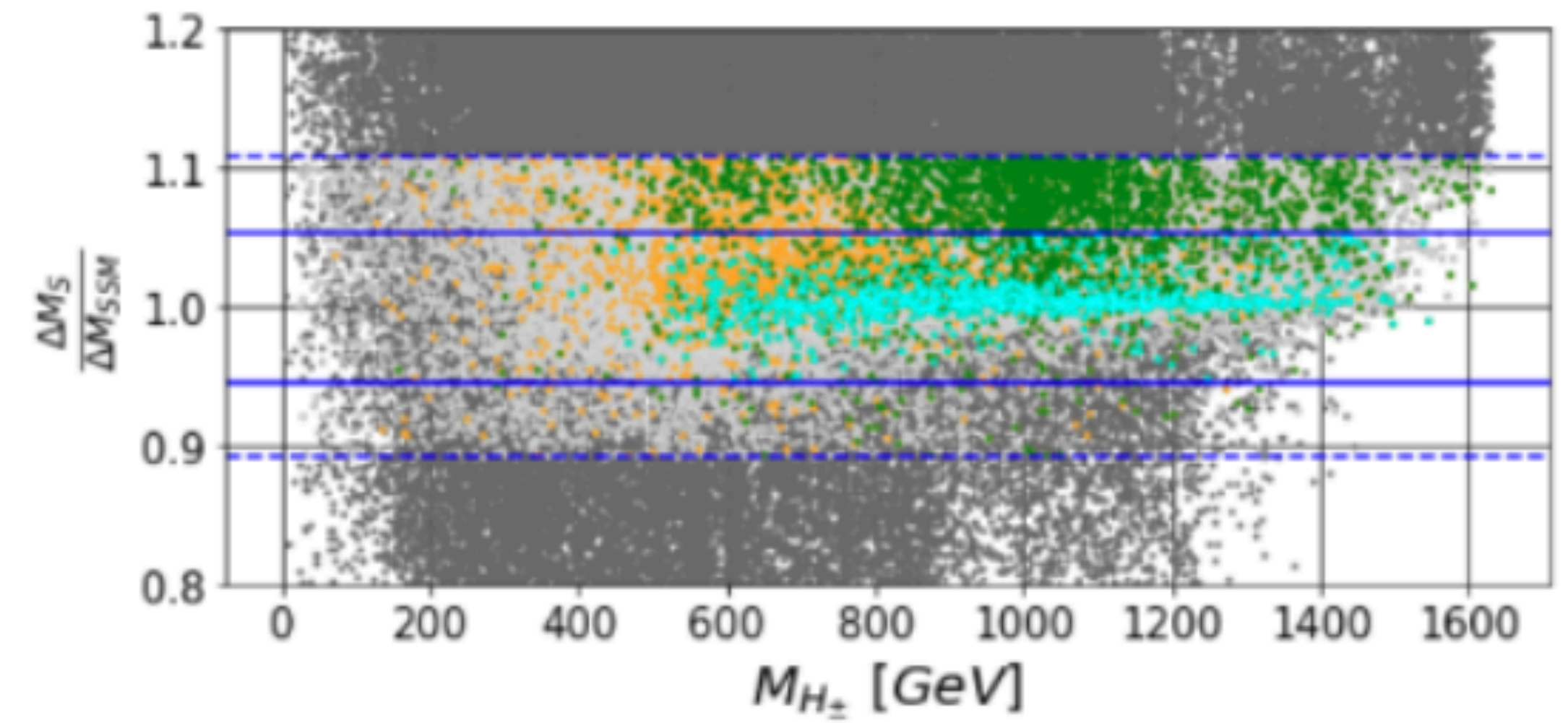
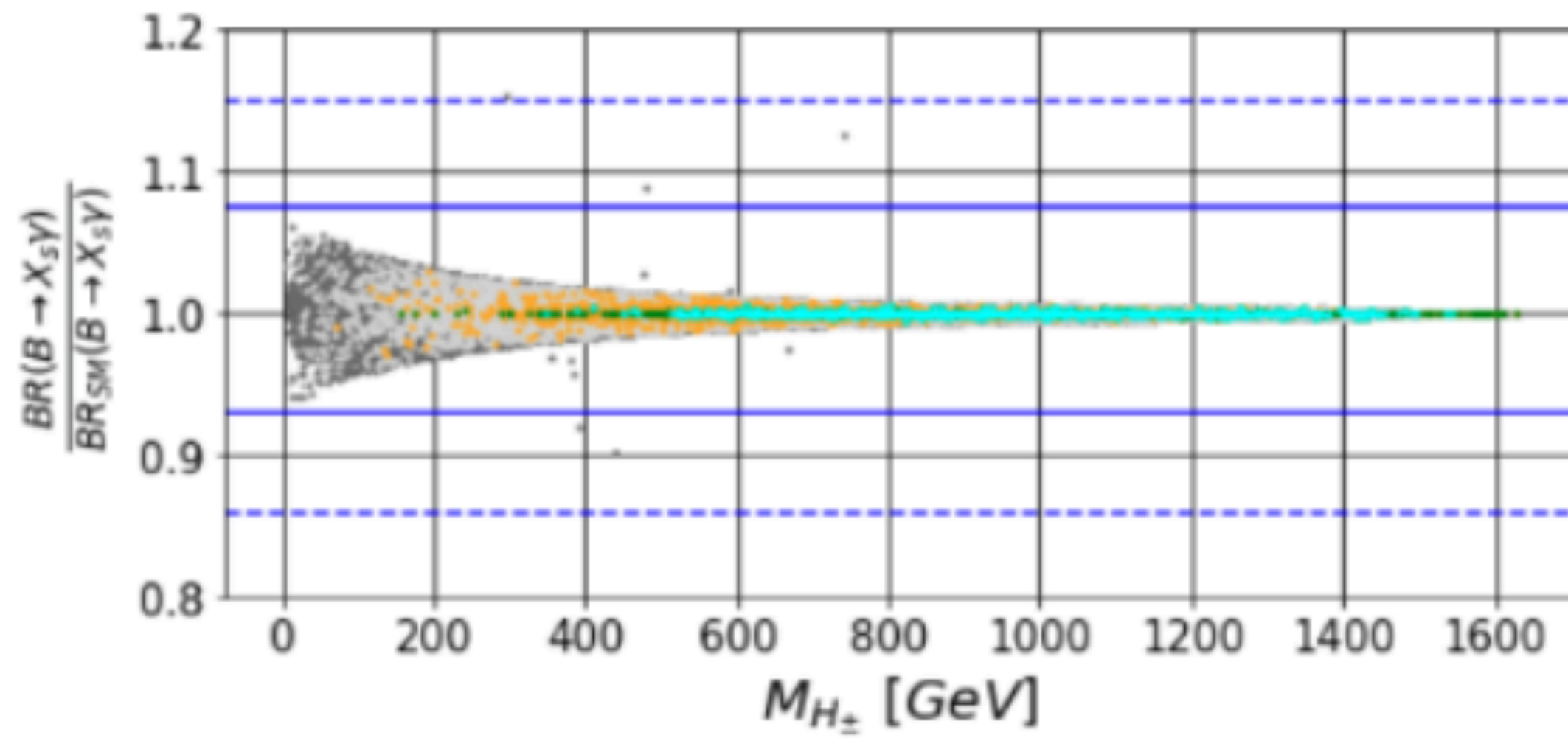


