Flavor-nondiagonal neutral Higgs Yukawa couplings revisited

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Introduction

In the Standard Model (SM), the Higgs-fermion Yukawa coupling matrices are proportional to the corresponding diagonal fermion mass matrices.

- This is a very good feature of the SM, since experimental data reveals that flavor-changing neutral currents (FCNC) are highly suppressed.

The absence of tree-level Higgs-mediated FCNCs is not a generic feature of extended Higgs sectors.

- For example, it is standard practice introduce a symmetry of the two-Higgs doublet model (2HDM) Lagrangian to provide a natural explanation for the absence of tree-level Higgs-mediated FCNCs.

Cheng and Sher[1] advocated for a mechanism that replicated the hierarchies of the quark masses and CKM angles in the structure of the Yukawa matrices.

- As a consequence of the Cheng-Sher ansatz, the tree-level off-diagonal neutral Higgs–fermion couplings are suppressed (but not set to zero).[2]

It is sometimes asserted that the Cheng-Sher ansatz is no longer viable in light of the most recent collider data. Joseph M. Connell and I have revisited the Cheng-Sher ansatz in the context of the basis-independent approach to the 2HDM[3] (and a recent update of the Fritzsch textures for the quark mass matrices).

[1] T.P. Cheng and M. Sher, Phys. Rev. D **35**, 3484 (1987)

[2] The detailed phenomenology of this proposal was further investigated in a series of papers by J.L. Díaz-Cruz, R. Noriega-Papaqui, and A. Rosado, Phys. Rev. D **69**, 095002 (2004); **71**, 015014 (2005) [with follow up works by M.A. Arroyo-Ureña, J.L. Díaz-Cruz and collaborators), and in a series of papers by M. Gómez-Bock and collaborators. Additional works by J. Hernández-Sánchez, S. Moretti, and collaborators are also noteworthy.

[3] H.E. Haber and D. O'Neil, Phys. Rev. D **74**, 015018 (2006); **D83**, 055017 (2011).

Lightning review of the 2HDM

In a general 2HDM, there are only two meaningful choices for the basis of Higgs doublet fields:

- charged Higgs basis fields $\mathcal{H}_1$ and $\mathcal{H}_2$ such that $\langle\mathcal{H}_1^0\rangle = v/\sqrt{2}$, where $v \equiv (\sqrt{2}G_F)^{-1/2} \simeq 246$ GeV, and $\langle\mathcal{H}_2^0\rangle = 0$

- mass basis for the neutral Higgs bosons

Physical neutral scalars: h_1, h_2 and h_3 obtained by diagonalizing the neutral scalar squared-mass matrix

$$R\mathcal{M}^2 R^{\mathsf{T}} = \mathrm{diag}\,(m_1^2\,,\, m_2^2\,,\, m_3^2)\,,$$

where $R \equiv R_{12}R_{13}R_{23}$ is the product of three rotation matrices parametrized by θ_{12}, θ_{13} and θ_{23}, respectively.

The physical neutral mass-eigenstate scalar fields are

$$h_k = q_{k1}\left(\sqrt{2}\,\mathrm{Re}\,\mathcal{H}_1^0 - v\right) + \frac{1}{\sqrt{2}}\left(q_{k2}^*\mathcal{H}_2^0 e^{i\theta_{23}} + \mathrm{h.c.}\right),$$

where the q_{k1} and q_{k2} are exhibited in table below (where $s_{ij} \equiv \sin\theta_{ij}$ and $c_{ij} \equiv \cos\theta_{ij}$).

k	q_{k1}	q_{k2}
1	$c_{12}c_{13}$	$-s_{12} - ic_{12}s_{13}$
2	$s_{12}c_{13}$	$c_{12} - is_{12}s_{13}$
3	s_{13}	ic_{13}

Without loss of generality, one can set $\theta_{23} = 0$ by rephasing $\mathcal{H}_2^0$.

2HDM Yukawa couplings

In the Higgs basis,

$$-\mathscr{L}_Y = \sum_{i,m,m} \Big\{ (\widehat{\boldsymbol{\kappa}}^{\boldsymbol{U}})_{mn} \mathcal{H}_1^{0\dagger}\, \widehat{\overline{u}}_{mL} \widehat{u}_{nR} + (\widehat{\boldsymbol{\rho}}^{\boldsymbol{U}})_{mn} \mathcal{H}_2^{0\dagger}\, \widehat{\overline{u}}_{mL} \widehat{u}_{nR} + \text{h.c.} \Big\}$$
$$+ \Big\{ (\widehat{\boldsymbol{\kappa}}^{\boldsymbol{D}\,\dagger})_{mn} \mathcal{H}_1^{0}\, \widehat{\overline{d}}_{mL} \widehat{d}_{nR} + (\widehat{\boldsymbol{\rho}}^{\boldsymbol{D}\,\dagger})_{mn} \mathcal{H}_2^{0}\, \widehat{\overline{d}}_{mL} \widehat{d}_{nR} + \text{h.c.} \Big\}$$
$$+ \Big\{ (\widehat{\boldsymbol{\kappa}}^{\boldsymbol{E}\,\dagger})_{mn} \mathcal{H}_1^{0}\, \widehat{\overline{e}}_{mL} \widehat{e}_{nR} + (\widehat{\boldsymbol{\rho}}^{\boldsymbol{E}\,\dagger})_{mn} \mathcal{H}_2^{0}\, \widehat{\overline{e}}_{mL} \widehat{e}_{nR} \text{h.c.} \Big\},$$

where $f_R \equiv \frac{1}{2}(1+\gamma_5)f$ and $f_L \equiv \frac{1}{2}(1-\gamma_5)f$ [with four-component fermion fields $f = u, d, \nu, e$]. The hatted fields correspond to the fermion interaction-eigenstate fields. Setting $\mathcal{H}_1^0 = \mathcal{H}_1^{0\,\dagger} = v/\sqrt{2}$ yields the fermion mass matrices

$$(\widehat{\boldsymbol{M}}_{\boldsymbol{U}})_{mn} = \frac{v}{\sqrt{2}} (\widehat{\boldsymbol{\kappa}}^{\boldsymbol{U}})_{mn}\,, \qquad (\widehat{\boldsymbol{M}}_{\boldsymbol{D,E}})_{mn} = \frac{v}{\sqrt{2}} (\widehat{\boldsymbol{\kappa}}^{\boldsymbol{D,E}\,\dagger})_{mn}\,.$$

The singular value decompositions of $\widehat{\boldsymbol{M}}_U$ and $\widehat{\boldsymbol{M}}_D$ yield:

$$L_u^\dagger \, \widehat{\boldsymbol{M}}_{\boldsymbol{U}} \, R_u \equiv \boldsymbol{M}_{\boldsymbol{U}} \,, \qquad L_d^\dagger \, \widehat{\boldsymbol{M}}_{\boldsymbol{D}} \, R_d \equiv \boldsymbol{M}_{\boldsymbol{D}}$$

where M_U and M_D are diagonal up- and down-type quark mass matrices with real and nonnegative diagonal elements, and the unitary matrices L_f and R_f $(f = u, d)$ relate hatted interaction-eigenstate fermion fields with unhatted mass-eigenstate fields,

$$\widehat{f}_{mL} = (L_f)_{mn} f_{nL} \,, \qquad \widehat{f}_{mR} = (R_f)_{mn} f_{nR} \,.$$

The Cabibbo-Kobayashi-Maskawa (CKM) matrix is $\boldsymbol{K} \equiv L_u^\dagger L_d$.

The physical ρ-type Yukawa couplings are complex matrices that yield off-diagonal neutral Higgs–fermion interactions,

$$\boldsymbol{\rho}^{\boldsymbol{U}} \equiv L_u^\dagger \, \widehat{\boldsymbol{\rho}}^{\boldsymbol{U}} R_u \,, \qquad \boldsymbol{\rho}^{\boldsymbol{D}\dagger} \equiv L_d^\dagger \, \widehat{\boldsymbol{\rho}}^{\boldsymbol{D}\dagger} R_d \,.$$

The corresponding Higgs–fermion interactions involving mass-eigenstate scalar and fermion fields are given by

$$
\begin{aligned}
-\mathscr{L}_Y = \overline{U}\Big\{\frac{\boldsymbol{M_U}}{v}q_{k1} + \frac{1}{\sqrt{2}}\Big[q^*_{k2}\,\boldsymbol{\rho^U}P_R + q_{k2}\,\boldsymbol{\rho^{U\dagger}}P_L\Big]\Big\}Uh_k \\
+\overline{D}\Big\{\frac{\boldsymbol{M_D}}{v}q_{k1} + \frac{1}{\sqrt{2}}\Big[q_{k2}\,\boldsymbol{\rho^{D\dagger}}P_R + q^*_{k2}\,\boldsymbol{\rho^D}P_L\Big]\Big\}Dh_k \\
+\overline{E}\Big\{\frac{\boldsymbol{M_E}}{v}q_{k1} + \frac{1}{\sqrt{2}}\Big[q_{k2}\,\boldsymbol{\rho^{E\dagger}}P_R + q^*_{k2}\,\boldsymbol{\rho^E}P_L\Big]\Big\}Eh_k \\
+\Big\{\overline{U}\Big[\boldsymbol{K}\boldsymbol{\rho^{D\dagger}}P_R - \boldsymbol{\rho^{U\dagger}}\boldsymbol{K}P_L\Big]DH^+ + \overline{N}\boldsymbol{\rho^{E\dagger}}P_R E H^+ + \text{h.c.}\Big\},
\end{aligned}
$$

where $P_R \equiv \frac{1}{2}(1+\gamma_5)$, $P_L \equiv \frac{1}{2}(1-\gamma_5)$, and the mass-eigenstate fields of the down-type quarks, the up-type quarks, the charged leptons and the neutrinos are $D = (d,s,b)^{\mathsf{T}}$, $U \equiv (u,c,t)^{\mathsf{T}}$, $E = (e,\mu,\tau)^{\mathsf{T}}$, and $N = (\nu_e,\nu_\mu,\nu_\tau)^{\mathsf{T}}$, respectively.

Specializing to the CP-conserving 2HDM

Assume that the only source of CP-violation is the unremovable phase of the CKM matrix. Then, there exists a real Higgs basis, in which the all scalar potential parameters and the ρ^F are real matrices. For example,

$$\mathcal{V} \supset \tfrac{1}{2}Z_5 e^{-2i\eta}(\mathcal{H}_1^\dagger \mathcal{H}_2)^2 + \left[Z_6 e^{-i\eta}\mathcal{H}_1^\dagger \mathcal{H}_1 + Z_7 e^{-i\eta}\mathcal{H}_2^\dagger \mathcal{H}_2\right]\mathcal{H}_1^\dagger \mathcal{H}_2 + \text{h.c.}$$

where η can be chosen such that $Z_{5,6,7}$ are real. If $Z_6 \neq 0$ and/or $Z_7 \neq 0$, then the real Higgs basis is unique up to an overall sign, $\mathcal{H}_2 \to -\mathcal{H}_2$. For example,

$$\varepsilon \equiv \operatorname{sgn} Z_6\,,$$

changes sign when $\mathcal{H}_2 \to -\mathcal{H}_2$.

The physical neutral Higgs bosons consist of two CP-even scalars h and H (with $m_h < m_H$) and a CP-odd scalar A, which are related to the neutral fields of the Higgs basis via

$$\begin{pmatrix} H \\ h \end{pmatrix} = \begin{pmatrix} c_{\beta-\alpha} & -s_{\beta-\alpha} \\ s_{\beta-\alpha} & c_{\beta-\alpha} \end{pmatrix} \begin{pmatrix} \sqrt{2}\,\mathrm{Re}\,\mathcal{H}_1^0 - v \\ \varepsilon\sqrt{2}\,\mathrm{Re}\,\mathcal{H}_2^0 \end{pmatrix}, \qquad A = \varepsilon\sqrt{2}\,\mathrm{Re}\,\mathcal{H}_2^0\,,$$

and[4] $c_{\beta-\alpha} = -\varepsilon|Z_6|v^2/\sqrt{(m_H^2 - m_h^2)(m_H^2 - Z_1 v^2)}$.

h is SM-like when $\lvert c_{\beta-\alpha}\rvert \ll 1$ $0 \leq \beta - \alpha \leq \pi$			
k	h_k	q_{k1}	q_{k2}
1	h	$s_{\beta-\alpha}$	$\varepsilon\, c_{\beta-\alpha}$
2	$-\varepsilon\, H$	$-\varepsilon\, c_{\beta-\alpha}$	$s_{\beta-\alpha}$
3	$\varepsilon\, A$	0	i

If $|c_{\beta-\alpha}| \ll 1$ then h is SM-like (the so-called Higgs alignment limit).

[4] Z_1 is the coefficient of $\frac{1}{2}(\mathcal{H}_1^\dagger\mathcal{H}_1)^2$ in the scalar potential.

CP-conserving neutral Higgs-fermion Yukawa couplings:

$$
\begin{aligned}
-\mathscr{L}_Y = \overline{U}\biggl\{ &\Bigl[\frac{\boldsymbol{M_U}}{v}s_{\beta-\alpha} + \frac{1}{\sqrt{2}}\varepsilon c_{\beta-\alpha}\Bigl(\boldsymbol{\rho}^{\boldsymbol{U}}P_R + [\boldsymbol{\rho}^{\boldsymbol{U}}]^{\mathsf{T}}P_L\Bigr)\Bigr]h \\
&+\varepsilon\Bigl[\frac{\boldsymbol{M_U}}{v}\varepsilon c_{\beta-\alpha} - \frac{1}{\sqrt{2}}s_{\beta-\alpha}\Bigl(\boldsymbol{\rho}^{\boldsymbol{U}}P_R + [\boldsymbol{\rho}^{\boldsymbol{U}}]^{\mathsf{T}}P_L\Bigr)\Bigr]H - \frac{i}{\sqrt{2}}\varepsilon\Bigl(\boldsymbol{\rho}^{\boldsymbol{U}}P_R - [\boldsymbol{\rho}^{\boldsymbol{U}}]^{\mathsf{T}}P_L\Bigr)A\biggr\}U \\
+\sum_{F=D,E}\overline{F}\biggl\{ &\Bigl[\frac{\boldsymbol{M_F}}{v}s_{\beta-\alpha} + \frac{1}{\sqrt{2}}\varepsilon c_{\beta-\alpha}\Bigl([\boldsymbol{\rho}^{\boldsymbol{F}}]^{\mathsf{T}}P_R + \boldsymbol{\rho}^{\boldsymbol{F}}P_L\Bigr)\Bigr]h \\
&+\varepsilon\Bigl[\frac{\boldsymbol{M_F}}{v}\varepsilon c_{\beta-\alpha} - \frac{1}{\sqrt{2}}s_{\beta-\alpha}\Bigl([\boldsymbol{\rho}^{\boldsymbol{F}}]^{\mathsf{T}}P_R + \boldsymbol{\rho}^{\boldsymbol{F}}P_L\Bigr)\Bigr]H + \frac{i}{\sqrt{2}}\varepsilon\Bigl([\boldsymbol{\rho}^{\boldsymbol{F}}]^{\mathsf{T}}P_R - \boldsymbol{\rho}^{\boldsymbol{F}}P_L\Bigr)A\biggr\}F\,.
\end{aligned}
$$

Note that: (i) $\varepsilon c_{\beta-\alpha} = -|c_{\beta-\alpha}|$; (ii) no separate $\tan\beta$ dependence.

