Dark Coloured Scalars Impact on Single and Di-Higgs Production at the LHC

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with
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Introduction

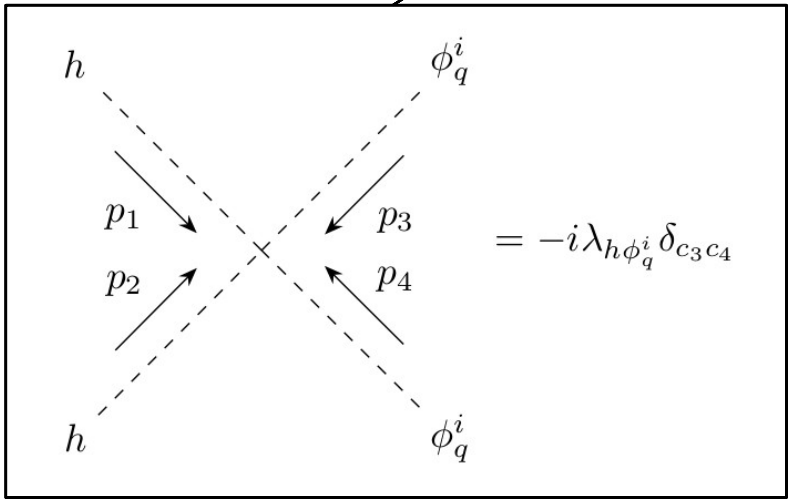
- This work was inspired by models [2.1] which provide a DM candidate while solving the muon $g-2$ and some B-physics anomalies, and in this process introduce a multiplet of colored scalars,
- However we take the colored scalars as independent fields, allowing for the results to be applied to any model,
- We focus on the impact of the number of colored scalars,
- We look at both single and double Higgs production,
- Colored Scalars contribute to Higgs production through gluon fusion [2.2] at loop level.

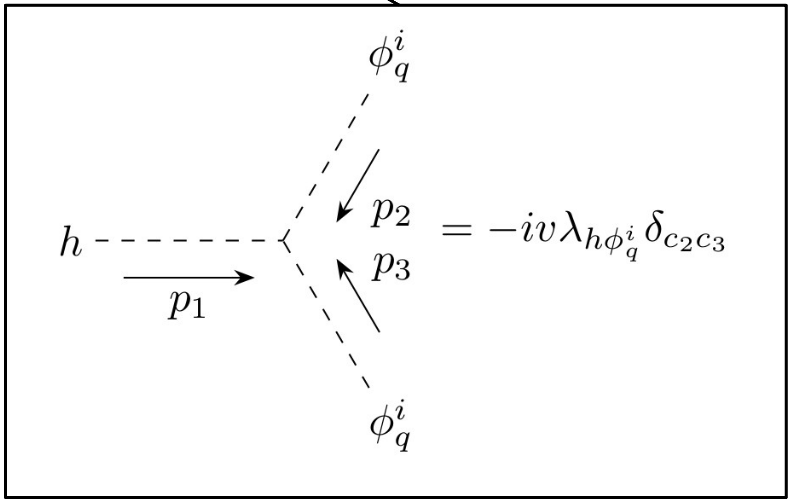
[2.1] H. M. Georgi, et al., Phys. Rev. Lett. 40 (1978) 692
[2.2] D. G. Cerdeño, et al., Eur. Phys. J. C 79 (2019) 517

Lagrangian for N Independent Colored Scalars

After EWSB we get the following potential:

$$V = \sum_{i=1}^{N} \Big[\underbrace{\left(\mu^2_{\phi^i_q} + \frac{v^2}{2}\lambda_{h\phi^i_q}\right)}_{m^2_{\phi^i_q}} |\phi^i_q|^2 + \underline{\frac{1}{2}\lambda_{h\phi^i_q} h^2 |\phi^i_q|^2} + \underline{v\lambda_{h\phi^i_q} h |\phi^i_q|^2} + \lambda_{\phi^i_q} |\phi^i_q|^4 + \ldots \Big] + \ldots$$

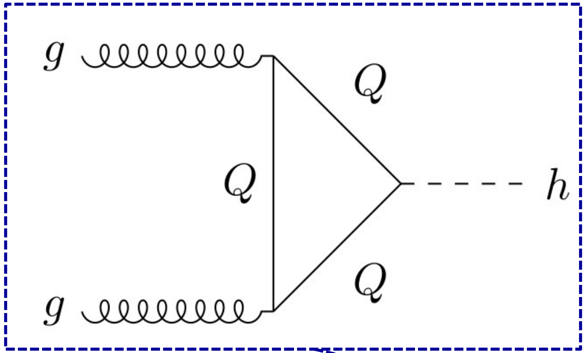


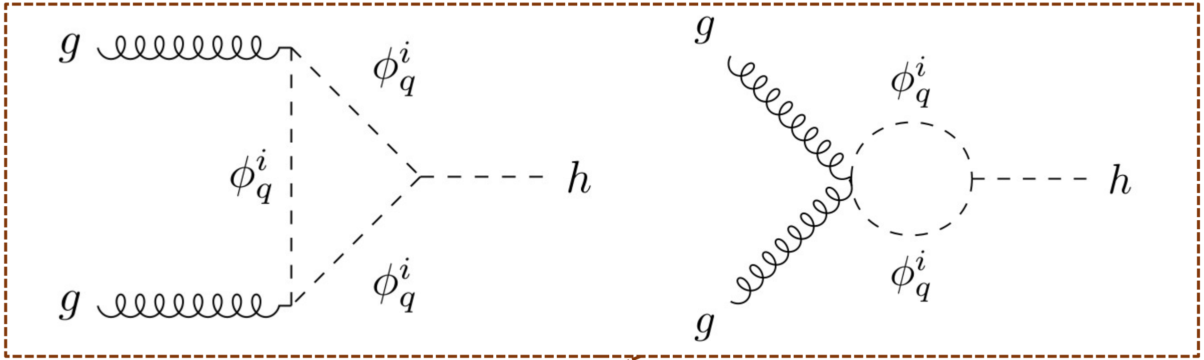


With quartic interactions to the higgs...

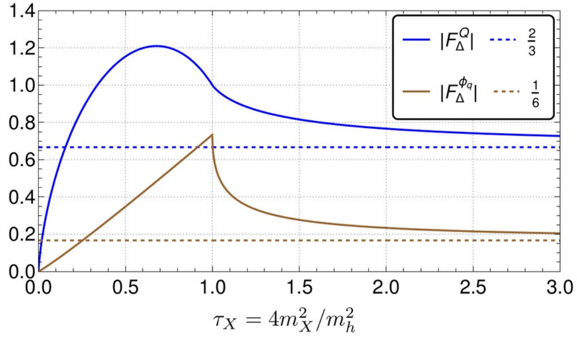
…and also triple interactions with the same parameter: $\lambda_{h\phi^i_q}$

Single Higgs Production





$$\mathcal{M}_{\triangle}^{gg\to h} = \frac{g_s^2 m_h^2}{16\pi^2}\Big(\sum_Q g_Q^h F_\triangle^Q + \sum_{\phi_q} g^h_{\phi_q^i} F_\triangle^{\phi_q^i}\Big)_i A_{1\mu\nu}\ \epsilon_a^\mu \epsilon_b^\nu\ \delta_{ab}$$

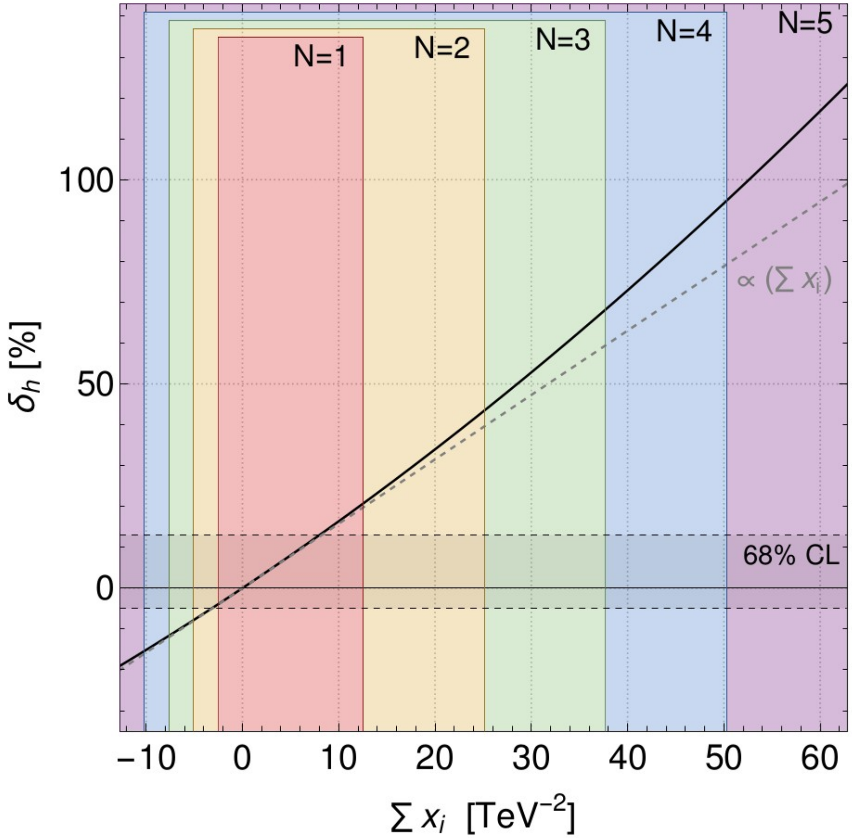


$$\lim_{m^2_{\phi_q^i}\to\infty} F_\triangle^{\phi_q^i} = \frac{1}{6}$$

The limit only carries at most a 0.2% error for masses above 1 TeV.

Single Higgs Production Results

$$\delta_h = \frac{\sigma_{\rm NP} - \sigma_{\rm SM}}{\sigma_{\rm SM}} = \frac{1}{\left|\sum_Q F_\triangle^Q\right|}\frac{v^2}{6}\underbrace{\sum_{\phi_q^i}\frac{\lambda_{h\phi_q^i}}{m^2_{\phi_q^i}}}_{(\sum_i x_i)} + \frac{1}{\left|\sum_Q F_\triangle^Q\right|^2}\frac{v^4}{144}\underbrace{\left(\sum_{\phi_q^i}\frac{\lambda_{h\phi_q^i}}{m^2_{\phi_q^i}}\right)^2}_{(\sum_i x_i)^2}$$



$$\sum_i x_i = \sum_{\phi_q^i}\frac{\lambda_{h\phi_q^i}}{m^2_{\phi_q^i}}$$

BFB and perturbative limits: $-\frac{m_h}{v}\sqrt{2\lambda_{\phi_q^i}} \leq \lambda_{h\phi_q^i} \leq 4\pi$

Minimum mass: $m_{\phi_q^i} \geq 1$ TeV

$$-\sqrt{8\pi}\frac{m_h}{v}N \leq \sum_i x_i \leq 4\pi N$$

$$-\sqrt{\frac{\pi}{2}}v m_h N + \frac{\pi}{8}v^2 m_h^2 N^2 \leq \delta_h \leq \pi v^2 N + \frac{\pi^2}{4}v^4 N^2$$

$N < 64$

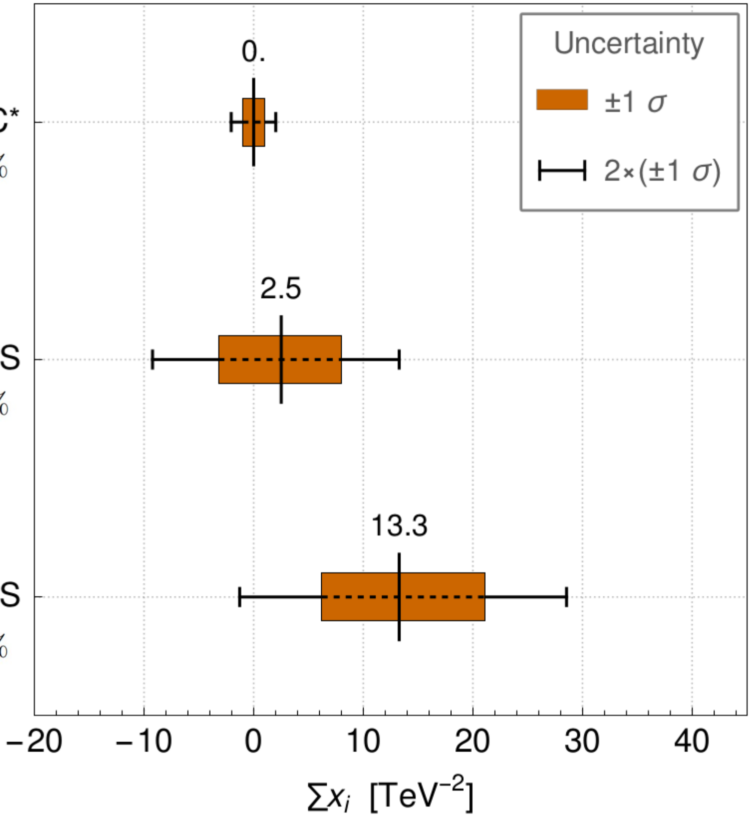
Experimental Constraints

We can determine the values for the sum $\sum_i x_i$ that comply to the experimental values for δ_h^{exp}

[6.1] HL–LHC*
$\delta_h^{\text{exp}} = 0 \pm 1.6\ \%$

[6.2] ATLAS
$\delta_h^{\text{exp}} = 4 \pm 9\ \%$

[6.3] CMS
$\delta_h^{\text{exp}} = 22^{+14}_{-12}\ \%$



$$-1\ \text{TeV}^{-2} \leq \sum_i x_i \leq 1\ \text{TeV}^{-2}$$

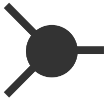
$$-3.2\ \text{TeV}^{-2} \leq \sum_i x_i \leq 8.0\ \text{TeV}^{-2}$$

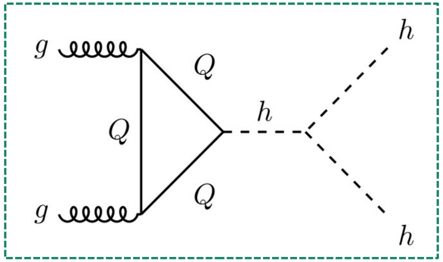
$$6.2\ \text{TeV}^{-2} \leq \sum_i x_i \leq 21.1\ \text{TeV}^{-2}$$

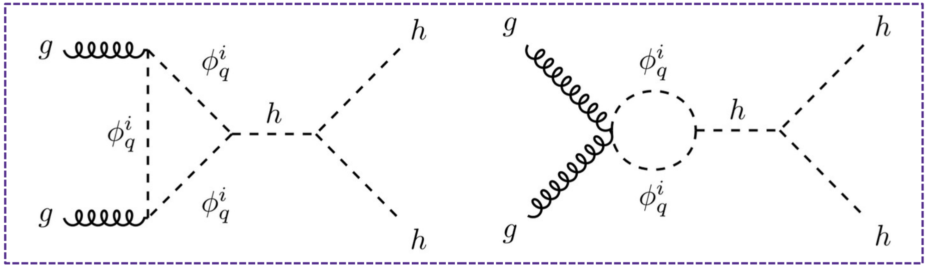
[6.1] M. Cepeda et al.,CERN Yellow Rep. Monogr. 7 (2019) 221–584
[6.2] ATLAS collaboration, Phys. Rev. D 101 (2020) 012002
[6.3] CMS collaboration, Eur. Phys. J. C 79 (2019) 421

Double Higgs Production

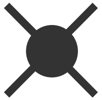
Triangle Diagrams

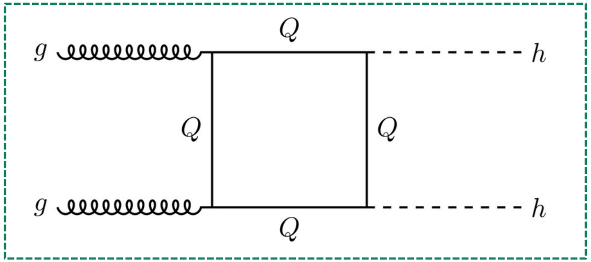


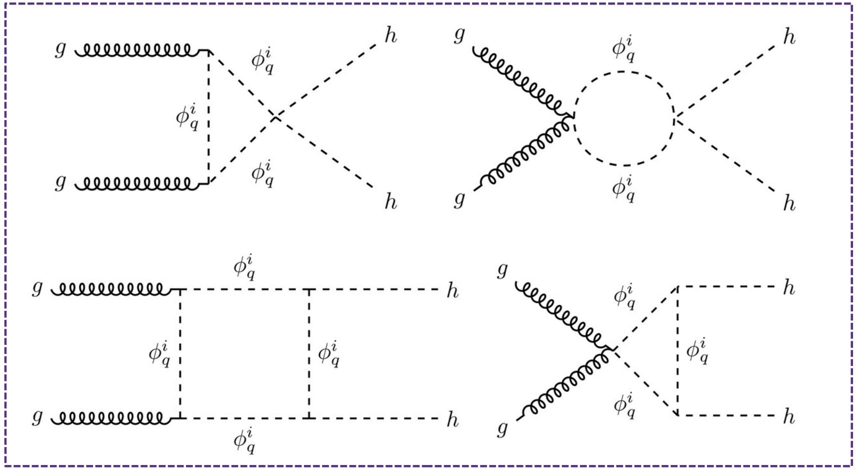




Box Diagrams







LHC Production (gluon fusion)

$$\sigma(pp \to hh) = \int_{4m_h^2/w}^{1} \mathrm{d}\tau_h \frac{\mathrm{d}\mathcal{L}^{gg}}{\mathrm{d}\tau_h} \sigma_0^{hh}(s = \tau_h w) \ , \qquad \sigma_0^{hh}(s) = \left|\mathcal{M}^{gg\to hh}\right|^2 = \left|\mathcal{M}_F^{gg\to hh}\right|^2 + \left|\mathcal{M}_G^{gg\to hh}\right|^2 \ ,$$

$$\delta_{hh} = \frac{\sigma_{NP} - \sigma_{\mathrm{SM}}}{\sigma_{\mathrm{SM}}}$$

Need to perform the gluon luminosity and form factor coeficients integrations.

