Probing the early Universe using BSMPT v3

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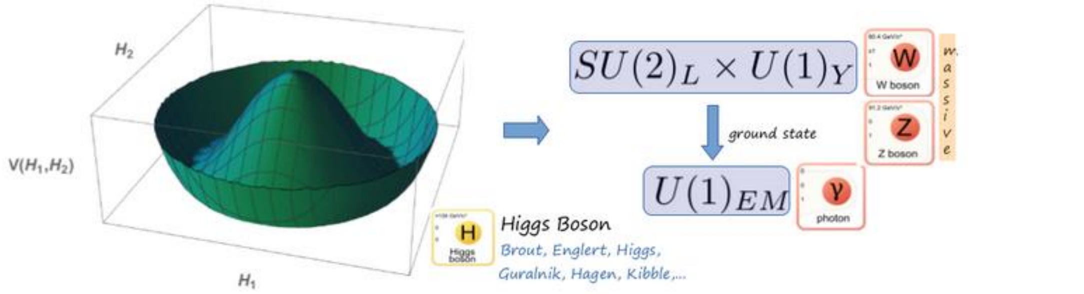
Electroweak phase transition

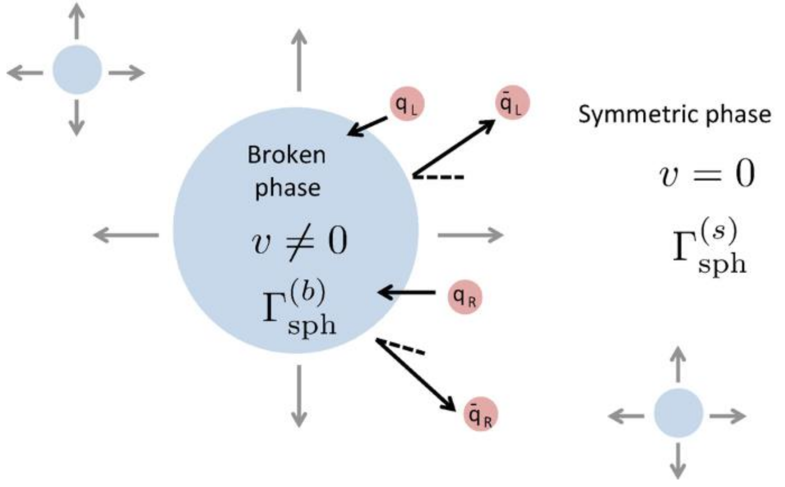
Vacuum expectation values (VEV) generated in the early Universe broke the electroweak gauge group. We call this **electroweak phase transition.**



$V(H) = \mu^2|H|^2 + \lambda|H|^4,\ \mu^2 < 0, \lambda > 0 \Rightarrow$ V_{min} not at origin → full symmetry not manifest after choosing ground state

https://www.mpi-hd.mpg.de/mpi/de/forschung/abteilungen-und-gruppen/unabhaengige-forschungsgruppen/newfo/forschung/elektroschwache-symmetriebrechung-und-das-higgs-potential

If **first order**, it can produce detectable **gravitational waves** in upcomming experiments, e.g. **LISA**. ← **BSMPT v3**



