

#### **Recent** $B^+ \rightarrow K^+ \nu \overline{\nu}$ **Excess & Muon** g - 2**Illuminating Light DM with Higgs Portal** Shu-Yu HO (KIAS)

Based on arXiv : 2401.10112

In collaboration with J. Kim (CAU) & P. Ko (KIAS)

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#### **Evidence of dark matter**

There are undeniable evidences for dark matter in a wide range of distance scales.



#### <u>Measurement of $B^+ \rightarrow K^+ \nu \overline{\nu}$ at Belle II</u>



Prediction of  $B^+ \rightarrow K^+ \nu \bar{\nu}$  by the SMThe decay rate of the  $B^+ \rightarrow K^+ \nu \bar{\nu}$  process is calculated with<br/>high accuracy in the SM.



 $\mathcal{B}(B^+ \to K^+ \nu \bar{\nu})_{\mathsf{SM}} = (4.97 \pm 0.37) \times 10^{-6}$  $\mathcal{B}(B^+ \to K^+ + \mathsf{missing energy}) = (1.8 \pm 0.7) \times 10^{-5}$ 

 $\implies B^+ \rightarrow K^+ + \text{light dark particles}?$ 



In this talk, we will consider 5.1  $\sigma$  is real to be explained by new physics BSM.

#### A new gauge boson?

The simplest way to resolve the muon g-2 anomaly is to introduce a new gauge boson which couples to the muon



 $=m_{Z'}\!\sim\!10^{(1-2)}\,{
m MeV}\,\&\,g_{\sf D}^{}\,\sim\!10^{-4}$  can explain  $\Delta a_{\mu}$  anomaly

### <u>Gauged</u> $U(1)_{L_{\mu}-L_{\tau}}$ <u>Model</u>

Without new fermions, there are three anomaly-free models by gauging one of the three differences of lepton flavors

• 
$$L_e - L_\mu$$
 ,  $L_e - L_\tau$  ,  $L_\mu - L_\tau$ 

X. G. He et al, PRD 1991

- Symmetry including  $L_e$  is strongly constrained
- Charge assignments :  $\widehat{\mathcal{Q}}_{\mathsf{L}_{\mu}-\mathsf{L}_{ au}}(
  u_{\mu},
  u_{ au},\mu, au)=(1,-1,1,-1)$

Unavoidable kinetic mixing between Z' and  $\gamma$ 

$$\begin{split} \epsilon &= -\frac{eg_{\mu-\tau}}{2\pi^2} \int_0^1 dx \, x(1-x) \ln\left[\frac{m_\tau^2 - x(1-x)q^2}{m_\mu^2 - x(1-x)q^2}\right] \\ & \xrightarrow{q^2 \ll m_\mu^2} -\frac{eg_{\mu-\tau}}{12\pi^2} \ln\left(\frac{m_\tau^2}{m_\mu^2}\right) \simeq -\frac{g_{\mu-\tau}}{70} \end{split}$$

#### $g_X$ VS. $m_{Z'}$



7/20

 $U(1)_{L_{\mu}-L_{\tau}}$ -charged DM model + Dark Higgs Particle content & charge assignments  $\widehat{\mathcal{Q}}_{\mathsf{L}_{\mu}-\mathsf{L}_{ au}}(
u_{\mu},
u_{ au},\mu, au,\underbrace{X}, \Phi) = (1,-1,1,-1,\mathcal{Q}_{X},\mathcal{Q}_{\Phi})$ SM Higgs singlet complex singlet scalar DM The renormalizable and gauge invariant Lagrangian  $\mathcal{L} = |\mathcal{D}_
ho \Phi|^2 + |\mathcal{D}_
ho X|^2 - rac{1}{4} (\partial_
ho Z'_
ho - \partial_
ho Z'_
ho)^2 - m_X^2 |X|^2$  $- g_X \Big( ar{\mu} \gamma^
ho \mu - ar{ au} \gamma^
ho au + ar{
u}_{L\mu} \gamma^
ho 
u_{L\mu} - ar{
u}_{L au} \gamma^
ho 
u_{L au} \Big) Z_
ho^
ho 
u_{L au}$  $-\lambda_{\Phi X}|X|^2\Big(|\Phi|^2-rac{1}{2}v_{\Phi}^2\Big)-\lambda_{HX}|X|^2\Big(|\mathcal{H}|^2-rac{1}{2}v_H^2\Big)$  dark photon  $-\lambda_{\Phi H} \Big( |\Phi|^2 - rac{1}{2} v_{\Phi}^2 \Big) \Big( |\mathcal{H}|^2 - rac{1}{2} v_H^2 \Big) + \cdots \Big|_{\mathsf{SM Higgs doublet}}$ 8/20