Numerical integration with HPAIR [8.1]

$$G_{\Box}^{\phi_q^i} = \frac{4m_{\phi_q^i}^4}{s}\left(\frac{1}{tu - m_h^4}\right)\Bigg(s(t+u)C_{ab}^{m_{\phi_q^i}^2} + (2t)(t-m_h^2)C_{ac}^{m_{\phi_q^i}^2} + (2u)(u-m_h^2)C_{bc}^{m_{\phi_q^i}^2} - (t^2+u^2-2m_h^4)C_{cd}^{m_{\phi_q^i}^2}$$
$$- (st^2 + 2m_{\phi_q^i}^2(tu - m_h^4))D_{bac}^{m_{\phi_q^i}^2} - (su^2 + 2m_{\phi_q^i}^2(tu - m_h^4))D_{abc}^{m_{\phi_q^i}^2} - (2m_{\phi_q^i}^2(tu - m_h^4))D_{acb}^{m_{\phi_q^i}^2}\Bigg) \ ,$$

$$F_{\Box_1}^{\phi_q^i} = \frac{4m_{\phi_q^i}^4}{s}\left(\frac{2}{s}(t-m_h^2)C_{ac}^{m_{\phi_q^i}^2} + \frac{2}{s}(u-m_h^2)C_{bc}^{m_{\phi_q^i}^2} - (2m_{\phi_q^i}^2)(D_{abc}^{m_{\phi_q^i}^2} + D_{bac}^{m_{\phi_q^i}^2}) - (2m_{\phi_q^i}^2 + \frac{1}{s}(tu - m_h^4))D_{acb}^{m_{\phi_q^i}^2}\right) \ ,$$

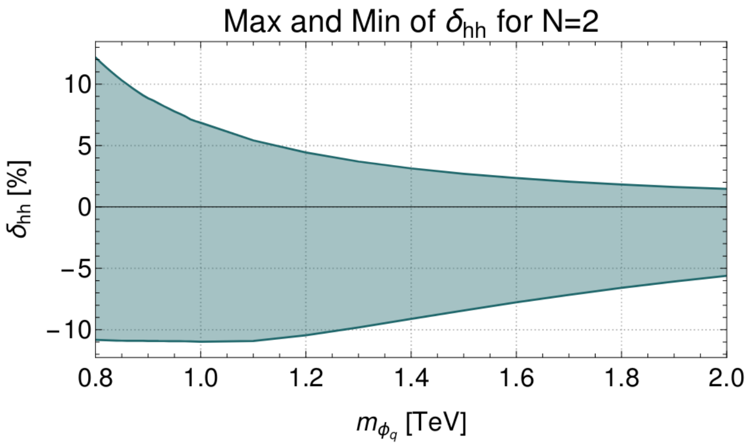
[8.1] T. Plehn, M. Spira and P. M. Zerwas, Nucl. Phys. B 479 (1996) 46–64

N = 2 Colored Scalars

We scan over all parameters for **N=2** and perform the calculations with **HPAIR**.

$$m_i \equiv m_{\phi_q^i}$$
$$\lambda_i \equiv \lambda_{h\phi_q^i}$$

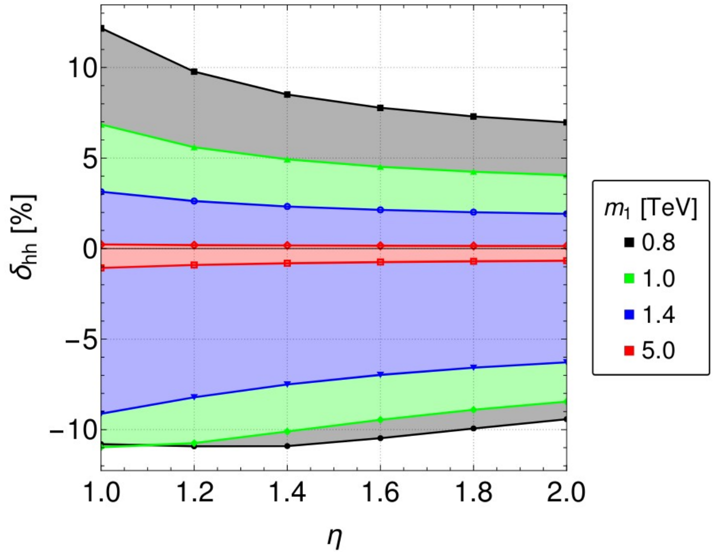
$$m_2 = m_1$$



When we fix the first mass

while increasing the second mass, the values decrease.

$$m_2 = \eta\ m_1$$



$$\delta_{hh}^{\max/\min}(m_{\phi_q^1}, m_{\phi_q^2}) \approx \left[\delta_{hh}^{\max/\min}(m_{\phi_q^1}, m_{\phi_q^1}) + \delta_{hh}^{\max/\min}(m_{\phi_q^2}, m_{\phi_q^2})\right]/2\,.$$

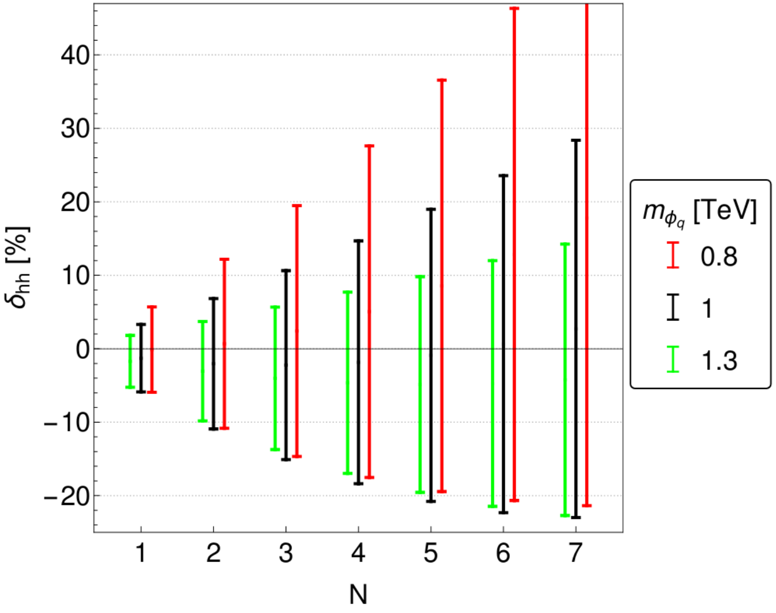
$$m_{\phi_q^i} = m_{\phi_q^k} \equiv m_{\phi_q}$$

For N > 1 we will take the masses always equal, assuming the interference between them is negligable

Double Higgs Production Results

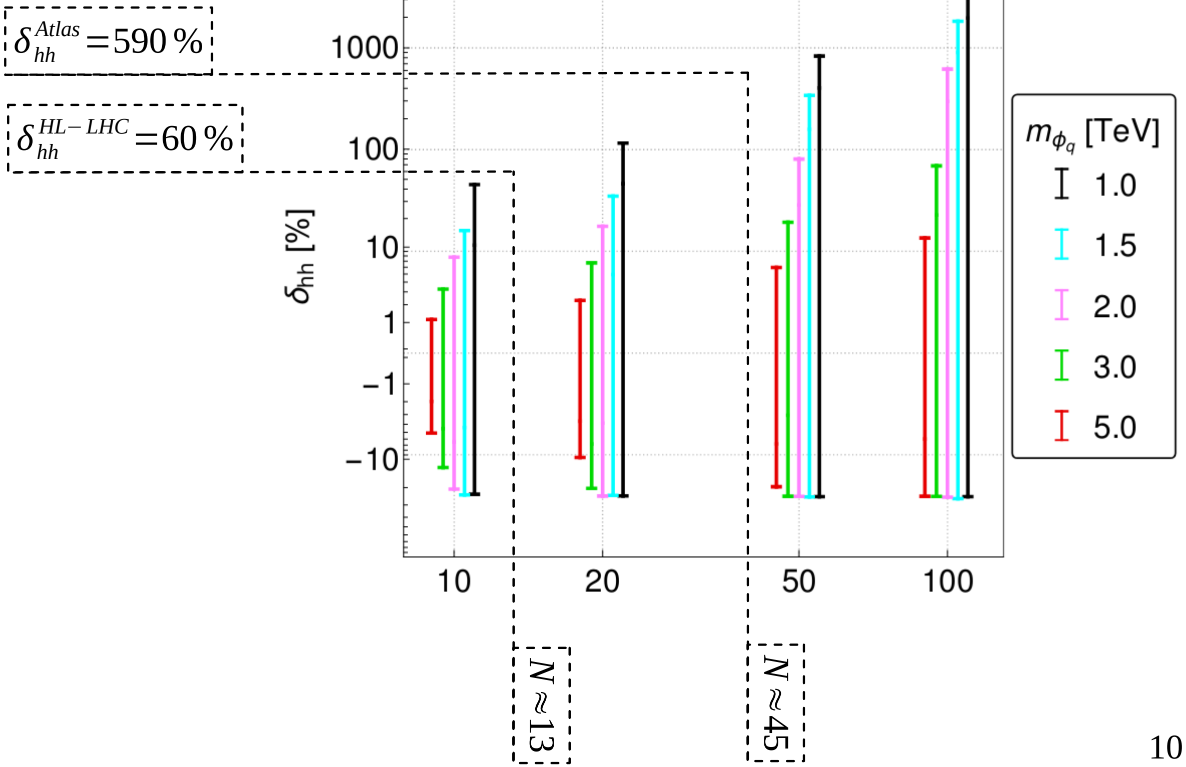
Larger values of N

The results for a few scalars are small when compared with the experimental values.



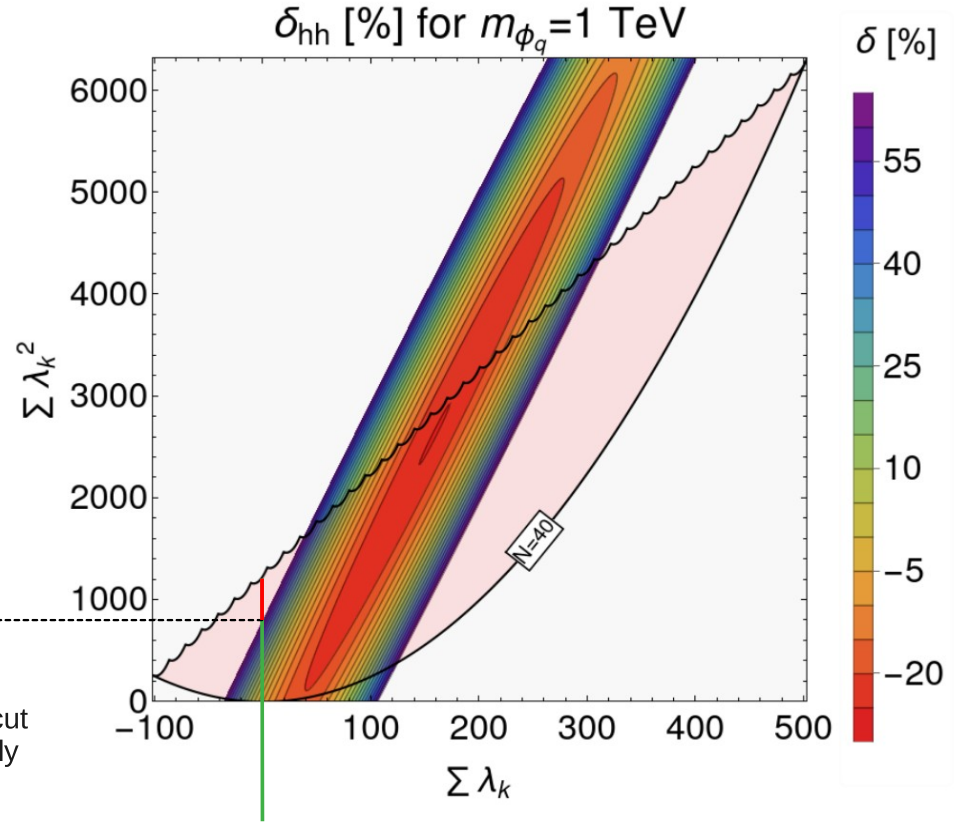
Only for higher values of N can double Higgs production offer any cuts in the parameters.

[10.1,10.2]



[10.1] ATLAS collaboration, Phys. Lett. B 800 (2020) 135103
[10.2] M. Cepeda et al., CERN Yellow Rep. Monogr. 7 (2019) 221–584

Single and Double Higgs Production Results



$$\sum \lambda_k^2 \leq 800$$

This dependence gives us a cut which is not possible with only single Higgs production.

$$-1 \leq \sum \lambda_k \leq 1$$

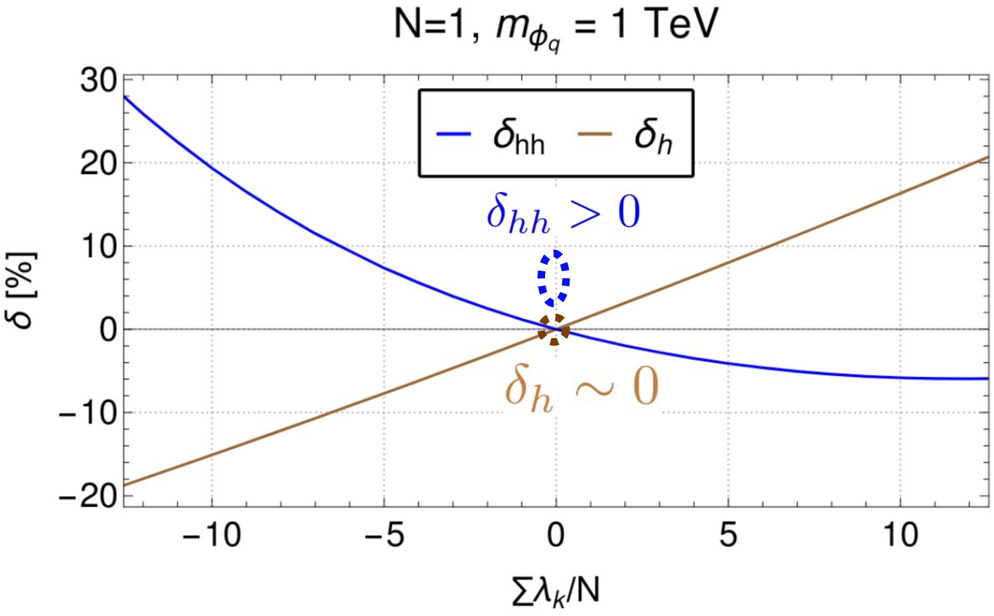
Constraints from **Single Higgs Production** (HL-LHC)

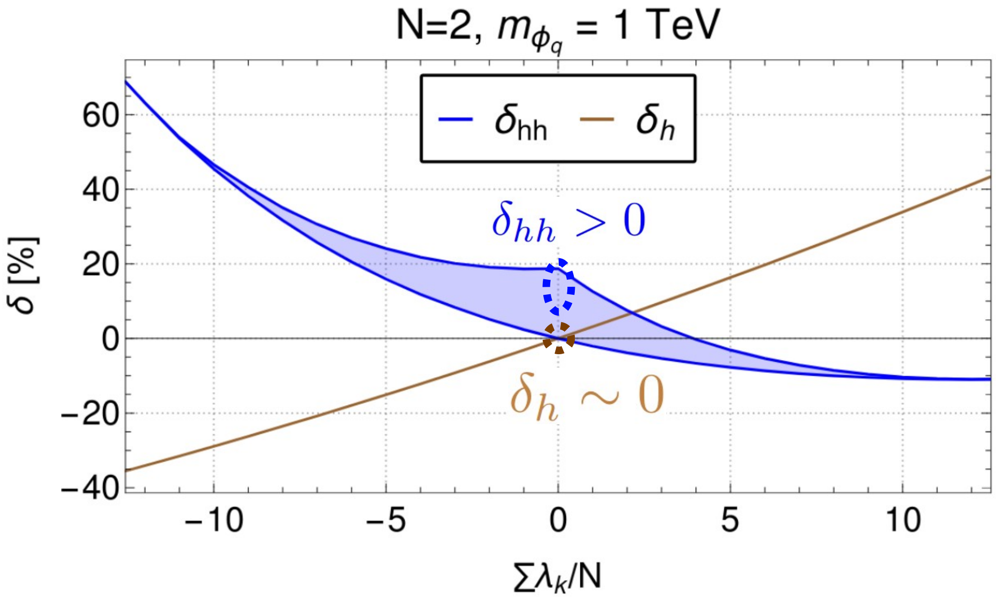
$$-35 \leq \sum \lambda_k \leq 310$$

Constraints from Double Higgs Production are less restrictive

Single and Double Higgs Production Results

Complementarity (no BFB)





$$(\delta_{hh} > 0) \wedge (\delta_h \sim 0) \longrightarrow N \geq 2$$

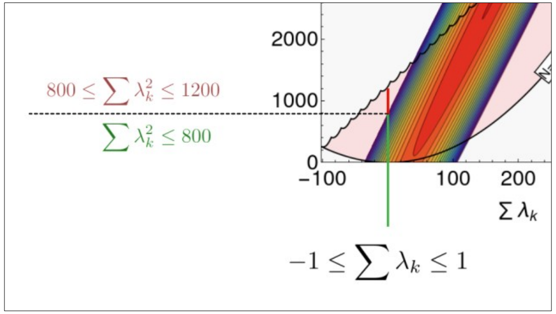
Only with both measurements can we make this conclusion about the number of scalars.

Conclusions

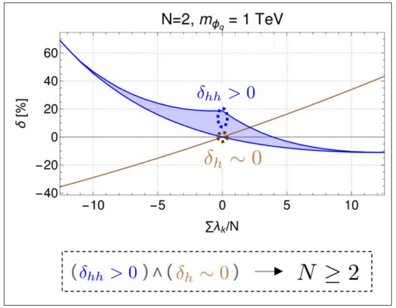
- Single Higgs production will offer a significant cut in the parameter space...

$$-1\ \text{TeV}^{-2} \leq \sum_i x_i \leq 1\ \text{TeV}^{-2}$$ (HL-LHC)

- ...while double Higgs production can offer cuts not possible with single Higgs (for large values of N)



- The combination of single and double Higgs production could provide insight in to the possible models.



- The colored scalar implementation of HPAIR (HPAIR-SCALARS) can be found at:

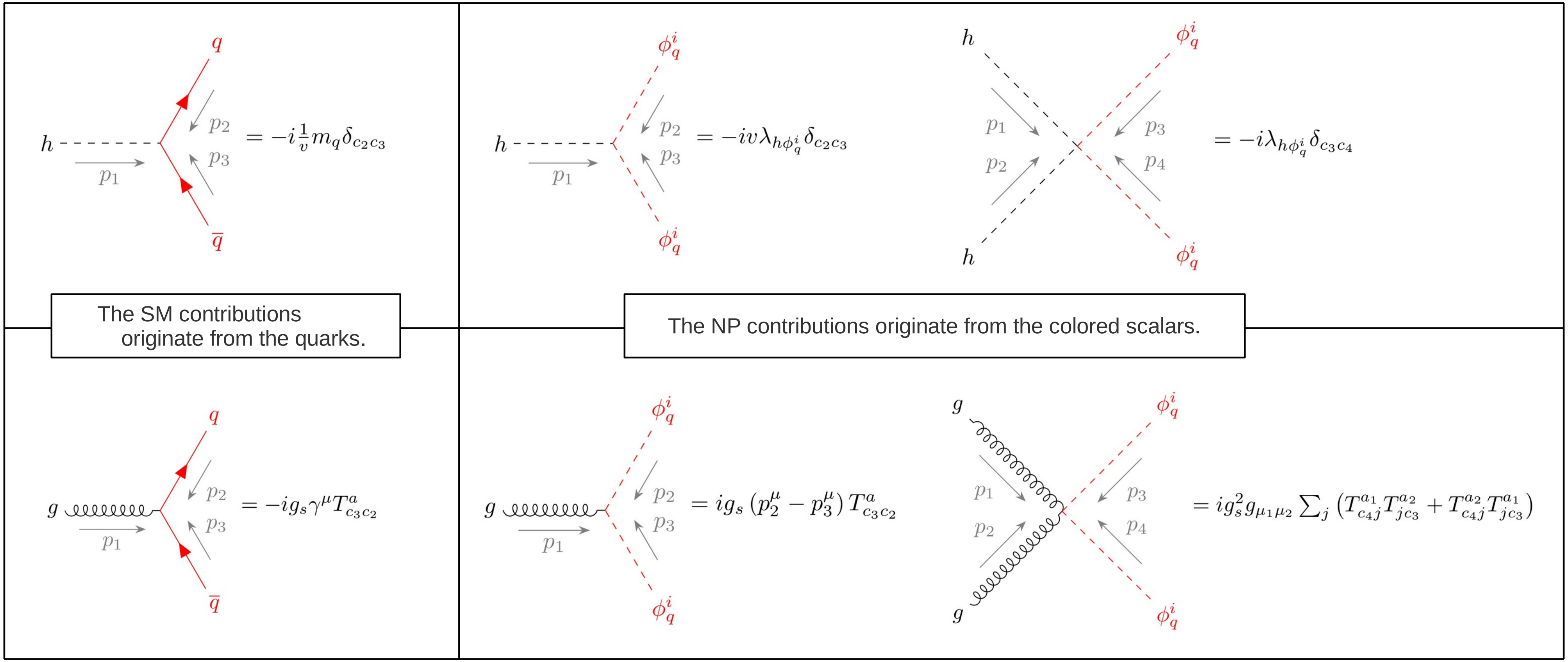
  https://gitlab.com/bdm-models/higgs-production/hpair-scalars

- Available online: arXiv:2308.07023 [hep-ph]

Thank you for listening!

