Constraints on large scalar multiplets added to the Standard Model

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We have considered a New-Physics model with a scalar sector consisting of the SM Higgs doublet H and its conjugate $\tilde{H}$,

$$H = \begin{pmatrix} a \\ b \end{pmatrix}, \quad \tilde{H} = \begin{pmatrix} b^* \\ -a^* \end{pmatrix}, \tag{1}$$

plus only one scalar multiplet χ with isospin J and arbitrary hypercharge Y, and its conjugate multiplet $\tilde{\chi}$:

$$\chi = \begin{pmatrix} \chi_J \\ \chi_{J-1} \\ \chi_{J-2} \\ \vdots \\ \chi_{-J} \end{pmatrix}, \quad \tilde{\chi} = \begin{pmatrix} \chi^*_{-J} \\ -\chi^*_{1-J} \\ \chi^*_{2-J} \\ \vdots \\ (-1)^{2J}\, \chi^*_J \end{pmatrix}. \tag{2}$$

(In the notation "χ_I," I is the third component of isospin.) According to Hally, Logan, and Pilkington (2012), there is a unitarity bound (from $VV \to \chi\chi$): J cannot be higher than $7/2$ and $|Y|$ has a J-dependent upper bound.

We have studied the unitarity (UNI) (from $\chi\chi \to \chi\chi$) and the bounded from below (BFB) constraints on this New Physics model, with their J- and Y-dependences.

The quartic part of the potential, V_4, has three types of terms:

- one term four-linear in H:

$$\frac{\lambda_1}{2}\left(F_1\right)^2, \quad F_1 \equiv |a|^2 + |b|^2, \tag{3}$$

  just as in the SM;

- two terms bilinear in both H and χ:

$$\lambda_3 F_1 F_2, \quad F_2 \equiv |\chi_J|^2 + |\chi_{J-1}|^2 + \cdots + |\chi_{-J}|^2, \tag{4}$$

  and

$$\lambda_4 F_4, \quad F_4 = \left[\left(H \otimes \tilde{H}\right)_{\mathbf{3}} \otimes \left(\chi \otimes \tilde{\chi}\right)_{\mathbf{3}}\right]_{\mathbf{1}}; \tag{5}$$

- one or more terms four-linear in χ:

$$\frac{\lambda_2}{2}\left(F_2\right)^2 + \sum_{i=5}^{t} \lambda_i F_i, \tag{6}$$

  $t = 5$ for $J = 1$ and $J = 3/2$;

  $t = 6$ for $J = 2$ and $J = 5/2$;

  $t = 7$ for $J = 3$ and $J = 7/2$.

The terms in Eq. (6) arise because $(\chi \otimes \chi)_{\text{symmetric}}$ contains several multiplets of $SU(2)$:

- $J = 1 \;\Rightarrow\; (\chi \otimes \chi)_{\text{symmetric}} = \mathbf{5} \oplus \mathbf{1}$;
- $J = 3/2 \;\Rightarrow\; (\chi \otimes \chi)_{\text{symmetric}} = \mathbf{7} \oplus \mathbf{3}$;
- $J = 2 \;\Rightarrow\; (\chi \otimes \chi)_{\text{symmetric}} = \mathbf{9} \oplus \mathbf{5} \oplus \mathbf{1}$;
- $J = 5/2 \;\Rightarrow\; (\chi \otimes \chi)_{\text{symmetric}} = \mathbf{11} \oplus \mathbf{7} \oplus \mathbf{3}$;
- $J = 3 \;\Rightarrow\; (\chi \otimes \chi)_{\text{symmetric}} = \mathbf{13} \oplus \mathbf{9} \oplus \mathbf{5} \oplus \mathbf{1}$;
- $J = 7/2 \;\Rightarrow\; (\chi \otimes \chi)_{\text{symmetric}} = \mathbf{15} \oplus \mathbf{11} \oplus \mathbf{7} \oplus \mathbf{3}$.

Taking the square moduli of all the $SU(2)$ multiplets, one obtains various terms (two for $J = 1$ or $J = 3/2$, three for $J = 2$ or $J = 5/2$, four for $J = 3$ or $J = 7/2$) that are equivalent to $(F_2)^2$ and the F_i $(i = 5, \ldots, t)$.