<u>Remark</u>: In the Higgs alignment limit, where $c_{\beta-\alpha} \to 0$, the Yukawa couplings of h coincide with those of the SM (and are hence flavor-diagonal). In contrast, H and A generically possess flavor-nondiagonal couplings in the Higgs alignment limit.

Flavor textures

A long-standing program initiated by H. Fritzsch[5] provides a phenomenological explanation of the quark mixing hierarchy based on a correlation with the quark mass hierarchy. Du and Xing subsequently proposed that[6]

$$\widehat{\boldsymbol{M}}_{\boldsymbol{F}} = \begin{pmatrix} 0 & C_F & 0 \\ C_F^* & \widetilde{B}_F & B_F \\ 0 & B_F^* & A_F \end{pmatrix}, \qquad F = U, D.$$

where A_F, $\widetilde{B}_F \in \mathbb{R}$ (with no loss of generality, one can take $A_F > 0$). In particular, by choosing $A_F \gg |B_F|$, $|\widetilde{B}_F|$, C_F, one can reproduce the hierarchy of quark masses and CKM angles.

[5] H. Fritzsch, "Calculating the Cabibbo angle," Phys. Lett. B **70** (1977) 436.

[6] D.-s. Du and Z.-z. Xing, Phys. Rev. D **48**, 2349 (1993).

This proposed form is called the four-zero texture of hermitian quark mass matrices, since there are a total of four independent zeros[7] in $\widetilde{M}_F$ for $F = U, D$.[8] A previous proposal in which $\widetilde{B}_F = 0$ (the six-zero texture) is no longer consistent with data.

Writing $B_F = |B_F| e^{i\phi_{B_F}}$ and defining $C_F = |C_F| e^{i\phi_{C_F}}$ and $P_F \equiv \mathrm{diag}\Big(1\,,\, e^{-i\phi_{C_F}}\,,\, e^{-i\left(\phi_{B_F}+\phi_{C_F}\right)}\Big)$, it is convenient to define:

$$\overline{\boldsymbol{M}}_{\boldsymbol{F}} = P_F^\dagger\,\widehat{\boldsymbol{M}}_{\boldsymbol{F}}\,P_F = \begin{pmatrix} 0 & |C_F| & 0 \\ |C_F| & \widetilde{B}_F & |B_F| \\ 0 & |B_F| & A_F \end{pmatrix} .$$

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[7]Due to the assumption of hermiticity, a pair of off-diagonal zeros is counted as one texture zero.

[8]The assertion that $\widehat{\boldsymbol{M}}_{\boldsymbol{U}}$ and $\widehat{\boldsymbol{M}}_{\boldsymbol{D}}$ are hermitian matrices with $(\widehat{\boldsymbol{M}}_{\boldsymbol{U}})_{11} = (\widehat{\boldsymbol{M}}_{\boldsymbol{D}})_{11} = (\widehat{\boldsymbol{M}}_{\boldsymbol{D}})_{13} = 0$ does not require an extra set of assumptions, since these conditions can always be achieved by an appropriately chosen weak-basis transformation [e.g., see G.C. Branco, D. Emmanuel-Costa, and R. González Felipe, Phys. Lett. B **477** (2000) 147 and **670** (2009) 340 (with H. Serôdio)]. The additional constraint of $(\widehat{\boldsymbol{M}}_{\boldsymbol{U}})_{13} = 0$ is chosen to provide a good fit to the CKM matrix elements as a function of the quark masses.

Since $\overline{\boldsymbol{M}_{\boldsymbol{F}}}$ is a real symmetric matrix, its eigenvalues (denoted by λ_i^F) are real numbers, denoted by λ_i^F $(i = 1, 2, 3)$

$$\lambda^3 - \lambda^2(A_F + \widetilde{B}_F) - \lambda(|C_F|^2 + |B_F|^2 - \widetilde{B}_F A_F) + |C_F|^2 A_F$$
$$= (\lambda - \lambda_1^F)(\lambda - \lambda_2^F)(\lambda - \lambda_3^F)\,.$$

in a convention where $|\lambda_1^F| < |\lambda_2^F| < |\lambda_3^F|$. The λ_i^F are related to the coefficients of the characteristic equation above,

$$\widetilde{B}_F = \lambda_1^F + \lambda_2^F + \lambda_3^F - A_F\,,$$

$$|B_F| = \sqrt{\frac{(A_F - \lambda_1^F)(A_F - \lambda_2^F)(\lambda_3^F - A_F)}{A_F}}\,,$$

$$|C_F| = \sqrt{\frac{-\lambda_1^F \lambda_2^F \lambda_3^F}{A_F}}\,.$$

Under the assumption that $A_F \gg |B_F|$, $|\widetilde{B}_F|$, C_F,

$$\lambda^F_{1,2} \simeq \frac{1}{2}\left[\widetilde{B}_F - \frac{|B_F|^2}{A_F} \pm \sqrt{\left(\widetilde{B}_F - \frac{|B_F|^2}{A_F}\right)^2 + 4|C_F|^2}\,\right]\,, \qquad \lambda^F_3 \simeq A_F + \frac{|B_F|^2}{A_F}\,,$$

where the maximal eigenvalue is denoted by λ^F_3 and terms of $\mathcal{O}(1/A_F^2)$ have been dropped. Since $A_F > 0$, it follows that $\lambda^F_1\lambda^F_2 < 0$ and $\lambda^F_3 > A_F$. It is convenient to adopt a convention where $|\lambda^F_1| < |\lambda^F_2| < \lambda^F_3$, with $\eta_F \equiv \operatorname{sgn}\lambda_2$. In particular,

$$\eta_F = \begin{cases} +1\,, & \text{if } \lambda^F_1 < 0 \text{ and } \lambda^F_2 > 0 \quad \Longrightarrow \quad |B_F|^2 < A_F\widetilde{B}_F \\ -1\,, & \text{if } \lambda^F_1 > 0 \text{ and } \lambda^F_2 < 0 \quad \Longrightarrow \quad |B_F|^2 > A_F\widetilde{B}_F \end{cases}$$

We now introduce the matrix $H_F = \operatorname{diag}\big(-\eta_F\,,\, \eta_F\,,\, 1\big)$.

Hence,

$$Q_F^\mathsf{T} P_F^\dagger \widehat{\boldsymbol{M}_{\boldsymbol{F}}} P_F Q_F H_F = \mathrm{diag}\big(m_1^F\,,\, m_2^F\,,\, m_3^F\big)\,,$$

where $m_i^F \equiv (-\eta_F\lambda_1^F\,,\,\eta_F\lambda_2^F\,,\,\lambda_3^F)$ and

$$Q_F = \begin{pmatrix} \sqrt{\frac{\lambda_2^F\lambda_3^F\left(A_F-\lambda_1^F\right)}{A_F\left(\lambda_2^F-\lambda_1^F\right)\left(\lambda_3^F-\lambda_1^F\right)}} & \eta_F\sqrt{\frac{\lambda_1^F\lambda_3^F\left(\lambda_2^F-A_F\right)}{A_F\left(\lambda_2^F-\lambda_1^F\right)\left(\lambda_3^F-\lambda_2^F\right)}} & \sqrt{\frac{\lambda_1^F\lambda_2^F\left(A_F-\lambda_3^F\right)}{A_F\left(\lambda_3^F-\lambda_1^F\right)\left(\lambda_3^F-\lambda_2^F\right)}} \\ -\eta_F\sqrt{\frac{\lambda_1^F\left(\lambda_1^F-A_F\right)}{\left(\lambda_2^F-\lambda_1^F\right)\left(\lambda_3^F-\lambda_1^F\right)}} & \sqrt{\frac{\lambda_2^F\left(A_F-\lambda_2^F\right)}{\left(\lambda_2^F-\lambda_1^F\right)\left(\lambda_3^F-\lambda_2^F\right)}} & \sqrt{\frac{\lambda_3^F\left(\lambda_3^F-A_F\right)}{\left(\lambda_3^F-\lambda_1^F\right)\left(\lambda_3^F-\lambda_2^F\right)}} \\ \eta_F\sqrt{\frac{\lambda_1^F\left(A_F-\lambda_2^F\right)\left(A_F-\lambda_3^F\right)}{A_F\left(\lambda_2^F-\lambda_1^F\right)\left(\lambda_3^F-\lambda_1^F\right)}} & -\sqrt{\frac{\lambda_2^F\left(A_F-\lambda_1^F\right)\left(\lambda_3^F-A_F\right)}{A_F\left(\lambda_2^F-\lambda_1^F\right)\left(\lambda_3^F-\lambda_2^F\right)}} & \sqrt{\frac{\lambda_3^F\left(A_F-\lambda_1^F\right)\left(A_F-\lambda_2^F\right)}{A_F\left(\lambda_3^F-\lambda_1^F\right)\left(\lambda_3^F-\lambda_2^F\right)}} \end{pmatrix}.$$

That is, the singular value decomposition of the quark mass matrices can be achieved using

$$L_f = P_F Q_F\,, \qquad R_f = L_f H_F\,, \qquad \text{for } f = u, d.$$

A detailed analysis by H. Fritzsch, Z.-z. Xing and D. Zhang, Nucl. Phys. B **974** (2022) 115634, yields a very good fit to the CKM mixing angles and CP-violating phase by setting $A_U = A_D$. For example, in the case of $\eta_U = \eta_D = 1$, Fritzsch et al. obtain:

$$\widehat{\boldsymbol{M}}_{\boldsymbol{U}} \simeq m_t \begin{pmatrix} 0 & 0.00018 & 0 \\ 0.00018 & 0.18924 & 0.38787 \\ 0 & 0.38787 & 0.81444 \end{pmatrix},$$

$$\widehat{\boldsymbol{M}}_{\boldsymbol{D}} \simeq m_b \begin{pmatrix} 0 & 0.00465 & 0 \\ 0.00465 & 0.20335 & 0.38448 \\ 0 & 0.38448 & 0.81444 \end{pmatrix}.$$

with $\arg C_U - \arg C_D = 0.53216\pi$ and $\arg B_U - \arg B_D = 1.0313\pi$.

We shall extend the ansatz of Fritzsch et al. by setting

$$A_E = A_U = A_D\,.$$

An ansatz for the flavor structure of $\widehat{\boldsymbol{\rho}}^{\boldsymbol{F}}$

The ρ-type Yukawa coupling matrices in the fermion mass-eigenstate basis are given by

$$\boldsymbol{\rho}^{\boldsymbol{F}} = Q_F^\mathsf{T} P_F^\dagger \widehat{\boldsymbol{\rho}}^{\boldsymbol{F}} P_F Q_F H_F \,.$$

For simplicity we take $\boldsymbol{\rho}^{\boldsymbol{F}}$ by adopting the following ansatz:

$$P_F^\dagger \widehat{\boldsymbol{\rho}}^{\boldsymbol{F}} P_F = \frac{\sqrt{2}}{v} \begin{pmatrix} 0 & c_F|C_F| & 0 \\ c_F|C_F| & \widetilde{b}_F \widetilde{B}_F & b_F|B_F| \\ 0 & b_F|B_F| & a_F A_F \end{pmatrix} \,.$$

The a_F, b_F, $\widetilde{b}_F$ and c_F are real $\mathcal{O}(1)$ parameters (of either sign).

Note that if $a_F = b_F = \widetilde{b}_F = c_F$ then $\boldsymbol{\rho^F} = a_F \boldsymbol{\kappa^F}$, which corresponds to the flavor-aligned 2HDM. By taking a_F, b_F, $\widetilde{b}_F$ and c_F unequal, we inject the hierarchical structure of the fermion mass matrices into the $\boldsymbol{\rho^F}$, as originally proposed by T.P. Cheng and M. Sher.[9]

We assume that $A \sim \mathcal{O}(m_3)$ and $m_1 \ll m_2 \ll m_3$ (dropping the superscript F for convenience). To obtain accurate approximate expressions for the ρ_{ij}, the size of $m_3 - A$ is critical. Suppose that $m_3 - A \sim \mathcal{O}(m_2)$. In this case, we can write

$$A = \overline{\alpha} m_3 \,, \qquad\qquad m_3 - A = \overline{\beta} m_2 \,,$$

where $\overline{\alpha}$ and $\overline{\beta}$ are positive $\mathcal{O}(1)$ parameters.[10]

[9] The original Cheng-Sher ansatz was based on the six-zero texture scheme where $\widetilde{B}_F = 0$.

[10] These parameters should not be confused with α and β, which appear in $c_{\beta-\alpha}$.