Higgs and Gluon Interactions

Only the following interactions are important to Higgs production through gluon fusion:



Form Factors

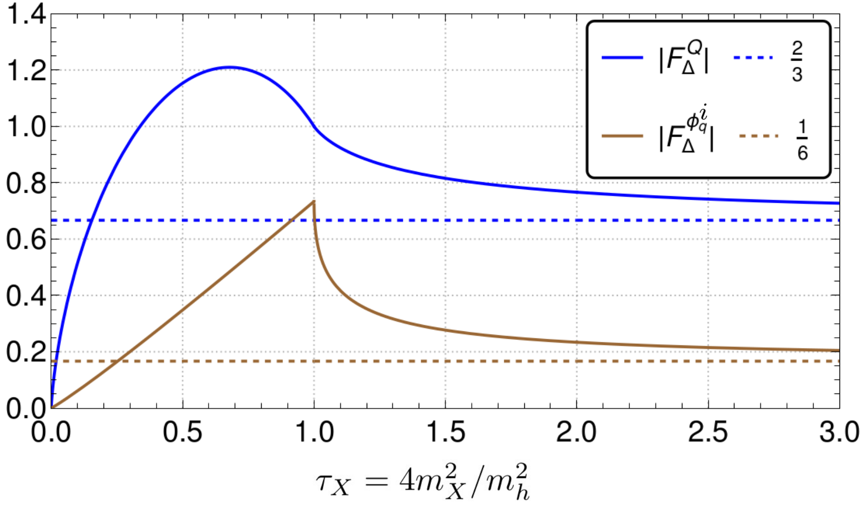
The form factors can be calculated analytically:

$$\mathcal{M}_{\triangle}^{gg\to h} = \frac{g_s^2 m_h^2}{16\pi^2}\Big(\sum_Q g_Q^h F_{\triangle}^{Q} + \sum_{\phi_q^i} g_{\phi_q^i}^h F_{\triangle}^{\phi_q^i}\Big)\; A_{1\mu\nu}\; \epsilon_a^{\mu}\epsilon_b^{\nu}\; \delta_{ab}$$

$$g_Q^h = \frac{1}{v}, \quad F_{\triangle}^{Q} = \tau_Q\left(1 + (1-\tau_Q) f(\tau_Q)\right),$$

$$g_{\phi_q^i}^h = \frac{\lambda_{h\phi_q^i} v}{2 m_{\phi_q^i}^2}, \quad F_{\triangle}^{\phi_q^i} = -\frac{1}{2}\tau_{\phi_q^i}\left(1 - \tau_{\phi_q^i} f(\tau_{\phi_q^i})\right),$$

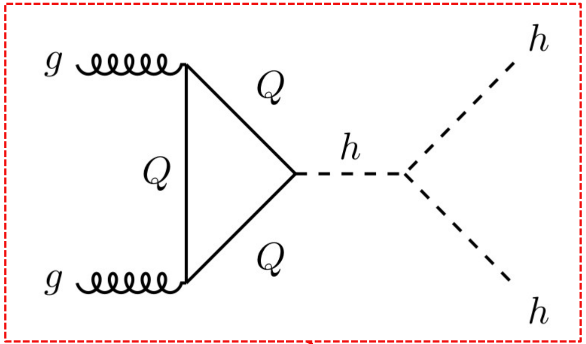
With our convention, both form factors have a non-zero constant limit for large masses:

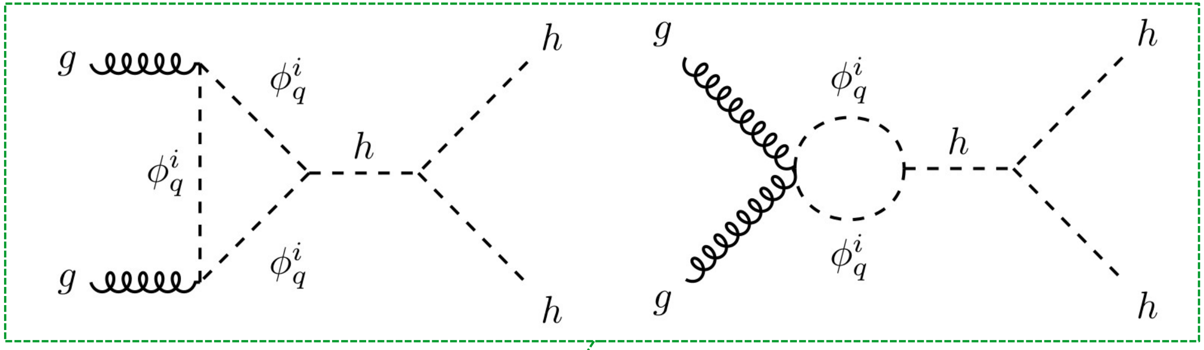


$$f(\tau) = \begin{cases} \arcsin\left(\frac{1}{\sqrt{\tau}}\right)^2, & \tau \geq 1 \\ -\frac{1}{4}\left[\log\left(\frac{1+\sqrt{1-\tau}}{1-\sqrt{1-\tau}}\right) - i\pi\right]^2, & \tau < 1 \end{cases}$$

Double Higgs Production

Triangle Diagrams





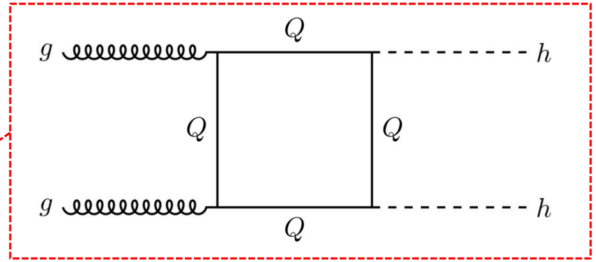
$$\mathcal{M}_{\triangle}^{gg\to hh} = \frac{g_s^2 s}{16\pi^2}\, C_{\triangle} \left(\sum_{Q} g_Q^h F_{\triangle}^{Q} + \sum_{\phi_q^i} g_{\phi_q^i}^h F_{\triangle}^{\phi_q^i} \right) A_{1\mu\nu}\epsilon_a^{\mu}\epsilon_b^{\nu}\ \delta_{ab}$$

Double Higgs Production

Box Diagrams

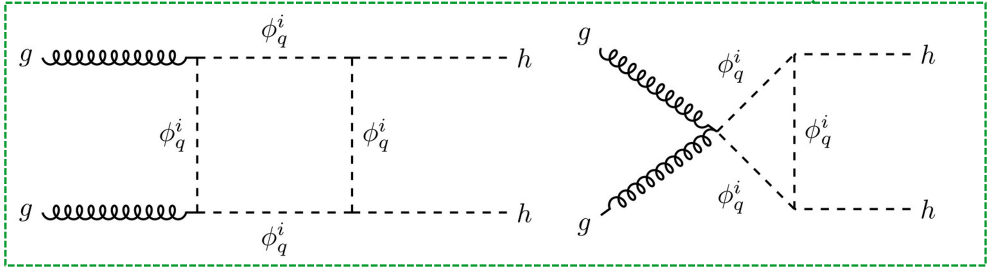
$$\mathcal{M}_{\square}^{gg\to hh} = \frac{g_s^2 s}{16\pi^2}\, C_{\square}\Bigg[\sum_{Q}\Big((g_Q^h)^2 G_{\square}^{Q} A_{2\mu\nu} + (g_Q^h)^2 F_{\square}^{Q} A_{1\mu\nu}\Big)$$

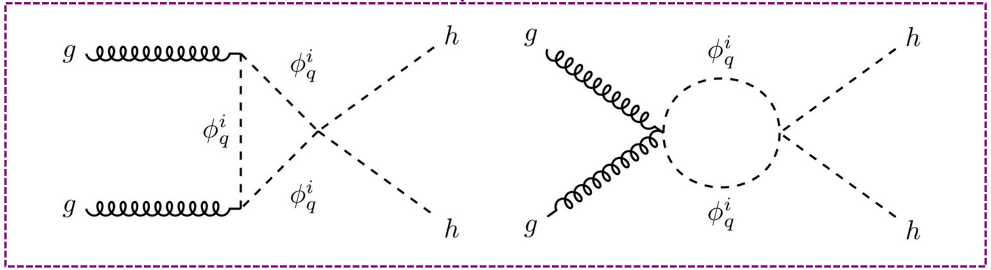
$$+\sum_{\phi_q^i}\Big((g_{\phi_q^i}^h)^2 G_{\square}^{\phi_q^i} A_{2\mu\nu} + \Big((g_{\phi_q^i}^h)^2 F_{\square_1}^{\phi_q^i} + (g_{\phi_q^i}^{hh}) F_{\square_2}^{\phi_q^i}\Big) A_{1\mu\nu}\Big)\Bigg]\epsilon_a^{\mu}\epsilon_b^{\nu}\ \delta_{ab}$$



$$g_{\phi_q^i}^h = \frac{\lambda_{h\phi_q^i} v}{2m_{\phi_q^i}^2},$$

$$g_{\phi_q^i}^{hh} = \frac{\lambda_{h\phi_q^i}}{2m_{\phi_q^i}^2}$$





LHC Production (gluon fusion)

$$\sigma(pp \to h) = \sigma_0^h \tau_h \frac{\mathrm{d}\mathcal{L}^{gg}}{\mathrm{d}\tau_h}, \qquad \sigma_0^h = \frac{\pi}{16 m_h^4} \left| \mathcal{M}_\triangle^{gg\to h} \right|^2,$$

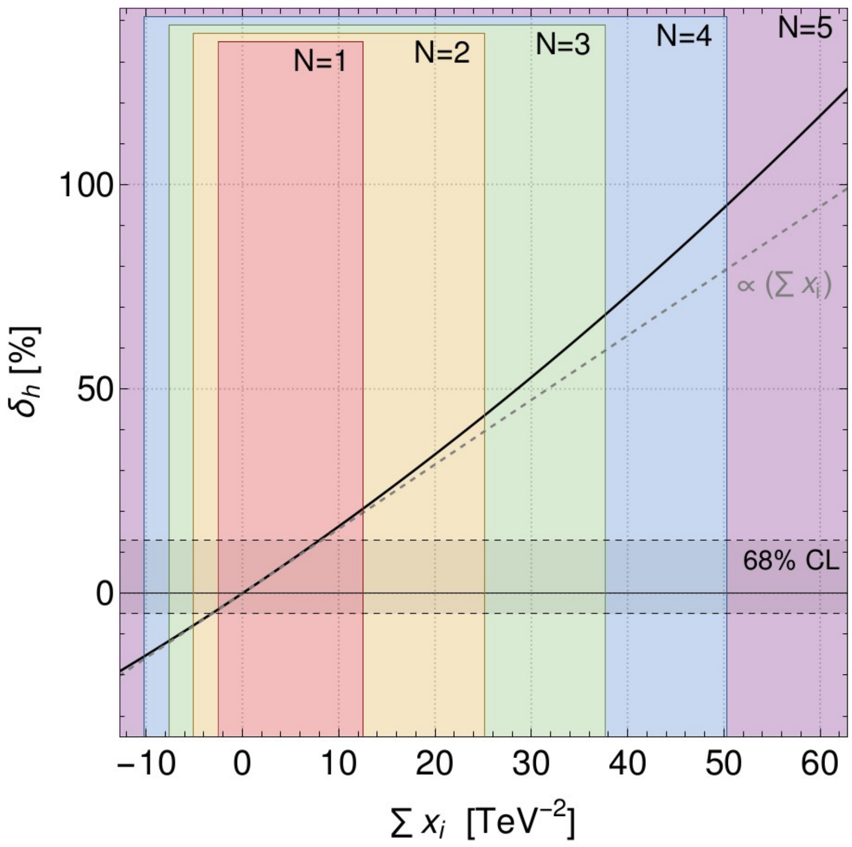
$$\delta_h = \frac{\sigma_{\mathrm{NP}} - \sigma_{\mathrm{SM}}}{\sigma_{\mathrm{SM}}}.$$

All the factors and gluon luminosity cancel out, leaving only the form factors.

$$\delta_h = 2v \sum_{\phi_q^i} g^h_{\phi_q^i} \,\mathrm{Re}\left[\frac{F_\triangle^{\phi_q^i}}{\sum_Q F_\triangle^Q}\right] + v^2 \frac{\left|\sum_{\phi_q^i} g^h_{\phi_q^i} F_\triangle^{\phi_q^i}\right|^2}{\left|\sum_Q F_\triangle^Q\right|^2} = \frac{1}{\left|\sum_Q F_\triangle^Q\right|} \frac{v^2}{6} \sum_{\phi_q^i} \frac{\lambda_{h\phi_q^i}}{m^2_{\phi_q^i}} + \frac{1}{\left|\sum_Q F_\triangle^Q\right|^2} \frac{v^4}{144} \left(\sum_{\phi_q^i} \frac{\lambda_{h\phi_q^i}}{m^2_{\phi_q^i}}\right)^2$$

$(\sum_i x_i)$ $\qquad$ $(\sum_i x_i)^2$

Single Higgs Production Results



$$\sum_i x_i = \sum_{\phi_q^i} \frac{\lambda_{h\phi_q^i}}{m^2_{\phi_q^i}}$$

BFB and perturbative limits: $-\frac{m_h}{v}\sqrt{2\lambda_{\phi_q^i}} \leq \lambda_{h\phi_q^i} \leq 4\pi$

Minimum mass: $m_{\phi_q^i} \geq 1\ \text{TeV}$

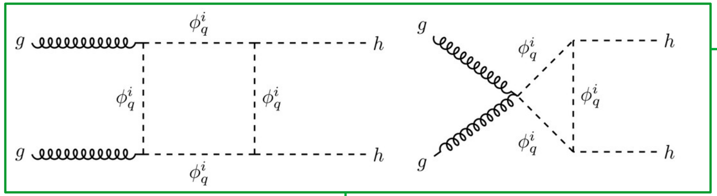
$$-\sqrt{8\pi}\,\frac{m_h}{v} N \leq \sum_i x_i \leq 4\pi N$$

$$-\sqrt{\frac{\pi}{2}} v m_h N + \frac{\pi}{8} v^2 m_h^2 N^2 \leq \delta_h \leq \pi v^2 N + \frac{\pi^2}{4} v^4 N^2$$

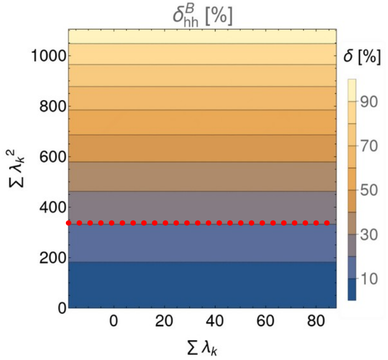
$N < 64$

Decrease computation time

We can split the diagrams based on their dependence on the couplings.



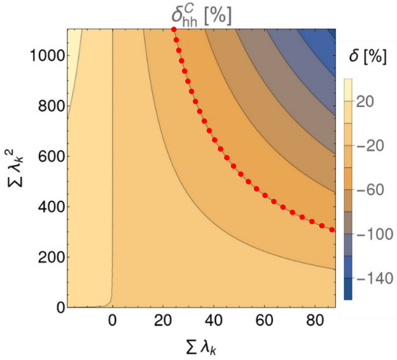
$\propto \sum \lambda_k^2$



We save a lot of time by calculating the individual contribution only once per contour.

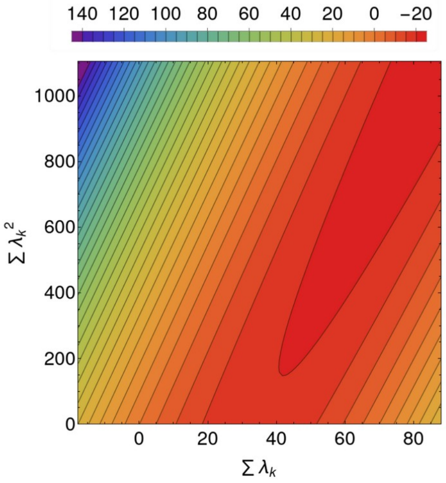
Interference Terms

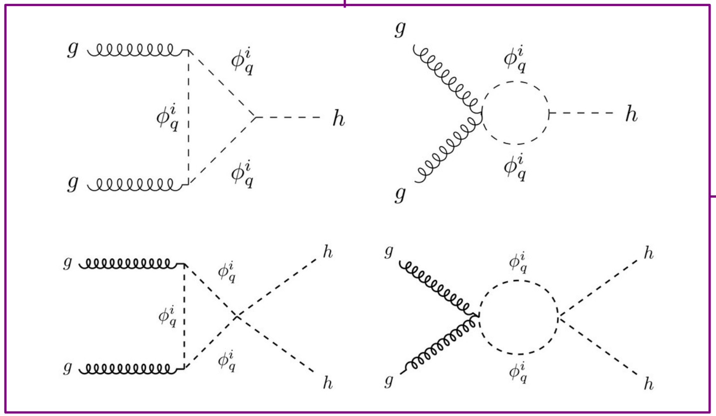
$\propto (\sum \lambda_k) \times (\sum \lambda_k^2)$



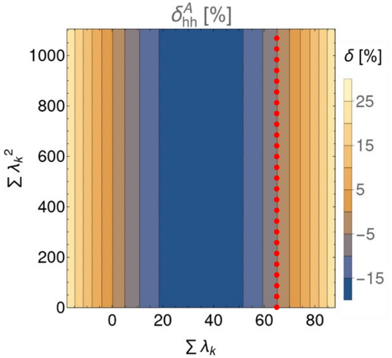
Splitting the contributions in these 3 components means less calculations.