$$U(1)_{L_{\mu}-L_{\tau}}$$
-charged DM model + Dark Higgs

After electroweak and  $U(1)_{L_{\mu}-L_{\tau}}$  symmetry breakings

$$\mathcal{H} = rac{1}{\sqrt{2}} (0 \ v_H + h)^{\mathsf{T}} \ , \ \Phi = rac{1}{\sqrt{2}} (v_\Phi + \phi) \ ,$$

The CP-even neutral components mix with each other

Dark photon mass :  $m_{Z'}^{}=g_X^{}|\mathcal{Q}_\Phi^{}|v_\Phi^{}$ 

Parameter set :  $iggl\{g_X, m_{Z'}, m_X, m_{H_1}, \lambda_{\Phi X}, \mathcal{Q}_{\Phi}, \sin hetaiggr\}$  9/20

#### Higgs invisible decay

The SM-like Higgs boson has additional decay processes

- $H_2 
  ightarrow H_1 H_1, Z'Z', XX^\dagger$  (invisible decay channels)
- SM Higgs mainly decays into dark photon & dark Higgs

$$\Gamma_{H_2 o H_1 H_1} \simeq \Gamma_{H_2 o Z' Z'} \propto rac{\sin^2 heta \, m_{H_2}^3}{v_{\Phi}^2} \gg \Gamma_{H_2 o X X^\dagger} \propto rac{\sin^2 heta \, \lambda_{\Phi X}^2 v_{\Phi}^2}{m_{H_2}}$$

The LHC provides a strong bound on these invisible decays

$$\mathcal{B}(H_2 
ightarrow \mathrm{Inv.}) < 0.13$$
 PDG 2022

Typically,  $\sin heta \lesssim 0.01$  in order to satisfy the Higgs invisible decays constraint.

#### DM relic abundance

#### Thermal WIMP DM relic density

$$egin{aligned} \Omega_{\mathsf{WIMP}} \hat{h}^2 &= 2 \Omega_X \hat{h}^2 \simeq rac{1.75 imes 10^{-10} \mathrm{GeV}^{-2} x_f}{\sqrt{g_*} \left< \sigma v \right>} & x_f \equiv rac{m_X}{T_f} \sim 12-19 \ g_* \sim 10 \end{aligned}$$

#### Annihilation processes:

#### **DM direct detection**

In the  $U(1)_{L_{\mu}-L_{\tau}}$  DM models without a dark Higgs boson, DM-nucleon/electron scattering is highly suppressed.

• 
$$\sigma_{\rm el}({\rm DM}-p)\sim 10^{-46}\,{\rm cm}^2\,,\,\sigma_{\rm el}({\rm DM}-e)\sim 10^{-51}\,{\rm cm}^2$$

Holst, D. Hooper, G. Krnjaic, PRL 2022

- In our model, we can have a sizable DM-nucleon scattering process thanks to the light dark Higgs boson exchange.
  - DM-nucleon elastic scattering cross section

$$\sigma_{\mathsf{el}} \simeq rac{4 \mu_n^2 f_n^2 \lambda_{\Phi \! X}^2}{\pi} \Bigl( rac{m_n}{m_X} \Bigr)^{\!\!2} \Bigl( rac{\upsilon_{\Phi}}{\upsilon_H} \Bigr)^{\!\!2} \Bigl( rac{1}{m_{H_1}^2} - rac{1}{m_{H_2}^2} \Bigr)^{\!\!2}$$

# **CMB constraint on light DM mass** For $m_{\text{DM}} \lesssim \mathcal{O}(10)$ GeV, the CMB gives a stringent bound on thermal DM freeze-out determined by s-wave annihilation.

$$m_{\mathsf{DM}}\gtrsim \mathcal{O}(10\,\mathrm{GeV})igg(rac{\langle \sigma v 
angle}{3 imes 10^{-26}\mathrm{cm}^3\,\mathrm{sec}^{-1}}igg)$$