We then obtain:

$$\rho_{11} \simeq \frac{\sqrt{2}\,\eta\, m_1}{v}\left[\overline{\alpha}\overline{\beta}(2b-a-\widetilde{b})+\eta\big(2c-(1-\overline{\alpha})(2b-a)-\overline{\alpha}\widetilde{b}\big)\right],$$

$$\rho_{12}=-\rho_{21} \simeq \frac{\sqrt{2m_1m_2}}{v}\left[\overline{\alpha}\overline{\beta}(2b-a-\widetilde{b})+\eta\big(c-(1-\overline{\alpha})b-\overline{\alpha}\widetilde{b}\big)\right],$$

$$\rho_{13}=-\eta\rho_{31} \simeq \frac{\sqrt{2m_1m_3}\,\sqrt{\overline{\alpha}\overline{\beta}}}{v}\left[\overline{\alpha}(a-b)+(1-\overline{\alpha})(b-\widetilde{b})\right],$$

$$\rho_{22} \simeq \frac{\sqrt{2}\,\eta\, m_2}{v}\left[-\overline{\alpha}\overline{\beta}(2b-a-\widetilde{b})+\eta\big((2\overline{\alpha}-1)\widetilde{b}+2(1-\overline{\alpha})b\big)\right],$$

$$\rho_{23}=\eta\rho_{32} \simeq \frac{\sqrt{2m_2m_3}\,\sqrt{\overline{\alpha}\overline{\beta}}}{v}\left[\overline{\alpha}(b-a)-(1-\overline{\alpha})(b-\widetilde{b})\right],$$

$$\rho_{33} \simeq \frac{\sqrt{2}m_3}{v}\left[\overline{\alpha}^2a+2\overline{\alpha}(1-\overline{\alpha})b+(1-\overline{\alpha})^2\widetilde{b}\right],$$

where terms of $\mathcal{O}(m_1/m_{2,3})$ and $\mathcal{O}(m_2/m_3)$ have been dropped.[11]

[11] Note that in an approximation where $m_2 \ll m_3$ and $\overline{\beta} \sim \mathcal{O}(1)$, one can also drop all terms that are proportional to $1-\overline{\alpha}=\overline{\beta}m_2/m_3$.

In particular, taking $\overline{\beta} \sim \mathcal{O}(1)$ yields the Cheng-Sher ansatz

$$\rho_{ij} = k_{ij} \frac{\sqrt{m_i m_j}}{v}\,, \qquad \text{where } k_{ij} \sim \mathcal{O}(1).$$

However, in light of the analysis by Fritzsch et al. previously cited, $A_F/m_f = 0.81444$, which yields

$$\overline{\beta} \simeq 0.18556\, m_3/m_2\,.$$

Using $\overline{\mathrm{MS}}$ quark masses evaluated at m_Z and the lepton masses yield: $m_t/m_c \simeq 271$, $m_b/m_s \simeq 53.4$, and $m_\tau/m_\mu = 16.81$. Hence,

$$\overline{\beta}_U \simeq 50\,, \qquad \overline{\beta}_D \simeq 10\,, \qquad \overline{\beta}_E = 3.12\,.$$

That is, k_{11}, k_{12}, k_{21}, and k_{22} are enhanced by an $\mathcal{O}(\overline{\beta})$ factor, while k_{13}, k_{31}, k_{23}, and k_{32} are enhanced by an $\mathcal{O}(\overline{\beta}^{1/2})$ factor.

The (modified) Cheng-Sher ansatz in light of LHC Higgs data

In the absence of FCNC phenomena mediated by the scalars of the 2HDM, one can ascertain an upper limit for $|c_{\beta-\alpha}|$ and lower limits for the masses of H, A, and $H^\pm$, assuming the ansatz for the flavor structure of $\widehat{\rho}^F$ adopted above.

Preliminary results for our analysis are shown below, where we have fixed $m_h = 125$ GeV and $m_H \sim m_A \sim m_{H^\pm} \sim 800$ GeV.[12]

[12]With these masses, one-loop FCNC phenomena mediated by $H^\pm$ (such as $b \to s + \gamma$) yield only small corrections to the corresponding contributions mediated by $W^\pm$ and cannot be ruled out by current experimental data.

Constraints imposed on our parameter scans

We scan over the $\mathcal{O}(1)$ parameters that define the $\boldsymbol{\rho^F}$ and $|c_{\beta-\alpha}|$ [the latter determines the parameter $|Z_6|$) subject to the following constraints:

- The scalar potential is bounded from below.
- Tree-level unitarity and perturbativity.
- precision electroweak constraints on S, T, and U.
- precision LHC Higgs data (h BRs and cross sections)

Constraints are checked using the public codes `2HDMC` and `HiggsTools`. We exclude points with $\Delta\chi^2 \gtrsim 6$ as provided by `HiggsSignals` (corresponding to a 95% CL exclusion limit for the joint estimation of two parameters).

Experimental limits on lepton-flavor violating decays of the Higgs boson

CMS Collaboration, *Phys. Rev. D 104 (2021) 032013*

The observed (expected) upper limits on the branching fractions are, respectively, $B(H\to\mu\tau) < 0.15\ (0.15)\%$ and $B(H\to e\tau) < 0.22\ (0.16)\%$ at 95% confidence level.

CMS Collaboration, *Phys. Rev. D 108 (2023) 072004*

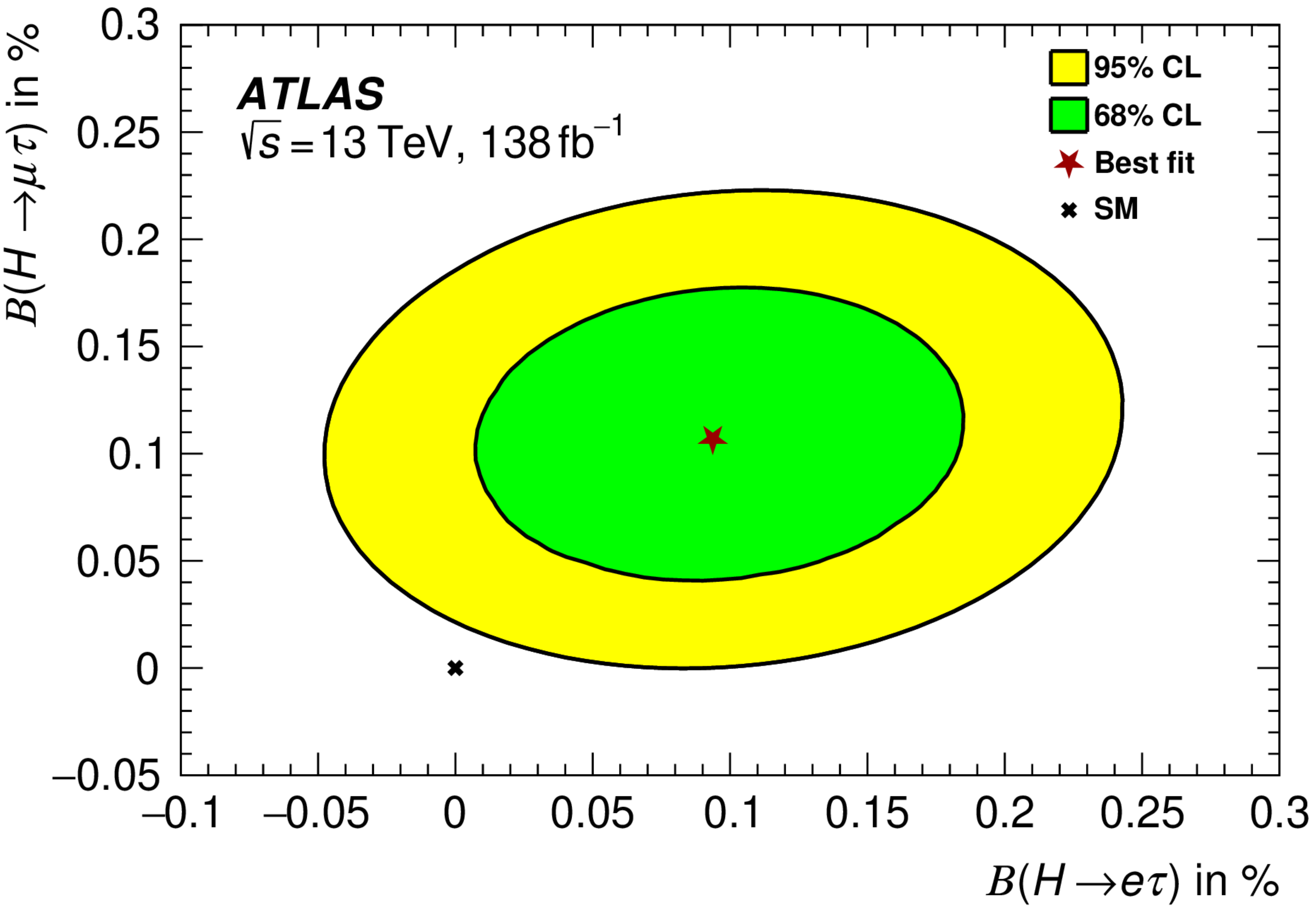
The observed (expected) upper limit on the $e^{\pm}\mu^{\mp}$ branching fraction for it is determined to be $4.4\ (4.7) \times 10^{-5}$ at 95% confidence level, the most stringent limit set thus far from direct searches. The largest excess of events over the expected background in the full mass range of the search is observed at an $e^{\pm}\mu^{\mp}$ invariant mass of approximately 146 GeV with a local (global) significance of 3.8 (2.8) standard deviations.

ATLAS Collaboration, **JHEP 07 (2023) 166**

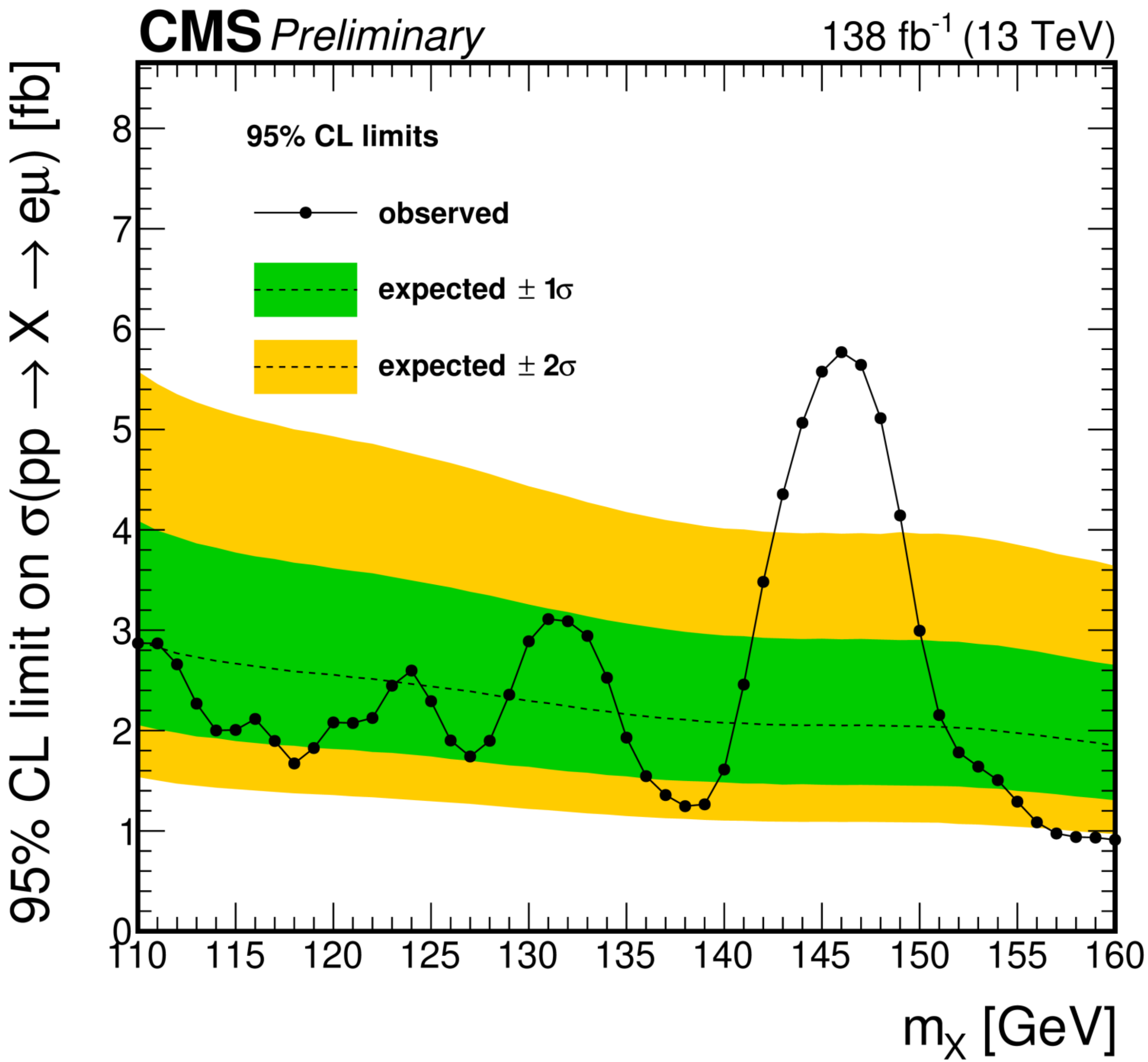
The observed (expected) upper limits set on the branching ratios at 95% confidence level, $B(H \to e\tau) < 0.20\%\ (0.12\%)$ and $B(H \to \mu\tau) < 0.18\%\ (0.09\%)$, are obtained with the MC-template method from a simultaneous measurement of potential $H \to e\tau$ and $H \to \mu\tau$ signals. The best-fit branching ratio difference, $B(H \to \mu\tau) - B(H \to e\tau)$, measured with the Symmetry method in the channel where the τ-lepton decays to leptons, is $(0.25 \pm 0.10)\%$, compatible with a value of zero within 2.5σ.