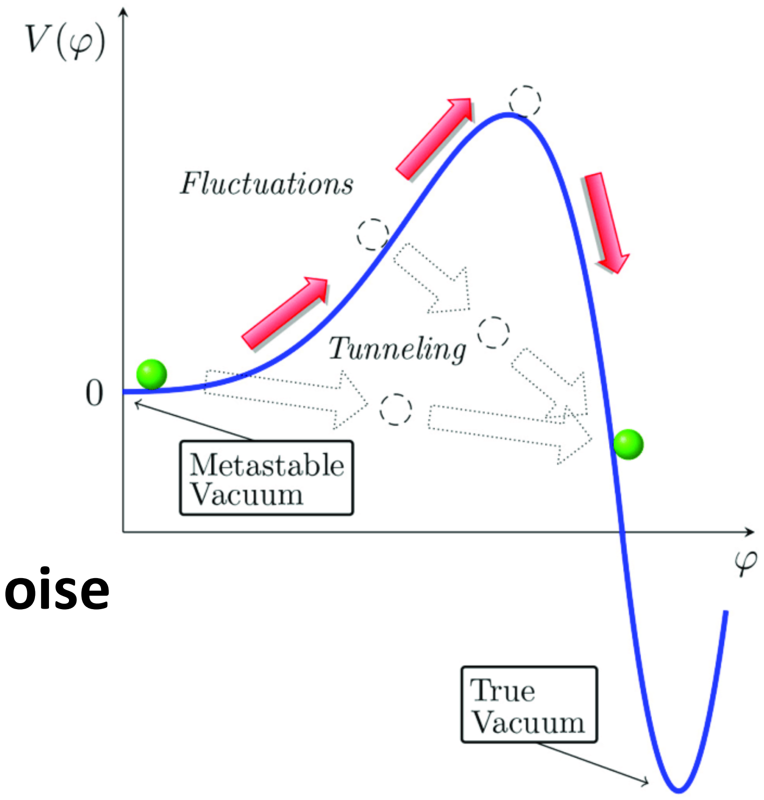
DOI:10.1007/978-981-13-1008-9_2

Interactions out of thermal equilibrium in early Universe.
If the model has **CP-violation** then all **Sakharov conditions** for **baryogenesis** are fulfilled. ← **Next version of BSMPT**

BSMPT v3

Recently released, **BSMPT v3** (e-Print: [2404.19037] [hep-ph])

- Calculates and tracks the **minima** of a model.
- Solves the bounce equation - **tunneling rate**.
- Calculates the **vacuum history** of the Universe
- Calculates the **gravitational wave spectrum** and **signal-to-noise ratio in LISA.**



DOI:10.3389/fspas.2018.00040

BSMPT v3 is shipped with SM scalar extensions but more can be implemented.

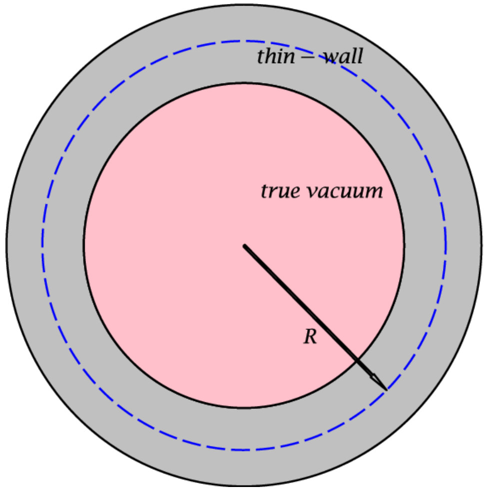
A comprehensive talk about **BSMPT v3** will be given by **Maggie** on Friday.

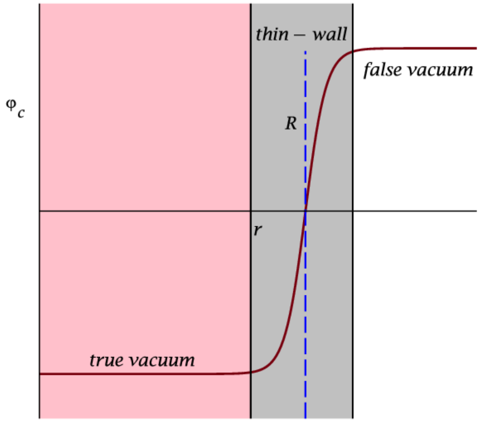
Stay tuned!

Baryogenesis on BSMPT v2

BSMPT v2 already has the **baryonic asymmetry of the Universe (BAU)** calculation implemented, but is uses a few approximations

- **Kink solution**, an interpolated tunneling path between false and true vacuum.
- Assumes a **low wall velocity.**
- Only works for the **C2HDM** (complex 2HDM).





DOI:10.48550/arXiv.1903.10864

BSMPT v2 has the **Fromme-Huber (FH) method** [10.1088/1126-6708/2007/03/049] and **VEV insertion approximation (VIA) method** [10.1103/PhysRevD.53.5834] implemented. Recent results [10.1007/JHEP12(2022)121] showed that the source term in the **VIA method** vanishes. For this reason, we will not consider it in **BSMPT v3** until the source term is properly calculated.