$$\delta_{hh} = \delta^A_{hh} + \delta^B_{hh} + \delta^C_{hh}$$



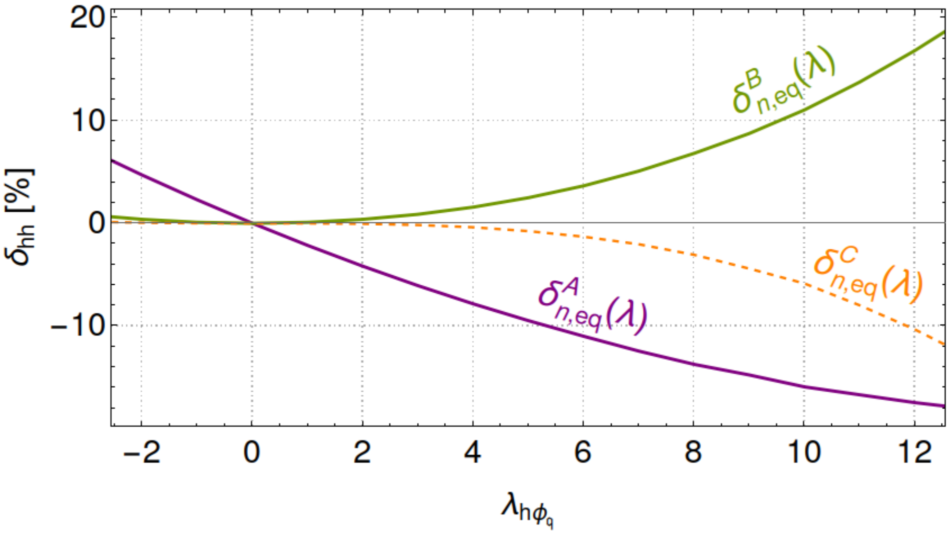


$\propto \sum \lambda_k$

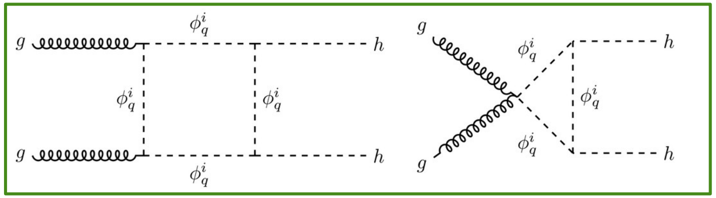


N Colored Scalars

Separated Diagrams

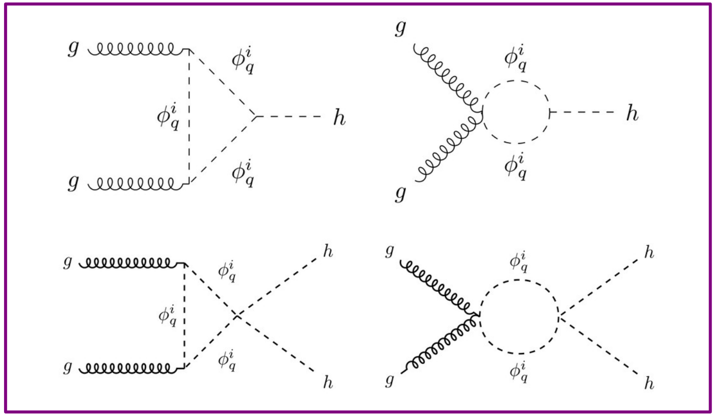


$F_{\square_1}+G_{\square}$



$F_{\Delta}+F_{\square_2}$ and $F_{\square_1}+G_{\square}$ Interference

$F_{\Delta}+F_{\square_2}$

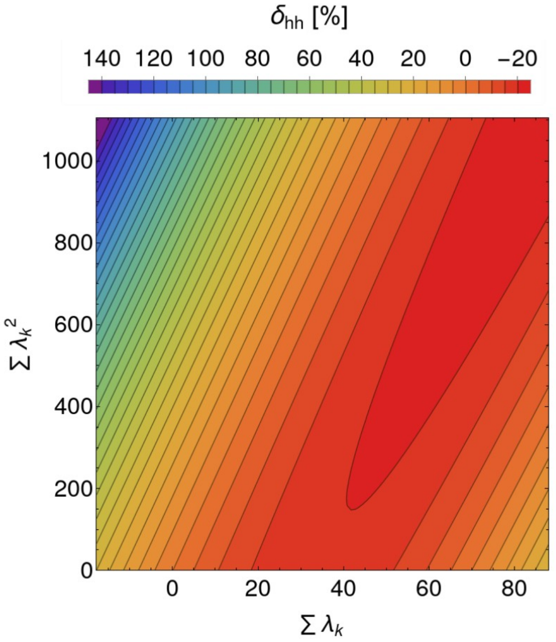


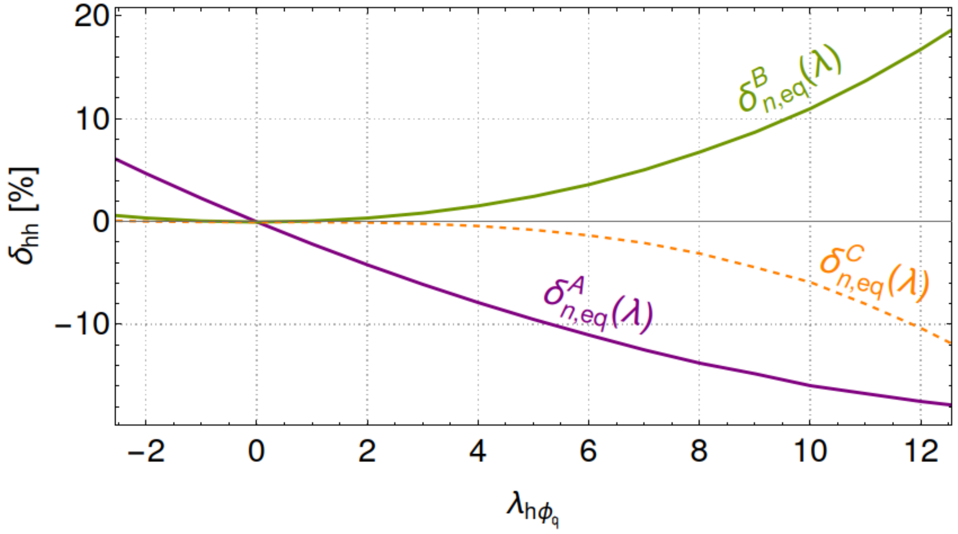
We calculate the three components for a model with n scalars and all the couplings equal.

$$\lambda_{h\phi_q^i} = \lambda_{h\phi_q^k} \equiv \lambda_{h\phi_q}$$

N Colored Scalars

Separated Diagrams Formula





With this method, the three functions on the right are enough to calculate the full results on the left for any other number of scalars, N.

$$\delta^N_{hh}(\{\lambda'_1, \ldots, \lambda'_N\}) = \delta^A_n(\lambda)\Big|_{\lambda=\frac{1}{n}\sum\lambda'_k} + \delta^B_n(\lambda)\Big|_{\lambda=\sqrt{\frac{1}{n}\sum\lambda'^2_k}} + \delta^C_n(\lambda)\Big|_{\lambda=\sqrt[3]{\left(\frac{1}{n}\sum\lambda'_k\right)\left(\frac{1}{n}\sum\lambda'^2_k\right)}}$$

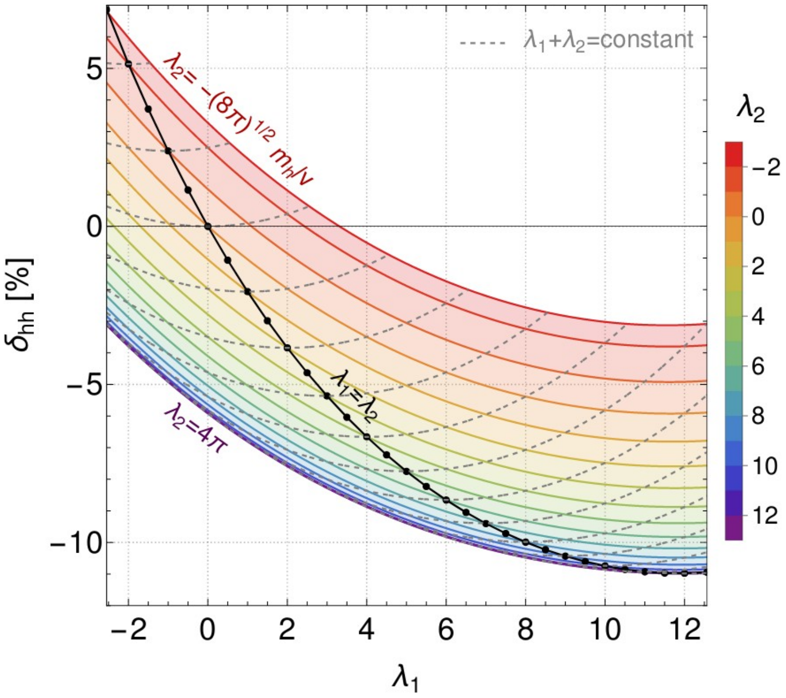
N = 2 Colored Scalars

We scan over all parameters for N=2 with a fixed mass of 1 TeV and perform the calculations with HPAIR.

m_1 = 1. TeV

$$m_i \equiv m_{\phi_q^i}$$
$$\lambda_i \equiv \lambda_{h\phi_q^i}$$

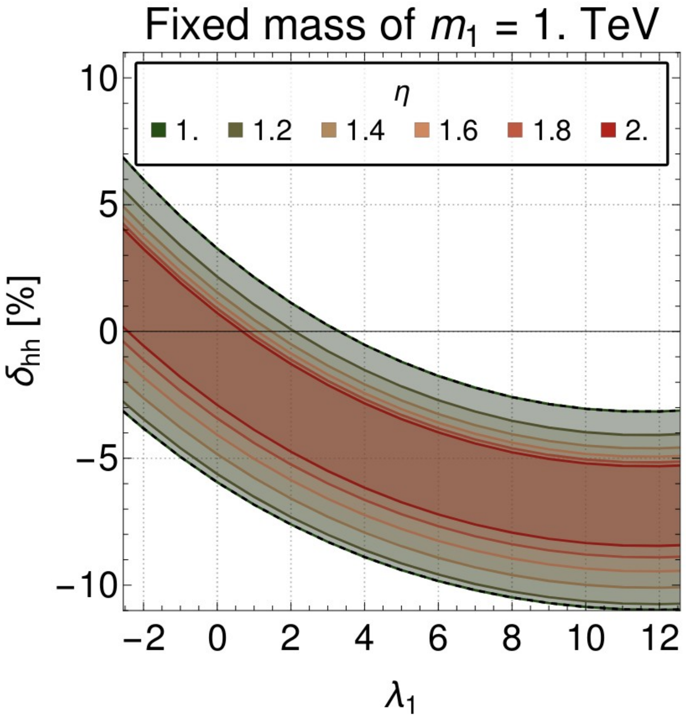
$$m_2 = m_1$$



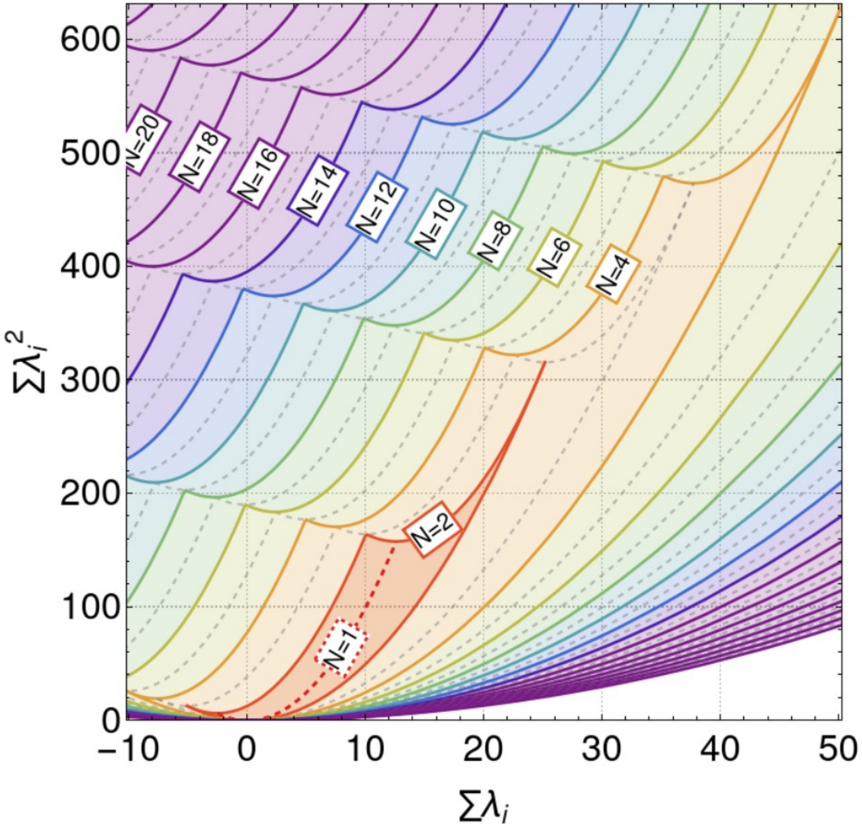
When we fix the first mass

while increasing the second mass, the values decrease.

$$m_2 = \eta\, m_1$$



N Colored Scalars
Model Regions



For a model with N scalars and all masses equal,
the remaining two parameters can be constrained by the following equations:

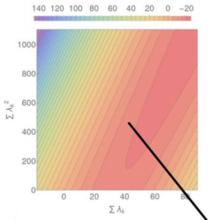
$$N\lambda_{\min} \leq \sum \lambda_k \leq N\lambda_{\max}$$

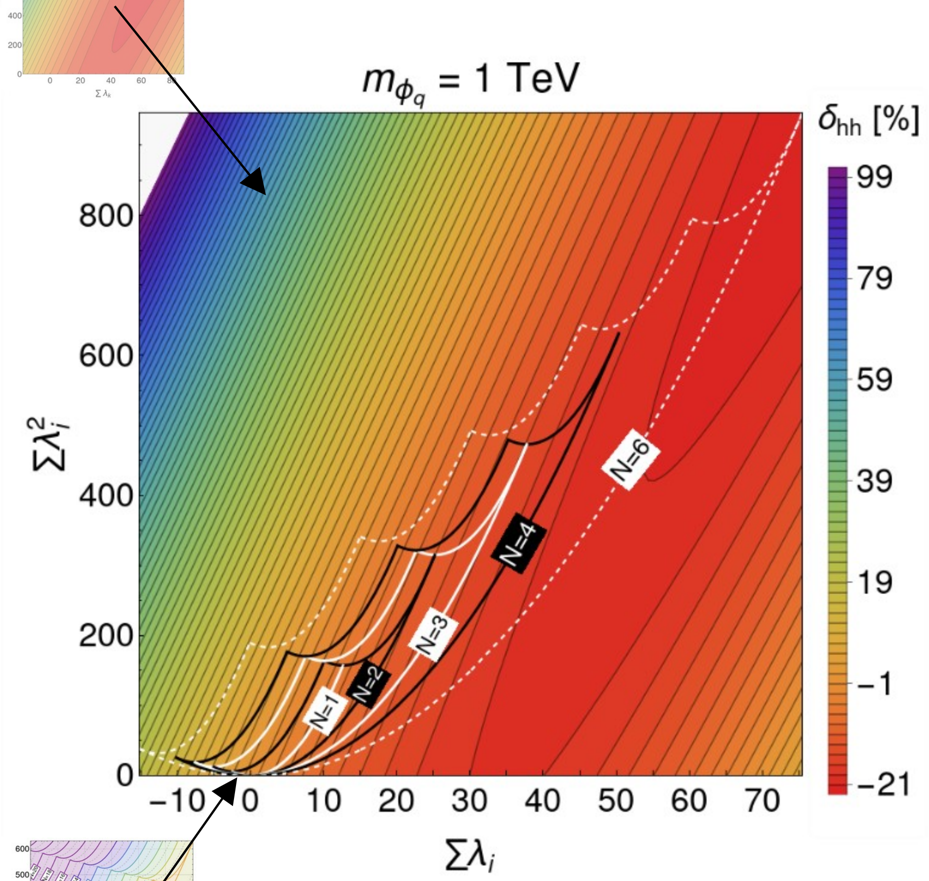
$$\sum \lambda_k^2 \geq \frac{\left(\sum \lambda_k\right)^2}{N}$$

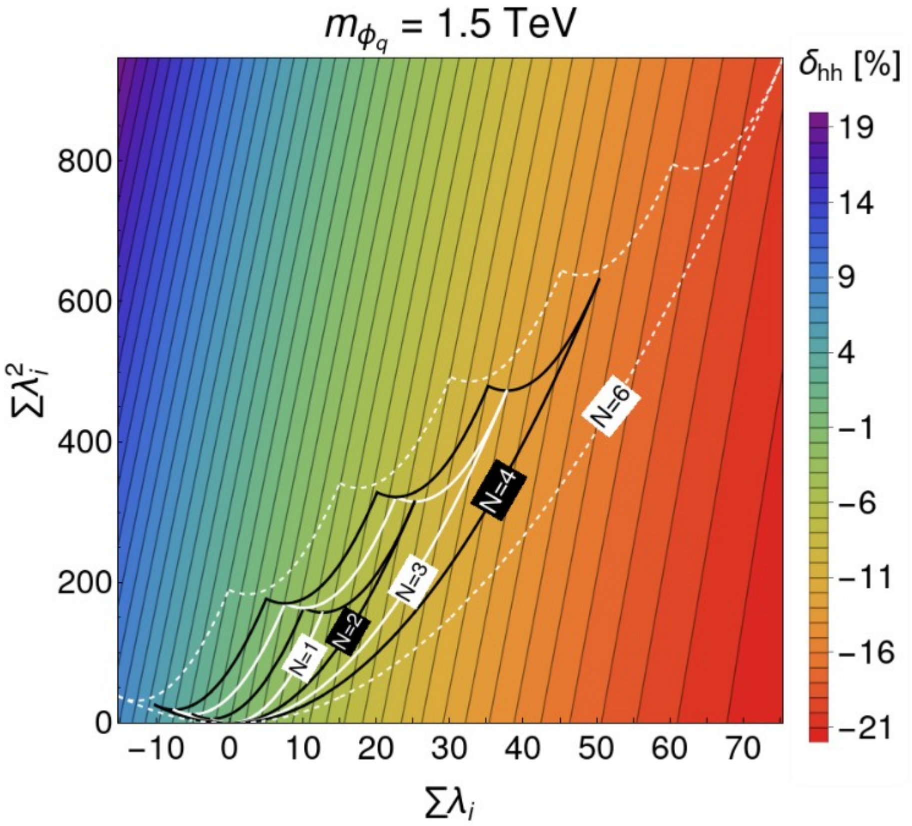
$$\sum \lambda_k^2 \leq \sum_{n=0}^{N-1} \left\{ \left[n\lambda_{\max}^2 + (N-1-n)\,\lambda_{\min}^2 + \left(\sum \lambda_k - (n\lambda_{\max} + (N-1-n)\,\lambda_{\min}) \right)^2 \right] \times \left[\Theta\left(\sum \tilde{\lambda}_k - n \right) - \Theta\left(\sum \tilde{\lambda}_k - (n+1) \right) \right] \right\}$$

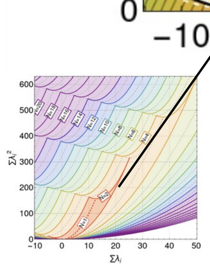
These will determine the models' allowed parameter region.