ATLAS Collaboration, **Phys. Lett. B 801 (2020) 135148**

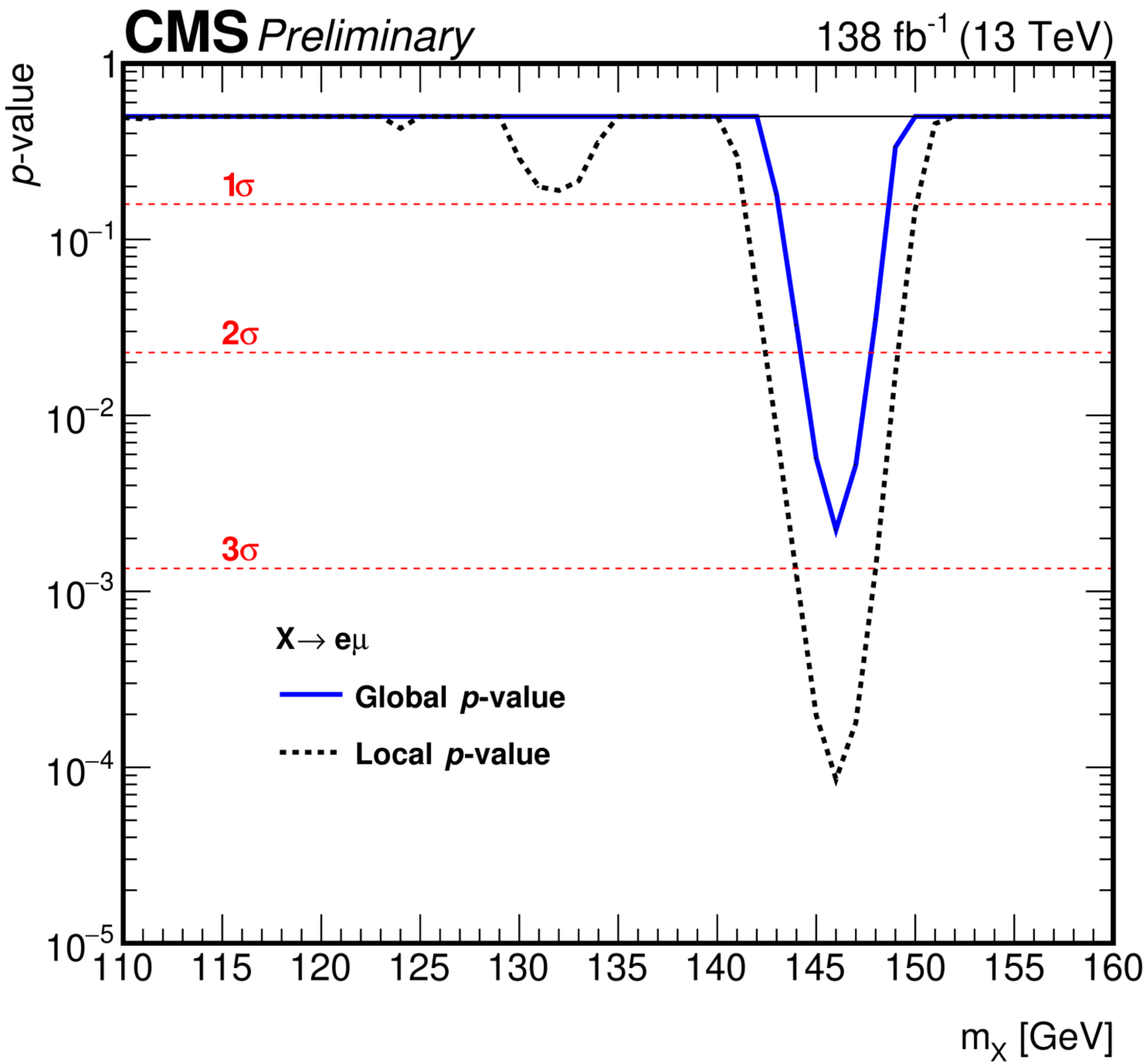
For a Higgs boson mass of 125 GeV, the observed (expected) upper limit at the 95% confidence level on the branching fraction $B(H \to e\mu)$ is 6.1×10^{-5} (5.8×10^{-5}). This results represent an improvement by a factor of about six on the previous best limit on $B(H \to e\mu)$.



ATLAS
$\sqrt{s} = 13$ TeV, 138 fb^{-1}
95% CL
68% CL
Best fit
SM
$B(H \to \mu\tau)$ in %
$B(H \to e\tau)$ in %
0.3
0.25
0.2
0.15
0.1
0.05
0
−0.05
−0.1



Taken from CMS Collaboration, *Phys. Rev. D 108 (2023) 072004*

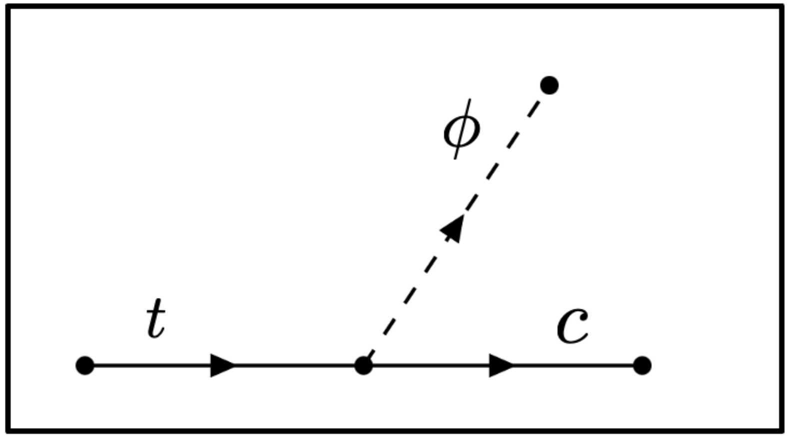


Taken from CMS Collaboration,
Phys. Rev. D 108 (2023) 072004

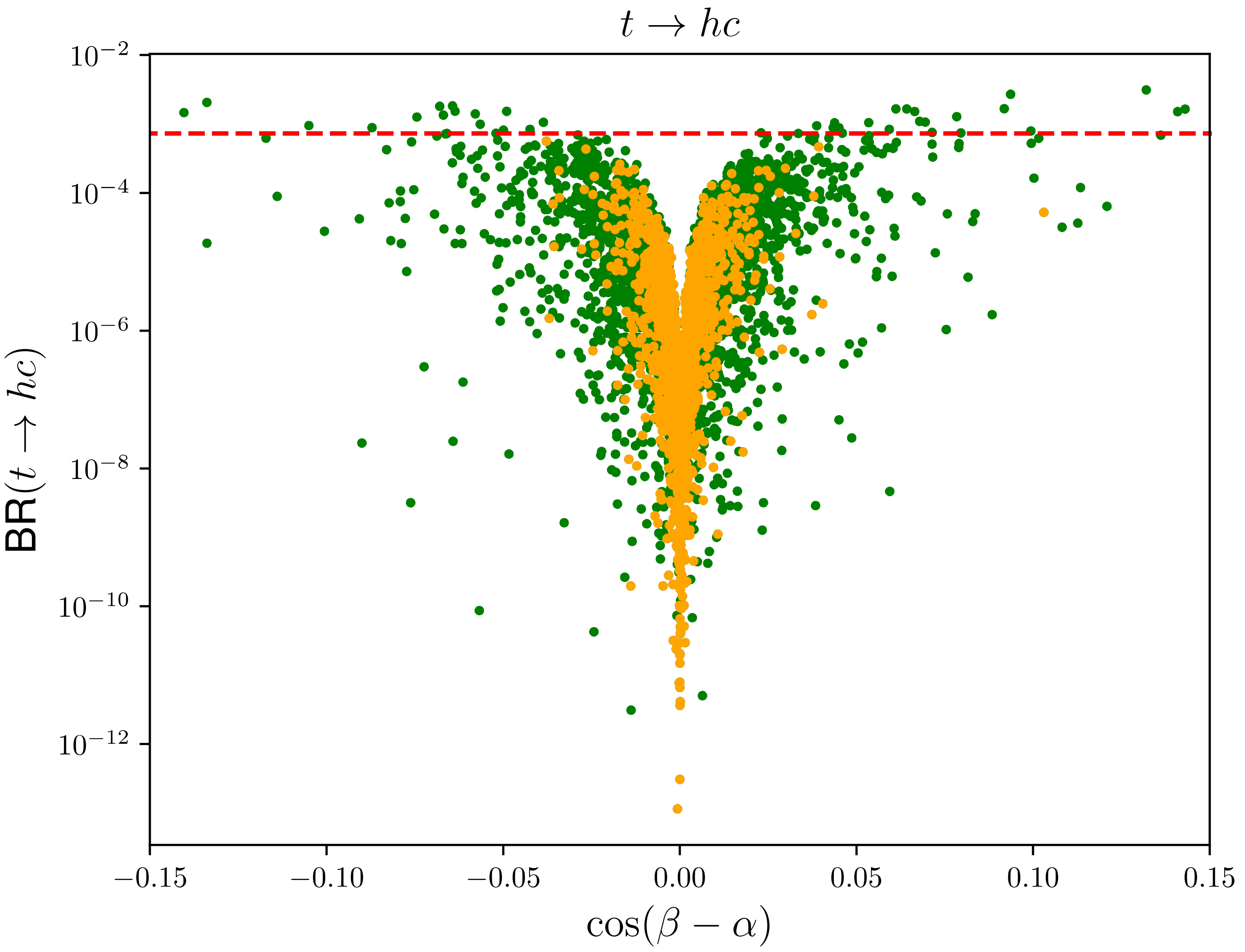
Flavor-changing processes mediated by neutral scalars

1. $t \to hc$
2. $h \to \tau^\pm \mu^\mp$, $h \to \tau^\pm e^\mp$, $h \to \mu^\pm e^\mp$
3. $\tau^\pm \to \mu^\pm \gamma$, $\tau^\pm \to e^\pm \gamma$, $\mu^\pm \to e^\pm \gamma$
4. $\tau^- \to \mu^-\mu^+\mu^-$, $\mu^-e^+e^-$, $e^-\mu^+\mu^-$, $\mu^- \to e^-e^+e^-$
5. K^0–$\bar{K}^0$ mixing
6. $P^0_{s,d}$–$\bar{P}_{s,d}$ mixing $(P = B,\ D)$
7. $B^0_{s,d} \to \mu^+\mu^-$, $\tau^+\tau^-$
8. $b \to s\mu^+\mu^-$, $s\tau^+\tau^-$
9. $\mu \to e$ conversion

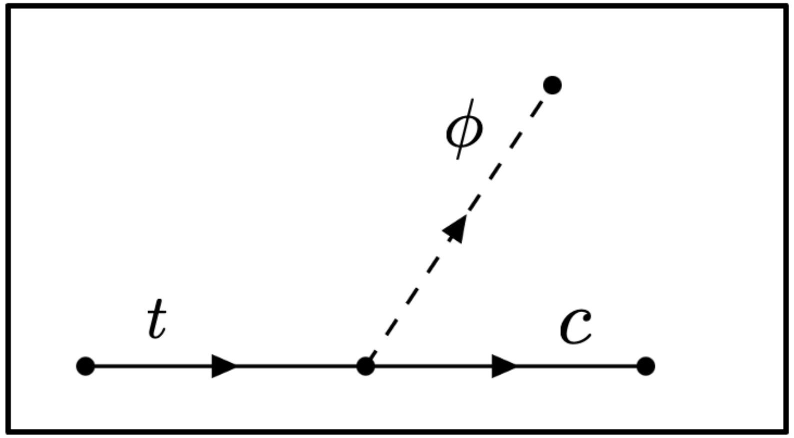
All plots shown below are preliminary.



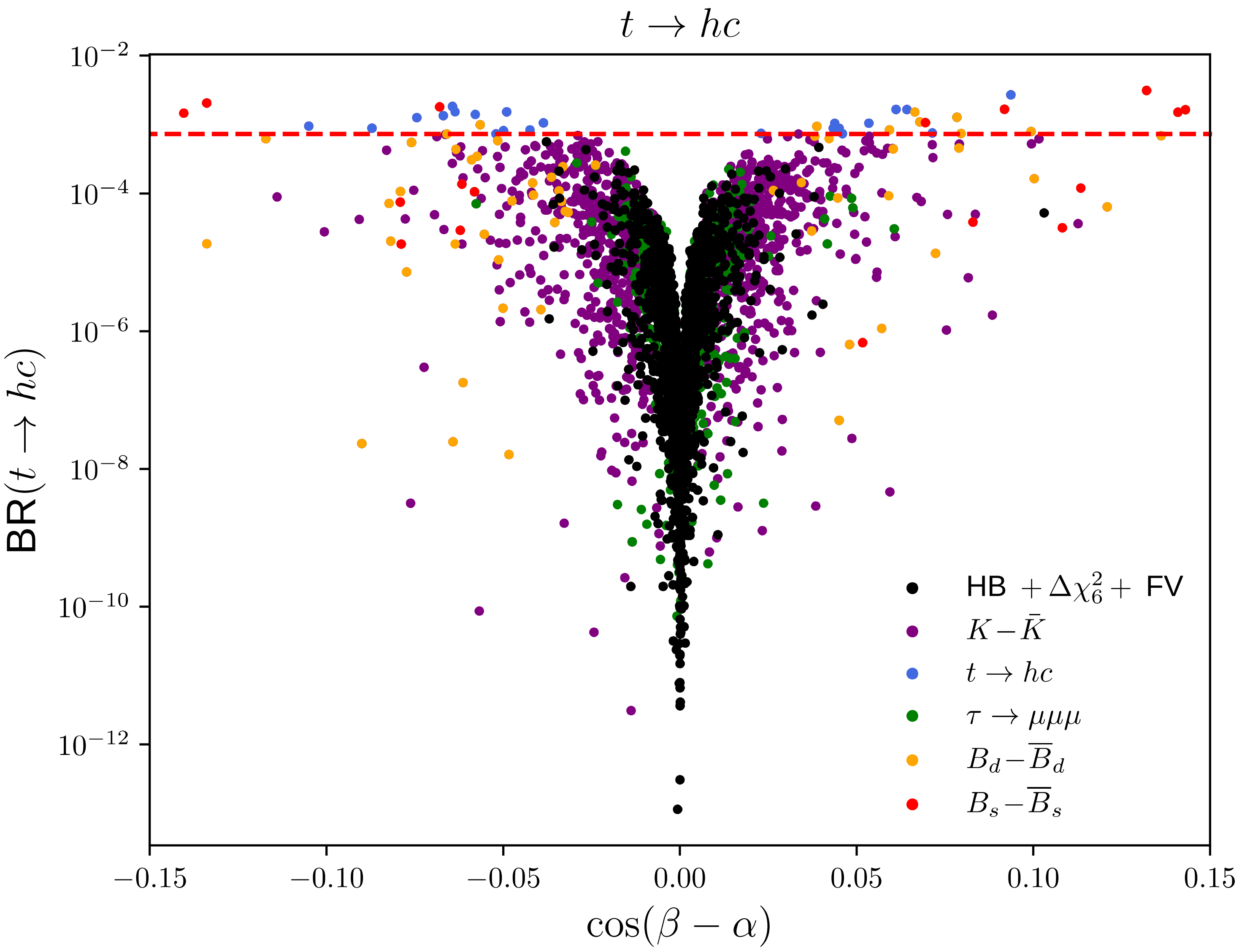
ϕ
t
c



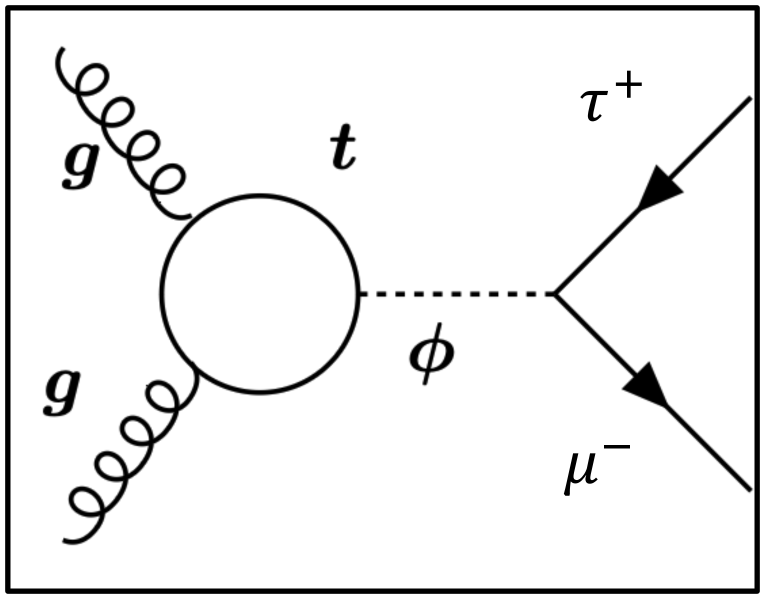
$t \to hc$
10^{-2}
10^{-4}
10^{-6}
10^{-8}
10^{-10}
10^{-12}
BR$(t \to hc)$
-0.15
-0.10
-0.05
0.00
0.05
0.10
0.15
$\cos(\beta - \alpha)$