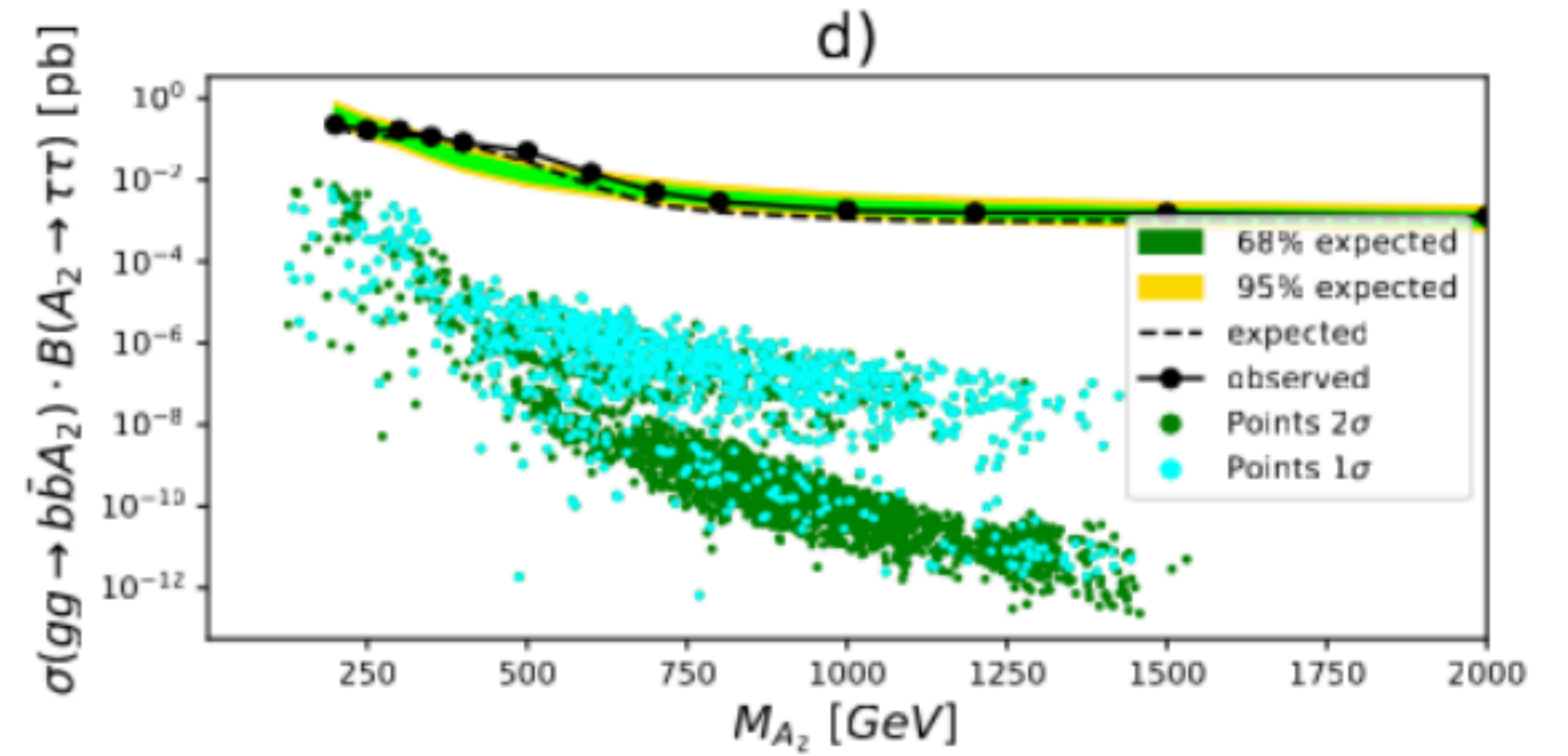
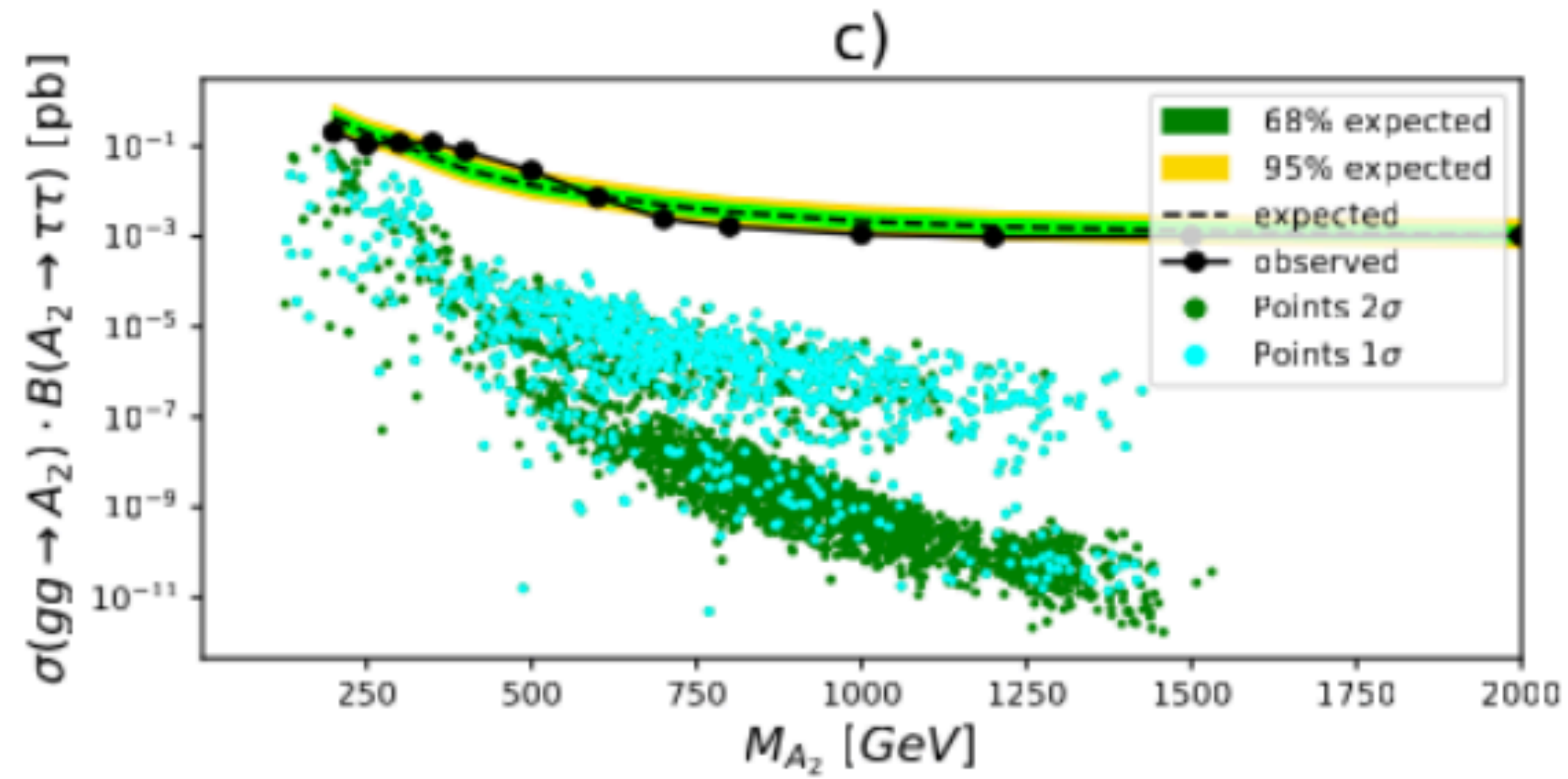
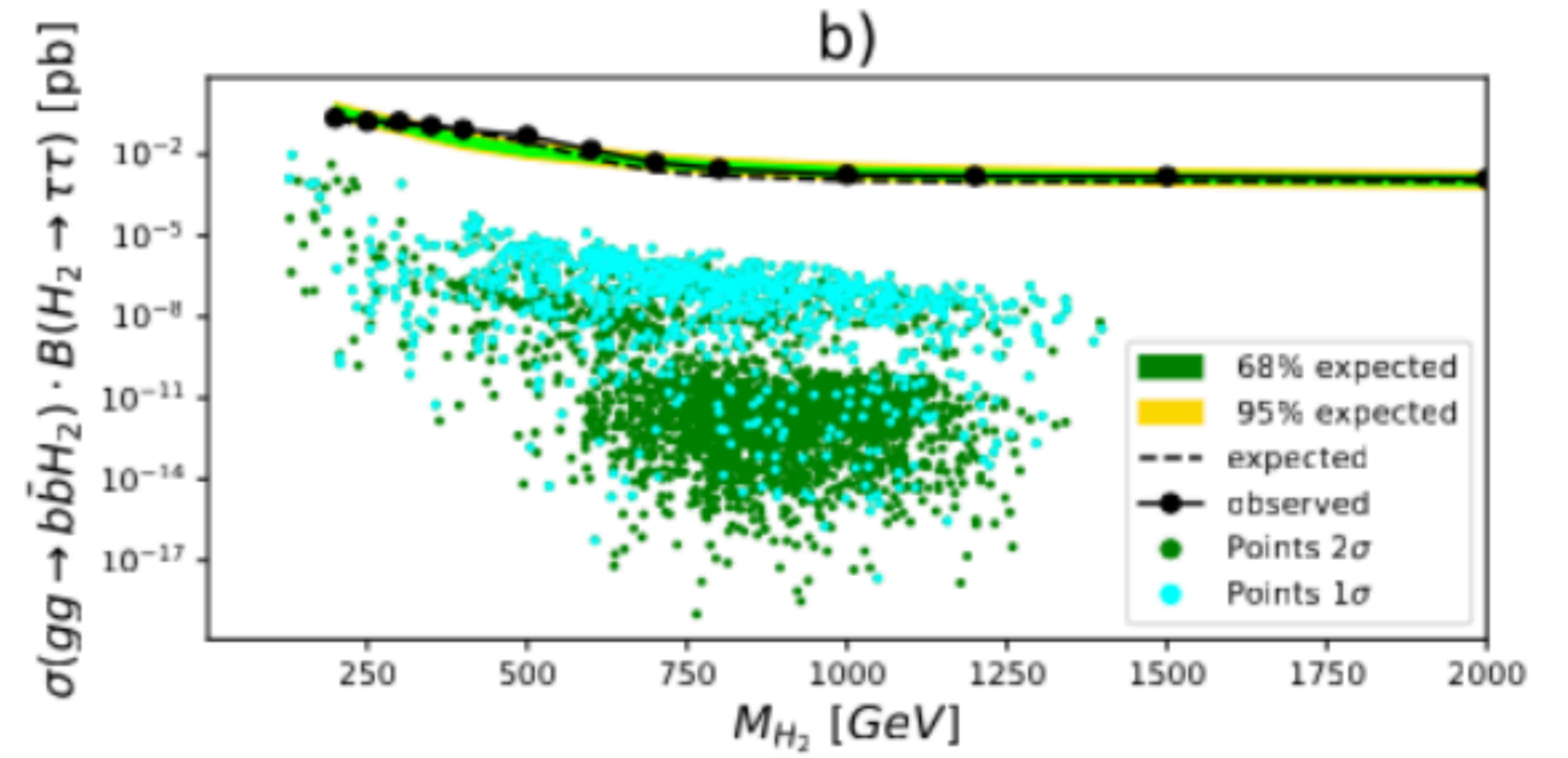
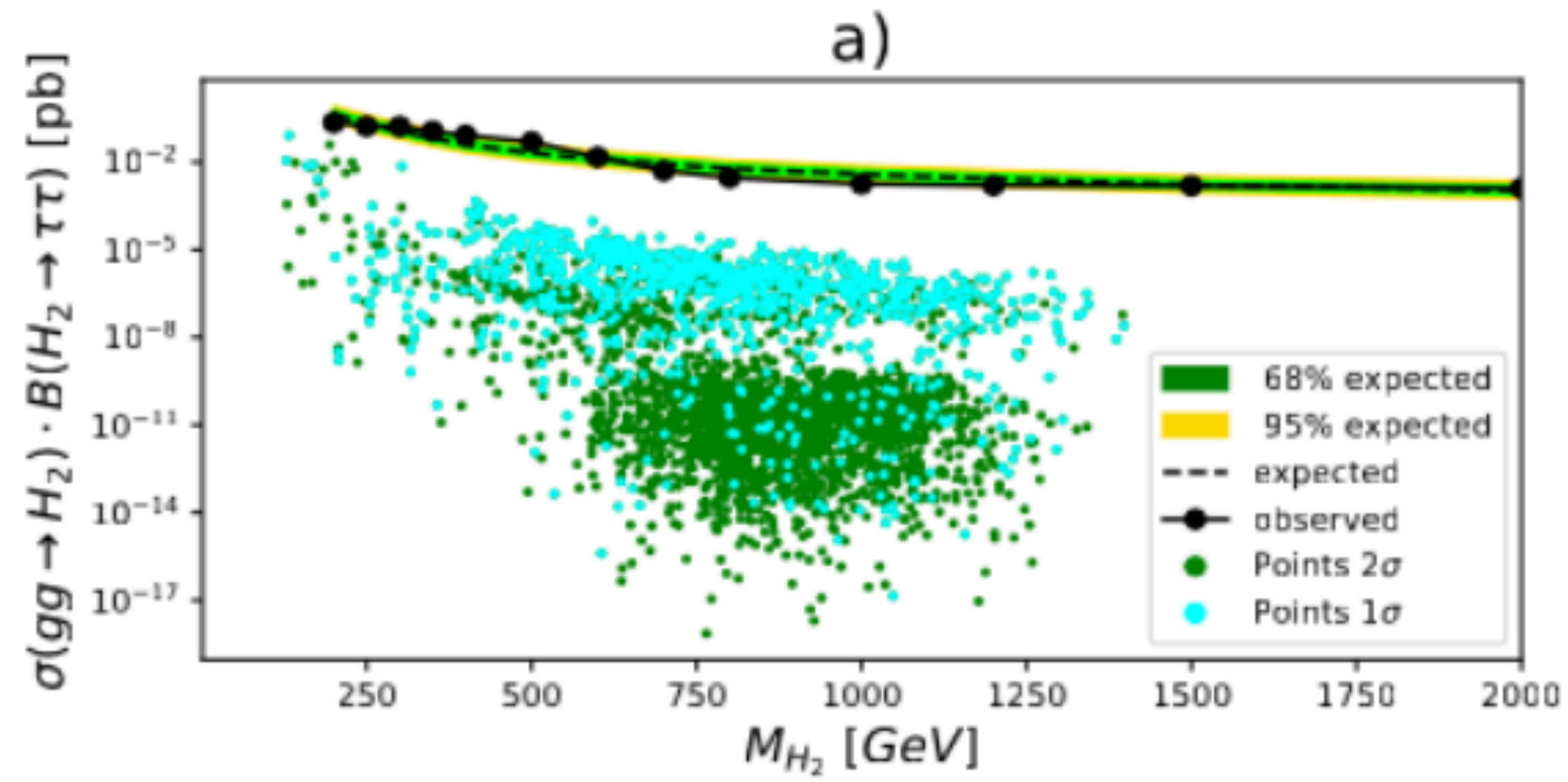