<u>alternative</u> asymmetric DM p-wave annihilation forbidden DM



#### CMB constraints in our model

Dominant DM annihilation channel

- $XX^\dagger\!
  ightarrow\!Z'Z', H_1H_1$  (s-wave)  $XX^\dagger\!
  ightarrow\!Z'H_1$  (p-wave)
- Dark photon decay
  - $Z' \rightarrow \nu \bar{\nu} \ (m_{Z'} = 11.5 \, {\rm MeV}, g_X = 5 \times 10^{-4})$
  - $\mathcal{B}(Z' \rightarrow e^+ e^-) \simeq 10^{-5} \ (\mathcal{L} = -\epsilon \, g_e \bar{e} \gamma^{\rho} e Z'_{\rho}, \epsilon \simeq -g_X/70)$
- Dark Higgs boson decay
  - $H_1 \! 
    ightarrow \! X X^\dagger \,, \, H_1 \! 
    ightarrow \! Z' Z' \! 
    ightarrow \! 4 
    u$
  - $H_1 
    ightarrow \ell^+ \ell^-$  (suppressed by small Yukawa coupling & mixing angle)
- We can naturally avoid the stringent CMB bound thanks to invisible decay of these dark particles.

#### Effective number of neutrino species

Considering the modification  $N_{\rm eff}$  of via light DM s-wave annihilation to neutrinos, complex scalar DM mass below 8.2 MeV is disfavored.



15/20

#### Two- or Three-body decays at Belle II

- Belle II provides information on the  $q^2$  spectrum
- A peak localized around  $q^2 = 4 \text{ GeV}^2$
- Two body decay : $B^+\!
  ightarrow K^+X, m_{_X}=2\,{
  m GeV}$

W. Altmannshofer et al, 2311.14629

• Three body decay :  $B^+ 
ightarrow K^+ XX, m_X^- < 0.6~{
m GeV}$ K. Fridell et al, 2312.12507



 $\frac{\text{Two- or Three-body decays at Belle II}}{\text{When }m_{B^+}-m_{K^+}>m_{H_1}}, \text{ the }B^+\text{ meson goes through a two body decay}}$ 

$$egin{split} \Gamma_{B^+ o K^+ H_1} &\simeq rac{|\kappa_{cb}|^2 {
m sin}^2 heta}{64 \pi m_{B^+}^3} \Big( rac{m_{B^+}^2 - m_{K^+}^2}{m_b - m_s} \Big)^2 rac{ig[ f_0(m_{H_1}^2) ig]^2}{{
m form \ factor}} &\sin heta \ll 1 \ & imes \sqrt{\mathcal{K}ig(m_{B^+}^2, m_{K^+}^2, m_{H_1}^2)} & {
m form \ factor} \end{split}$$



 $|\kappa_{cb}|\simeq 6.7 imes 10^{-6} \quad \mathcal{K}(a,b,c)=a^2+b^2+c^2-2(ab+bc+ca)$ 

When  $m_{H_1} > m_{B^+} - m_{K^+} > 2m_X$ , the  $B^+$  meson goes through a three body decay

$$egin{split} \Gamma_{B^+ o K^+ X X^\dagger} &\simeq rac{\lambda_{\Phi X}^2 v_\Phi^2 |\kappa_{cb}|^2 {
m sin}^2 heta}{1024 \pi^3 m_{B^+}^3} igg( rac{m_{B^+}^2 - m_{K^+}^2}{m_b - m_s} igg)^2 igg( m_{H_1}^2 - m_{H_2}^2 igg)^2 \ & imes \int_{4m_X^2}^{(m_{B^+} - m_{K^+})^2} {
m d}q^2 rac{\sqrt{1 - 4m_X^2/q^2}}{(q^2 - m_{H_1}^2)^2 (q^2 - m_{H_2}^2)^2} igg[ f_0(q^2) igg]^2 \end{split}$$







#### **Conclusions**

In this work, we have studied the simplest UV-completion gauged  $U(1)_{L_{\mu}-L_{\tau}}$ -charged complex scalar DM model with the dark Higgs mechanism.