The term

$$\begin{aligned} F_4 \;=\; & \frac{|a|^2 - |b|^2}{2} \sum_{I=-J}^{J} I \left|\chi_I\right|^2 \\ & + \Re\left(ab^* \sum_{I=1-J}^{J} \chi_I^* \chi_{I-1} \sqrt{J^2 - I^2 + J + I} \right) \end{aligned} \quad (7)$$

is especially relevant because, when b acquires vacuum expectation value (VEV) v—the multiplet χ is supposed not to acquire VEV, lest there is a Goldstone boson because Y is arbitrary and lest $m_W \neq c_w m_Z$—it generates a mass-squared splitting between any two components of χ that have I differing by one unit:

$$\Delta m^2 = \frac{\left|\lambda_4 v^2\right|}{2}. \tag{8}$$

How large can $|\lambda_4|$, and hence Δm^2, be? The answer depends on the UNI but also on the BFB conditions.

Because $v = 174\,\mathrm{GeV}$ and the mass of the Higgs boson in $125\,\mathrm{GeV}$, $\lambda_1 = 0.258$ just as in the SM.

In the following we neglect the terms $\left(F_2\right)^2$ and F_i in V_4.

The UNI conditions that we consider in this work arise from the scattering of a pair of scalars to another pair of scalars with equal total values of I and Y. From the cases where the two scalars are either $(\chi_0 a,\ \chi_1 b)$ or $(\chi_1 a^*,\ \chi_0 b^*)$ for integer J, or $\left(\chi_{-1/2} a,\ \chi_{1/2} b\right)$ or $\left(\chi_{1/2} a^*,\ \chi_{-1/2} b^*\right)$ for half-integer J, one derives

$$\left|\lambda_3\right| + \frac{J+1}{2}\left|\lambda_4\right| < 8\pi. \tag{9}$$

From the case where the two scalars are $a a^*,\ b b^*,\ \chi_J \chi_J^*,\ \chi_{J-1} \chi_{J-1}^*,\ \ldots,\ \chi_{-J} \chi_{-J}^*$ one obtains

$$3\left|\lambda_1\right| + \sqrt{9\lambda_1^2 + 8\left(2J+1\right)\lambda_3^2} < 16\pi, \tag{10}$$

$$\left|\lambda_1\right| + \sqrt{\lambda_1^2 + \left(2/3\right) J\left(J+1\right)\left(2J+1\right)\lambda_4^2} < 16\pi. \tag{11}$$

It turns out that Eqs. (9), (10) and (11) are *the strongest* UNI conditions (i.e. all the other scatterings produce weaker constraints).

The BFB conditions are best obtained in the gauge where $b = 0$. There,

$$V_4 = \frac{\lambda_1}{2} |a|^4 + \lambda_3 |a|^2 \sum_{I=-J}^{J} |\chi_I|^2 + \frac{\lambda_4}{2} |a|^2 \sum_{I=-J}^{J} I |\chi_I|^2 . \tag{12}$$

This is necessarily larger than

$$\frac{\lambda_1}{2} |a|^4 + \lambda_3 |a|^2 \sum_{I=-J}^{J} |\chi_I|^2 - \frac{J |\lambda_4|}{2} |a|^2 \sum_{I=-J}^{J} |\chi_I|^2 . \tag{13}$$