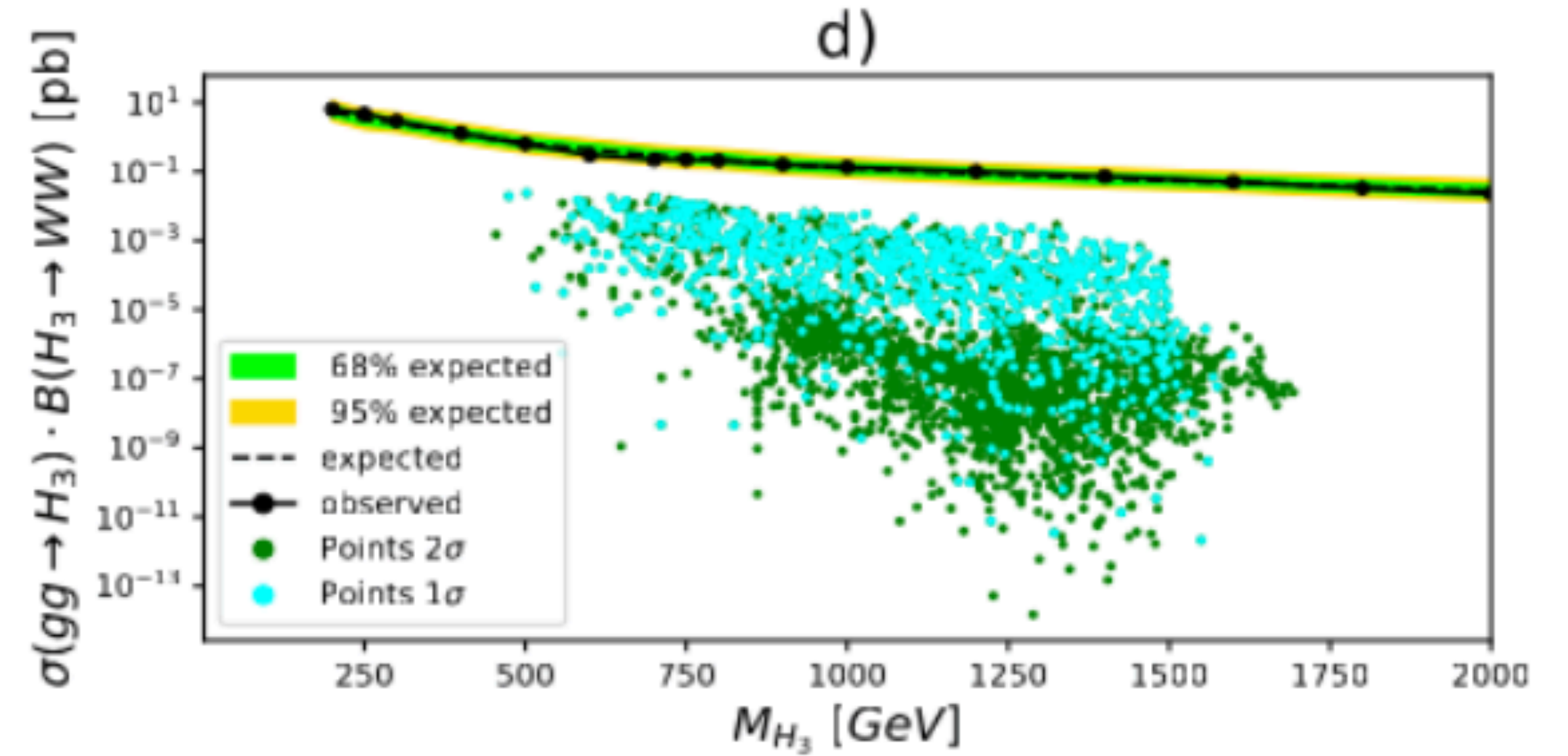
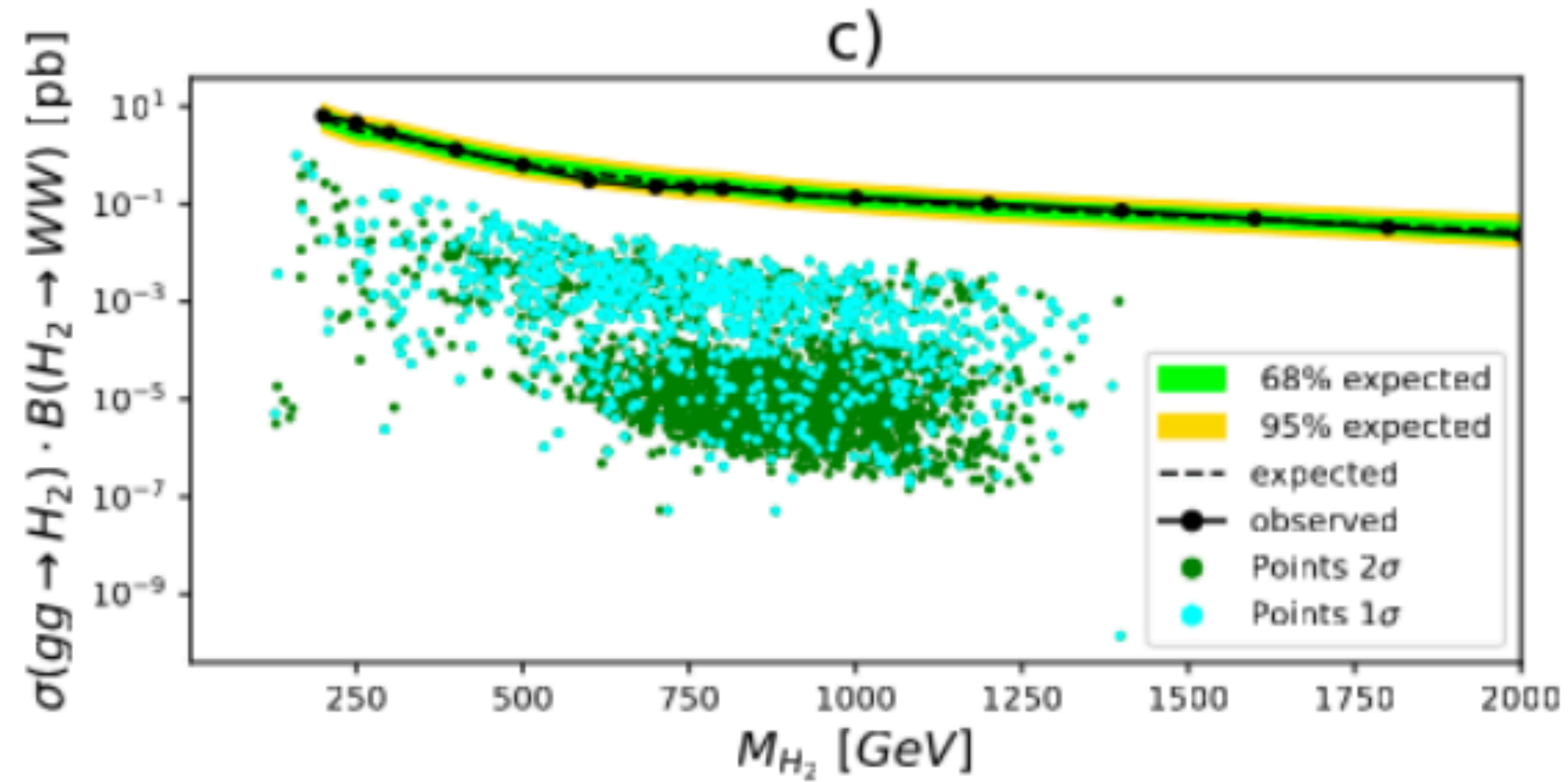
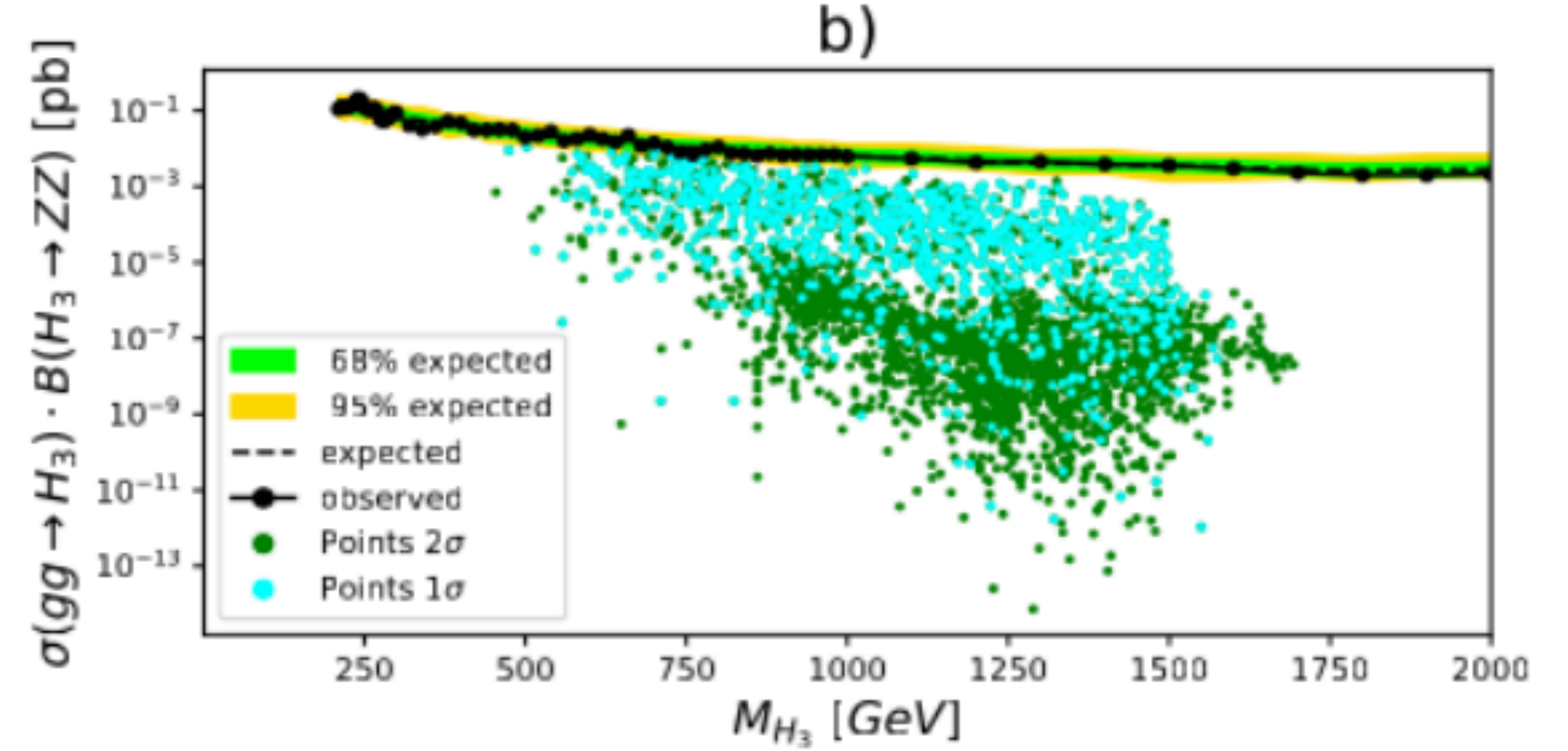
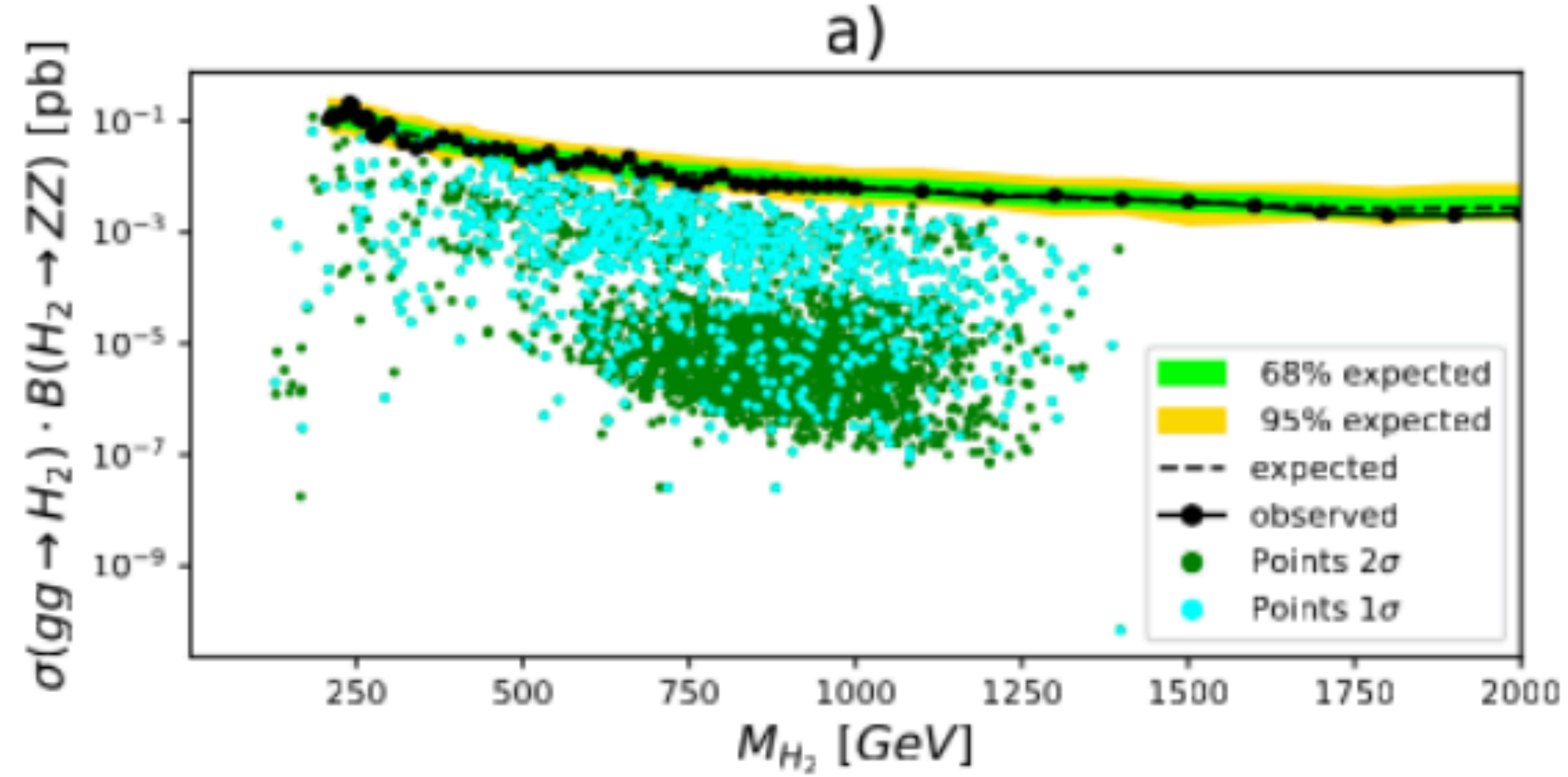
Set of QFV observables	Acceptance ratio
$\text{BR}(B \rightarrow \chi_s \gamma)$	100.0%
$\text{BR}(B_s \rightarrow \mu\mu)$	35.0%
ΔM_d (GeV)	48.0%
ΔM_s (GeV)	26.0%
ϵ_K (GeV)	100.0%
$\text{BR}(B_s \rightarrow \mu\mu) \ \& \ \Delta M_s$	9.39%
$\text{BR}(B_s \rightarrow \mu\mu) \ \& \ \Delta M_d$	22.21%
$\Delta M_s \ \& \ \Delta M_d$	25.57%