Double Higgs Production Results





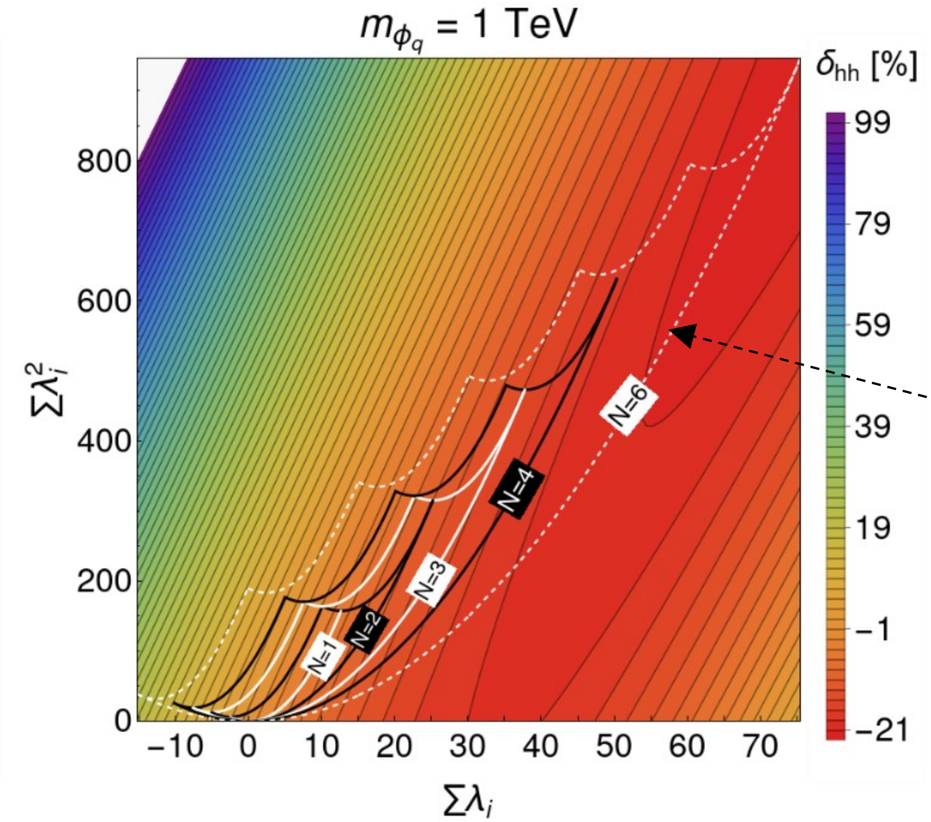




The model limits are superimposed over the final values for δ_{hh} [%].

Double Higgs Production Results

δ_{hh} [%] is calculated for a fixed mass (all equal) in function of the sums of couplings and squares.



For a model with N scalars and all masses equal,
the remaining two parameters can be constrained by the following equations:

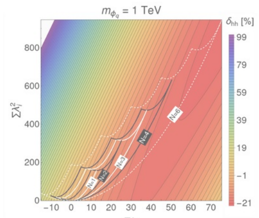
$$N\lambda_{\text{min}} \leq \sum \lambda_k \leq N\lambda_{\text{max}}$$

$$\sum \lambda_k^2 \geq \frac{\left(\sum \lambda_k\right)^2}{N}$$

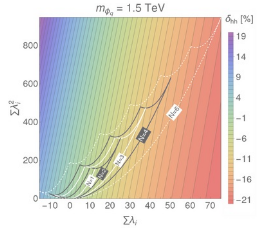
$$\sum \lambda_k^2 \leq \sum_{n=0}^{N-1} \left\{ \left[n\lambda_{\text{max}}^2 + (N-1-n)\,\lambda_{\text{min}}^2 + \left(\sum \lambda_k - \left(n\lambda_{\text{max}} + (N-1-n)\,\lambda_{\text{min}}\right) \right)^2 \right] \times \left[\Theta\left(\sum \tilde{\lambda}_k - n\right) - \Theta\left(\sum \tilde{\lambda}_k - (n+1)\right) \right] \right\}$$

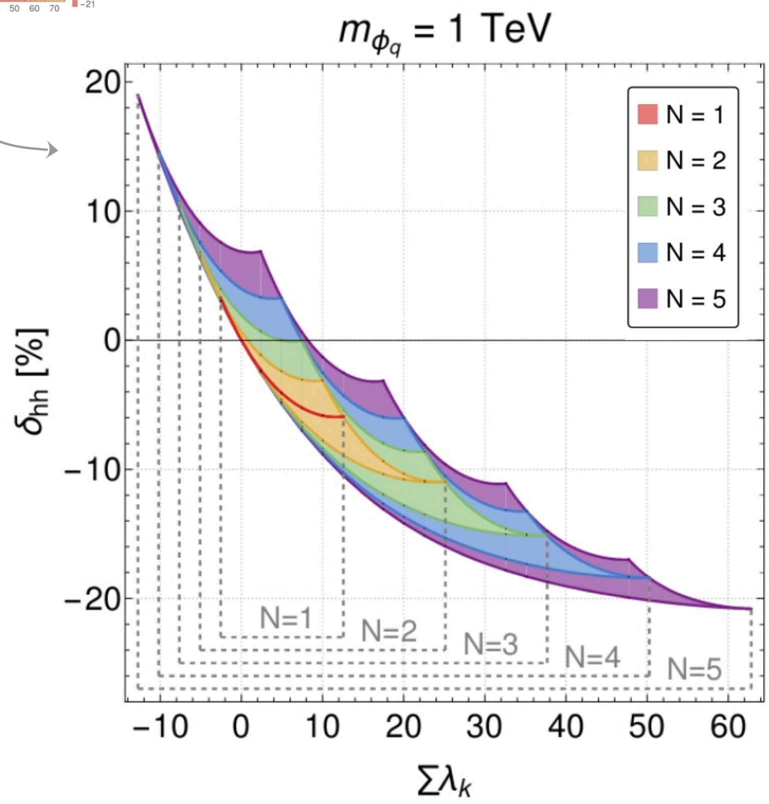
The model limits for a certain N are superimposed over the final values for δ_{hh} [%] .

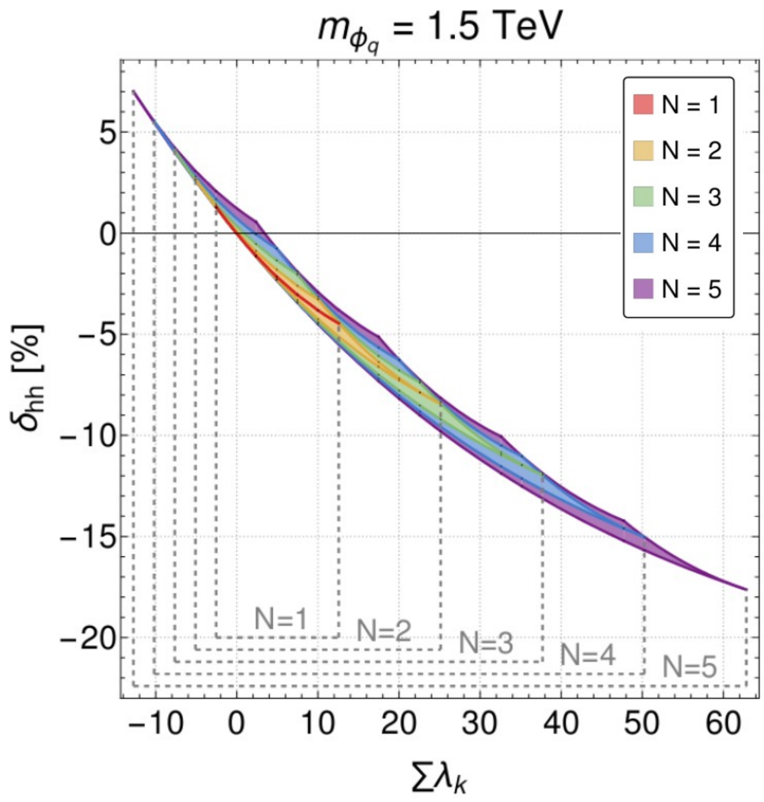
Double Higgs Production Results



From the previous results we can calculate the final ranges in function of the sum only.
This will allow us to compare with the single Higgs production results.

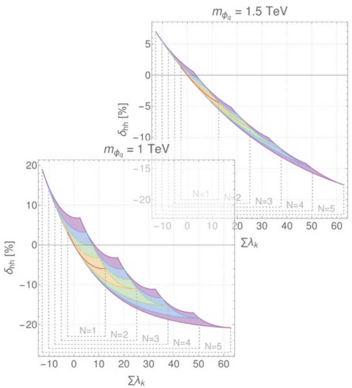




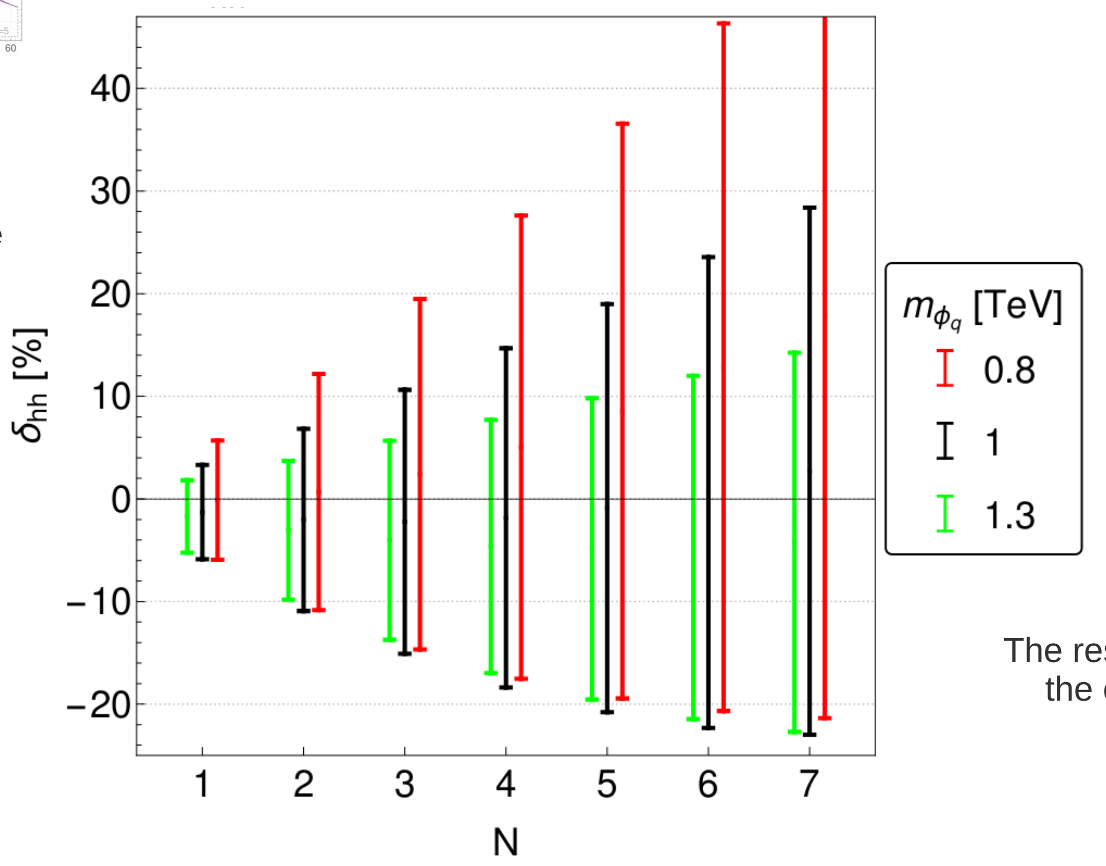


The area comes from the freedom in $\sum \lambda_k^2$ ... → ...which vanishes for larger masses since the related terms are supressed by m^{-4} .

Double Higgs Production Results



We can also calculate the ranges in function of N only.



$$\delta_{hh}^{Atlas} = 590\,\%$$ [31.1]

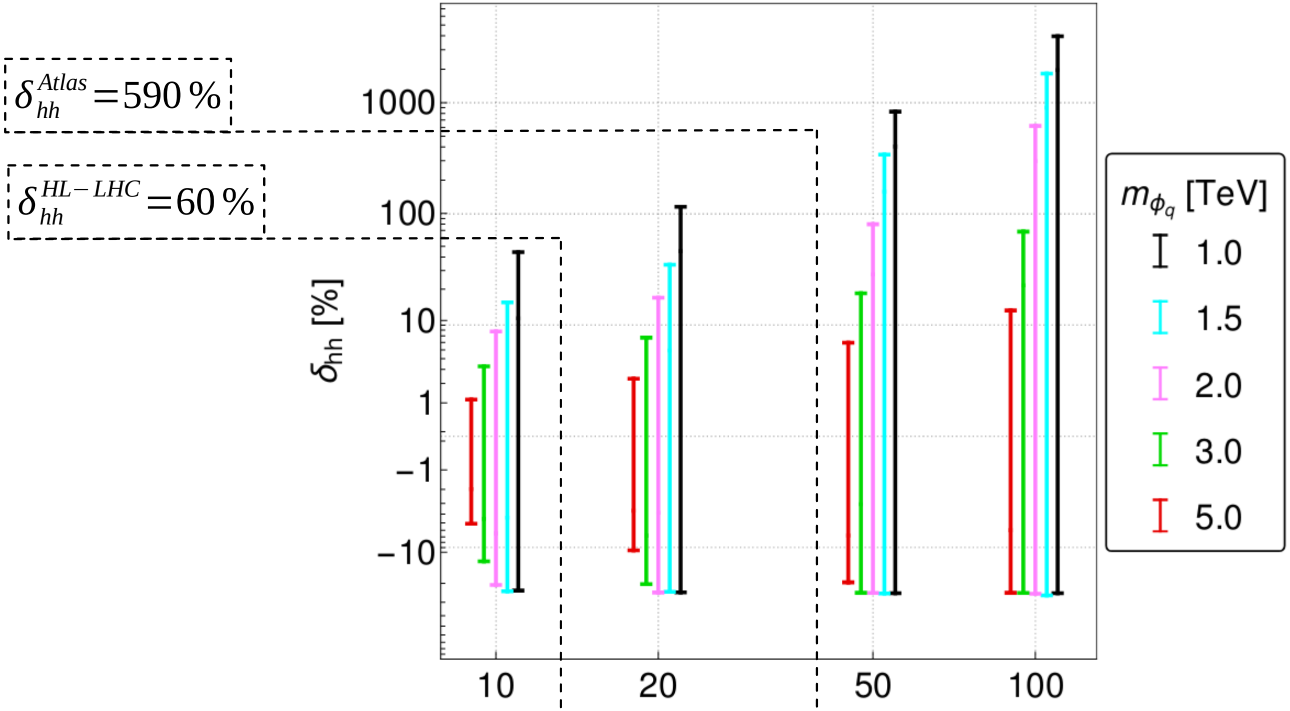
$$\delta_{hh}^{HL-LHC} = 60\,\%$$ [31.2]

The results are small when compared with the experimental values so we will need even larger values of N.

[31.1] ATLAS collaboration, Phys. Lett. B 800 (2020) 135103
[31.2] M. Cepeda et al., CERN Yellow Rep. Monogr. 7 (2019) 221–584

Double Higgs Production Results

Larger values of N



$\delta_{hh}^{Atlas} = 590\,\%$ [32.1]

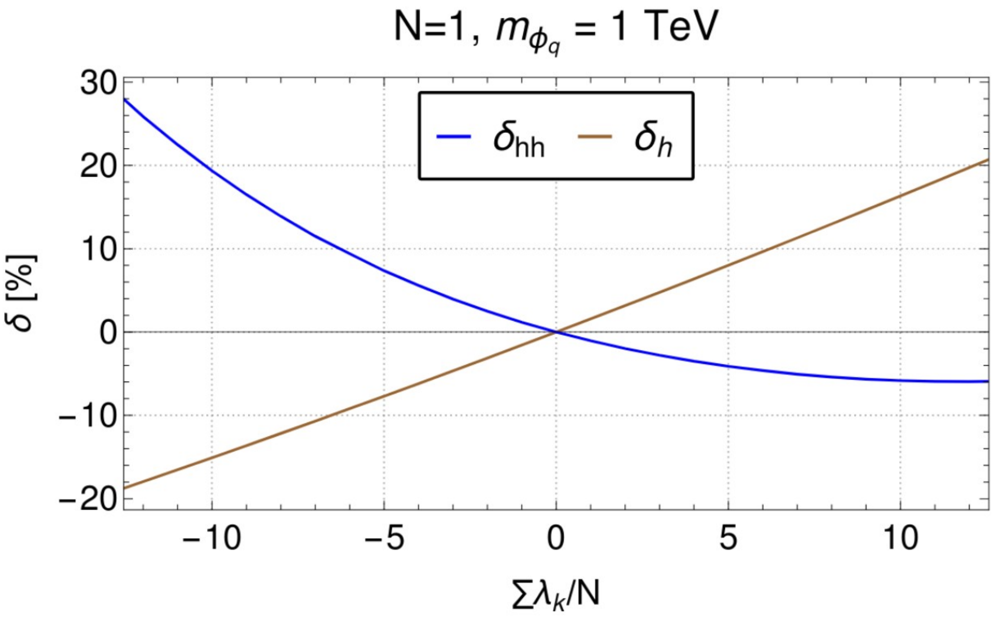
$\delta_{hh}^{HL-LHC} = 60\,\%$ [32.2]

Only above these values of N can double Higgs production offer any cuts in the parameters.

[32.1] ATLAS collaboration, Phys. Lett. B 800 (2020) 135103
[32.2] M. Cepeda et al., CERN Yellow Rep. Monogr. 7 (2019) 221–584

Single and Double Higgs Production Results

Complementarity (no BFB)



Single Higgs Production Form Factors

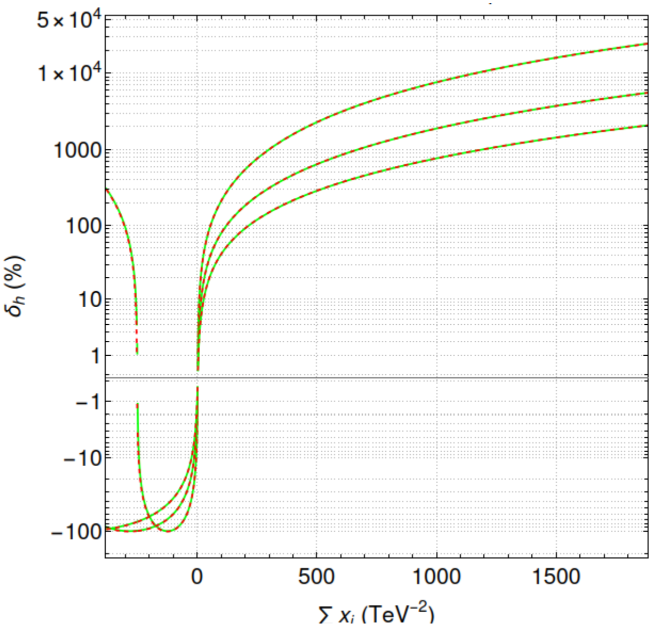
$$\begin{cases} F_\triangle^Q = +\frac{2}{3} + \mathcal{O}(m_Q^{-2}) \\ F_\triangle^{\phi_q^i} = +\frac{1}{6} + \mathcal{O}(m_{\phi_q^i}^{-2}) \end{cases}$$

Double Higgs Production Form Factors

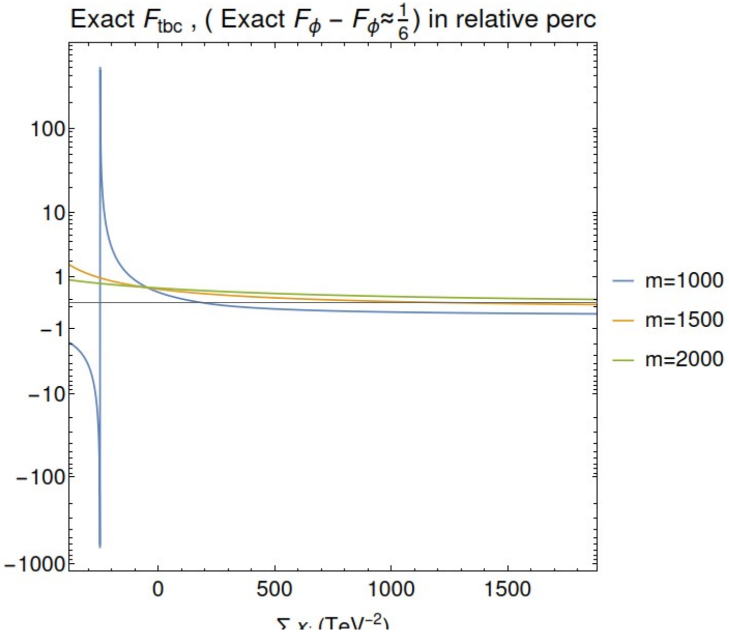
$$\begin{cases} F_\triangle^Q = +\frac{2}{3} + \mathcal{O}(m_Q^{-2}) \\ F_{\square_2}^{\phi_q^i} = F_\triangle^{\phi_q^i} = +\frac{1}{6} + \mathcal{O}(m_{\phi_q^i}^{-2}) \end{cases} \qquad \begin{cases} F_\square^Q = -\frac{2}{3} + \mathcal{O}(m_Q^{-2}) \\ F_{\square_1}^{\phi_q^i} = -c + \mathcal{O}(m_{\phi_q^i}^{-2}) \end{cases}$$

For double Higgs production we have negative interference for positive couplings.