We plan to **improve** the implementation as well as **generalize** it for **any model**.

FHCK / Semiclassical force method

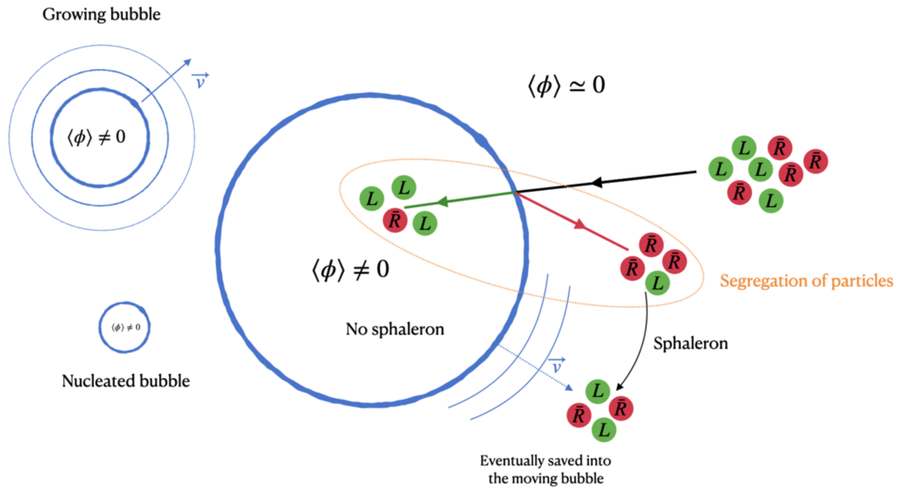
Our work is based on **J. Cline** and **K. Kainulainen's** method [10.1103/PhysRevD.101.063525], the generalization of the **FH method** for any bubble wall velocity.

Model with a **CP-violating complex fermionic mass**

$$\mathcal{M} = m(z)e^{i\theta(z)}$$

Using the **Wentzel–Kramers–Brillouin (WKB) ansatz** on the Dirac equation

$$\Psi \sim e^{-i\omega t + i\int^z p_{cz}(z')dz'}$$



DOI:10.3389/0.3390/galaxies10060116

We get the **semiclassical group velocity v_g** and **force F** given by

$$v_g = \frac{p_z}{E} + s_h s_{k_0}\frac{m^2\theta'}{2E^2E_z} \quad \Bigg| \quad F = -\frac{(m^2)'}{2E} + s_h s_{k_0}\left(\frac{(m^2\theta')'}{2EE_z} - \frac{m^2(m^2)'\theta'}{4E^3E_z}\right) \quad \Bigg| \quad s_h = h\gamma_{||}\frac{p_z}{|\mathbf{p}|} \equiv hs_{\mathrm{p}}$$

with $s_{k_0} = 1(-1)$ for particles (anti-particles) and $s = \pm 1$ for spin eigenstates in the z −direction (bubble wall). **Particles and anti-particles "feel" a different force.**

Liouville and collision operator

δf does not change local particle density.

The **distribution function** is parameterized as

$$f = \frac{1}{e^{\beta[\gamma_w(E_w + v_w p_z) - \mu]} \pm 1} + \delta f$$, where μ is the chemical potential and $\int \mathrm{d}^3 p \delta f = 0.$

The **Boltzmann equation** acting on the distribution function, reads

$$L[\mu_h, \delta f_h] = \mathcal{S}_h + \delta \mathcal{C}_h$$

- $L[\mu_h, \delta f_h]$: CP-conserving interactions with the bubble wall
- $\mathcal{S}_h$: CP-violating interactions with the bubble wall
- $\delta \mathcal{C}_h$: Interaction between particles

where the **Liouville operator** L and **source term** $\mathcal{S}_h$ are defined as

$$L[\mu, \delta f] \equiv -\frac{p_z}{E} f'_{0w} \partial_z \mu + v_w \gamma_w \frac{(m^2)'}{2E} f''_{0w} \mu + \frac{p_z}{E} \partial_z \delta f - \frac{(m^2)'}{2E} \partial_{p_z} \delta f,$$

$$\mathcal{S}_h = - v_w \gamma_w h s_\mathrm{p} \frac{\left(m^2 \theta'\right)'}{2 E E_z} f'_{0w} + v_w \gamma_w h s_p \frac{m^2 \left(m^2\right)' \theta'}{4 E^2 E_z} \left(\frac{f'_{0w}}{E} - \gamma_w f''_{0w} \right)$$