Figure 5: Histograms containing points that survive STU, HS, HB and a given QFV (or pair of) in bins of the A_2 mass. The most restrictive is coloured in *blue*.

Results









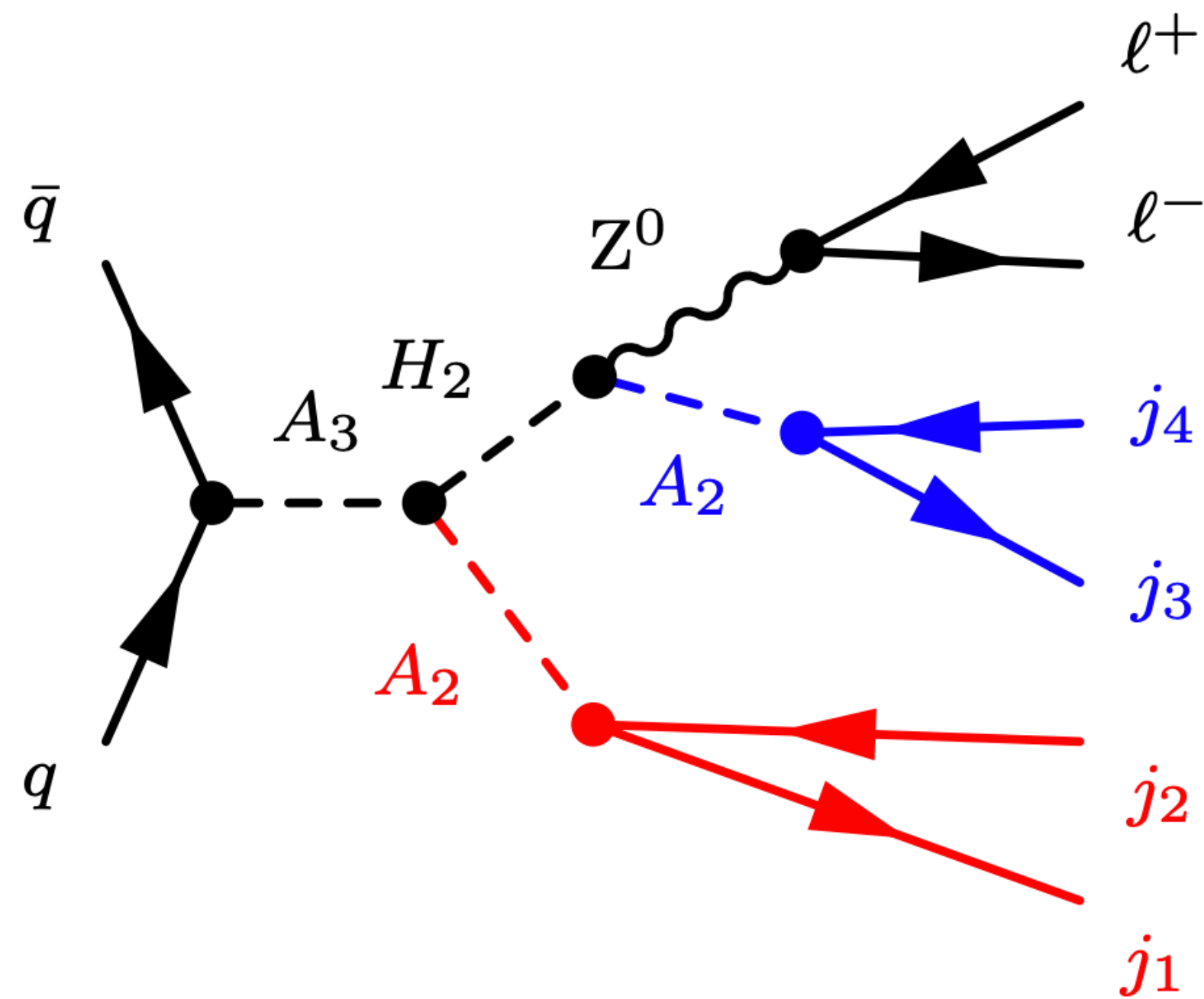
ϕ	ID	Mass (GeV)	$\text{BR}(\phi \rightarrow \tau^+\tau^-)$	$\sigma(gg \rightarrow \phi) \cdot \text{BR} \text{ (pb)}$	$\sigma(gg \rightarrow b\bar{b}\phi) \cdot \text{BR} \text{ (pb)}$	Maximum BR
H_2	BP1	160.21	4.99×10^{-3}	0.007354	8.01×10^{-4}	$\text{BR}(W^+W^*) = 0.881$
	BP2	347.99	6.65×10^{-7}	3.25×10^{-8}	3.67×10^{-9}	$\text{BR}(H_1H_1) = 0.611$
	BP3	129.26	0.0127	3.59×10^{-4}	4.3×10^{-5}	$\text{BR}(W^+W^*) = 0.377$
	BP4	132.27	0.0357	0.0830	0.00967	$\text{BR}(b\bar{b}) = 0.590$
	BP5	668.49	6.14×10^{-6}	7.42×10^{-8}	1.01×10^{-8}	$\text{BR}(A_2Z^0) = 0.75$
A_2	BP1	194.99	0.0336	0.0546	0.00532	$\text{BR}(b\bar{b}) = 0.553$
	BP2	173.55	0.0264	0.0874	0.008249	$\text{BR}(b\bar{b}) = 0.432$
	BP3	1077.21	2.65×10^{-5}	1.0×10^{-6}	1.62×10^{-7}	$\text{BR}(t\bar{t}) = 0.711$
	BP4	937.61	3.49×10^{-5}	6.36×10^{-9}	8.66×10^{-10}	$\text{BR}(t\bar{t}) = 0.918$
	BP5	126.74	2.48×10^{-3}	3.2×10^{-5}	3.0×10^{-6}	$\text{BR}(c\bar{c}) = 0.922$

ϕ	ID	Mass (GeV)	$\text{BR}(\phi \rightarrow W^+W^-)$	$\text{BR}(\phi \rightarrow Z^0Z^0)$	$\sigma(gg \rightarrow \phi \rightarrow W^+W^-) \text{ (pb)}$	$\sigma(gg \rightarrow \phi \rightarrow Z^0Z^0) \text{ (pb)}$	Maximum BR
H_2	BP1	160.21	0.881	0.0210	1.0032	0.00054	$\text{BR}(W^+W^*) = 0.881$
	BP2	347.99	0.129	0.0589	0.00722	0.003319	$\text{BR}(H_1H_1) = 0.611$
	BP3	129.26	0.377	0.0449	3.0×10^{-6}	1.0×10^{-6}	$\text{BR}(W^+W^*) = 0.377$
	BP4	132.27	0.239	0.0299	0.00366	0.001426	$\text{BR}(b\bar{b}) = 0.590$
	BP5	668.49	0.416	0.203	0.00506	0.00244	$\text{BR}(A_2Z^0) = 0.75$
H_3	BP1	929.20	0.124	0.0616	0.00341	0.00167	$\text{BR}(H_1H_1) = 0.456$
	BP2	823.89	0.0922	0.0455	1.36×10^{-4}	6.7×10^{-5}	$\text{BR}(A_2A_2) = 0.228$
	BP3	1093.01	0.166	0.0824	0.002964	0.001468	$\text{BR}(H_1H_2) = 0.531$
	BP4	1156.49	0.0598	0.0297	0.000192	9.4×10^{-5}	$\text{BR}(t\bar{t}) = 0.617$
	BP5	754.64	0.0517	0.0254	8.4×10^{-4}	4.1×10^{-4}	$\text{BR}(t\bar{t}) = 0.591$



The next two slides concerning a follow up work that we expect to be uploaded to arXiv very soon, and has already been presented by João Gonçalves in FLASY2022 & ICHEP.

The next two slides are a summary of João talks.



- Mass information can be use to match pairs of jets to original scalars fields
- $\Delta M = M(j_1, j_2) - M(j_3, j_4) < \varepsilon$
 - **Signal:** small ε
 - **Background** Arbitrary ε
- Loop over all possible combinations of jets and select the pairs with smallest ε

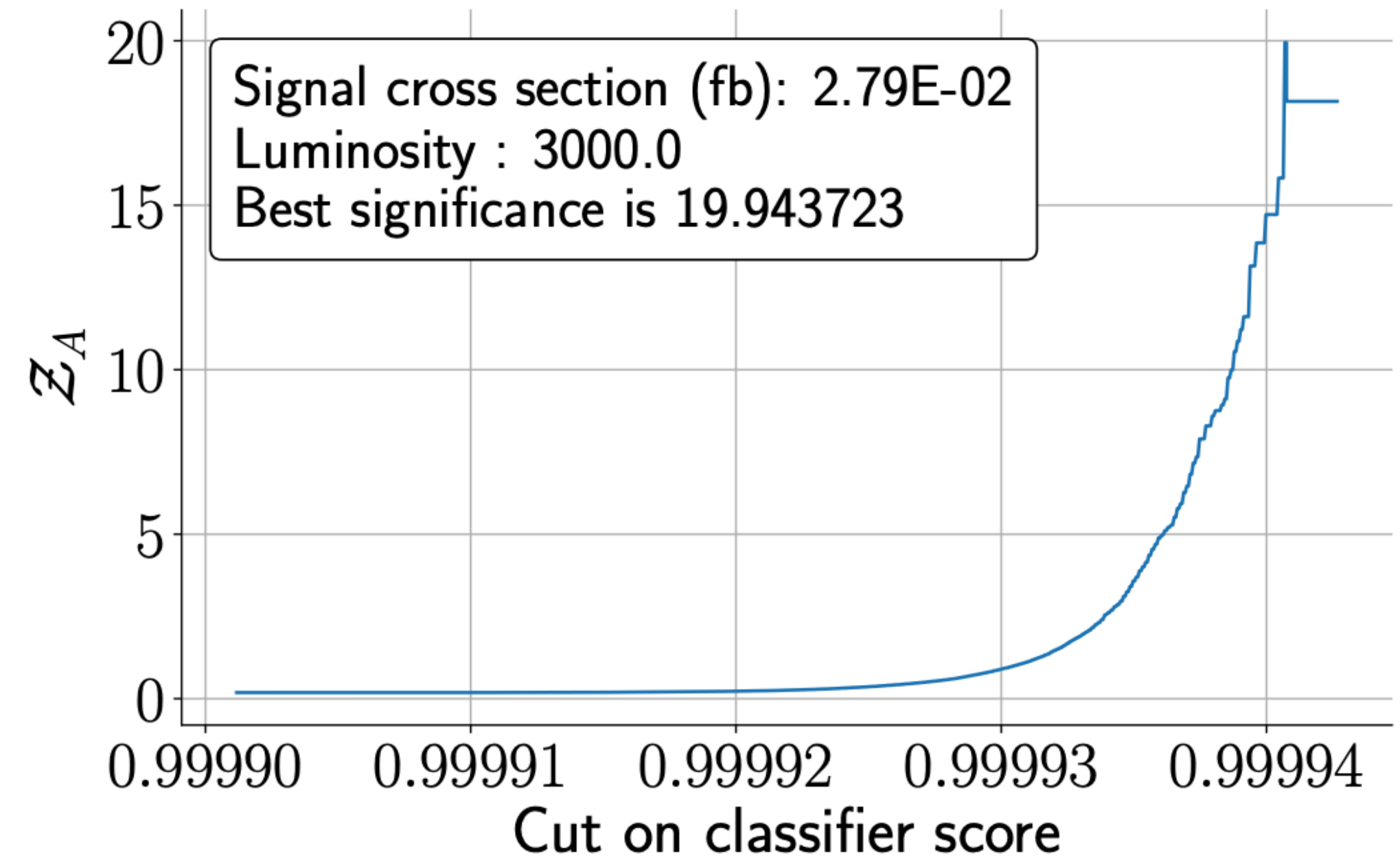
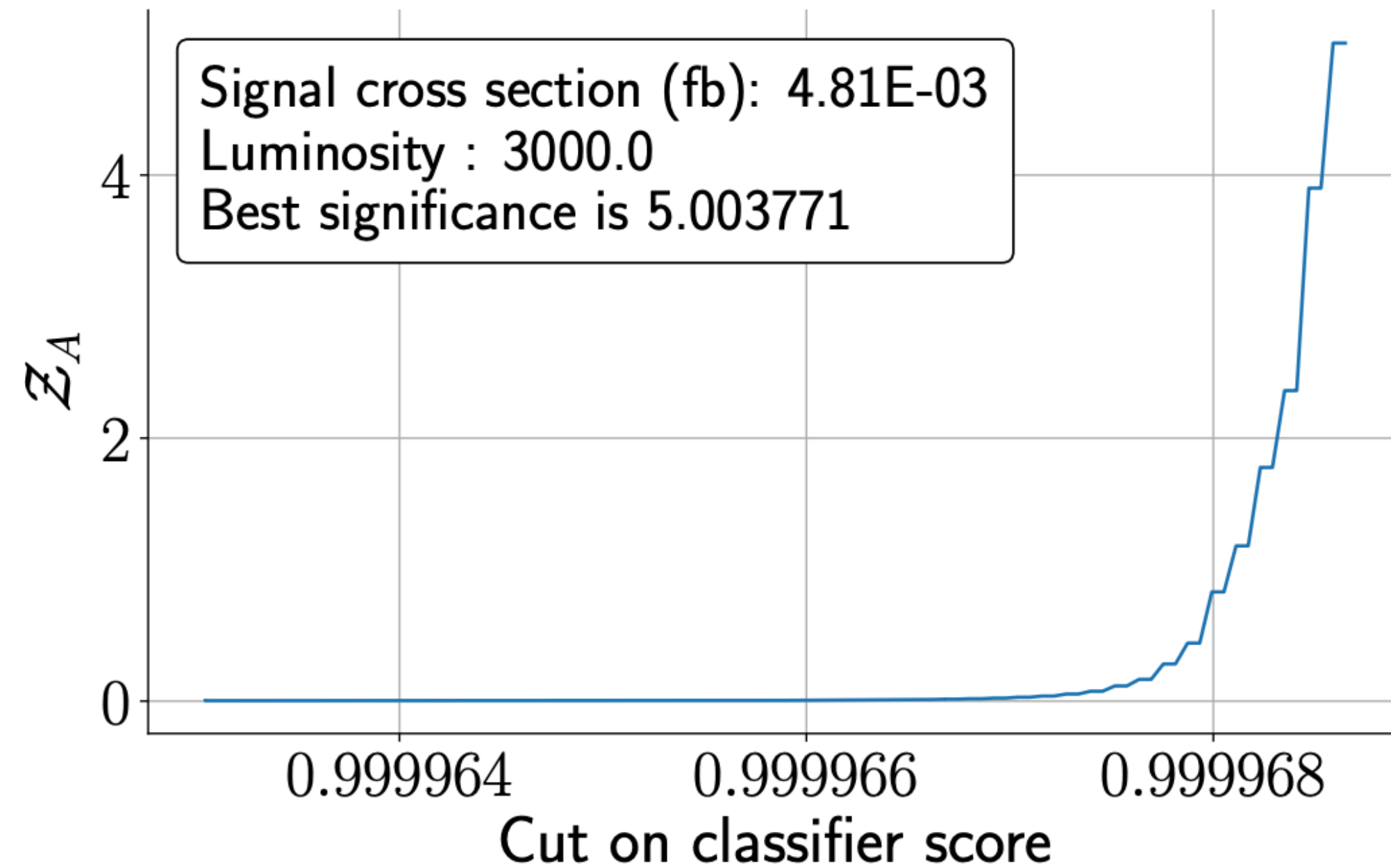
Match jets to H_2 scalar: $\min(|M(j_n, j_m) - M(Z^0) - M(H_2)|)$