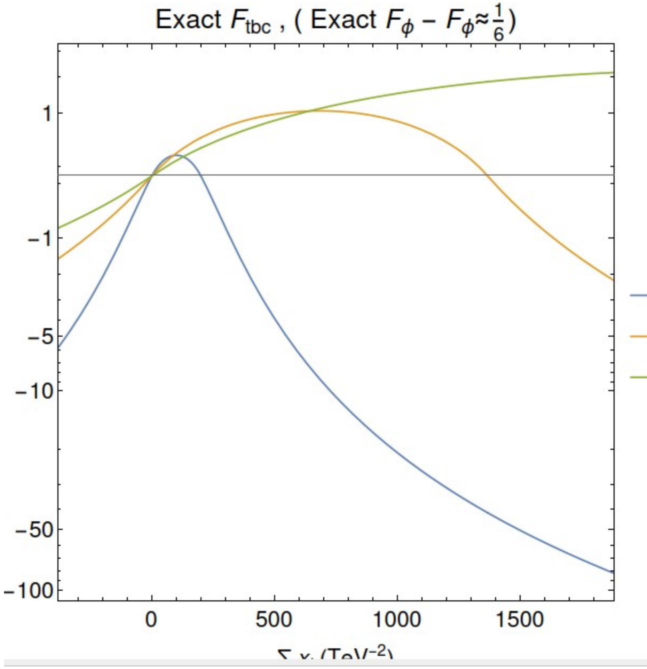
Which is one of the reasons behind the low values we obtained for this process.



5 × 10^4
1 × 10^4
1000
100
10
1
−1
−10
−100
δ_h (%)
0
500
1000
1500
Σ x_i (TeV^−2)

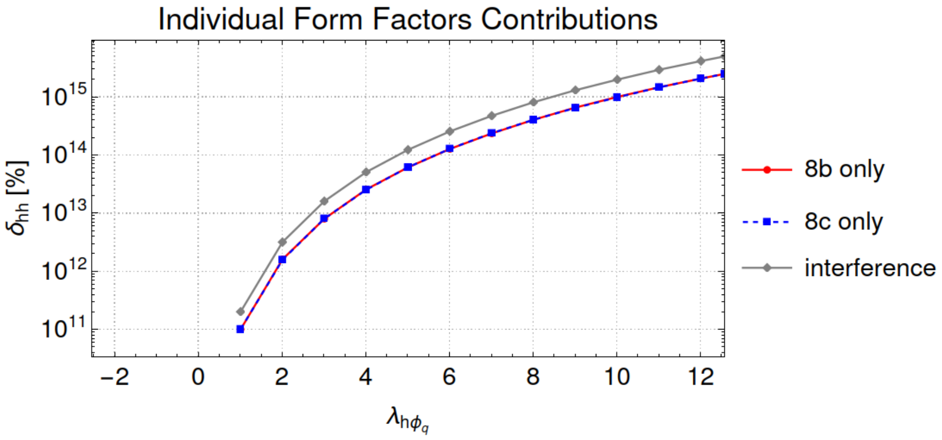


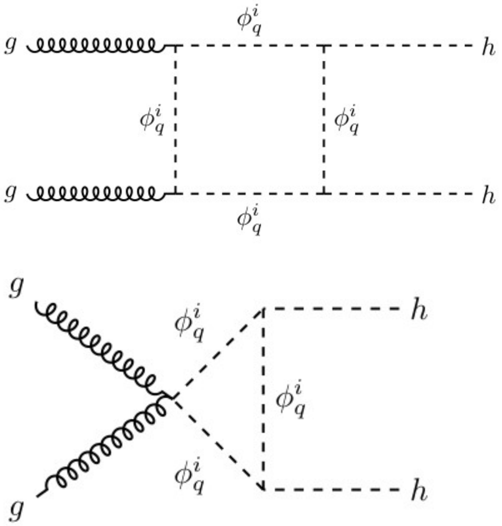
Exact F_tbc , (Exact F_ϕ − F_ϕ≈1/6) in relative perc
100
10
1
−1
−10
−100
−1000
0
500
1000
1500
m=1000
m=1500
m=2000



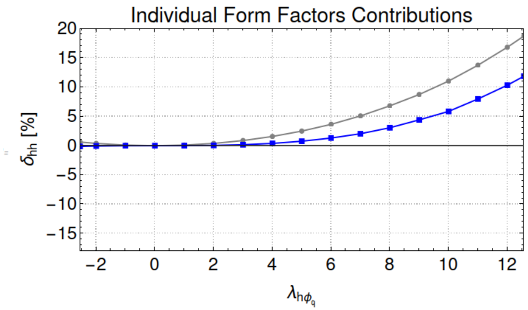
Exact F_tbc , (Exact F_ϕ − F_ϕ≈1/6)
1
−1
−5
−10
−50
−100
0
500
1000
1500

Diagrams gauge unitarity

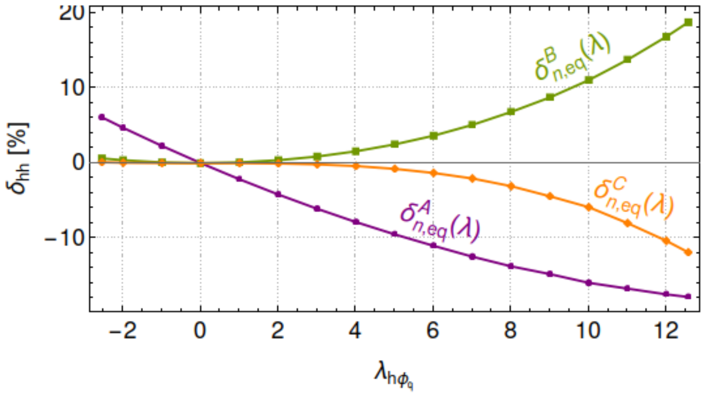


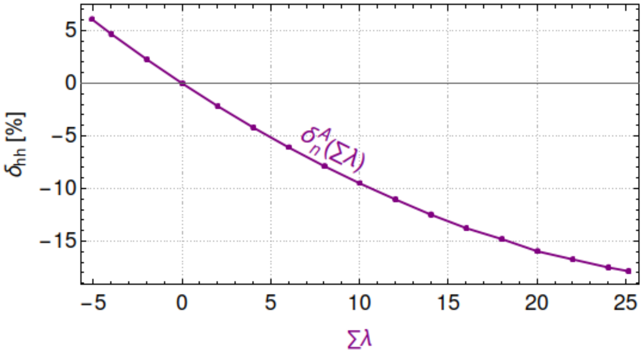


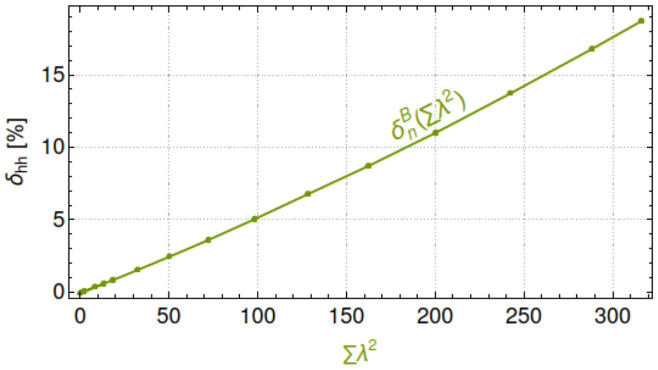
(c)

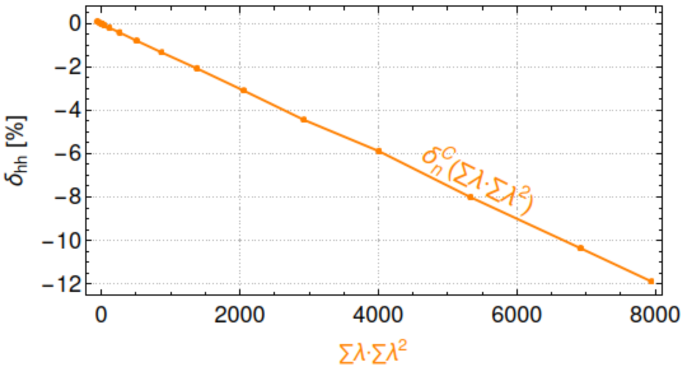


Blue is the sum


δ_{hh} [%]
$\lambda_{h\phi_q}$
$\delta^B_{n,eq}(\lambda)$
$\delta^C_{n,eq}(\lambda)$
$\delta^A_{n,eq}(\lambda)$

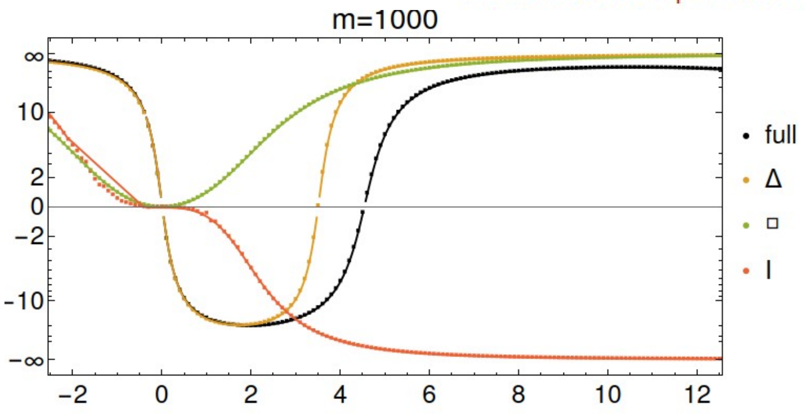

δ_{hh} [%]
$\Sigma\lambda$
$\delta^A_n(\Sigma\lambda)$

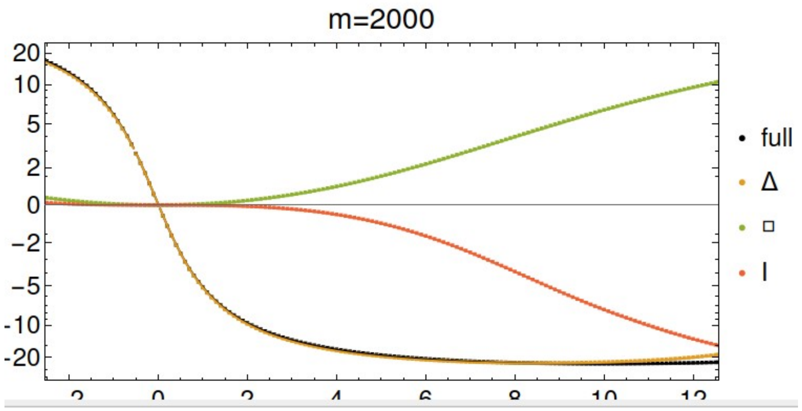

δ_{hh} [%]
$\Sigma\lambda^2$
$\delta^B_n(\Sigma\lambda^2)$


δ_{hh} [%]
$\Sigma\lambda\cdot\Sigma\lambda^2$
$\delta^C_n(\Sigma\lambda\cdot\Sigma\lambda^2)$

Example of fits on the three functions

Model N=20 for up-to N=20 for several Masses [-2.5, 4π]






```
temp = (MinMax[#["FitResiduals"]] & /@ sigTriMasses20)ᵀ;
{MinMax[temp[[1]] (*mins*)], MinMax[temp[[1]] (*maxs*)]}
temp // MatrixForm
temp = (MinMax[#["FitResiduals"]] & /@ sigSqrMasses20)ᵀ;
{MinMax[temp[[1]] (*mins*)], MinMax[temp[[1]] (*maxs*)]}
temp // MatrixForm
temp = (MinMax[#["FitResiduals"]] & /@ sigIntMasses20)ᵀ;
{MinMax[temp[[1]] (*mins*)], MinMax[temp[[1]] (*maxs*)]}
temp // MatrixForm
```

{{-0.308618, -0.000690544}, {-0.308618, -0.000690544}}

MatrixForm=

-0.254016	-0.308618	-0.208572	-0.0339305	-0.00155734	-0.000690544
0.173661	0.226038	0.0975679	0.0443487	0.00132904	0.000565515

{{-0.261006, -1.21258×10^{-6}}, {-0.261006, -1.21258×10^{-6}}}

MatrixForm=

-0.256461	-0.261006	-0.0355293	-0.000203253	-0.0000271382	-1.21258×10^{-6}
0.296687	0.235944	0.0175593	0.000564388	0.0000459105	2.34835×10^{-6}

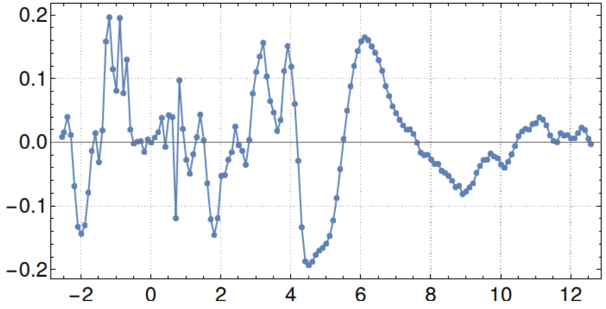
{{-0.357617, -0.000113487}, {-0.357617, -0.000113487}}

MatrixForm=

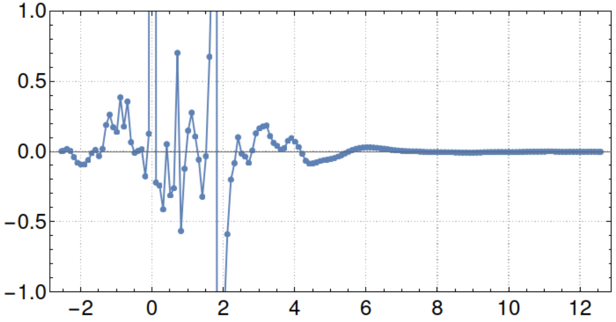
-0.357617	-0.320825	-0.213062	-0.0650669	-0.000776799	-0.000113487
0.354495	0.325755	0.288073	0.0578677	0.00044557	0.0000862761

Example of fits on the three functions

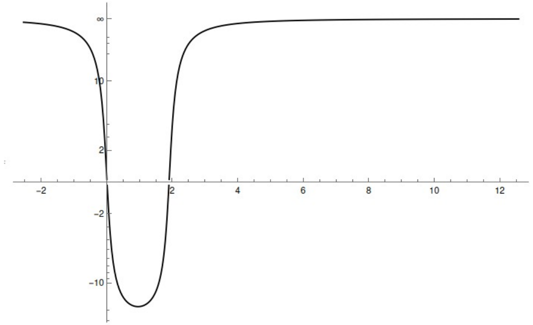
```
ListPlot[{lambdaExtra, sigTriExtra["FitResiduals"]}ᵀ, PlotRange → Full, Joined → Tru
ListPlot[{lambdaExtra, sigTriExtra["FitResiduals"] 100 / percExtraIndividualB1B6T1T2[[All (*lambda*), 1 (*mass*), 1]]}ᵀ
Plot[{sigTriExtra[λ]}, {λ, Min[lambdaExtra], Max[lambdaExtra]}, PlotRange → Full, P
```



Abs Error



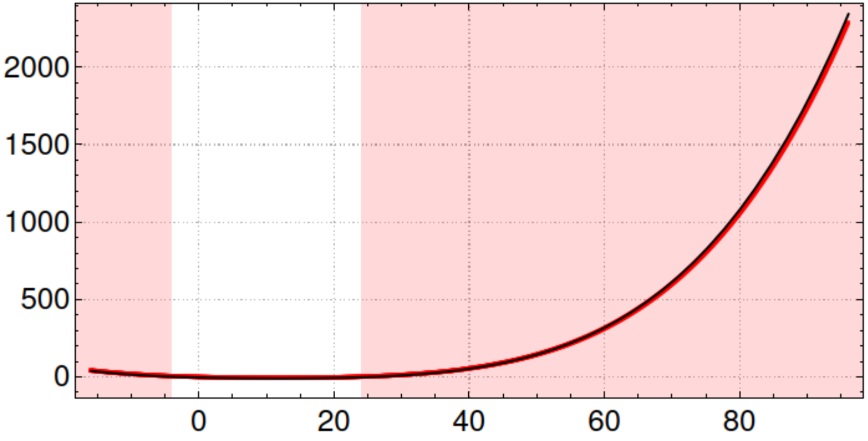
Rel Error %



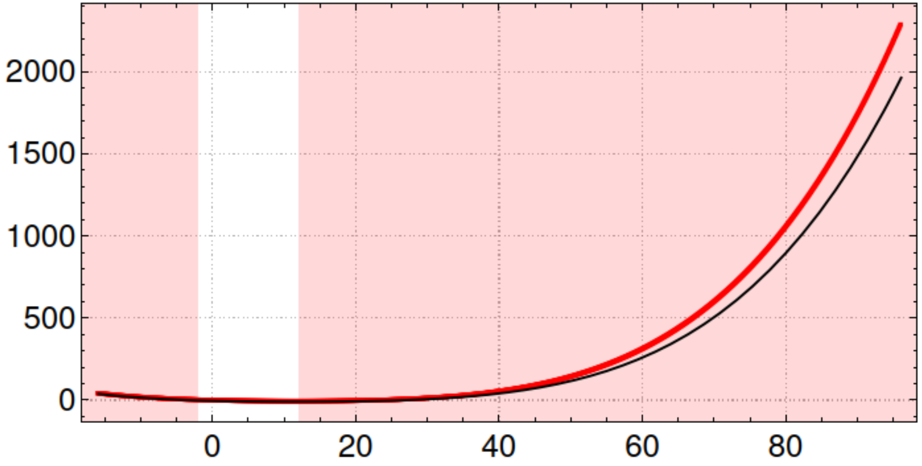
Delta Tri

The functions are well behaved

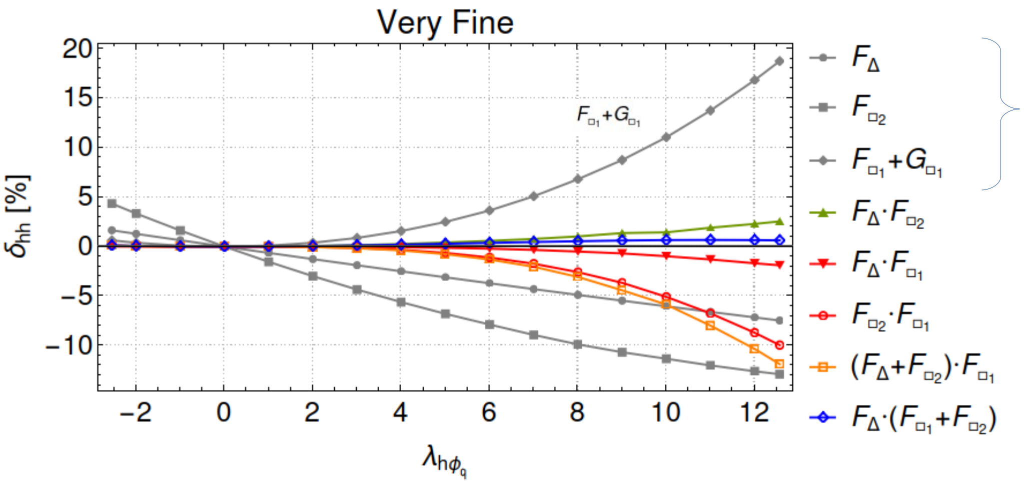
N=2 is already pretty good to extrapolate all the way up to N=7



