Generalized CP Symmetries in Three-Higgs-Doublet Models

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Introduction

GCP transformations and the 2HDM

- Generalized CP (GCP) transformations combine unitary transformations with the usual charge-parity conjugation

GCP Transformation

$$\Phi_a \to X_{ab}\Phi_b^*$$
$$X \in SU(n_H)$$

2HDM case

- Three classes of scalar potentials
- Extending to Yukawa sector, only two models are consistent with non zero quark masses

Scalar Potential

Imposing GCP symmetry

- $V_H = Y_{ij}(\Phi_i^\dagger \Phi_j) + Z_{ij,kl}(\Phi_i^\dagger \Phi_j)(\Phi_k^\dagger \Phi_l)$

GCP Transformation

$$\Phi_a \to X_{ab}\Phi_b^*$$
$$X \in SU(n_H)$$

Symmetry conditions

$$Y_{ab}^* = X_{\alpha a}^* Y_{\alpha\beta} X_{\beta b} = (X^\dagger \, Y \, X)_{ab}$$

$$Z_{ab,cd}^* = X_{\alpha a}^* X_{\gamma c}^* Z_{\alpha\beta,\gamma\delta} X_{\beta b} X_{\delta d} \, .$$

- X_θ can always be brought to a basis where $X_\theta = \begin{bmatrix} c_\theta & s_\theta & 0 \\ -s_\theta & c_\theta & 0 \\ 0 & 0 & 1 \end{bmatrix} \equiv R_\theta \oplus 1 \; , \quad 0 \leq \theta \leq \pi/2$

[Ecker, Grimus, Neufeld]

Scalar Potential

Brief Mathematical Detour

- $$\begin{pmatrix} a \\ b \end{pmatrix}^* = R_\theta^\top \begin{pmatrix} a \\ b \end{pmatrix} \iff \begin{cases} \theta = 2k\pi\,, & a,b \in \mathbb{R} \\ \theta = (2k+1)\pi\,, & a,b \in \mathbb{I} \\ \text{else}\,, & a,b = 0 \end{cases}$$

- $$A^* = R_{\theta_1}^\top \otimes R_{\theta_2}^\top \otimes \cdots \otimes R_{\theta_n}^\top A$$

$$(C_n A)^* = R_{\omega_0^n} \oplus R_{\omega_1^n} \oplus \cdots \oplus R_{\omega_{2^n-1}^n-1} (C_n A)$$

$$C_n \bigotimes_{q=1}^{n} R_{\theta_q} C_n^\top = \bigoplus_{p=1}^{2^{n-1}} R_{\omega_p^n}$$

$R_{\omega_0^n}$ → linear combination of θ_1

Scalar Potential

Brief Mathematical Detour

- Example:

$$\begin{pmatrix} a \\ b \\ c \\ d \end{pmatrix}^* = \underbrace{\begin{pmatrix} c_\alpha c_\beta & -c_\alpha s_\beta & -c_\beta s_\alpha & s_\alpha s_\beta \\ c_\alpha s_\beta & c_\alpha c_\beta & -s_\alpha s_\beta & -c_\beta s_\alpha \\ c_\beta s_\alpha & -s_\alpha s_\beta & c_\alpha s_\beta & -c_\alpha c_\beta \\ s_\alpha s_\beta & c_\beta s_\alpha & c_\alpha s_\beta & c_\alpha c_\beta \end{pmatrix}}_{R_\alpha^\top \otimes R_\beta^\top} \begin{pmatrix} a \\ b \\ c \\ d \end{pmatrix}$$

$$C_n \bigotimes_{q=1}^{n} R_{\theta_q} C_n^\top = \bigoplus_{p=1}^{2^{n-1}} R_{\omega_p^n}$$

$$\underbrace{\begin{pmatrix} a-d \\ b+c \\ a+d \\ b-c \end{pmatrix}^*}_{C_2 A} = \begin{pmatrix} R_{\alpha+\beta}^\top & O \\ O & R_{\beta-\alpha}^\top \end{pmatrix} \begin{pmatrix} a-d \\ b+c \\ a+d \\ b-c \end{pmatrix}$$

! Turns $2^n \times 2^n$ system into 2^{n-1} 2×2 systems

Scalar Potential

Imposing GCP symmetry

- Splitting the potentials parameters into relevant blocks, we can solve the symmetry conditions

e.g. $Y^*_{\{m3\}} = R_\theta^\top Y_{\{m3\}} \longrightarrow \theta = 0$

$$Z^*_{\{ij33\}} = R_\theta^{\top\otimes 2} Z_{\{ij33\}} \longrightarrow \theta = 0\,, \quad \theta = \pi/2$$

$$Z^*_{\{i3kl\}} = R_\theta^{\top\otimes 3} Z_{\{i3kl\}} \longrightarrow \theta = 0\,, \quad \theta = \pi/3.$$