If the minimum is for pair (j_3, j_4) , then this is matched to the **blue leg** and the pair (j_1, j_2) is matched to the **red leg**.

Since ε is expected to be arbitrary, the matching procedure can help reduce backgrounds for small values of ε .



$$M(j) > 10 \text{ GeV and } \Delta M < 35 \text{ GeV}$$



(a) $M_{A_2} = 215 \text{ GeV} / M_{H_2} = 400 \text{ GeV}$

(b) $M_{A_2} = 300 \text{ GeV} / M_{H_2} = 600 \text{ GeV}$

Relaxed constraints on jet mass distributions increases the significance. Particularly helpful for lower mass scalar fields. Still, **high cuts** on data for optimal results.



An anomaly-free implementations of a NTHDM-BGL model with three generations of right-handed neutrinos

Constrained by: 1) STU, 2) Higgs, 3) flavour observables

We have successfully assessed the viability of the low mass region and found that even for a number of scenarios with new scalars around the EW scale, the vBGL-I model remains unconstrained

The majority of the excluded scenarios came from ΔM_S and $\text{BR}(B_S \rightarrow \mu\mu)$ QFV observables, which have eliminated approximately 90.6% of the sampled points.

All points are consistent with existing LHC constrains for

$$gg \rightarrow H_2 \rightarrow \tau\tau, gg \rightarrow A_2 \rightarrow \tau\tau, gg \rightarrow b\bar{b} H_2 \cdot H_2 \rightarrow \tau\tau, gg \rightarrow b\bar{b} A_2 \cdot A_2 \rightarrow \tau\tau$$
$$gg \rightarrow H_2 \rightarrow ZZ, gg \rightarrow H_2 \rightarrow WW, gg \rightarrow H_3 \rightarrow ZZ, gg \rightarrow H_3 \rightarrow WW$$



Thank you very much!



Backup slides



Anomaly cancellation

This work was inspired considering local $U(1)'$ symmetry where gauge anomalies are forbidden. With this in mind, and with the purpose of making the considered model consistent with a gauged version (to be studied elsewhere), one must also include a set of restrictions that incorporate the $U(1)'$.

Anomaly cancellation conditions

The set of restrictions for the gauge anomalies of the $U(1)'$ charges are the following triangle anomalies

$$\begin{aligned} & [U(1)']^3, \quad U(1)'[\text{Gravity}]^2, \\ & U(1)'[U(1)_Y]^2, \quad U(1)'[SU(2)_L]^2 \\ & U(1)'[SU(3)_C]^2, \quad [U(1)']^2 U(1)_Y. \end{aligned}$$



Anomaly-free conditions

$$\begin{aligned}
 -\mathcal{L}_{\text{Yukawa}} = & \overline{q_L^0} \Gamma_a \Phi^a d_R^0 + \overline{q_L^0} \Delta_a \tilde{\Phi}^a u_R^0 + \overline{\ell_L^0} \Pi_a \Phi^a e_R^0 + \overline{\ell_L^0} \Sigma_a \tilde{\Phi}^a \nu_R^0 \\
 & + \frac{1}{2} \overline{\nu_R^{c0}} (A + BS + CS^*) \nu_R^0 + \text{h.c.},
 \end{aligned}$$

Based on the BGL quark structure we have 36 constrains:

$$\begin{aligned}
 X_{q_{1,2}} - X_{d_{1,2,3}} &= X_{\Phi_1}, \quad X_{q_3} - X_{d_{1,2,3}} \neq X_{\Phi_1}, \\
 X_{q_3} - X_{d_{1,2,3}} &= X_{\Phi_2}, \quad X_{q_{1,2}} - X_{d_{1,2,3}} \neq X_{\Phi_2}, \\
 X_{q_{1,2}} - X_{u_{1,2}} &= -X_{\Phi_1}, \quad X_{q_3} - X_{u_{1,2,3}} \neq -X_{\Phi_1}, \\
 X_{q_{1,2}} - X_{u_3} &\neq -X_{\Phi_1}, \quad X_{q_3} - X_{u_3} = -X_{\Phi_2}, \\
 X_{q_{1,2}} - X_{u_{1,2,3}} &\neq -X_{\Phi_2}, \quad X_{q_3} - X_{u_{1,2}} \neq -X_{\Phi_2}.
 \end{aligned}$$



Anomaly-free conditions

For the lepton and neutrino

- Three massive charged leptons $\det M_e \neq 0$
- Three generations of massive neutrinos $\det M_\nu \neq 0$;
- A non-zero complex phase in the PMNS matrix $\det[M_e M_e^\dagger] \neq 0$ and $\det[M_\nu M_\nu^\dagger] \neq 0$

There are 11 minimal textures for A, B and C that fulfil this constraints. Also, in the presence of the $U(1)'$ flavour symmetry one must fulfil the transformation laws

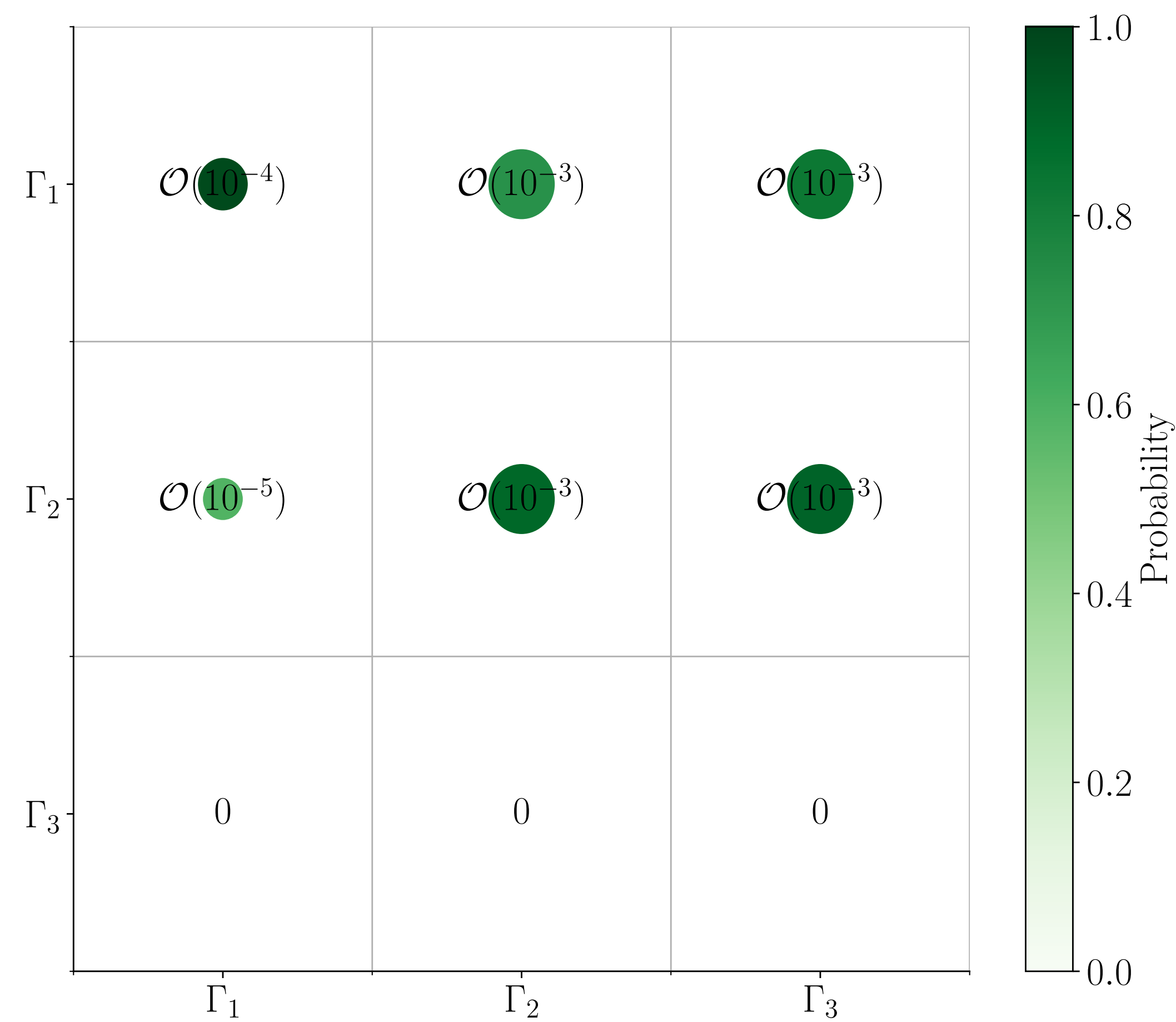
$$A_{ij} = e^{i\alpha(X_{\nu_i} + X_{\nu_j})} A_{ij}, \quad B_{ij} = e^{i\alpha(X_{\nu_i} + X_{\nu_j} + X_S)} B_{ij}, \quad C_{ij} = e^{i\alpha(X_{\nu_i} + X_{\nu_j} - X_S)} C_{ij}.$$