We have found the dark Higgs boson mass,

the complex scalar DM mass,

 $(\Delta N_{
m eff}) \ \ 10 \ {
m MeV} \lesssim m_X \lesssim 10 \ {
m GeV} \ \ + \ {
m DM} \ {
m direct} \ {
m detection}$ 

and the dark photon mass  ${m_{Z'}\sim 10\,{
m MeV}}\,,\;g_X\sim 5 imes 10^{-4}$  (muon g-2 anomaly)

With these light dark particles, we can have the integrated solution of  $B^+ \rightarrow K^+ \nu \bar{\nu}$  excess,  $\Delta a_\mu$  anomaly, and DM. 20/20

# Backup

# The Standard Model (SM) is GOOD, but .....

=1.275 GeV/c2 =173.07 GeV/c2 ≈126 GeV/c2 mass → ≈2.3 MeV/c<sup>2</sup> 0 charge → 2/3 2/3 2/3 0 u С t g 1/2 0 1/2 spin  $\rightarrow$  1/2 Higgs boson gluon charm up top ≈4.8 MeV/c<sup>2</sup> ≈95 MeV/c<sup>2</sup> ≈4.18 GeV/c<sup>2</sup> 0 UARKS -1/3 -1/3 0 -1/3 γ S b C 1/2 1/2 1/2 down strange bottom photon 0.511 MeV/c2 105.7 MeV/c2 1.777 GeV/c2 91.2 GeV/c<sup>2</sup> -1 -1 e μ τ 1/2 1/2 ONS 1/2 Z boson electron muon tau Ś 0 <0.17 MeV/c<sup>2</sup> <15.5 MeV/c<sup>2</sup> 80.4 GeV/c<sup>2</sup> <2.2 eV/c2 S 8 EPTON 0 0 0 ±1  $\mathcal{V}_{\mathrm{e}}$ GAUGE  $\mathcal{V}_{\mu}$  $\mathcal{V}_{\tau}$ 1/2 1/2 1/2 electron tau muon W boson neutrino neutrino neutrino

Unsolved problems

- Dark Matter (DM)
- Dark energy
- Neutrino mass
- Baryon asymmetry
- Gravity

....

We need new physics beyond the SM (BSM)!!!

## $U(1)_{L_{\mu}-L_{\tau}}$ -charged DM model

Conventional  $U(1)_{L_{\mu}-L_{\tau}}$ -charged fermionic DM ( $\chi$ ) model

$$egin{split} \mathcal{L} &= \mathcal{L}_{\mathsf{SM}} - rac{1}{4} Z'_{
ho\omega} Z'^{
ho\omega} + rac{1}{2} m_{Z'}^2 Z'_{
ho} Z'^{
ho} + \overline{\chi} ig( i \gamma^{
ho} \partial_{
ho} - m_{\chi} ig) \chi \ &+ g_X Z'_{
ho} ig( \mathcal{Q}_\chi \overline{\chi} \gamma^{
ho} \chi + \sum_{\ell=\mu, au,
u_\mu,
u_ au} \mathcal{Q}_\ell \, \overline{\ell} \gamma^{
ho} \ell ig) \end{split}$$

- The dark photon  $Z^\prime$  plays a role of a messenger particle between DM and the SM leptons.
- The dark photon mass  $m_{Z'}$  is put by hand or is generated by the Stueckelberg mechanism. H. Ruegg, M. Ruiz-Altaba (2003)

## $U(1)_{L_{\mu}-L_{\tau}}$ -charged DM model

Dominant annihilation channels :  $\chi ar{\chi} o Z^{'*} o \ell^+ \ell^-, 
u ar{
u}$ 

• To explain the muon g-2,  $g_X \sim 10^{-4}$ , which is too small to get

the correct DM relic abundance.

• Only  $m_{Z'}\sim 2m_{\chi}$  can give the right DM relic abundance.



 $U(1)_{L_{\mu}-L_{\tau}}$ -charged DM model + Dark Higgs Particle content & charge assignments  $\widehat{\mathcal{Q}}_{\mathsf{L}_{\mu}-\mathsf{L}_{\tau}}(\nu_{\mu},\nu_{\tau},\mu,\tau,X,\Phi) = (1,-1,1,-1,\mathcal{Q}_{X},\mathcal{Q}_{\Phi})$ SM Higgs singlet complex singlet scalar DM To make DM absolutely stable, we choose  $\mathcal{Q}_{\Phi} \neq \frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \cdots$ • e.g.  ${\cal O}^{(5)}=rac{1}{{}_{\!\!\!\Lambda}}X\Phi^{\dagger4}\,$  (dark matter decays) We also choose  $\mathcal{Q}_{\Phi} 
eq 2$  to avoid the mass splitting of X•  $\mu \Big( X^2 \Phi^\dagger + ext{h.c.} \Big) \supset rac{1}{2} \mu v_\Phi ig( X_R^2 - X_I^2 ig)$ Baek, JK, Ko, 2204.04889

#### The Hubble tension

There is a large difference between the early- and late-time determinations of Hubble constant  $H_0$ .