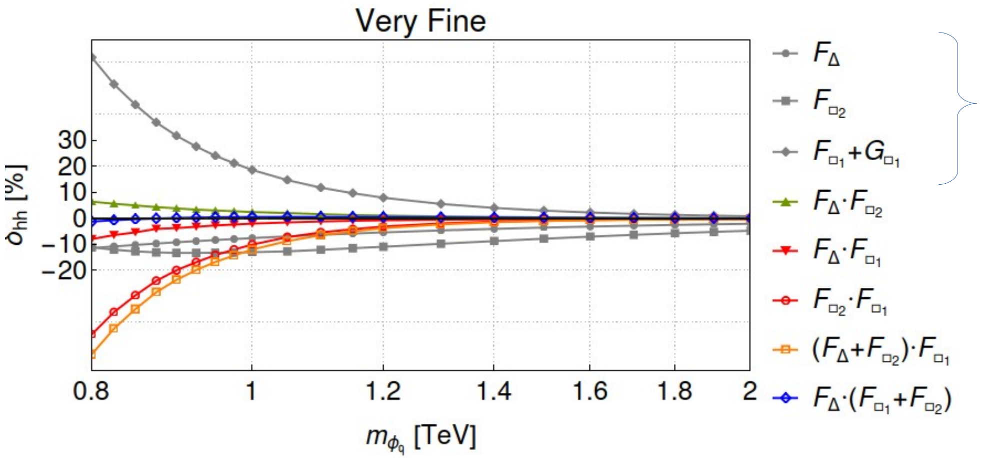
Less so with N=1 but still not too bad



N=2, signs of terms

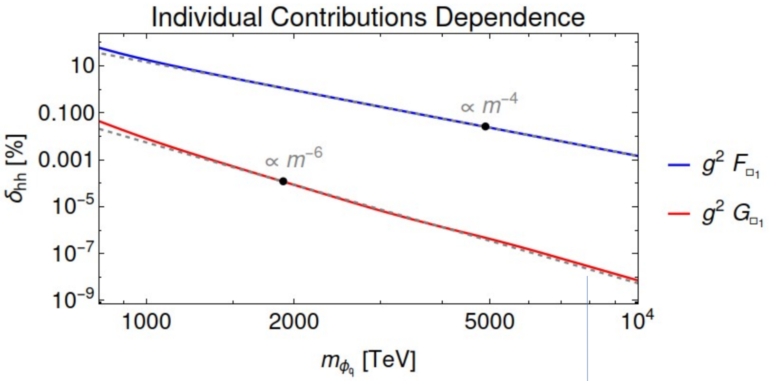


This is interference with SM

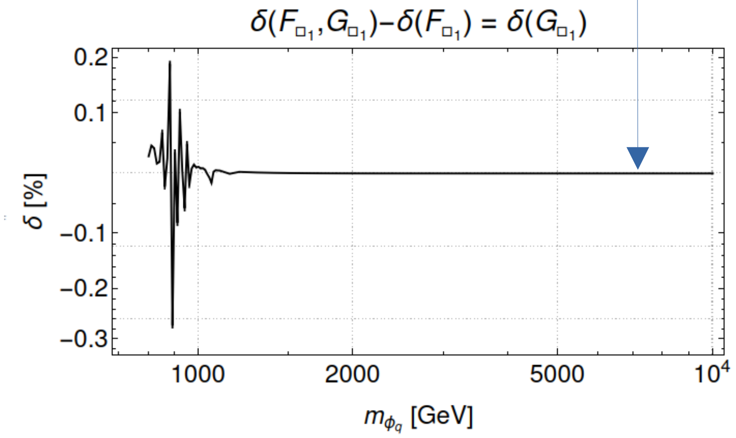


This is interference with SM

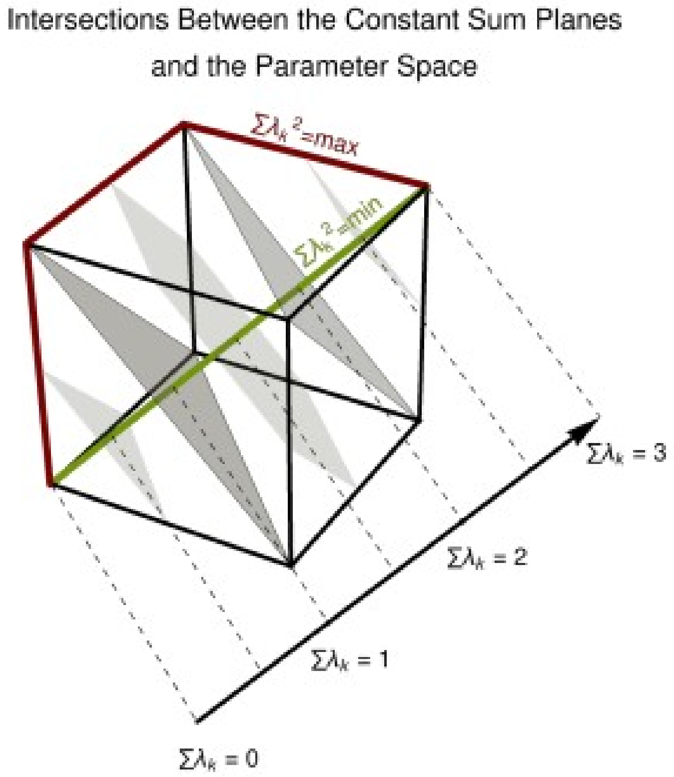
G and Fsq separated test lamda = 4 pi

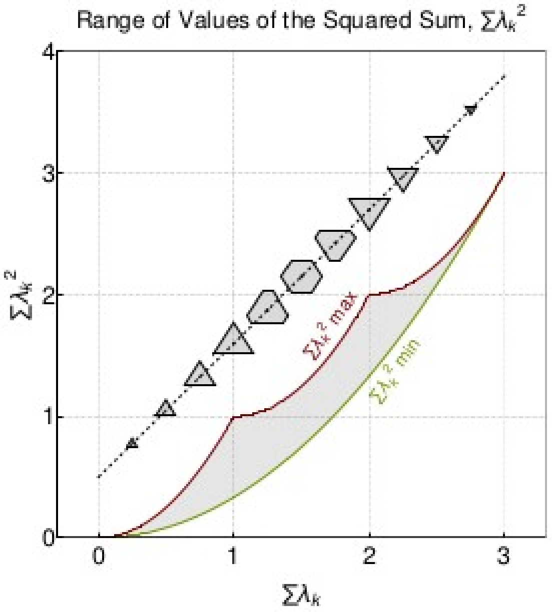


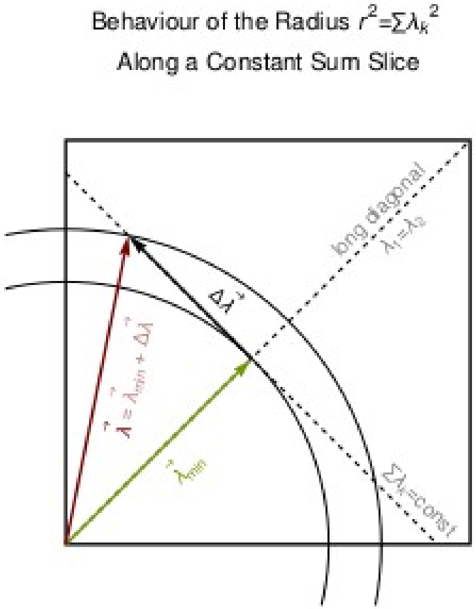
percBoxF[[All,	percBoxG[[All, Al	(100 percBoxF / percBoxG)	(100 percBoxG / percBoxF)
{{62.044, 57.!	{{0.0459745, 0.0	{{134953., 138 1	{0.0740999, 0
17.0421, 16	0.00956928, 0.	232789., 238 :	0.0471964, 0
0.648115, 0	0.00136289, 0.	1.09482×10^6,	0.0239982, 0
0.0650757,	0.0000166305,	3.17348×10^6,	0.00427605,
0.0164381,	1.83338×10^{-6},	4.9649×10^6, 5	0.00217386,
0.00591746,	3.92581×10^{-7},	7.35849×10^6,	0.00144222,
0.00274865,	1.07827×10^{-7},	1.11033×10^7,	0.000925316,
	3.84976×10^{-8},	1.56499×10^7,	0.000653632,
	1.60201×10^{-8},		

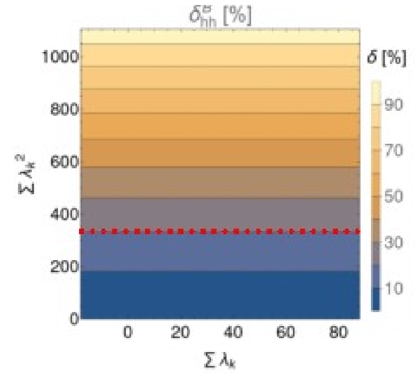


Should have gotten the contributions from G in this plot but the values for G are too small, só when calculating F+G they are lost in the accuracy. Potentially even in the printing of the values

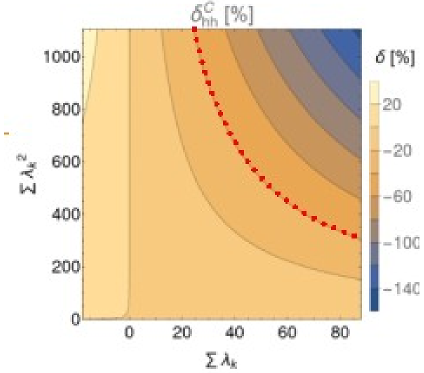

Intersections Between the Constant Sum Planes
and the Parameter Space
Σλk²=max
Σλk²=min
Σλk = 3
Σλk = 2
Σλk = 1
Σλk = 0


Range of Values of the Squared Sum, Σλk²
Σλk² max
Σλk² min
Σλk²
Σλk
0
1
2
3
4

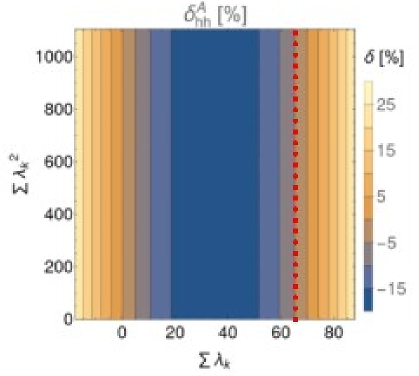

Behaviour of the Radius r²=Σλk²
Along a Constant Sum Slice
long diagonal
Δλ
λmin
Σλk=const



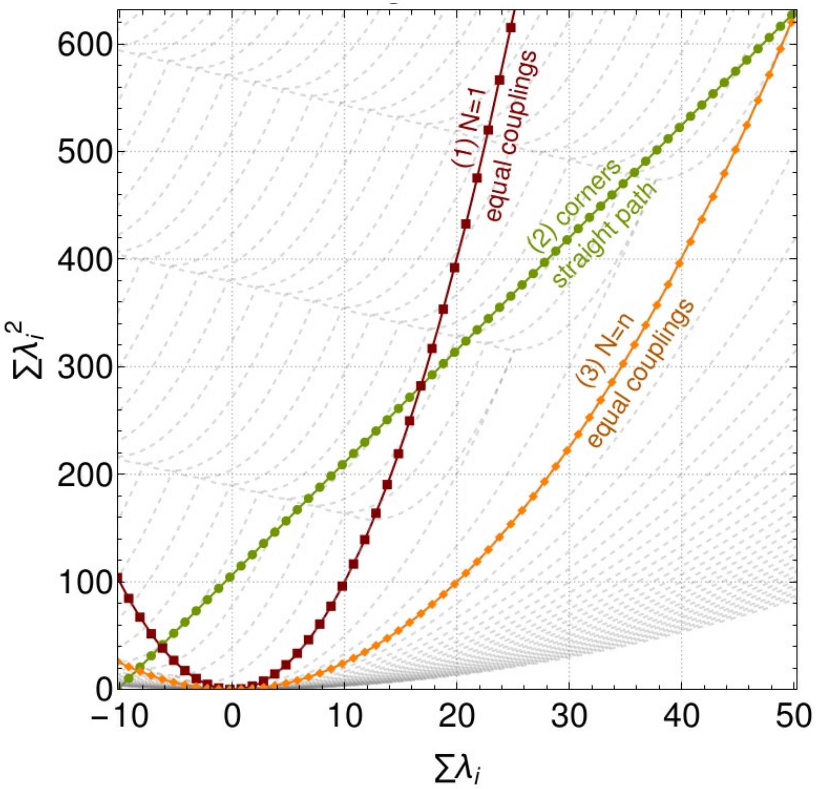
$\delta_{hh}^{\square}$ [%]
δ [%]
Σ λ_k
Σ λ_k²