Last, from the potential V_1 and the terms $a_{1,2,3,4}$ we extra the conditions

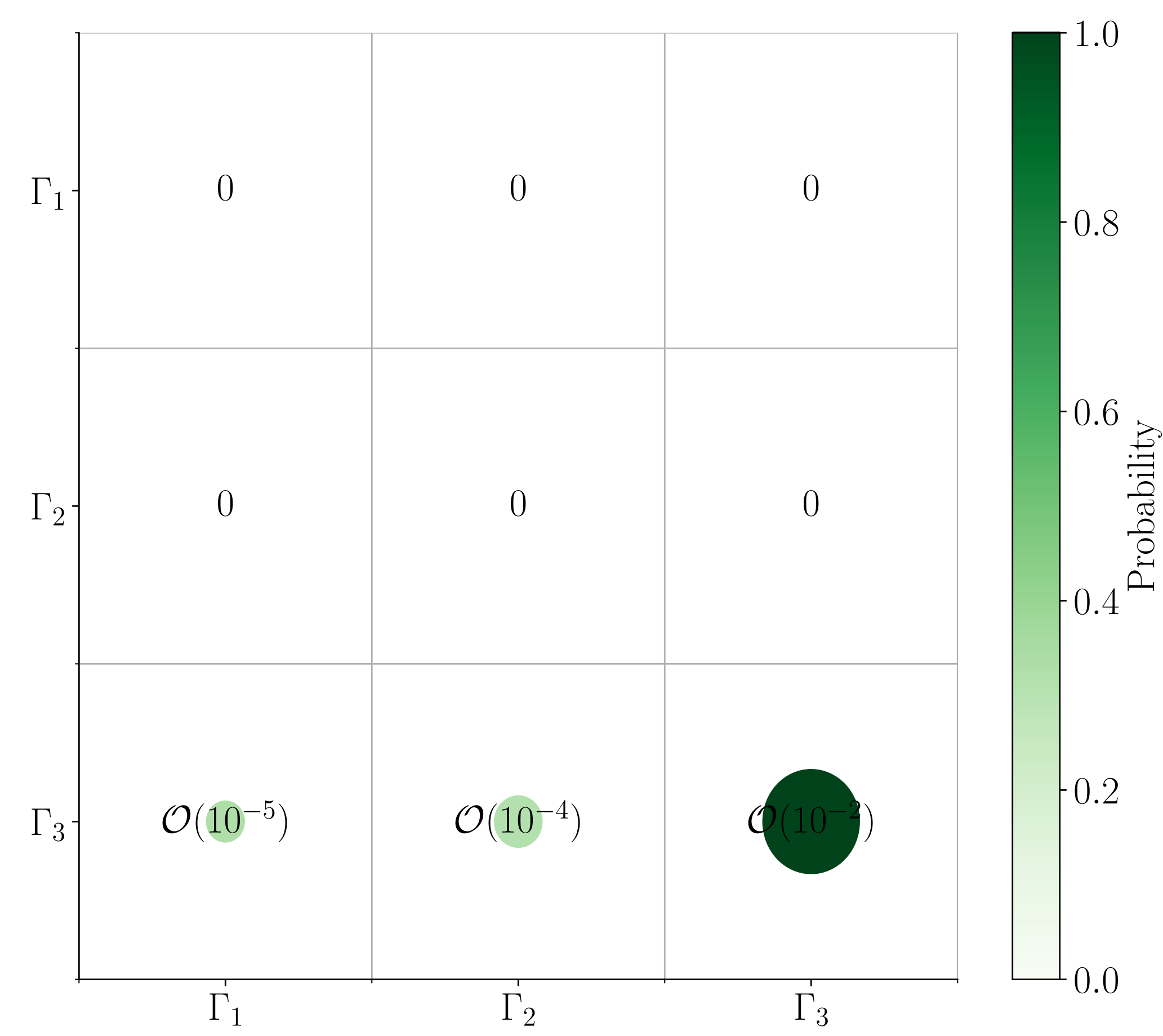
$$X_S = \pm \left(X_{\Phi_1} - X_{\Phi_2} \right), \quad X_S = \pm \frac{1}{2} \left(X_{\Phi_1} - X_{\Phi_2} \right),$$

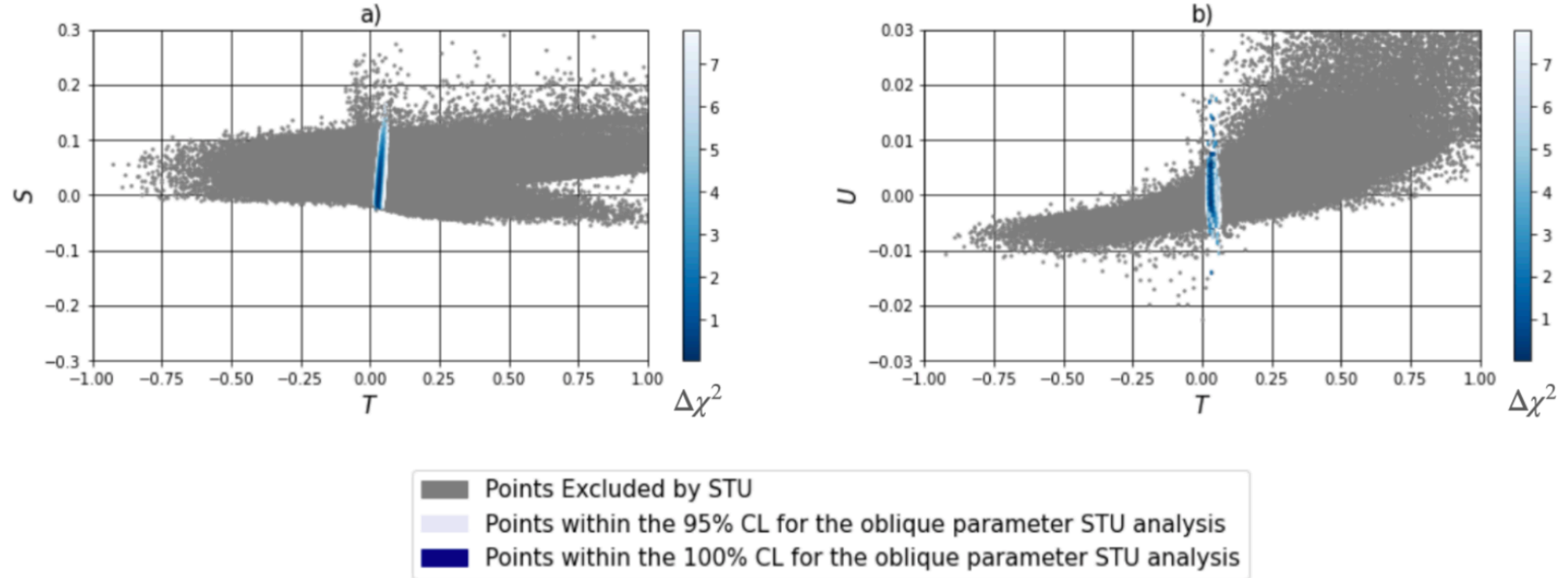


Values of Γ_1 matrix

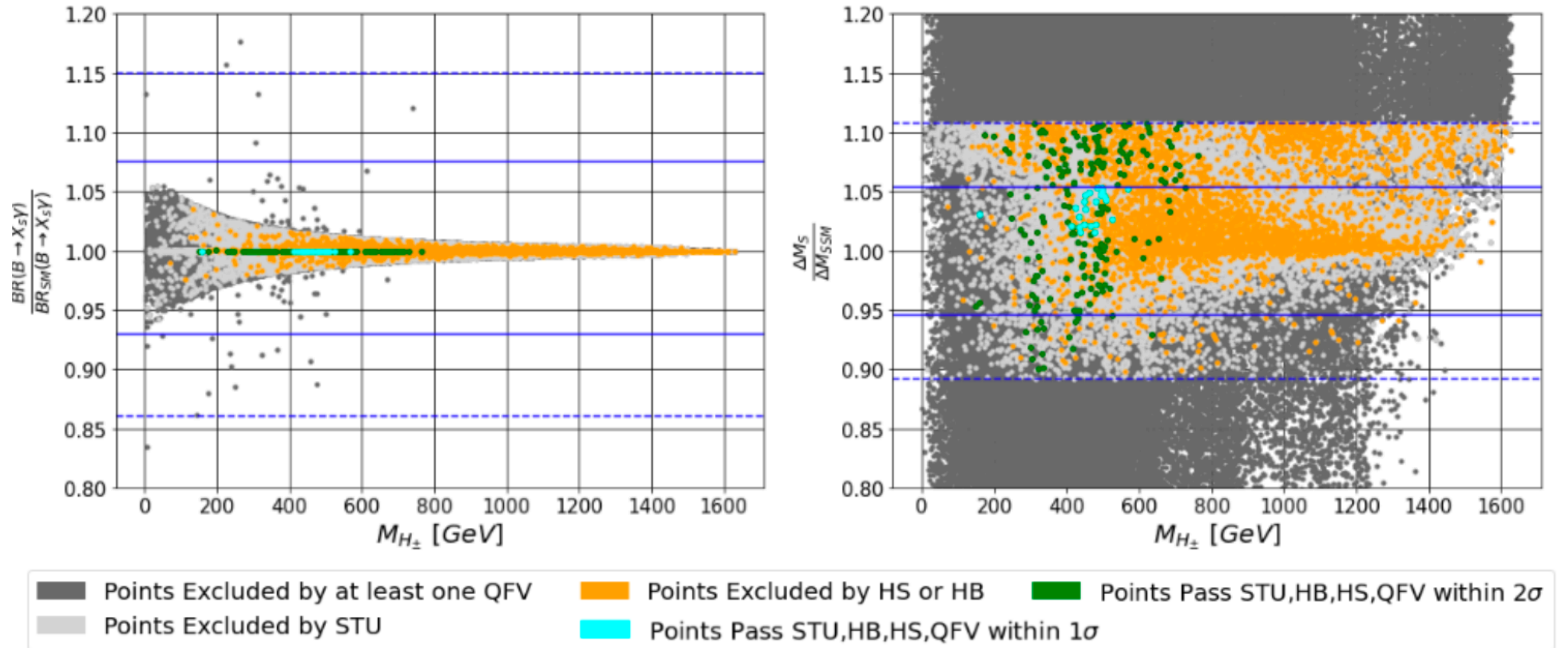


Values of Γ_2 matrix





Electroweak precision observables for all simulated points. The colored points are those who pass the STU analysis with a confidence level (CL) of at least 95%. Grey points are excluded by precision EW fit data.