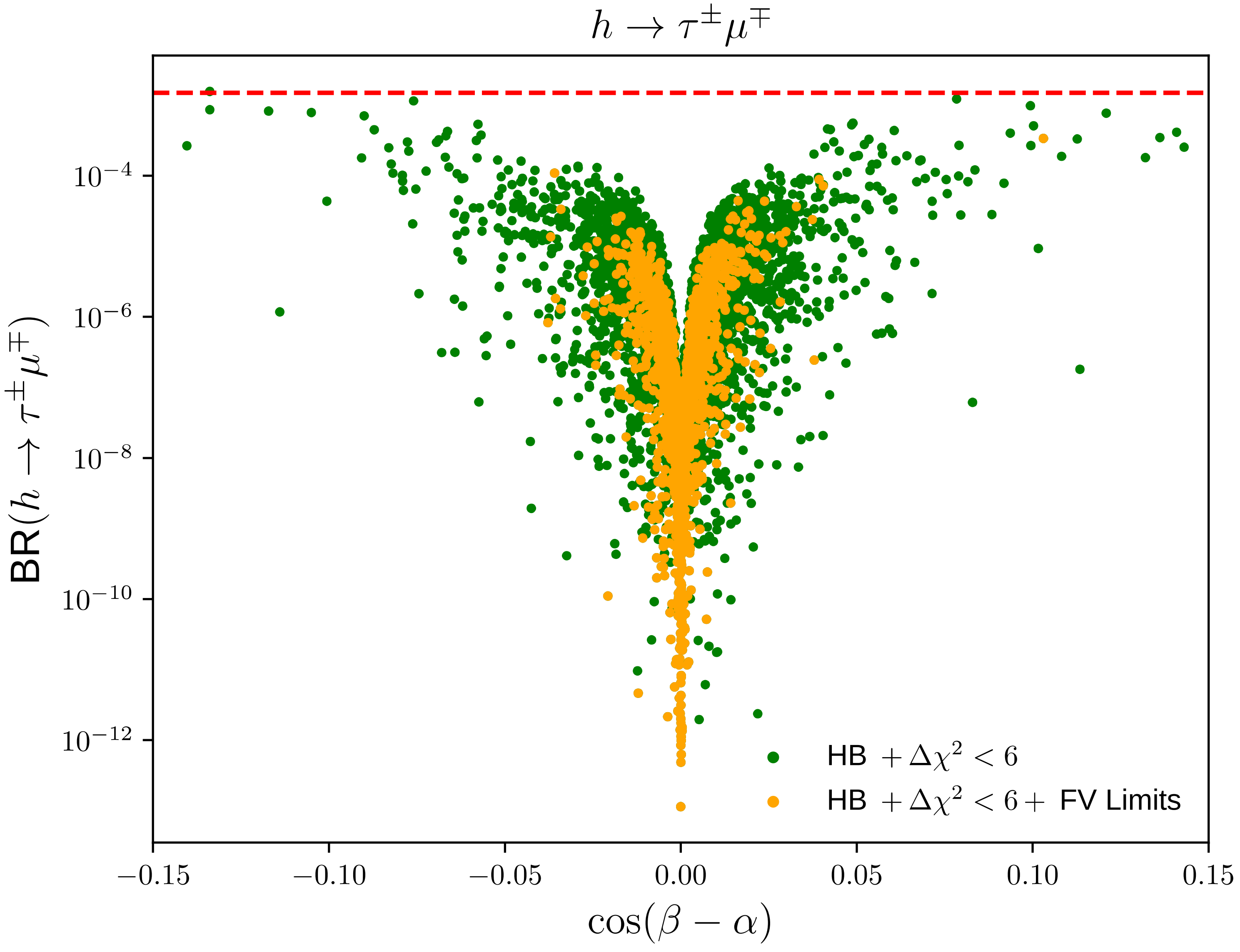
ϕ
t
c



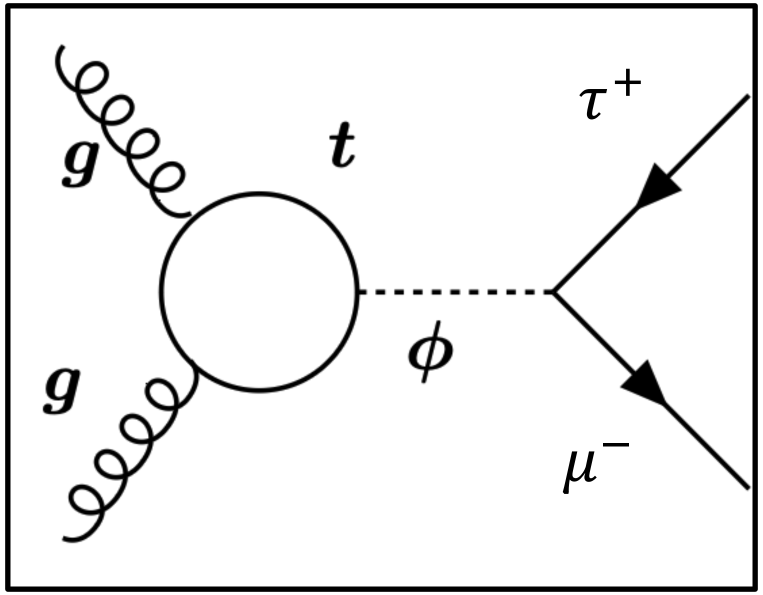
$t \to hc$
BR$(t \to hc)$
$\cos(\beta - \alpha)$
10^{-2}
10^{-4}
10^{-6}
10^{-8}
10^{-10}
10^{-12}
-0.15
-0.10
-0.05
0.00
0.05
0.10
0.15
HB $+\Delta\chi^2_6+$ FV
$K-\bar{K}$
$t \to hc$
$\tau \to \mu\mu\mu$
$B_d-\overline{B}_d$
$B_s-\overline{B}_s$



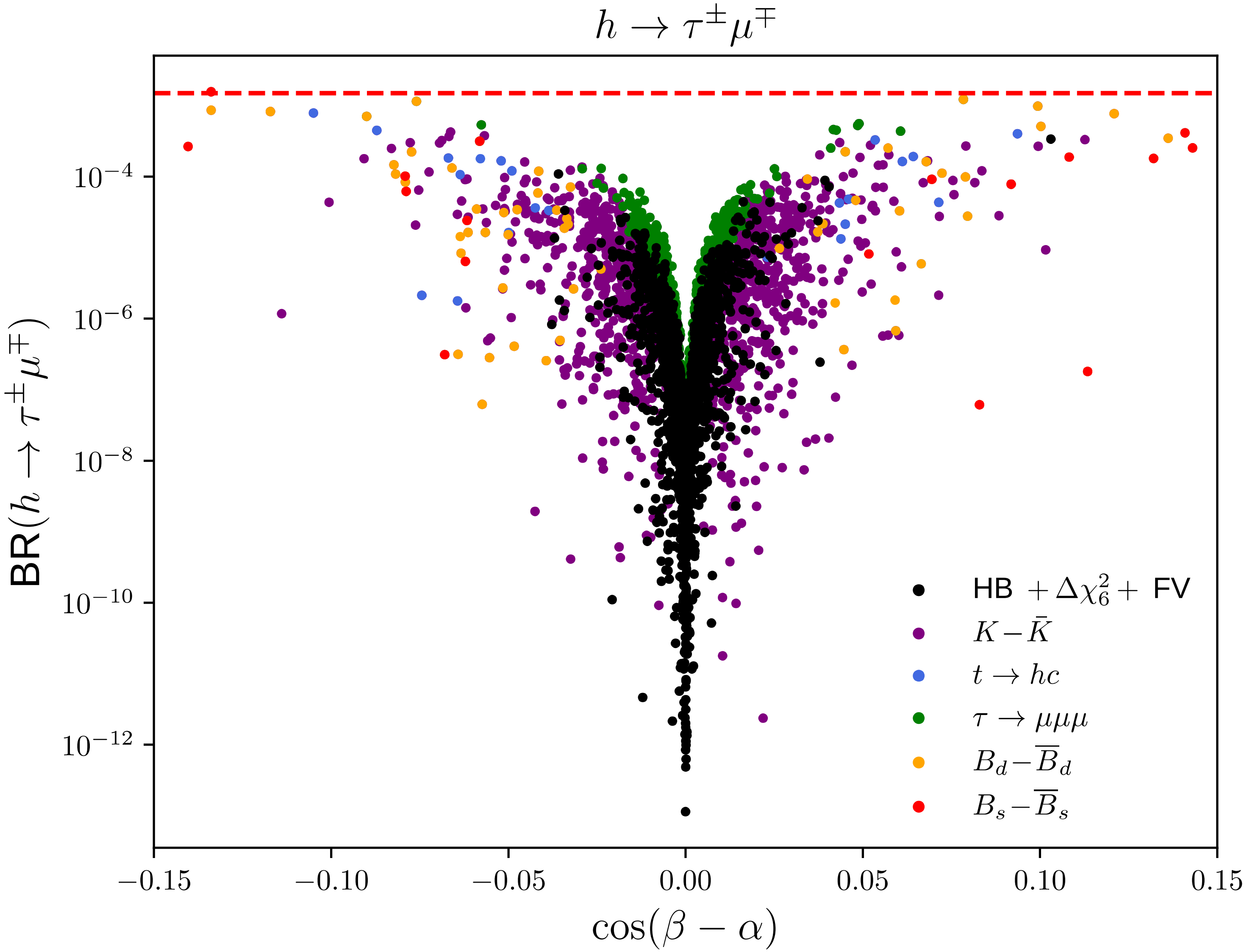
g
g
t
ϕ
τ^+
μ^-



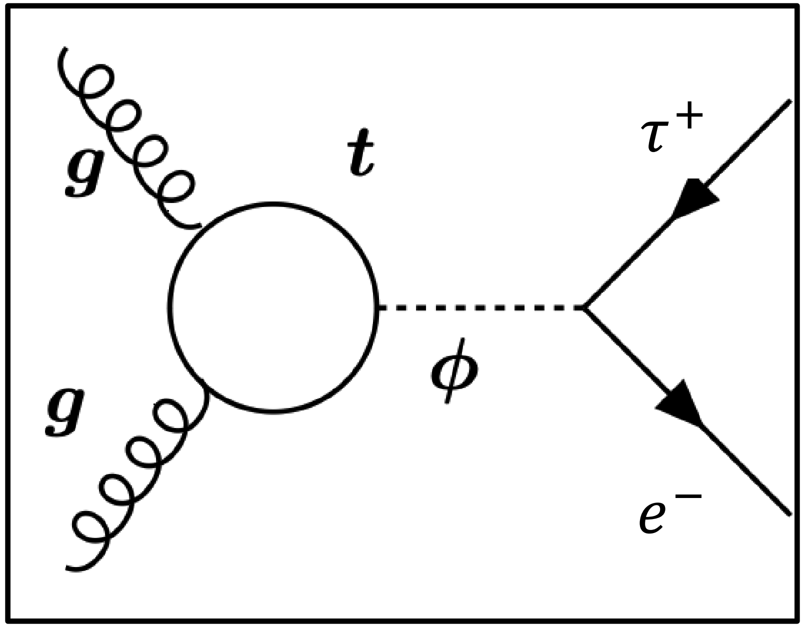
$h \to \tau^\pm \mu^\mp$
BR$(h \to \tau^\pm \mu^\mp)$
$\cos(\beta - \alpha)$
10^{-4}
10^{-6}
10^{-8}
10^{-10}
10^{-12}
-0.15
-0.10
-0.05
0.00
0.05
0.10
0.15
HB $+\Delta\chi^2 < 6$
HB $+\Delta\chi^2 < 6 +$ FV Limits