Early-time measurement

 $H_0=67.4\pm0.5\,\mathrm{km/s/Mpc}$ 

Late-time measurement

 $H_0 = 73.2 \pm 1.3 \, {\rm km/s/Mpc}$ 

This discrepancy can arise either b/c

- Our distance measurements are incorrect ( $\Delta G_N$ ).
- Cosmological model we use to fit all those distances is wrong ( $\Delta N_{
  m eff}$ ).

 $\Delta N_{
m eff}$  v.s.  $\Delta G_N$ ?

#### What we know about dark matter

Dark matter as a particle must be

- Massive : gravitationally interact with ordinary matter
- Cold : non-relativistic at the time of structures formation
- Electric neutral : Almost no electromagnetic interaction
- Stable or with lifetime longer than the age of Universe
- Non-baryonic matter
- Making up about a quarter of the energy density of the present universe

#### What we don't know about dark matter

- Unknown particle nature of dark matter
  - Mass :  $10^{-31} M_{
    m proton} < M_{
    m DM} < 5 M_{\odot}$
  - Spin : Scalar or Vector Boson? Dirac or Majorana Fermion?
  - Number of species : There may exist more than one kind of dark matter in the universe. (Occam's razor?)
  - Interactions : Dark matter may have interactions with ordinary matter or itself (SIDM) other than the gravitational interaction.
- Unknown origin of dark matter
  - Thermal : Relic produced from the SM thermal plasma
  - Non-thermal : e.g. coherent oscillation, topological defect, .....

#### **Dark matter candidates**

Thermal production B.W. Lee Weakly interacting massive particles (WIMP) & S. Weinberg (1977) Strongly interacting massive particles (SIMP) Y. Hochberg, etal (2014) Elastically decoupling relic (ELDER) E. Kuflik, etal (2016) R. T. D'Agnolo, Forbidden dark matter & J. T. Ruderman (2015) . . . . . . . . . Non-thermal production The QCD axion/axion-like particles (ALP) P. Arias, et al. (2012) Feebly interacting massive particles (FIMP) L. J. Hall (2009) Hidden monopole dark matter H. Murayama, J. Shu (2009) Primordial black hole (PBH) Ya.B. Zel'dovich and I.D. Novikov (1967)

#### Weakly Interacting Massive Particle (WIMP) DM



# $\begin{array}{|c|c|c|c|c|} \hline \hline Two- \ or \ Three-body \ decays \ at \ Belle \ II \\ \hline When \ m_{B^+} - m_{K^+} > m_{H_1}, \ the \ B^+ \ meson \ goes \ through \ a \ two \ body \ decay \end{array}$

$$egin{split} \Gamma_{B^+ o K^+ H_1} &\simeq rac{|\kappa_{cb}|^2 {
m sin}^2 heta}{64 \pi m_{B^+}^3} igg( rac{m_{B^+}^2 - m_{K^+}^2}{m_b - m_s} igg)^2 rac{igg[ f_0(m_{H_1}^2) igg]^2}{{
m form \ factor}} &\sin heta \ll 1 \ & imes \sqrt{\mathcal{K}(m_{B^+}^2, m_{K^+}^2, m_{H_1}^2)} & {
m form \ factor} \end{split}$$



$$|\kappa_{cb}|\simeq 6.7 imes 10^{-6} ~~ \mathcal{K}(a,b,c) = a^2 + b^2 + c^2 - 2(ab + bc + ca)$$



#### Weakly Interacting Massive Particle (WIMP) DM



Weakly Interacting Massive Particle (WIMP) DM
Assumptions for WIMP DM (2 to n annihilations)