Anomaly cancellation conditions

$$A_{U(1)'U(1)'U(1)'} \equiv \sum_{i=1}^3 \left(6X_{q_i}^3 + 2X_{l_i}^3 - 3X_{u_i}^3 - 3X_{d_i}^3 - X_{e_i}^3 - X_{\nu_i}^3 \right) = 0$$

$$A_{ggU(1)'} \equiv \sum_{i=1}^3 \left(6X_{q_i} + 2X_{l_i} - 3X_{u_i} - 3X_{d_i} - X_{e_i} - X_{\nu_i} \right) = 0,$$

$$A_{U(1)_Y U(1)_Y U(1)'} \equiv \sum_{i=1}^3 \left(X_{q_i} + 3X_{l_i} - 8X_{u_i} - 2X_{d_i} - 6X_{e_i} \right) = 0,$$

$$A_{U(1)_Y U(1)'U(1)'} \equiv \sum_{i=1}^3 \left(X_{q_i}^2 - X_{l_i}^2 - 2X_{u_i}^2 + X_{d_i}^2 + X_{e_i}^2 \right) = 0,$$

$$A_{SU(2)_L SU(2)_L U(1)'} \equiv \sum_{i=1}^3 \left(3X_{q_i} + X_{l_i} \right) = 0,$$

$$A_{SU(3)_C SU(3)_C U(1)'} \equiv \sum_{i=1}^3 \left(2X_{q_i} - X_{u_i} - X_{d_i} \right) = 0,$$



The model has a type-I seesaw mechanism, where the neutrino Lagrangian can be written as:

$$-\mathcal{L}_\nu^{\text{mass}} = \frac{1}{2} \overline{n_L^0} \mathcal{M} n_L^{0,c} + \text{h.c.} ,$$

Where

$$\mathcal{M} \equiv \begin{pmatrix} 0 & m_D \\ m_D^T & M_R \end{pmatrix}$$

Neutrino tree level masses:

$$m_D \equiv \frac{1}{\sqrt{2}} (v_1 \Sigma_1 + v_2 \Sigma_2) , \quad M_R \equiv A + \frac{v_S}{\sqrt{2}} (B + C)$$



Parameter space:

Parameter	$\alpha_2, \alpha_3, \gamma_1$	$\tan \beta$	δ	a_3
range	$[-\pi, \pi]$	$[0.5, 30]$	$[\frac{\pi}{2} - 1, \frac{\pi}{2} + 1]$	$[-1, 1]$

Parameter	M_{A_2}, M_{H^\pm}	M_{A_3}	M_{H_2}, M_{H_3}	a_1, a_2
range [GeV]	$[0.5, 1600]$	$[30, 2000]$	$[126, 1.800]$	$[-1, 1]$



Field expansion: $\Phi_a \equiv \frac{1}{\sqrt{2}} \begin{pmatrix} \sqrt{2}\phi_a^+ \\ v_a e^{i\varphi_a} + R_a + iI_a \end{pmatrix}, S \equiv \frac{1}{\sqrt{2}} (v_S e^{i\varphi_S} + \rho + i\eta)$

Tree level masses:

$$M_u^0 \equiv \frac{1}{\sqrt{2}} (v_1 \Delta_1 + v_2 \Delta_2),$$

$$M_d^0 = \frac{1}{\sqrt{2}} (v_1 \Gamma_1 + v_2 \Gamma_2),$$

$$M_e^0 = \frac{1}{\sqrt{2}} (v_1 \Pi_1 + v_2 \Pi_2).$$

Rotation to the mass basis

$$D_f = U_{fL}^\dagger M_f^0 U_{fR},$$

[....]



One can write the tree level mass matrices in the Higgs basis:

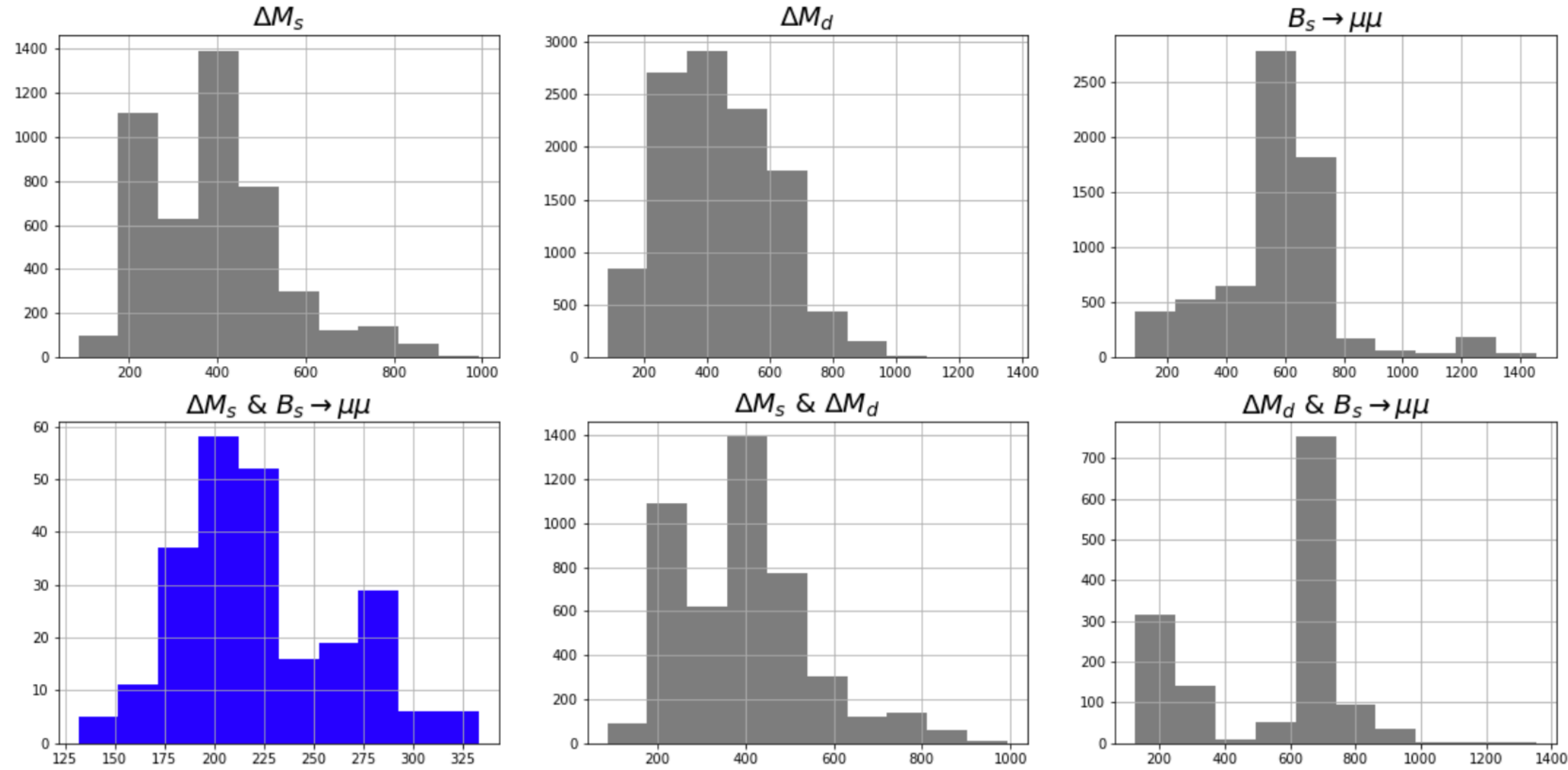
$$N_u^0 = \frac{1}{\sqrt{2}}(v_2\Delta_1 - v_1\Delta_2) \quad N_d^0 = \frac{1}{\sqrt{2}}(v_2\Gamma_1 - v_1\Gamma_2),$$

Whose off-diagonal elements are responsible for inducing tree-level FCNC interactions. One of the features of the BGL model is that those matrices can be re-expressed in terms of quark masses, CKM mixing elements and β angle

$$(N_u)_{ij} = \left(t_\beta \delta_{ij} - \left(t_\beta + t_\beta^{-1} \right) \delta_{ij} \delta_{j3} \right) m_{u_j},$$

$$(N_d)_{ij} = \left(t_\beta \delta_{ij} - \left(t_\beta + t_\beta^{-1} \right) V_{3i}^* V_{3j} \right) m_{d_j},$$

$$t_\beta = \tan \beta = v_1/v_2$$



Set of QFV observables	Acceptance ratio
$\text{BR}(B \rightarrow \chi_s \gamma)$	100.0%
$\text{BR}(B_s \rightarrow \mu\mu)$	13.0%
ΔM_d (GeV)	21.0%
ΔM_s (GeV)	9.0%
ϵ_K (GeV)	100.0%
$\text{BR}(B_s \rightarrow \mu\mu)$ & ΔM_s	0.457%
$\text{BR}(B_s \rightarrow \mu\mu)$ & ΔM_d	2.69%
ΔM_s & ΔM_d	8.788%

FIG. 5: Histograms containing points that survive STU, HS, HB and a given QFV (or pair of) in bins of the A_2 mass. The most restrictive is coloured in blue.