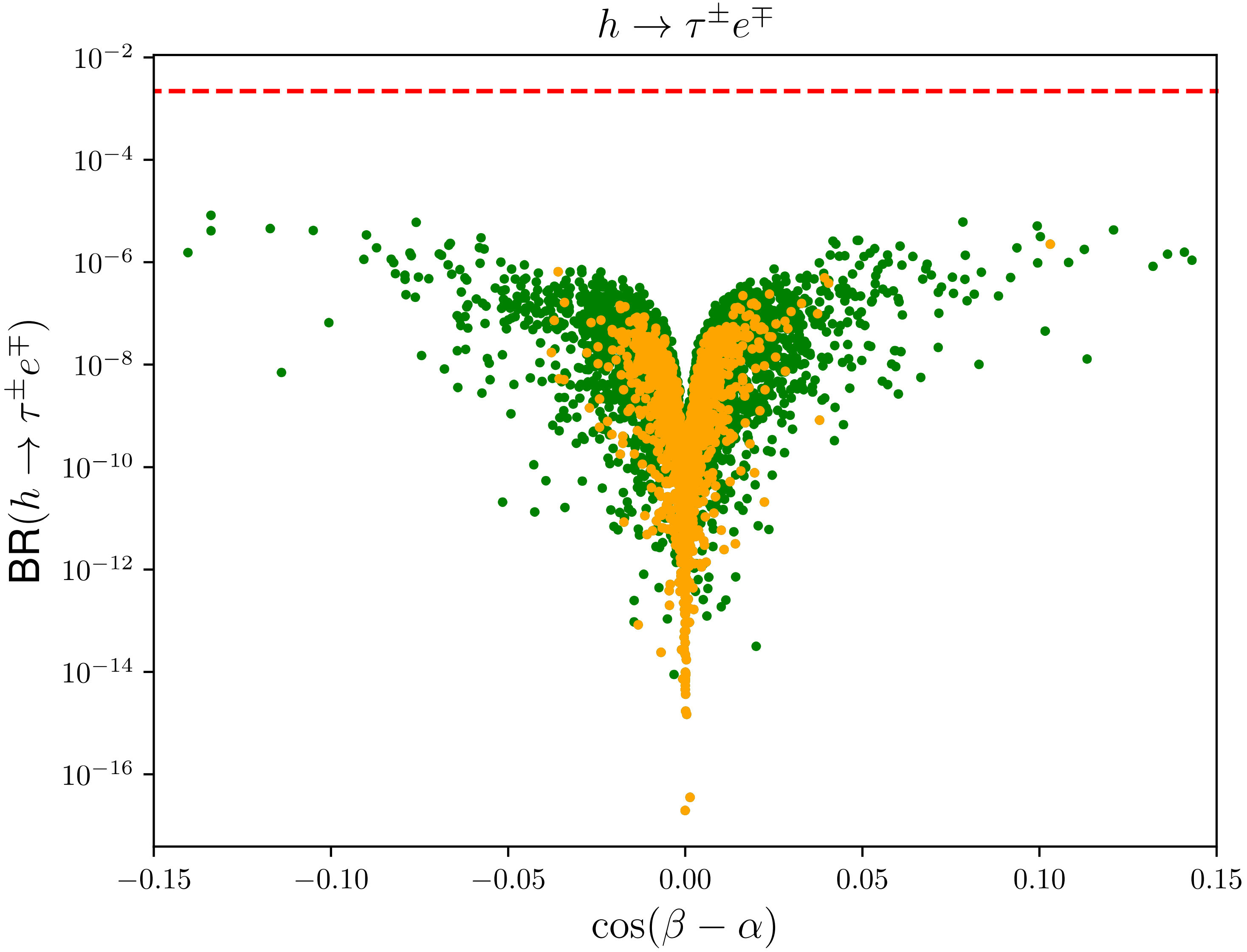


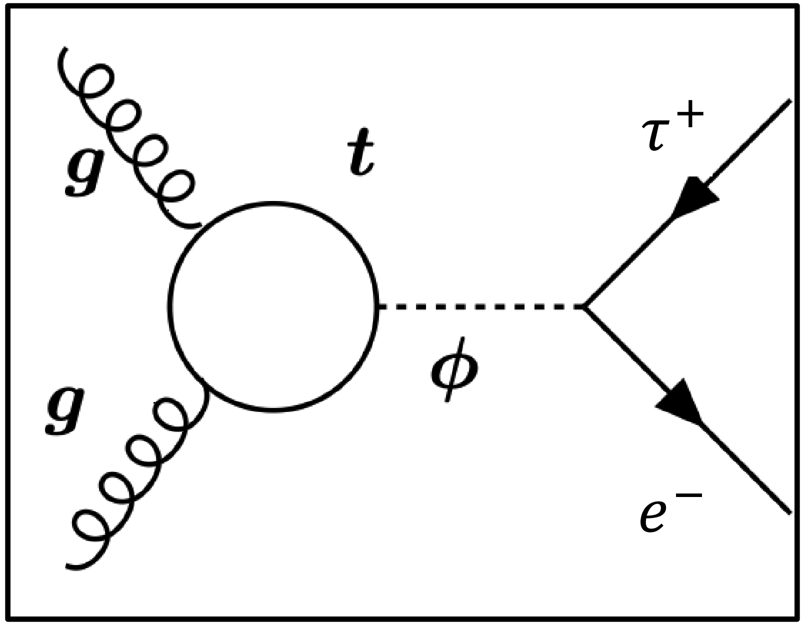
g
t
τ^+
g
ϕ
μ^-



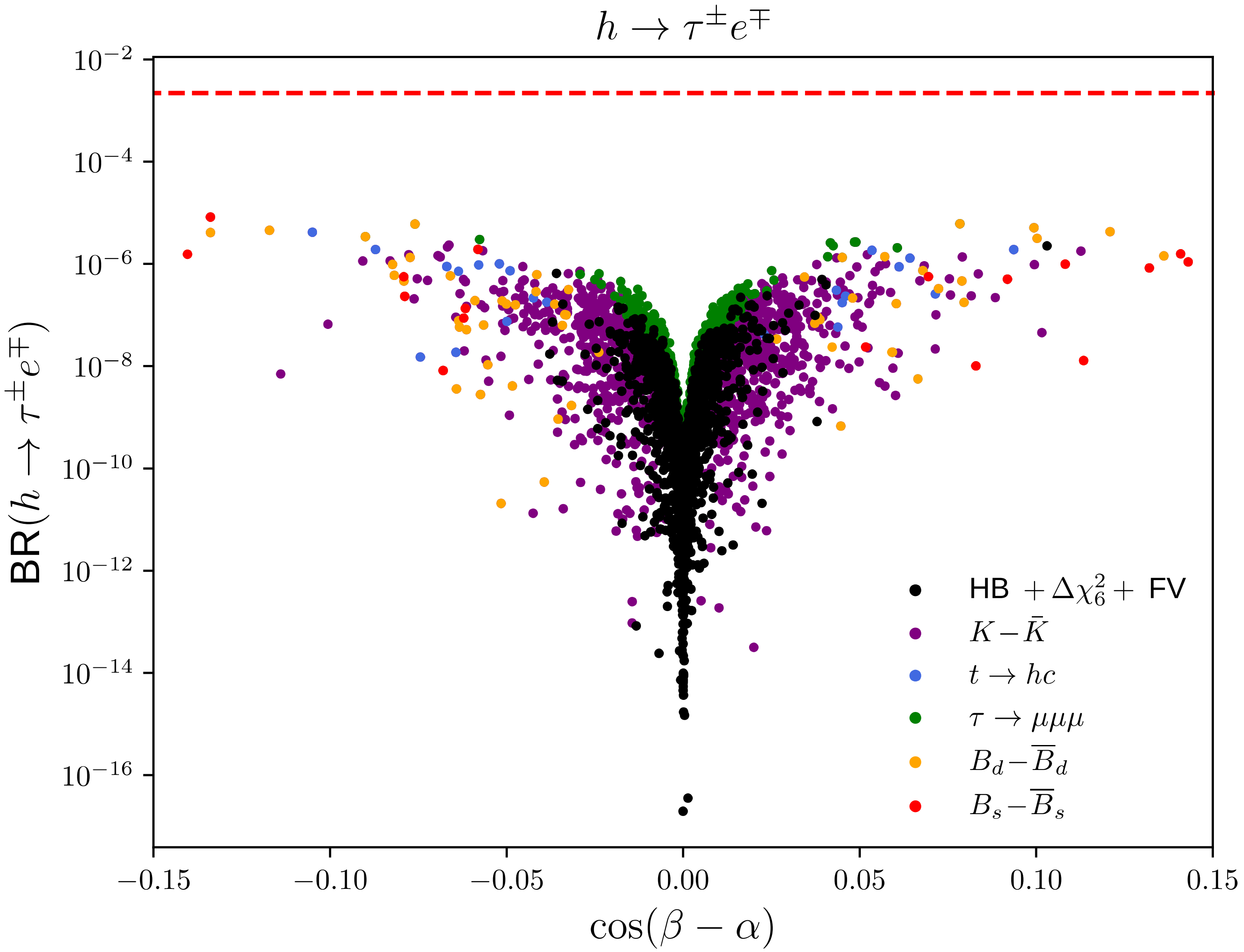
$h \to \tau^{\pm}\mu^{\mp}$
$\mathrm{BR}(h \to \tau^{\pm}\mu^{\mp})$
10^{-4}
10^{-6}
10^{-8}
10^{-10}
10^{-12}
-0.15
-0.10
-0.05
0.00
0.05
0.10
0.15
$\cos(\beta - \alpha)$
HB $+ \Delta\chi^2_6 +$ FV
$K - \bar{K}$
$t \to hc$
$\tau \to \mu\mu\mu$
$B_d - \overline{B}_d$
$B_s - \overline{B}_s$


g
t
τ^+
ϕ
g
e^-

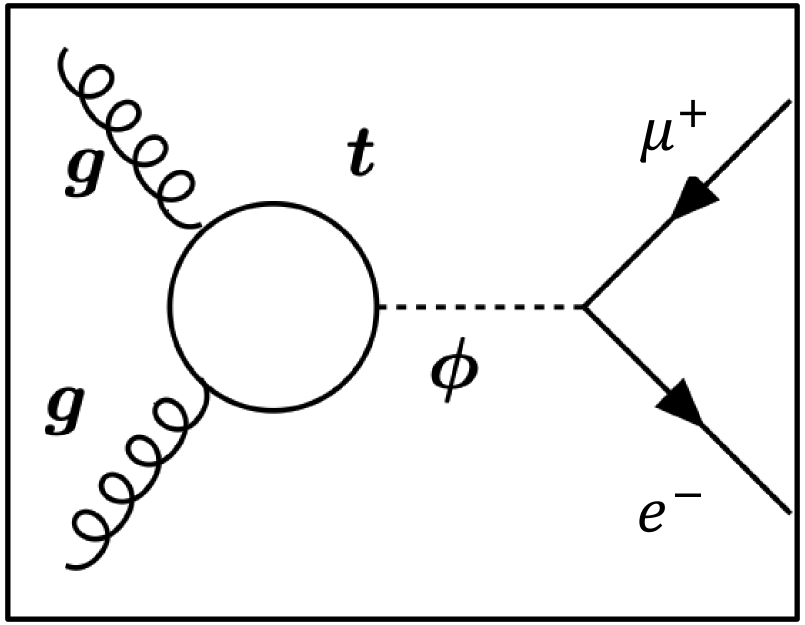

$h \to \tau^\pm e^\mp$
$\mathrm{BR}(h \to \tau^\pm e^\mp)$
$\cos(\beta - \alpha)$
10^{-2}
10^{-4}
10^{-6}
10^{-8}
10^{-10}
10^{-12}
10^{-14}
10^{-16}
-0.15
-0.10
-0.05
0.00
0.05
0.10
0.15