- $\mu_{\rm DM} = \mu_{\overline{\rm DM}} \ [ \mu_{\rm DM} \neq \mu_{\overline{\rm DM}} \Rightarrow {\rm asymmetric DM} \ ]$ & K. M. Zurek (2009)
- $m_{\rm DM} > m_{\rm SM}$  [  $m_{\rm DM} < m_{\rm SM} \Rightarrow$  forbidden DM ] & J. T. Ruderman (2015)
- $T_{\rm FO} < T_{\rm RH}$  [ $T_{\rm FO} > T_{\rm RH} \Rightarrow$  WIMPs during reheating ] Nicolás Bernal & Yong Xu (2022)
- Standard cosmology [  $ho_{\phi}(a) \propto a^{-(4+n)} \Rightarrow$  relentless DM ]
- Collisionless [  $\sigma_{\rm SI} \neq 0 \implies$  Self-interacting dark matter ]
- T invariance :  $|\mathcal{M}_{\text{DMDM} \rightarrow \text{SMSM}}|^2 = |\mathcal{M}_{\text{SMSM} \rightarrow \text{DMDM}}|^2$  (?)

#### WIMP dark matter direct detections

Null result of direct detections has cornered WIMP scenario



#### WIMP dark matter direct detections

Null result of direct detections has cornered WIMP scenario



#### Possible explanations

#### (1) DM-nucleon cross-section is below the neutrino floor

Theor. : pseudo-Nambu-Goldstone boson DM Exp. : directional DM search, e.g. Cygnus

- (2) DM only has gravitational interaction
- (3) DM mass is outside the current direct detection search range Low/Heavy mass DM?

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#### **Direct detections of light dark matter**



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#### Projected detections of light dark matter



#### Projected detections of light dark matter

- DM-electron interaction
  - DM scatters off electrons :  $E_{
    m dep.}^{
    m scat} \sim m_{
    m DM} v_{
    m DM}^2 \sim 10^{-6} m_{
    m DM}$
  - DM is absorbed by electrons :  $E^{
    m abs}_{
    m dep.} \sim m_{
    m DM} ~~ v_{
    m \tiny DM} \sim 10^{-3}$ c
- List of the potential material and their sensitivities

Material	Sensitivitiy	Dark matter mass
Superconductor	$\gtrsim$ meV	$\gtrsim$ keV (scattering) , $\gtrsim$ meV (absorption)
Superconducting nanowire	$\gtrsim { m eV}$	$\gtrsim$ MeV (scattering) , $\gtrsim$ eV (absorption)
2D material (e.g. graphene)	$\gtrsim { m eV}$	$\gtrsim$ MeV (scattering)
3D material (e.g. $ZrTe_5$ )	$\gtrsim$ meV	$\gtrsim$ keV (scattering) , $\gtrsim$ meV (absorption)
Semiconductor (e.g. Ge, Si)	$\gtrsim { m eV}$	$\gtrsim$ MeV (scattering)

#### Light particle emission from celestial bodies

Light (DM) particles can be produced copiously from some and hot dense celestial objects such as supernovae (SNe), neutron stars, and white dwarfs.



#### Probing light dark matter by B physics

Real singlet  $Z_2$  scalar dark matter model Bird et al, PRL 2004

 $-\mathcal{L}=rac{1}{4}\lambda_SS^4+rac{1}{2}m_S^2S^2+\lambda v_{\mathsf{EW}}hS^2+rac{1}{2}\lambda h^2S^2$ 



#### Probing light dark matter by B physics

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#### **CMB constraint on light DM mass**

- DM annihilation continues to take place after decoupling & cause significant effects on cosmology and astrophysics.
- Energy released per DM annihilation  $E_{\mathsf{DM}}pprox 2m_{\mathsf{DM}}$

$$\frac{\mathrm{d}E}{\mathrm{d}t\,\mathrm{d}V}\Big|_{\mathrm{inj.}}(z) = n_{\mathrm{DM}}^2(z)\langle\sigma v\rangle \big(2m_{\mathrm{DM}}\big) = \rho_{\mathrm{c}}^2\Omega_{\mathrm{DM,0}}^2(1+z)^6\bigg(\frac{\langle\sigma v\rangle}{m_{\mathrm{DM}}}\bigg)$$

$$n_{\rm DM}(z) = \rho_{\rm c} \Omega_{\rm DM}(z)/m_{\rm DM} = \rho_{\rm c} \Omega_{\rm DM,0} (1+z)^3/m_{\rm DM}$$

$$\begin{array}{l} \textbf{Planck} \implies \\ \langle \sigma v \rangle \leqslant \frac{4.1 \times 10^{-28} \, \mathrm{cm^3 \, sec^{-1}}}{f_{\mathrm{eff}}} \left( \frac{m_{\mathrm{DM}}}{\mathrm{GeV}} \right) \end{array}$$