$s \to s_h = h \gamma_{||} \frac{p_z}{|\mathbf{p}|} \equiv h s_\mathrm{p}$

h is helicity

v_w is the wall velocity

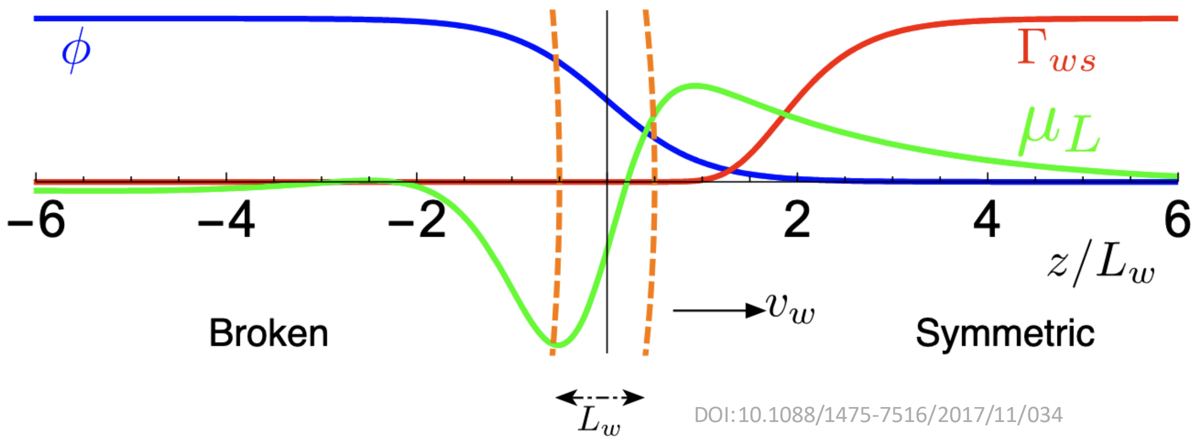
γ_w is the Lorentz factor

f_{0w} is f with $\delta f = \mu = 0$

The **collision operator** $\delta \mathcal{C}_h$ is model dependent.

BAU

We solve for the chemical potentials $\boldsymbol{\mu_L}$



DOI:10.1088/1475-7516/2017/11/034

- ϕ – EWPT order parameter
- L_w – Bubble wall thickness
- μ_L – Chemical potentials
- $\Gamma_{ws} = \Gamma_{sph}$ – EW sphaleron rate

To calculate **baryonic asymmetry in the Universe** we integrate the chemical potential

$$\eta_B = \frac{405\Gamma_{\rm sph}}{4\pi^2 v_w \gamma_w g_* T} \int_{-\infty}^{\infty} dz \mu_{B_L} f_{\rm sph} e^{-45\Gamma_{\rm sph}|z|/4v_w\gamma_w}$$

$$f_{\rm sph}(z) = \min\left(1, 2.4\frac{\Gamma_{\rm sph}}{T} e^{-40h(z)/T}\right)$$

f_{sph}(z) describes the sphaleron rate as a function of the distance to the bubble wall

The **experimental value** is given by

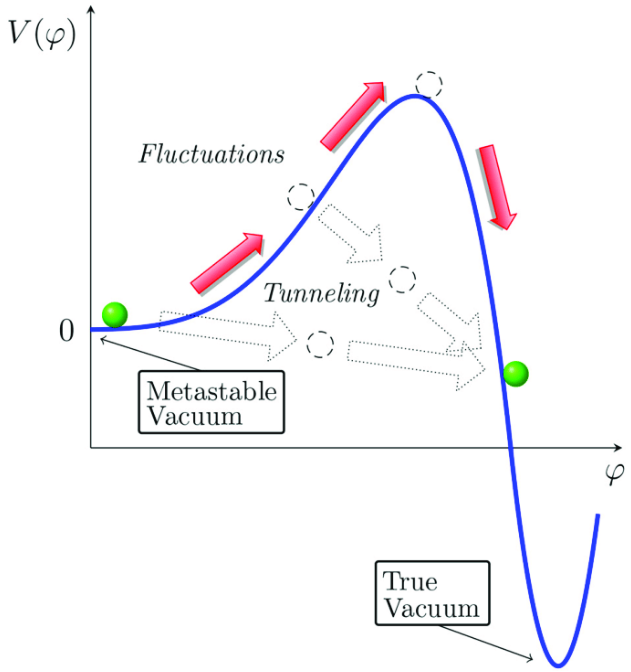
$$\eta_B \equiv \frac{n_B}{n_\gamma} = (6.2 \pm 0.4) \cdot 10^{-10}$$