- We conclude that there are **4** regions of interest:

[Ivanov, Varzielas]

CPa	$\theta = 0$	······	CP2
CPb	$\theta = \pi/2$	······	CP4
CPc	$\theta = \pi/3$	······	$S_3 \times GCP_{\theta=\pi}$
CPd	$\theta \neq 0, \pi/2, \pi/3$	······	$O(2) \times CP$

Yukawa Sector

Imposing GCP symmetry

- $-\mathcal{L}_Y = \bar{q}_L \Gamma_i \Phi_i n_R + \bar{q}_L \Delta_i \tilde{\Phi}_i p_R + \text{H.c.}$

similar for up case

Symmetry conditions

$$X_\alpha \Gamma_1^* - (c_\theta \Gamma_1 - s_\theta \Gamma_2) X_\beta = 0$$

$$X_\alpha \Gamma_2^* - (s_\theta \Gamma_1 + c_\theta \Gamma_2) X_\beta = 0$$

$$X_\alpha \Gamma_3^* - \Gamma_3 X_\beta = 0$$

GCP Transformations

$$q_L \to X_\alpha \gamma^0 C q_L^*$$

$$n_R \to X_\beta \gamma^0 C n_R^*$$

$$p_R \to X_\gamma \gamma^0 C p_R^*$$

- We will allow for general vevs, mapping out all possibilities

Yukawa Sector

Imposing GCP symmetry

- Using the mathematical result we previously introduced, the symmetry conditions yield

$$\Gamma^*_{\{3\}} = [\overbrace{(R_\alpha^\top \otimes R_\beta^\top)}^{4} \oplus \overbrace{(R_\alpha^\top)}^{2} \oplus \overbrace{(R_\beta^\top)}^{2} \oplus 1]\, \Gamma_{\{3\}}$$

15 + 4 + 4 + 4 + 2 + 2 + 2
= 33 conditions

$$\Gamma^*_{\{1,2\}} = [\underbrace{(R_\theta^\top \otimes R_\alpha^\top \otimes R_\beta^\top)}_{\text{15 conditions}} \oplus \underbrace{(R_\theta^\top \otimes R_\alpha^\top)}_{4} \oplus \underbrace{(R_\theta^\top \otimes R_\beta^\top)}_{4} \oplus \underbrace{R_\theta^\top}_{2}]\, \Gamma_{\{1,2\}}$$

e.g.

$$\theta + \alpha = \beta, \quad \beta \neq \{\theta, \alpha, \pi/2\}$$

or

$$\theta + \alpha + \beta = \pi, \quad \theta, \alpha, \beta \neq \pi/2$$

Yukawa Sector

Imposing GCP symmetry

- Combining these conditions and excluding combinations leading to null or degenerate masses, we obtain **27** cases

- Including the up sector and then excluding combinations leading to a block diagonal CKM matrix, we obtain **51** cases

excluding models with a CP conserving CKM matrix, i.e. $J = \mathrm{Tr}\left[H_u, H_d\right]^3 \propto J_{CP} = 0$

40 models

3HDM with GCP Symmetry

Example

Scalar potential

CPd $\theta \neq 0, \pi/2, \pi/3$

- $\{\theta, \alpha, \beta, \gamma\} = \left\{\frac{\pi}{5}, \frac{2\pi}{5}, \frac{\pi}{5}, \frac{2\pi}{5}\right\}$:

$$\Delta_1 = \begin{pmatrix} -ia & ib & 0 \\ ib & ia & 0 \\ 0 & 0 & 0 \end{pmatrix} \quad \Delta_2 = \begin{pmatrix} ib & ia & 0 \\ ia & -ib & 0 \\ 0 & 0 & 0 \end{pmatrix} \quad \Delta_3 = \begin{pmatrix} c & d & 0 \\ -d & c & 0 \\ 0 & 0 & e \end{pmatrix}$$

→ 5 real parameters

$$\Gamma_1 = \begin{pmatrix} f & g & 0 \\ -g & f & 0 \\ l & m & 0 \end{pmatrix} \quad \Gamma_2 = \begin{pmatrix} g & -f & 0 \\ f & g & 0 \\ -m & l & 0 \end{pmatrix} \quad \Gamma_3 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & n \end{pmatrix}$$

→ 5 real parameters

Final Remarks

- We found 4 classes of 3HDM GCP symmetric potentials, just one more than in the 2HDM, and were able to identify them as $\mathrm{CP2},\ \mathrm{CP4},\ S_3 \times GCP_{\theta=\pi},\ O(2) \times CP$

- We found that there are **40** different possible implementations of GCP symmetry in the Yukawa sector

  contrast with the 2HDM case, where only 2 scenarios exist

- 8 of these models entail only 10 parameters, which may be more easily tested against experimental constraints