The quantity (13) must always remain positive. Therefore, the BFB conditions are

$$\lambda_1 \geq 0, \tag{14}$$

$$\lambda_3 \geq 0, \tag{15}$$

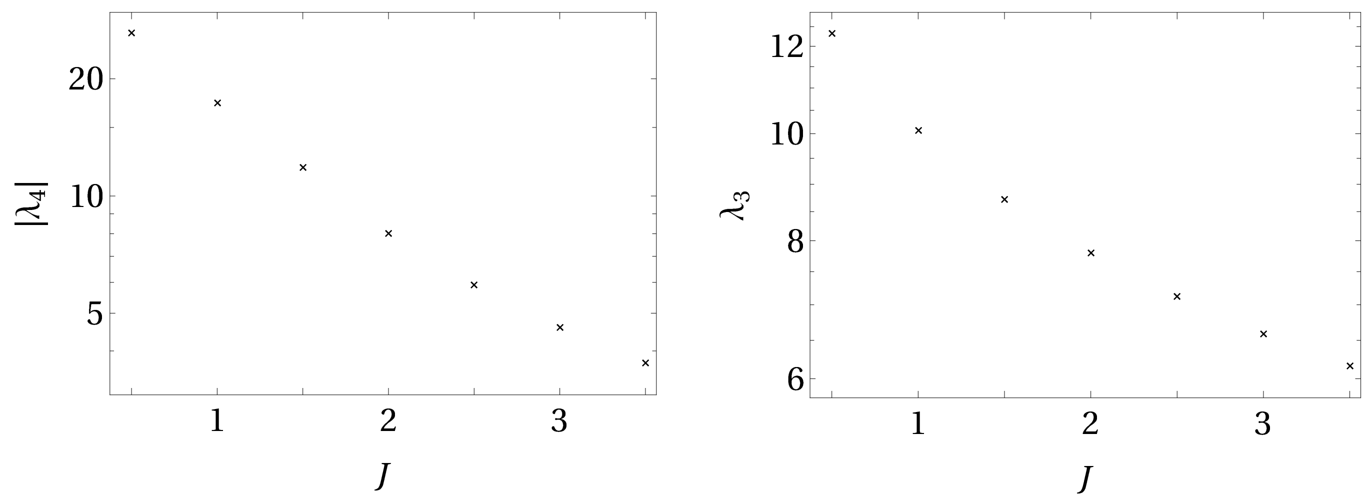
$$|\lambda_4| \leq \frac{2\lambda_3}{J}. \tag{16}$$

If the terms $(F_2)^2$ and F_i in V_4 are not neglected, then exact necessary and sufficient BFB conditions can still be derived by generalizing a method devised, for the special case $J = 1$, by Bonilla, Fonseca, and Valle (2015).

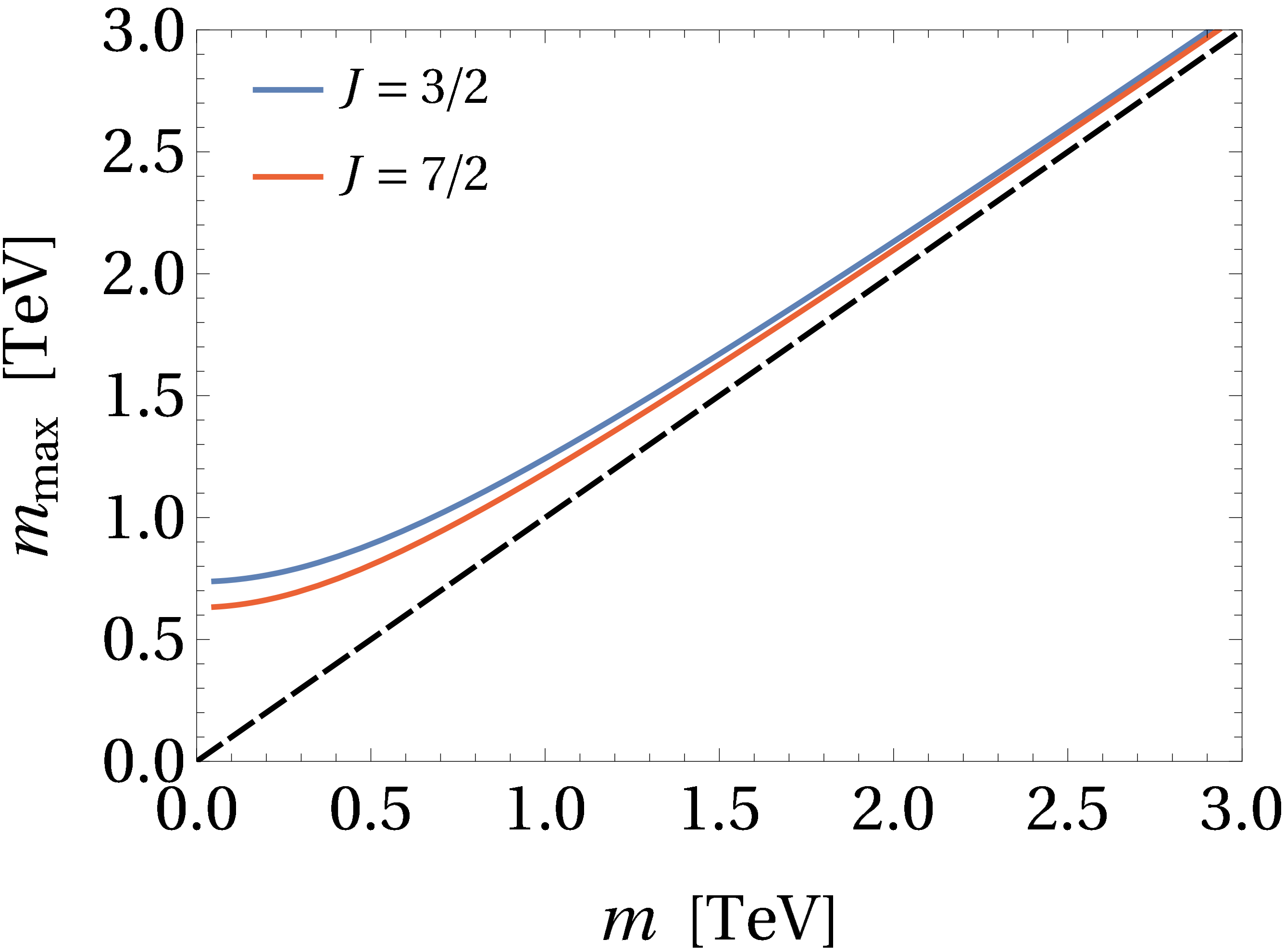
By putting together the UNI and the BFB conditions, one derives the maximum allowed value of $|\lambda_4|$, and the maximum (and minimum) allowed values of λ_3, given in the following table and figure for various values of J:

J	$1/2$	1	$3/2$	2
maximum $\|\lambda_4\|$	26.46	17.49	11.96	8.10
maximum λ_3	12.37	10.10	8.75	7.82
minimum λ_3	-1.46	-1.26	-1.13	-1.03

J	$5/2$	3	$7/2$
maximum $\|\lambda_4\|$	5.97	4.65	3.76
maximum λ_3	7.14	6.61	6.19
minimum λ_3	-0.95	-0.89	-0.84



In practice, if J is rather large, then $|\lambda_4|$ must be—because of UNI and BFB—so small that the multiplets are almost fully degenerate. This is illustrated in the figure below, where we depict the maximal mass of the heaviest particle in the multiplet *versus* the mass of the lightest particle.



The renormalization-group equations (RGEs) are of the form

$$16\pi^2 \mu \, \frac{\mathrm{d}\lambda}{\mathrm{d}\mu} = \beta_\lambda, \tag{17}$$

where μ is the energy scale at which the dimensionless coupling λ is measured. In order to derive the RGEs for our model—without the terms $(F_2)^2$ and F_i—we have used a feature of the software `SARAH`.

- We had to adapt that software, which only tolerates J through 3, in order to treat the case $J = 7/2$.
- The running time of the software increases exponentially with J.
- It is inconsistent to neglect λ_2 and the λ_i, because they have nonzero β functions, still we have done it.

We have obtained

$$\beta_{g_1} = \left(\frac{41}{10} + \frac{4}{5}\, Y^2 \right) g_1^3, \tag{18a}$$