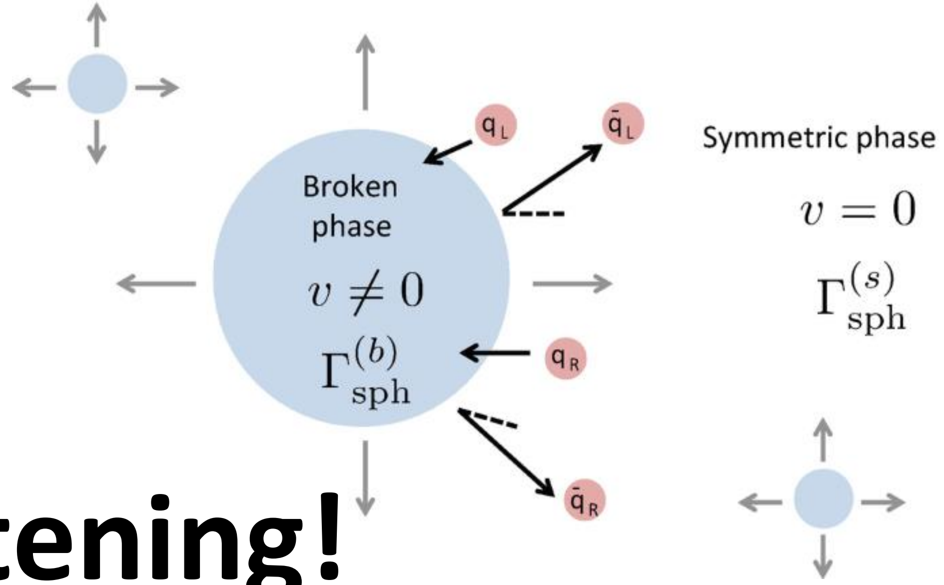
Computational problems

The **semi-classical force method** is straightforward to use but we face some numerical challenges

- The different scales of the involved **sources** might produce **stiff systems** (numerically unstable). We must find an **appropriate integration method**.
- The **thermal transport coefficients**, i.e. $D_l, Q_l, etc.$, are computationally expensive to calculate. We must find fast methods/approximations for them.
- **CP-violation** can be present **without an imaginary mass**, e.g. CKM phase. How can we have a general method for all cases.

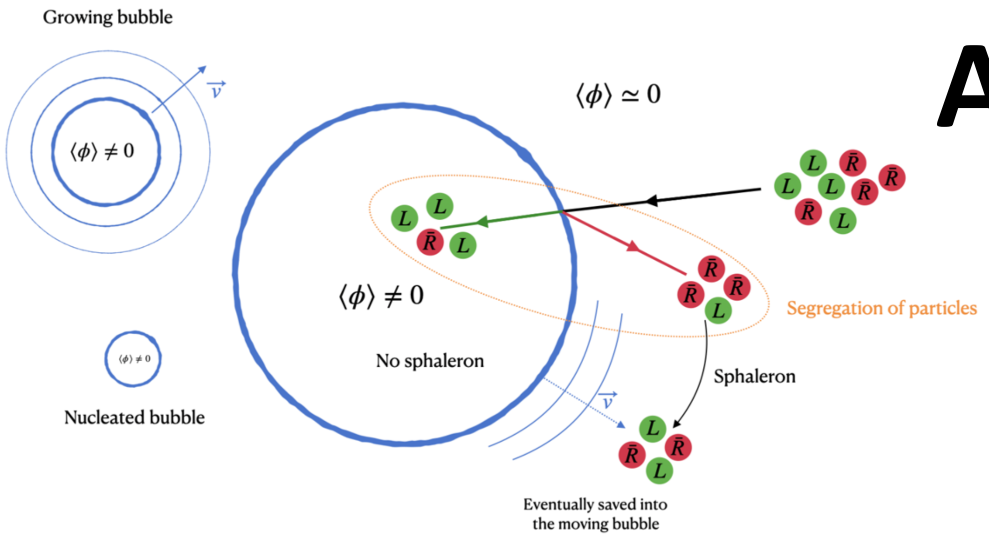
The project is still in its early days. Ideas/suggestions are welcome!