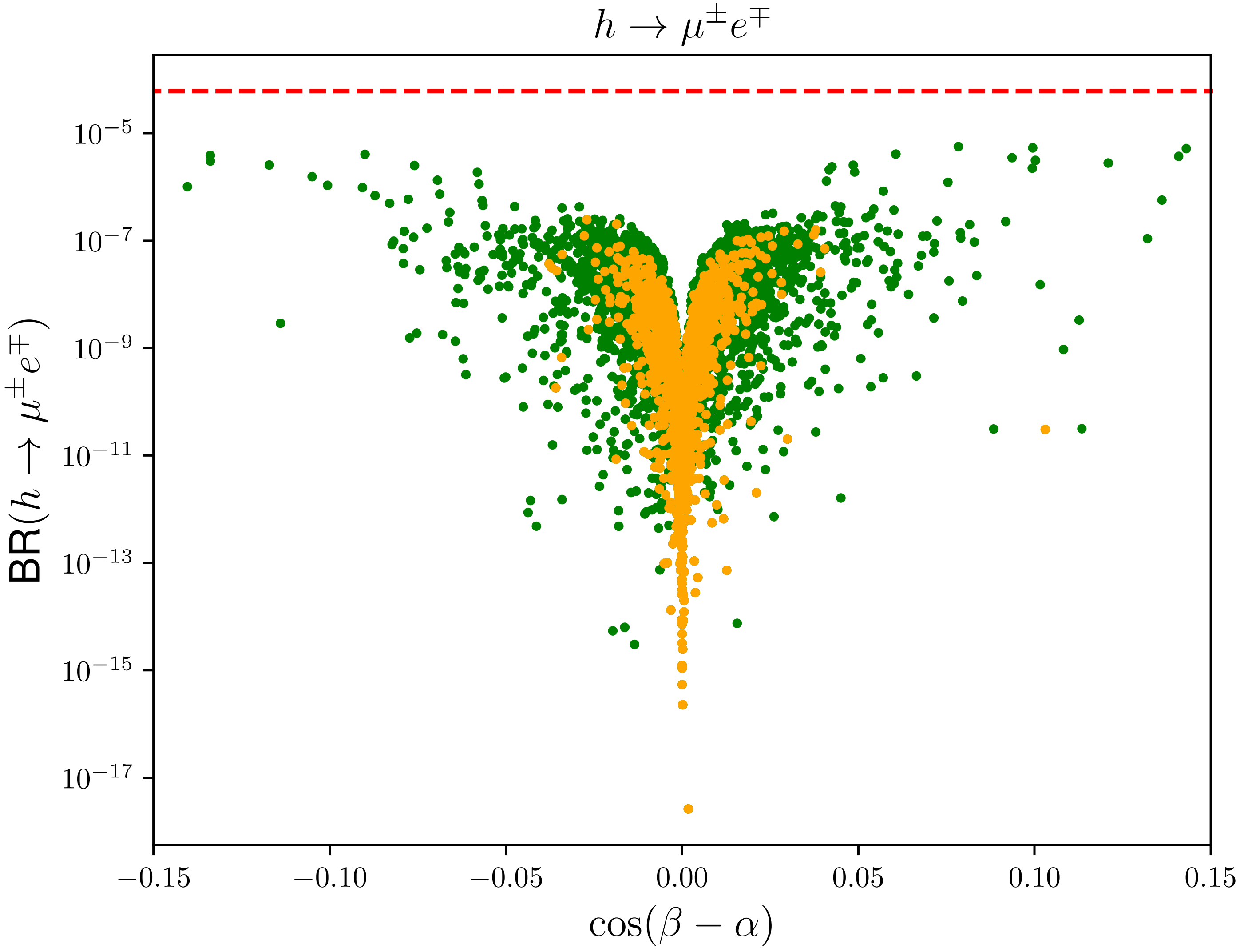


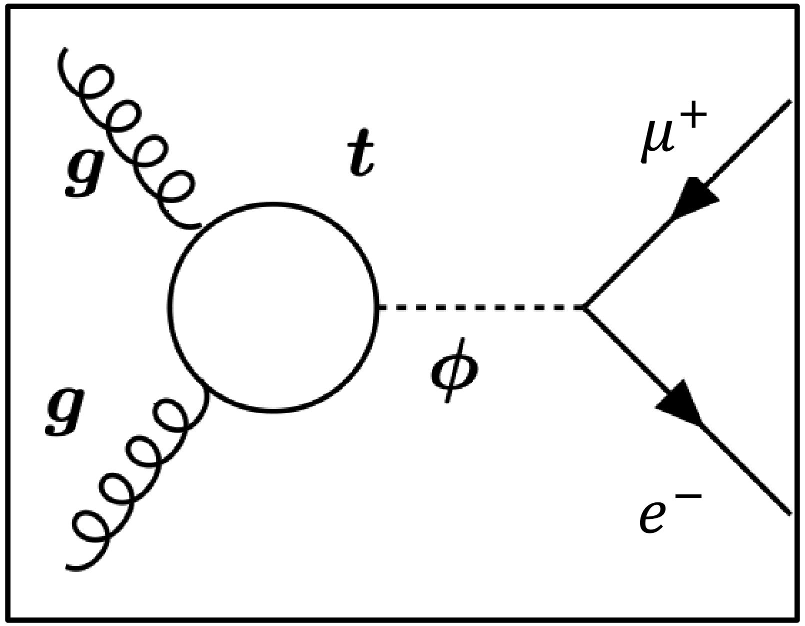
g
g
t
ϕ
τ^+
e^-



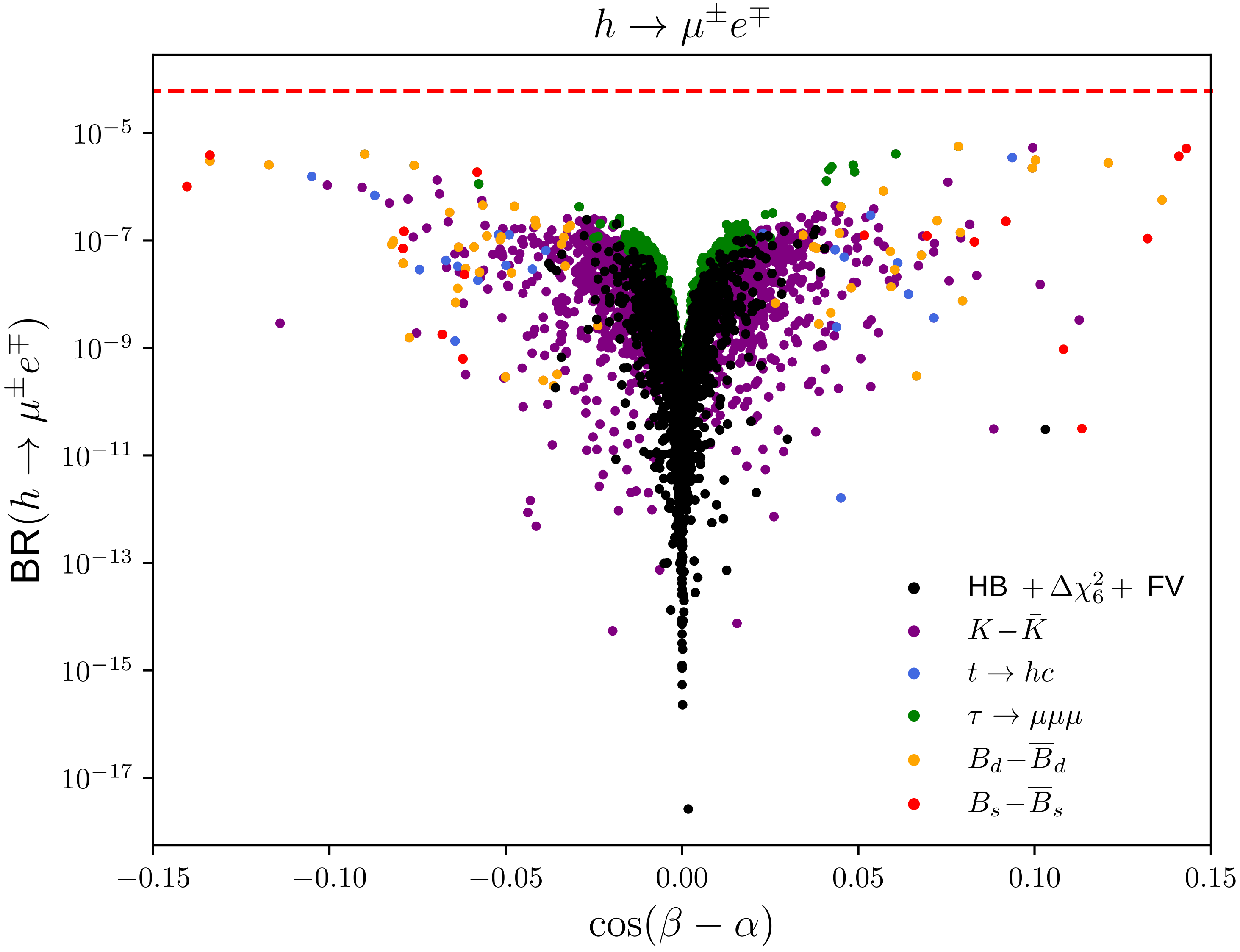
$h \to \tau^\pm e^\mp$
$\mathrm{BR}(h \to \tau^\pm e^\mp)$
$\cos(\beta-\alpha)$
10^{-2}
10^{-4}
10^{-6}
10^{-8}
10^{-10}
10^{-12}
10^{-14}
10^{-16}
−0.15
−0.10
−0.05
0.00
0.05
0.10
0.15
HB $+\Delta\chi^2_6+$ FV
$K-\bar{K}$
$t \to hc$
$\tau \to \mu\mu\mu$
$B_d-\overline{B}_d$
$B_s-\overline{B}_s$


g
t
μ+
g
φ
e−


h → μ±e∓
BR(h → μ±e∓)
10−5
10−7
10−9
10−11
10−13
10−15
10−17
−0.15
−0.10
−0.05
0.00
0.05
0.10
0.15
cos(β − α)

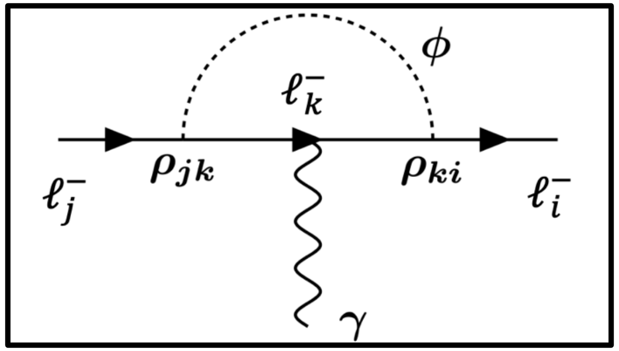


g
t
μ^+
g
ϕ
e^-

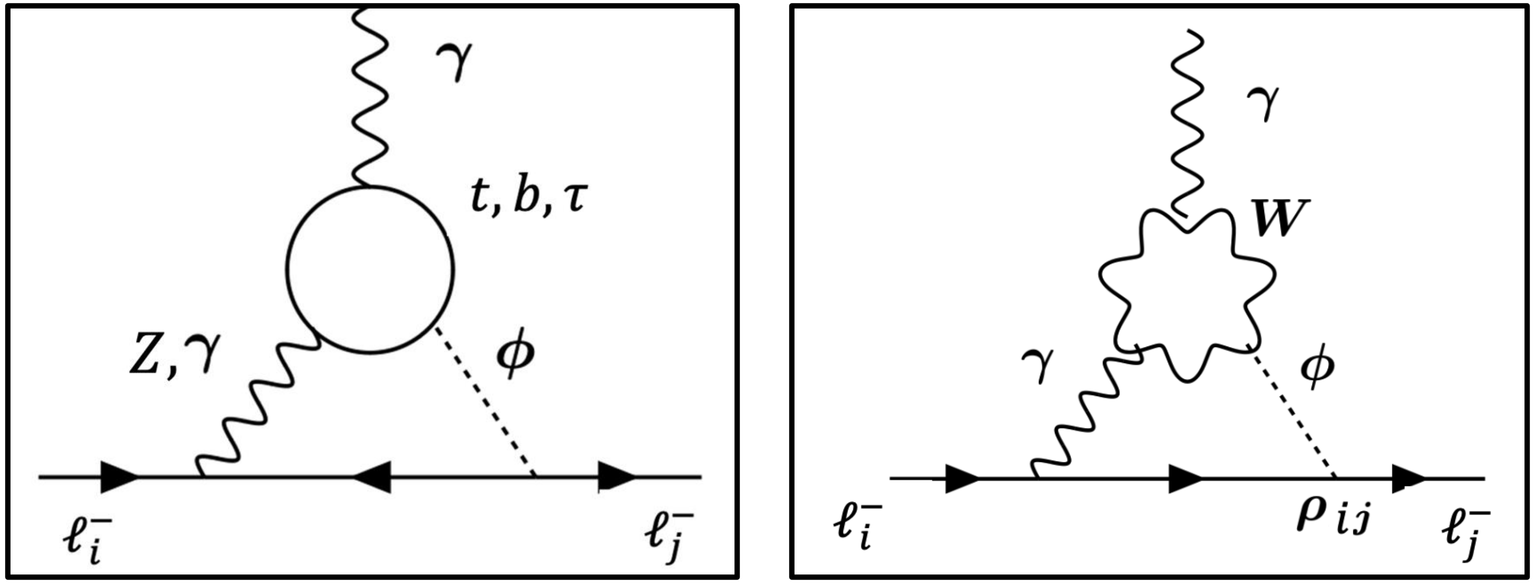


$h \to \mu^\pm e^\mp$
BR$(h \to \mu^\pm e^\mp)$
10^{-5}
10^{-7}
10^{-9}
10^{-11}
10^{-13}
10^{-15}
10^{-17}
-0.15
-0.10
-0.05
0.00
0.05
0.10
0.15
$\cos(\beta - \alpha)$
HB $+\Delta\chi^2_6+$ FV
$K-\bar{K}$
$t \to hc$
$\tau \to \mu\mu\mu$
$B_d-\overline{B}_d$
$B_s-\overline{B}_s$