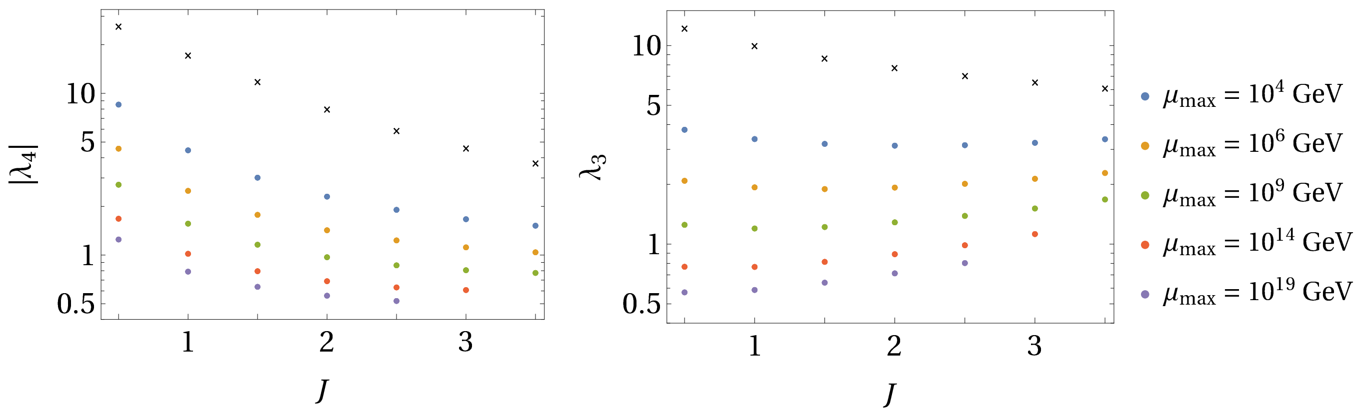
$$\beta_{g_2} = \left[-\frac{19}{6} + \frac{J\,(J+1)\,(2J+1)}{9} \right] g_2^3, \tag{18b}$$

$$\beta_{g_3} = -7 g_3^3; \tag{18c}$$

$$\beta_{y_t} = \left(\frac{9}{2}\, y_t^2 - \frac{17}{20}\, g_1^2 - \frac{9}{4}\, g_2^2 - 8 g_3^2 \right) y_t; \tag{19}$$

$$
\begin{aligned}
\beta_{\lambda_1} &= \frac{27}{100}\, g_1^4 + \frac{9}{10}\, g_1^2 g_2^2 + \frac{9}{4}\, g_2^4 + 12 y_t^2 \lambda_1 \\
&\quad + 12\lambda_1^2 + 2\left(2J+1\right)\lambda_3^2 + \frac{J\left(J+1\right)\left(2J+1\right)}{6}\,\lambda_4^2 \\
&\quad - \left(\frac{9}{5}\, g_1^2 + 9 g_2^2\right)\lambda_1 - 12 y_t^4, \qquad (20a)\\
\beta_{\lambda_3} &= \frac{27}{25}\left(Y g_1^2\right)^2 + 3J\left(J+1\right) g_2^4 + 6 y_t^2 \lambda_3 \\
&\quad + 6\lambda_1\lambda_3 + 4\lambda_3^2 + J\left(J+1\right)\lambda_4^2 \\
&\quad - \left(\frac{9}{10} + \frac{18}{5}\, Y^2\right) g_1^2 \lambda_3 \\
&\quad - \left[\frac{9}{2} + 6J\left(J+1\right)\right] g_2^2 \lambda_3, \qquad (20b)\\
\beta_{\lambda_4} &= \frac{36}{5}\, Y g_1^2 g_2^2 + 6 y_t^2 \lambda_4 + 2\lambda_1\lambda_4 + 8\lambda_3\lambda_4 \\
&\quad - \left(\frac{9}{10} + \frac{18}{5}\, Y^2\right) g_1^2 \lambda_4 \\
&\quad - \left[\frac{9}{2} + 6J\left(J+1\right)\right] g_2^2 \lambda_4. \qquad (20c)
\end{aligned}
$$

We have integrated these equations up to the scale $\mu = 10^{19}\,\mathrm{GeV}$, starting from allowed values of λ_3 and λ_4 at the scale $\mu = m_t$. We have required that both the UNI and BFB conditions are valid at every intermediate energy scale; this constrains the initial λ_3 and λ_4 even more strongly:



$|\lambda_4|$
10
5
1
0.5
1
2
3
J
λ_3
10
5
1
0.5
1
2
3
J
$\mu_{\max} = 10^4$ GeV
$\mu_{\max} = 10^6$ GeV
$\mu_{\max} = 10^9$ GeV
$\mu_{\max} = 10^{14}$ GeV
$\mu_{\max} = 10^{19}$ GeV