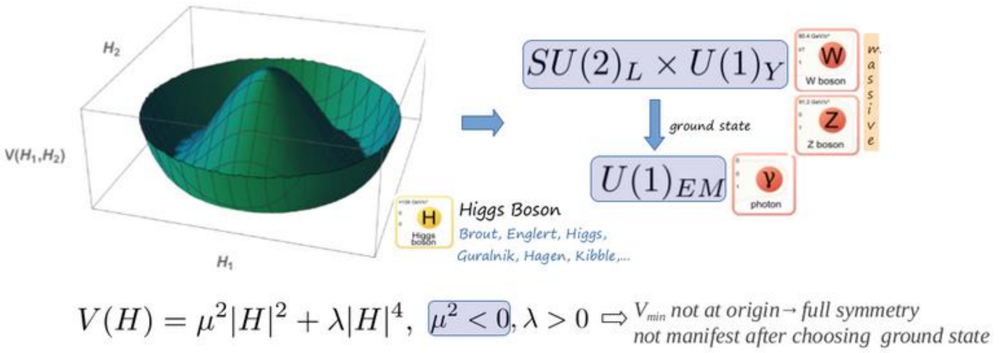




Thanks for listening!

Any questions?





Additional slides

Moment expansion

We define the **momentum averages** as

$$\langle X\rangle \equiv \frac{1}{N_1}\int \mathrm{d}^3 p X \qquad N_1 \equiv \int \mathrm{d}^3 p f'_{0w,\mathrm{FD}} = -\gamma_w \frac{2\pi^3}{3} T^2$$

$$[\mathcal{X}] \equiv \frac{1}{N_0}\int d^3 p \mathcal{X} f_{0w} \qquad N_0 = \int d^3 p f_{0w} = \gamma_w \int d^3 p f_0 \equiv \gamma_w \hat{N}_0$$

The **velocity pertubations** are defined as

$$u_\ell \equiv \left\langle \left(\frac{p_z}{E}\right)^\ell \delta f \right\rangle$$

The ***l*-th moment** is given by

$$\left\langle \left(\frac{p_z}{E}\right)^\ell L \right\rangle = \left\langle \left(\frac{p_z}{E}\right)^\ell (\mathcal{S} + \delta\mathcal{C}) \right\rangle$$

Moment expansion

The **Liouville term** is given by

$$\langle L \rangle = -D_1\mu' + u_1' + v_w\gamma_w (m^2)' Q_1 \mu,$$

$$\left\langle \frac{p_z}{E} L \right\rangle = -D_2\mu' + u_2' + v_w\gamma_w (m^2)' Q_2 \mu + (m^2)' \left\langle \frac{1}{2E^2} \delta f \right\rangle,$$

The **source term** is given by

$$S^o_{h\ell} = -v_w\gamma_w h[(m^2\theta')' Q^{8o}_\ell - (m^2)' m^2 \theta' Q^{9o}_\ell]$$

The **collision term** is given by

$$\delta C_1 \equiv \langle \delta \mathcal{C} \rangle \qquad \delta C_1 = K_0 \sum_i \Gamma_i \sum_j s_{ij} \frac{\mu_j}{T},$$

$$\delta C_2 \equiv \langle (p_z/E) \delta \mathcal{C} \rangle \qquad \delta C_2 = -\Gamma_{\text{tot}} u - v_w \delta C_1.$$