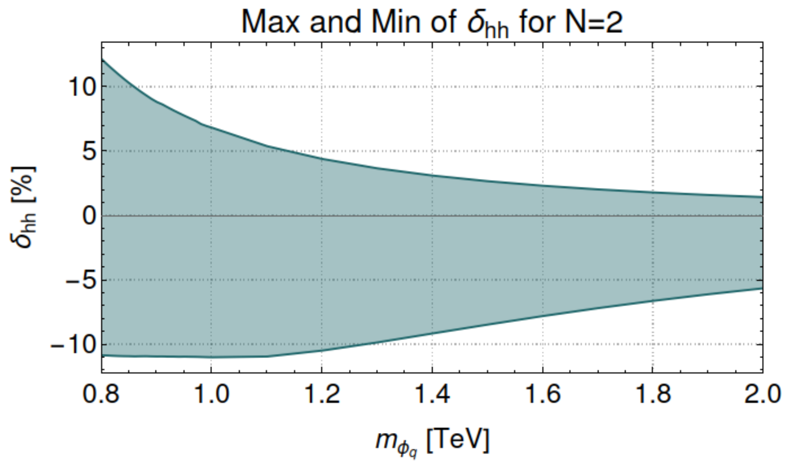
δ_{hh}^{C} [%]
δ [%]
Σ λ_k
Σ λ_k²

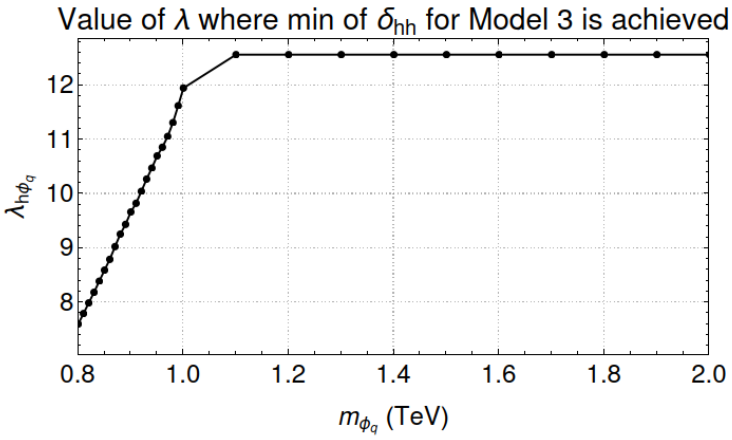


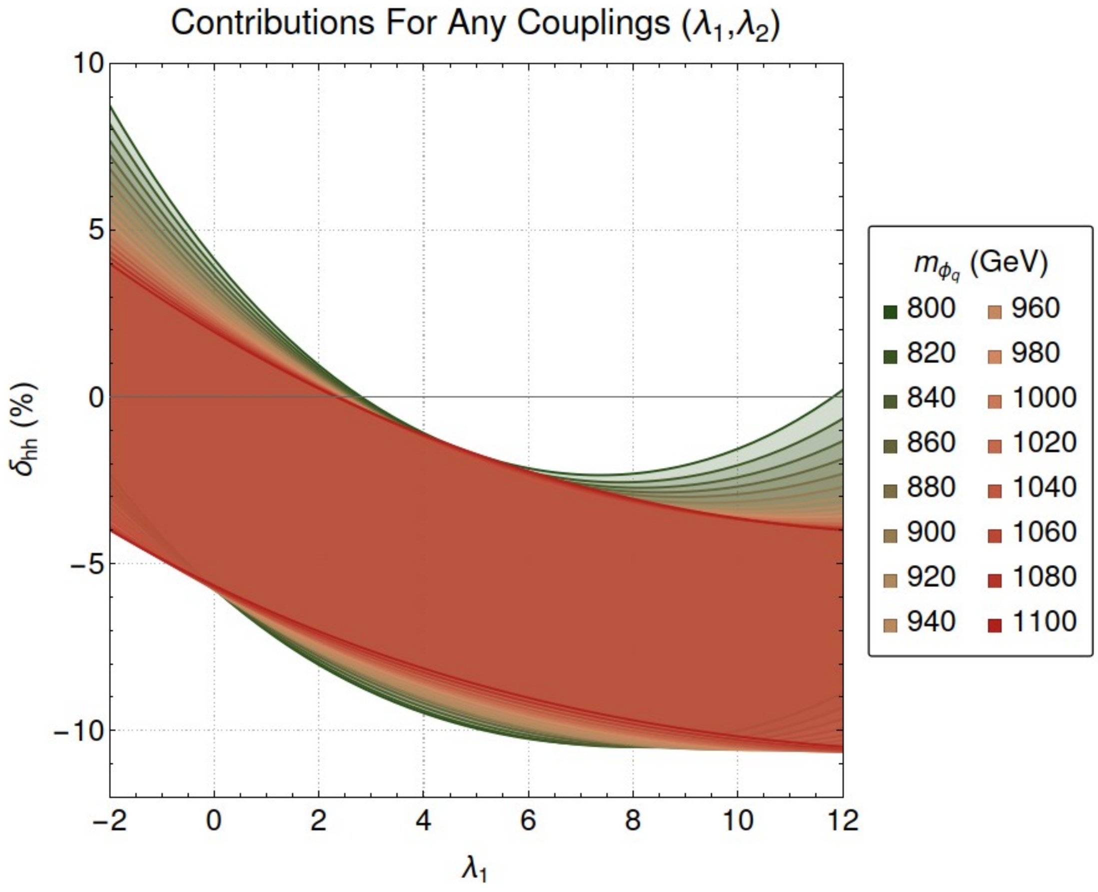
δ_{hh}^{A} [%]
δ [%]
Σ λ_k
Σ λ_k²

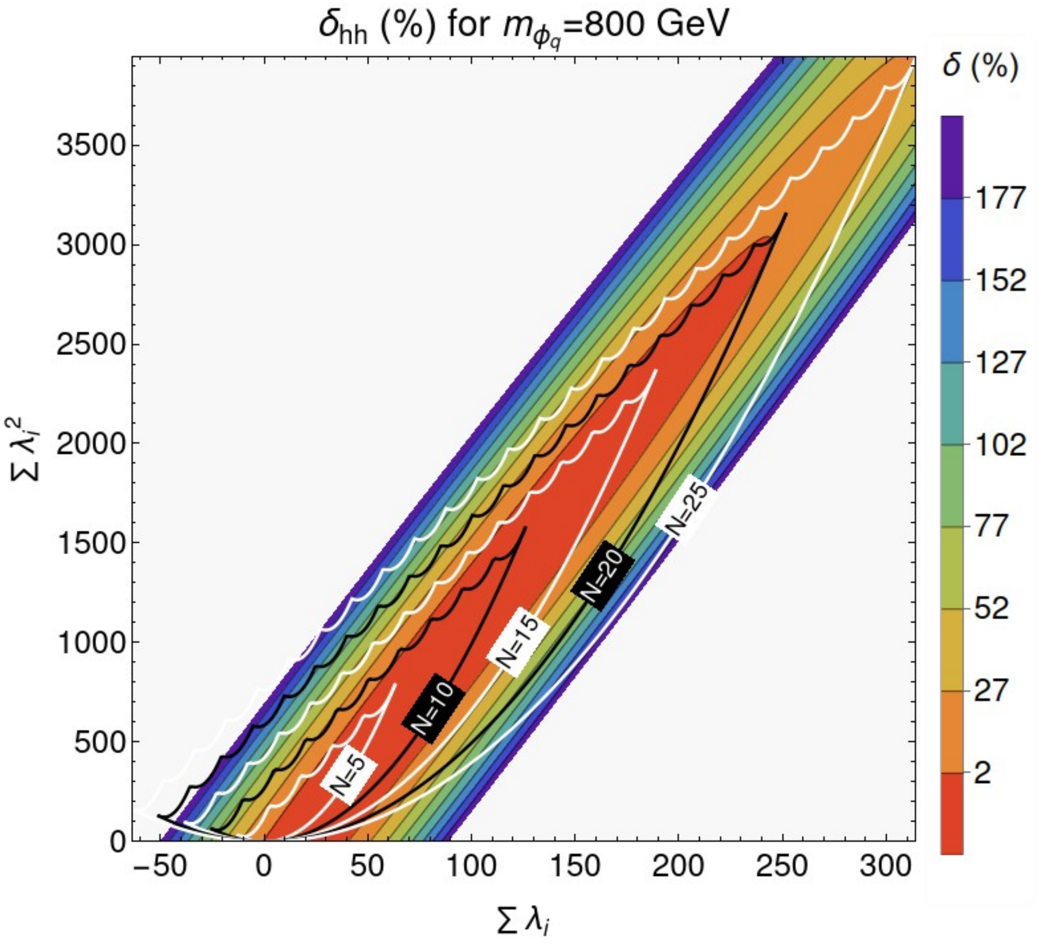


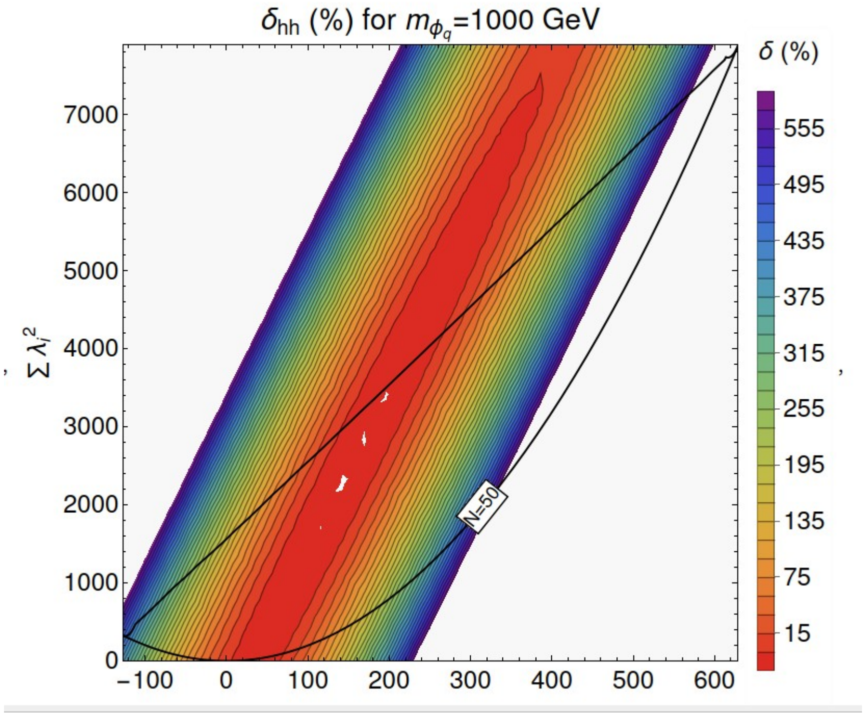
(1) N=1
equal couplings
(2) corners
straight path
(3) N=n
equal couplings
Σλ_i
Σλ_i²

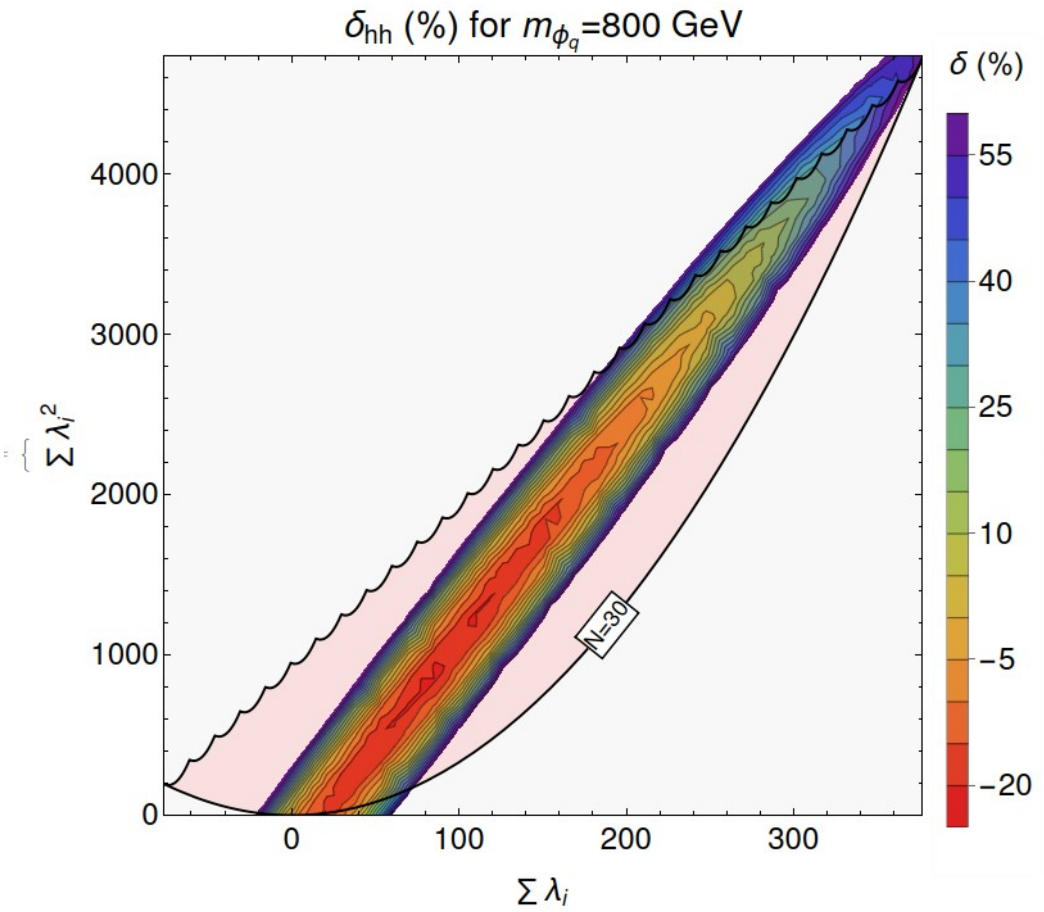

Max and Min of δ_{hh} for N=2
δ_{hh} [%]
10
5
0
−5
−10
0.8
1.0
1.2
1.4
1.6
1.8
2.0
m_{ϕ_q} [TeV]


Value of λ where min of δ_{hh} for Model 3 is achieved
$\lambda_{h\phi_q}$
12
11
10
9
8
0.8
1.0
1.2
1.4
1.6
1.8
2.0
m_{ϕ_q} (TeV)


Contributions For Any Couplings (λ_1,λ_2)
δ_{hh} (%)
10
5
0
−5
−10
−2
0
2
4
6
8
10
12
λ_1
m_{ϕ_q} (GeV)
800
820
840
860
880
900
920
940
960
980
1000
1020
1040
1060
1080
1100


δ_{hh} (%) for m_{ϕ_q}=800 GeV
δ (%)
177
152
127
102
77
52
27
2
3500
3000
2500
2000
1500
1000
500
0
−50
0
50
100
150
200
250
300
$\Sigma\,\lambda_i^2$
$\Sigma\,\lambda_i$
N=5
N=10
N=15
N=20
N=25


δ_{hh} (%) for m_{ϕ_q}=1000 GeV
δ (%)
555
495
435
375
315
255
195
135
75
15
7000
6000
5000
4000
3000
2000
1000
0
−100
0
100
200
300
400
500
600
$\Sigma\,\lambda_i^2$
N=50

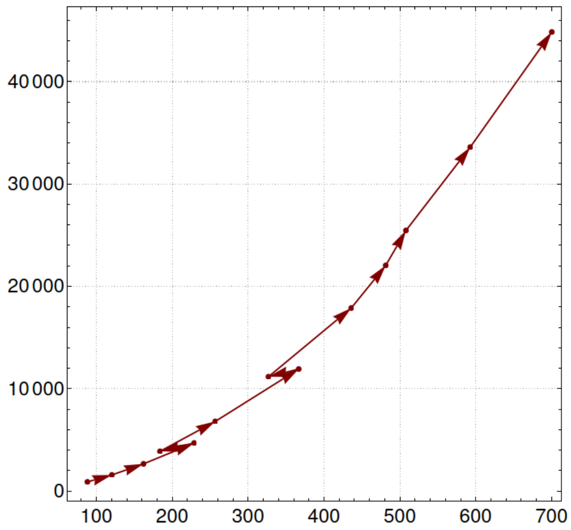


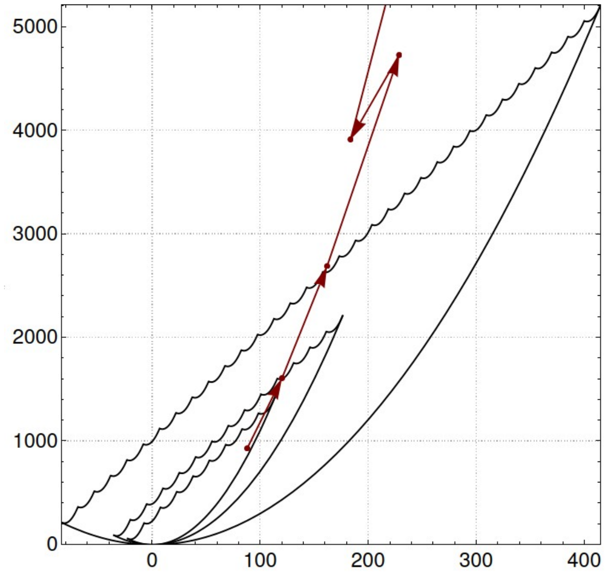
δ_{hh} (%) for m_{ϕ_q}=800 GeV
4000
3000
2000
1000
0
$\Sigma \lambda_i^2$
0
100
200
300
$\Sigma \lambda_i$
N=30
δ (%)
55
40
25
10
−5
−20

Minimum position with mass (from N = 7)

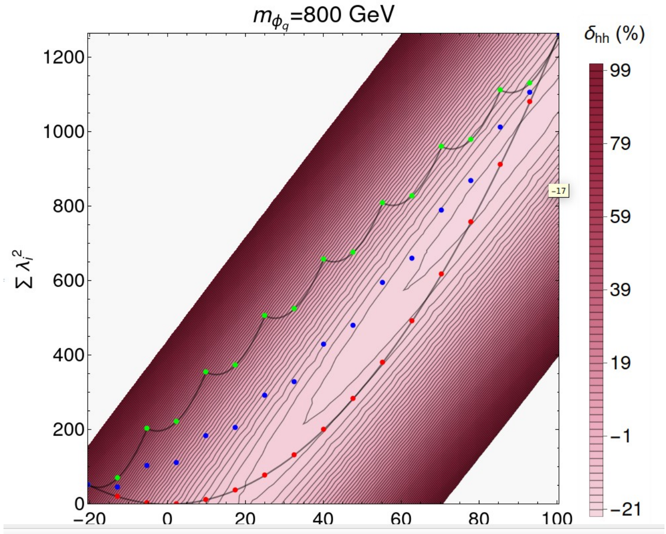
TableForm=

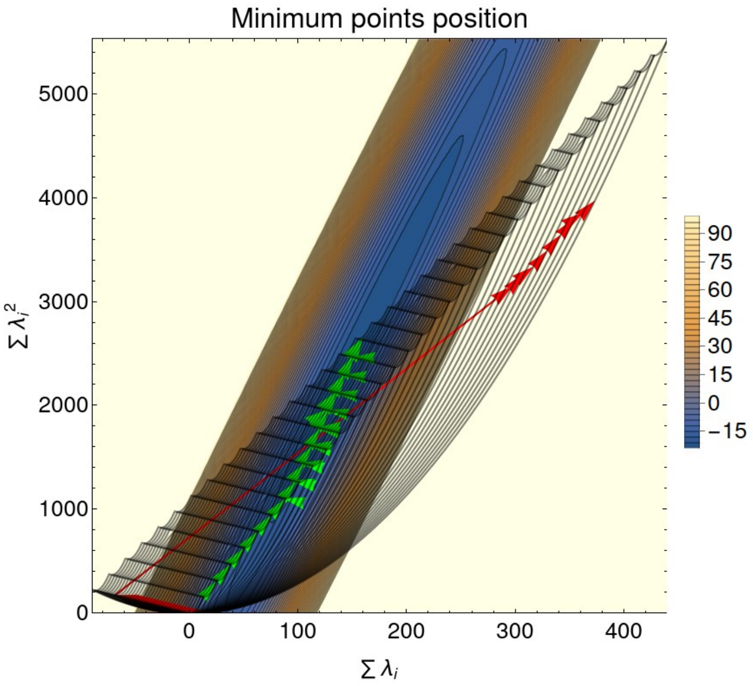
Mass	Min Unbounded
800	-21.6642 sum → 87.6176 sum2 → 930.837
900	-23.3758 sum → 119.862 sum2 → 1609.08
1000	-24.9223 sum → 161.581 sum2 → 2690.31
1100	-26.6087 sum → 228.278 sum2 → 4729.82
1200	-25.073 sum → 183.27 sum2 → 3914.66
1300	-26.548 sum → 256.061 sum2 → 6834.44
1400	-28.4977 sum → 366.129 sum2 → 11 956.3
1500	-27.0677 sum → 326.243 sum2 → 11 205.1
1600	-28.5371 sum → 435.393 sum2 → 17 918.9
1700	-28.6374 sum → 480.836 sum2 → 22 081.7
1800	-28.3501 sum → 507.51 sum2 → 25 499.3
1900	-28.7952 sum → 592.229 sum2 → 33 638.5
2000	-29.4376 sum → 699.736 sum2 → 44 886.2

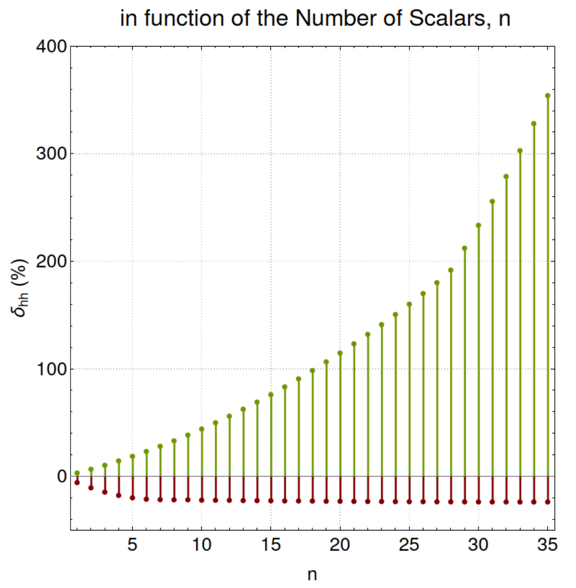


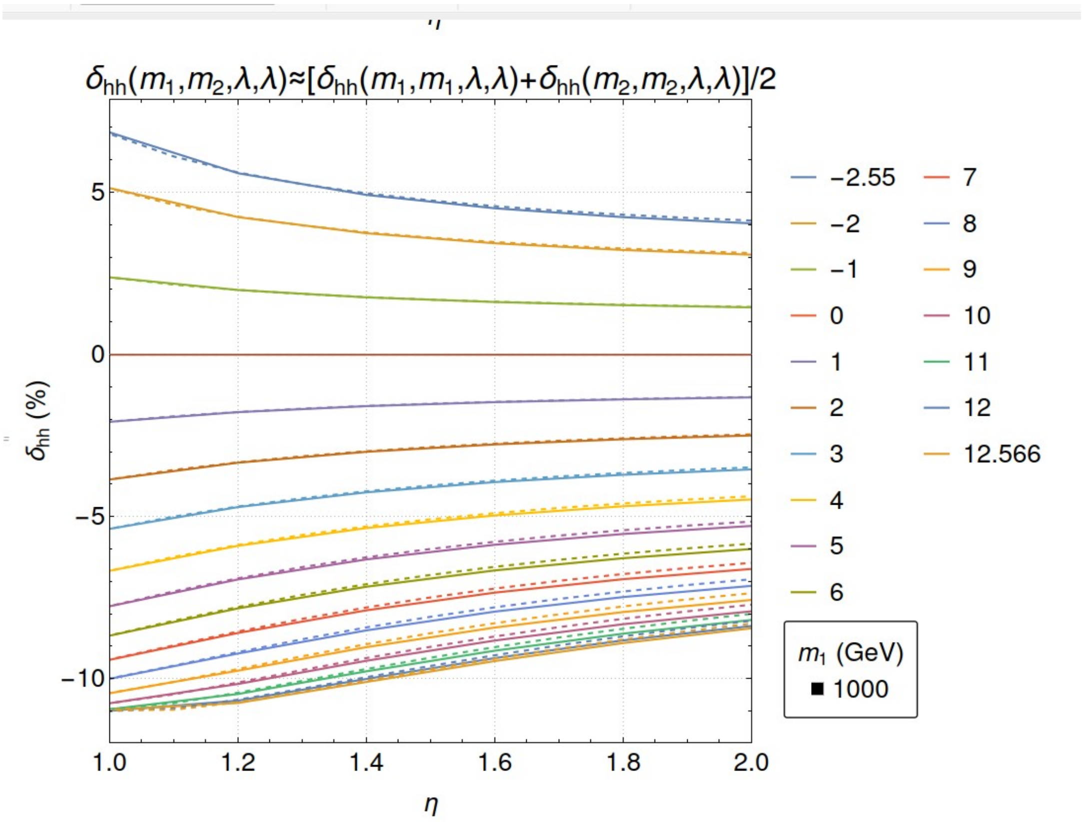


How I calculate the ranges









$\delta_{hh}(m_1,m_2,\lambda,\lambda)\approx[\delta_{hh}(m_1,m_1,\lambda,\lambda)+\delta_{hh}(m_2,m_2,\lambda,\lambda)]/2$
δ_{hh} (%)
η
5
0
−5
−10
1.0
1.2
1.4
1.6
1.8
2.0
−2.55
−2
−1
0
1
2
3
4
5
6
7
8
9
10
11
12
12.566
m_1 (GeV)
1000