BAU

Neglecting **electroweak sphalerons** we have

$$B = \sum_q (n_q - \bar{n}_q) = 0$$

so that the **baryonic chemical potential** is given by

$$\mu_{B_\mathrm{L}} = \frac{1}{2}(1 + 4D_0^t)\mu_{t_\mathrm{L}} + \frac{1}{2}(1 + 4D_0^b)\mu_{b_\mathrm{L}} + 2D_0^t \mu_{t_\mathrm{R}}$$

To calculate **baryonic asymmetry in the Universe** we integrate the chemical potential

$$\eta_B = \frac{405\Gamma_\mathrm{sph}}{4\pi^2 v_w \gamma_w g_* T} \int dz \mu_{B_\mathrm{L}} f_\mathrm{sph} e^{-45\Gamma_\mathrm{sph}|z|/4v_w\gamma_w} \qquad f_\mathrm{sph}(z) = \min(1, 2.4\frac{\Gamma_\mathrm{sph}}{T} e^{-40h(z)/T})$$

The **experimental value** is given by

$$\eta \equiv \frac{n_B}{n_\gamma} = (6.2 \pm 0.4) \cdot 10^{-10}$$

Collision terms

$$\delta\bar{C}_1^{t_L} = \Gamma_y(\mu_{t_L} - \mu_{t_R} + \mu_h) + \Gamma_m(\mu_{t_L} - \mu_{t_R}) + \Gamma_W(\mu_{t_L} - \mu_{b_L}) + \tilde{\Gamma}_{SS}[\mu_i],$$

$$\delta\bar{C}_1^{b_L} = \Gamma_y(\mu_{b_L} - \mu_{t_R} + \mu_h) + \Gamma_W(\mu_{b_L} - \mu_{t_L}) + \tilde{\Gamma}_{SS}[\mu_i],$$

$$\delta\bar{C}_1^{t_R} = -\Gamma_y(\mu_{t_L} + \mu_{b_L} - 2\mu_{t_R} + 2\mu_h) + \Gamma_m(\mu_{t_R} - \mu_{t_L}) - \tilde{\Gamma}_{SS}[\mu_i],$$

$$\delta\bar{C}_1^{h} = \tilde{\Gamma}_y(\mu_{t_L} + \mu_{b_L} - 2\mu_{t_R} + 2\mu_h) + \Gamma_h\mu_h$$

Strong sphaleron

$$\tilde{\Gamma}_{SS}[\mu_i] = \Gamma_{SS}((1 + 9D_0^t)\mu_{t_L} + (1 + 9D_0^b)\mu_{b_L} - (1 - 9D_0^t)\mu_{t_R}).$$

interaction	rate
$t_L \leftrightarrow t_R + h$ $b_L \leftrightarrow t_R + h$	$\Gamma_{y,t}$
$b_L \leftrightarrow b_R + h$ $c_L \leftrightarrow c_R + h$ $s_L \leftrightarrow s_R + h$ $t_L \leftrightarrow b_R + h$ $s_L \leftrightarrow c_R + h$ $c_L \leftrightarrow s_R + h$	$\Gamma_{y,b}$
$t_L \leftrightarrow t_R$	$2\Gamma_{m,t}$
$b_L \leftrightarrow b_R$ $c_L \leftrightarrow c_R$ $s_L \leftrightarrow s_R$	$2\Gamma_{m,b}$
$t_L \leftrightarrow b_L$ $c_L \leftrightarrow s_L$	Γ_W
$h \leftrightarrow 0$	Γ_h
all L $\leftrightarrow$ all R	Γ_{ss}

inelastic rates
$\Gamma_{y,t} = 4.2 \times 10^{-3}\, y_t^2\, T$
$\Gamma_{y,b} = 4.2 \times 10^{-3}\, y_b^2\, T$
$\Gamma_{m,t} = \frac{m_t^2}{63T}$
$\Gamma_{m,b} = \frac{m_b^2}{63T}$
$\Gamma_W = \frac{T}{60}$
$\Gamma_h = \frac{m_W^2}{50T}$
$\Gamma_{ss} = 4.9 \times 10^{-4} T$

elastic rates
$\Gamma_{tot,q} = \frac{T}{18}$
$\Gamma_{tot,h} = \frac{T}{60}$