$$\ell_1^{\pm} \to \ell_2^{\pm} + \gamma$$

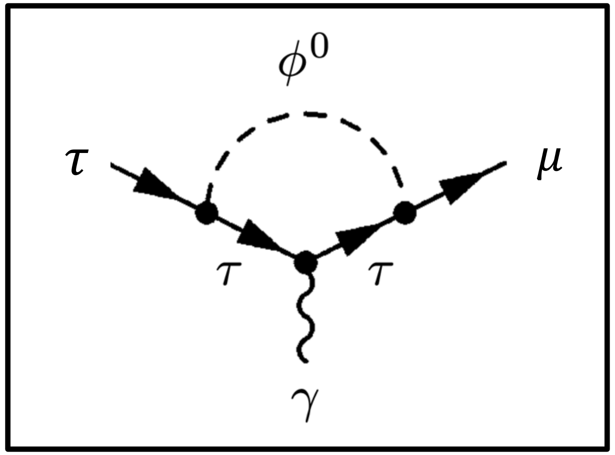


One-loop diagrams

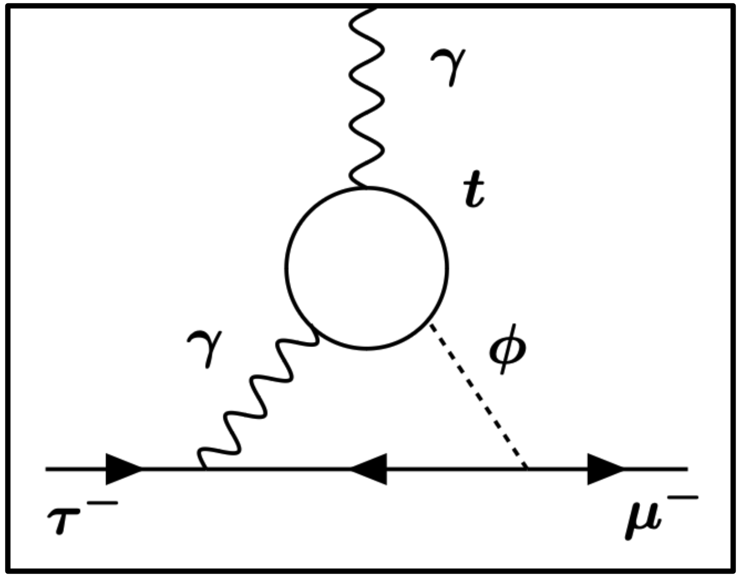


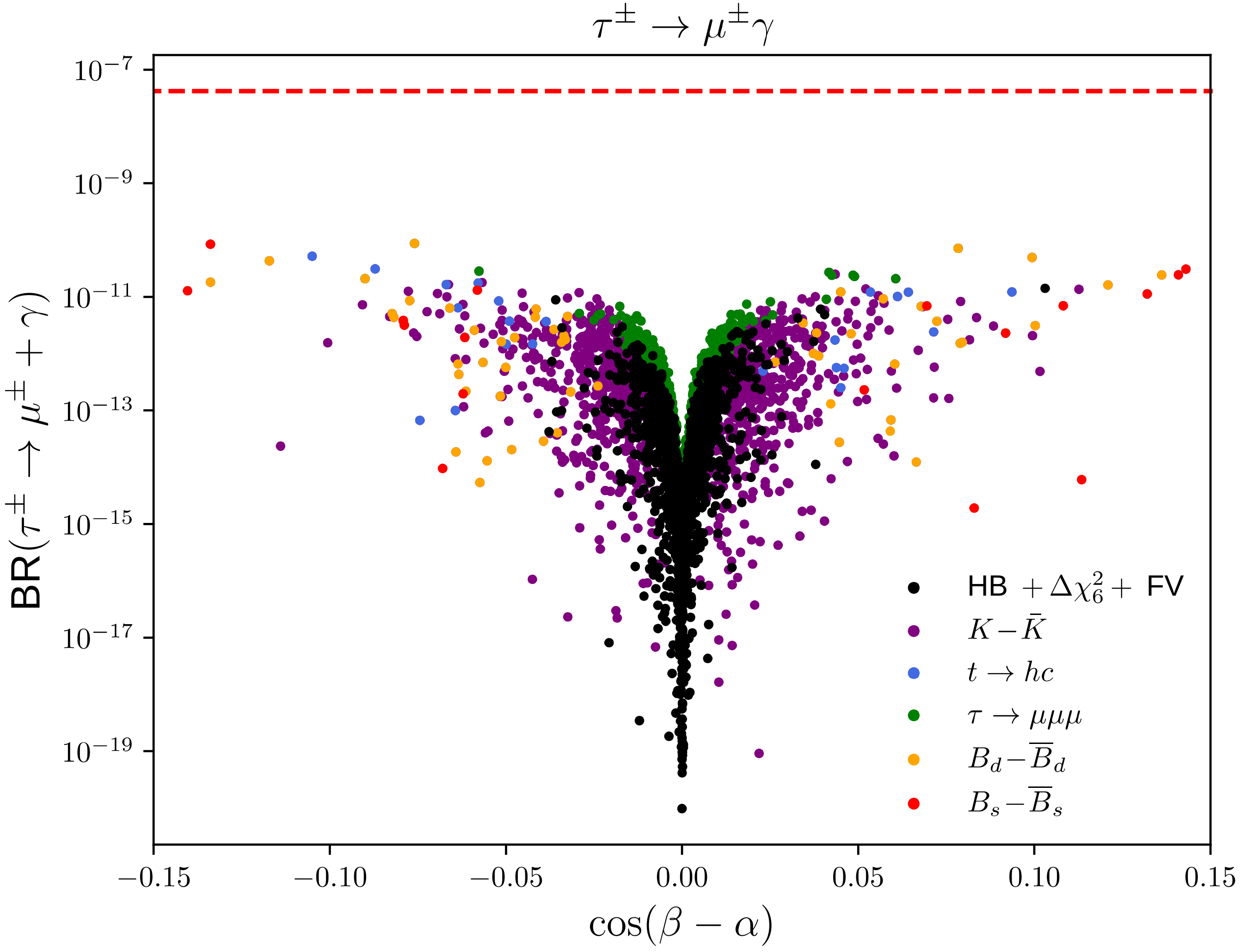
Two-loop Barr-Zee diagrams dominate

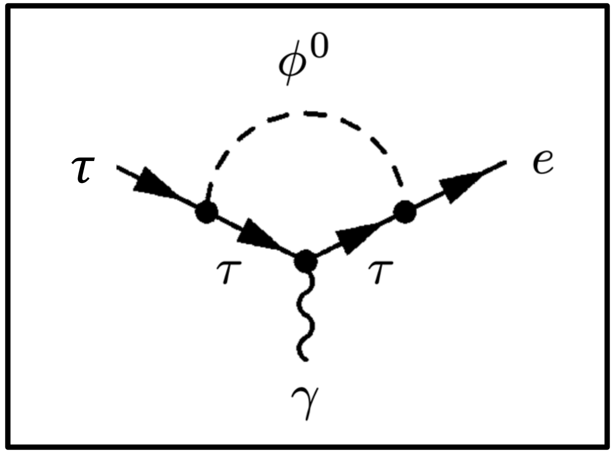
Heaviest loops dominate



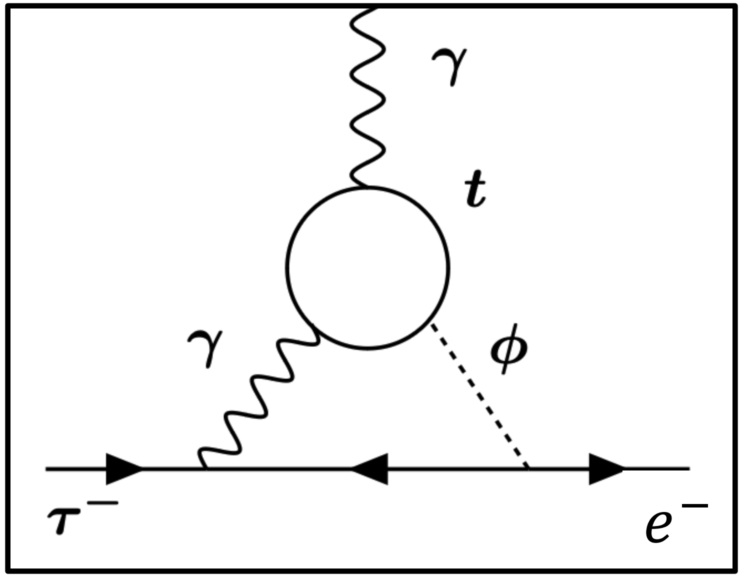
Two-loop diagrams dominate

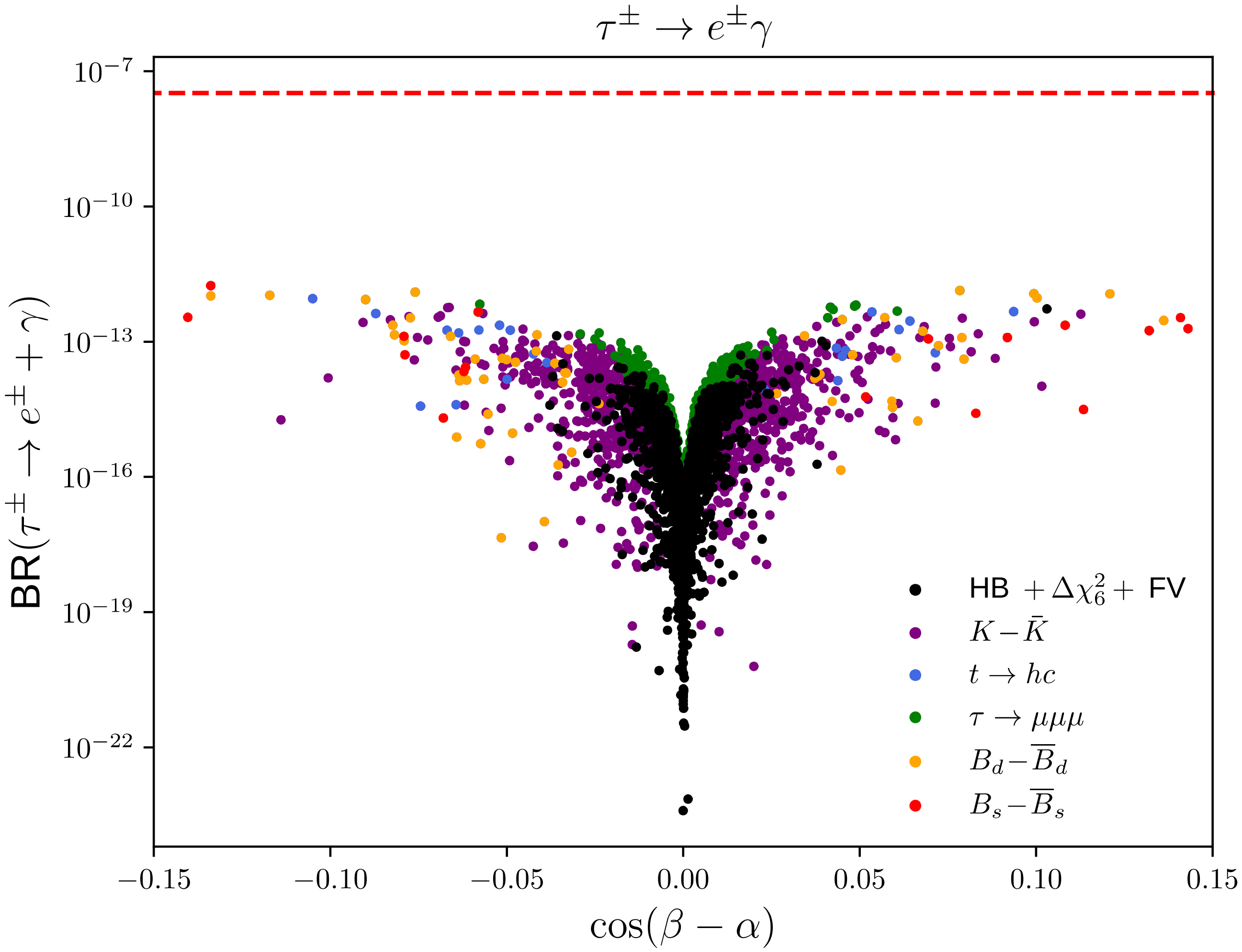






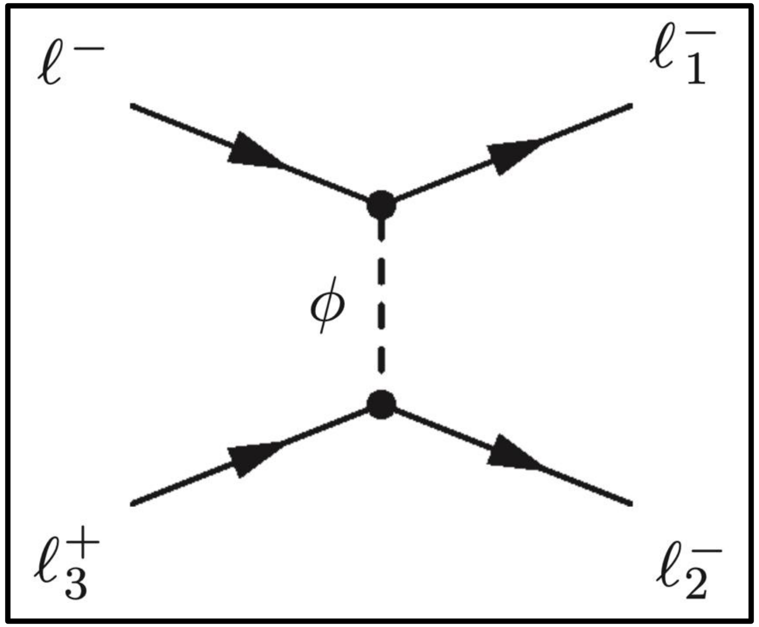
Two-loop
diagrams
dominate

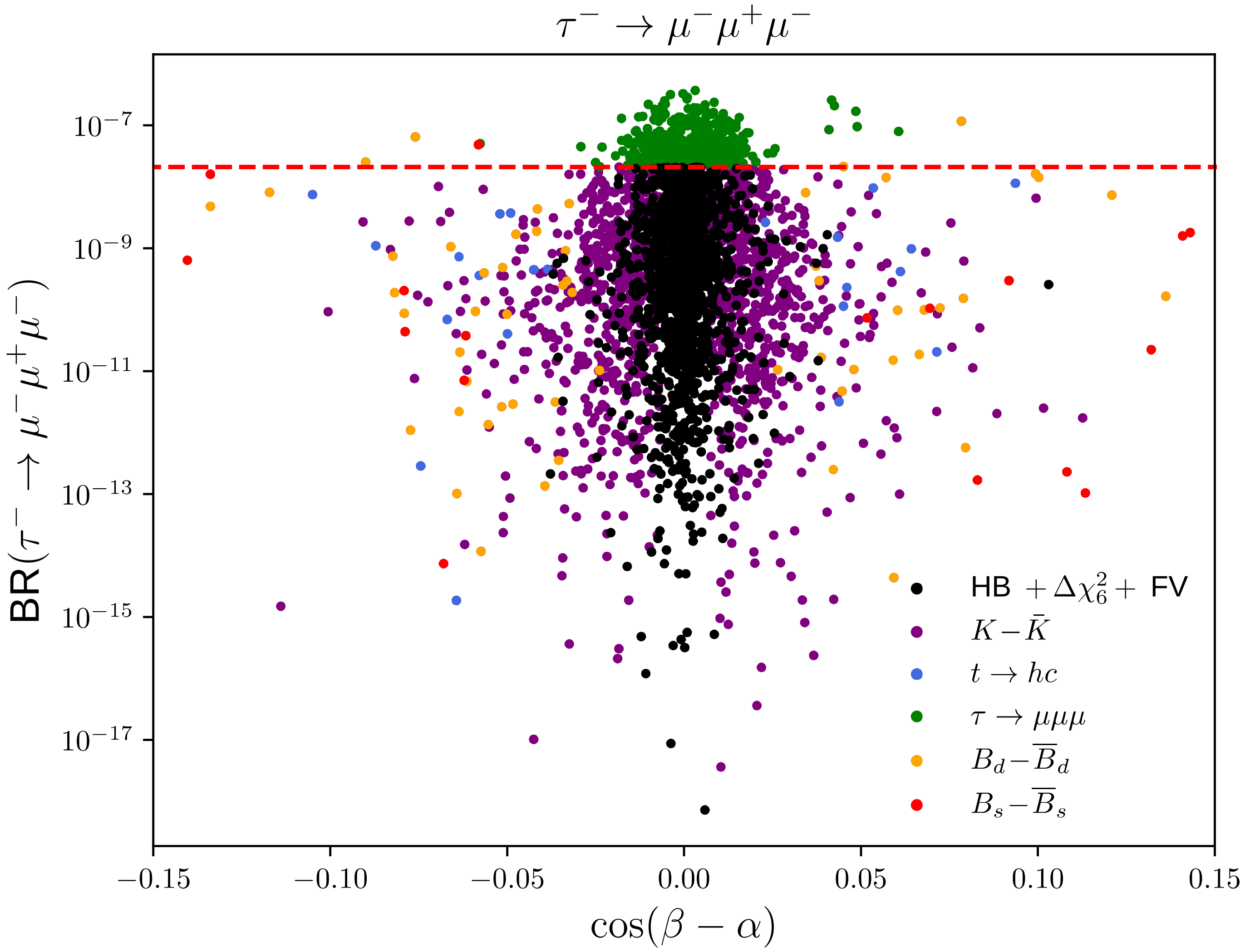




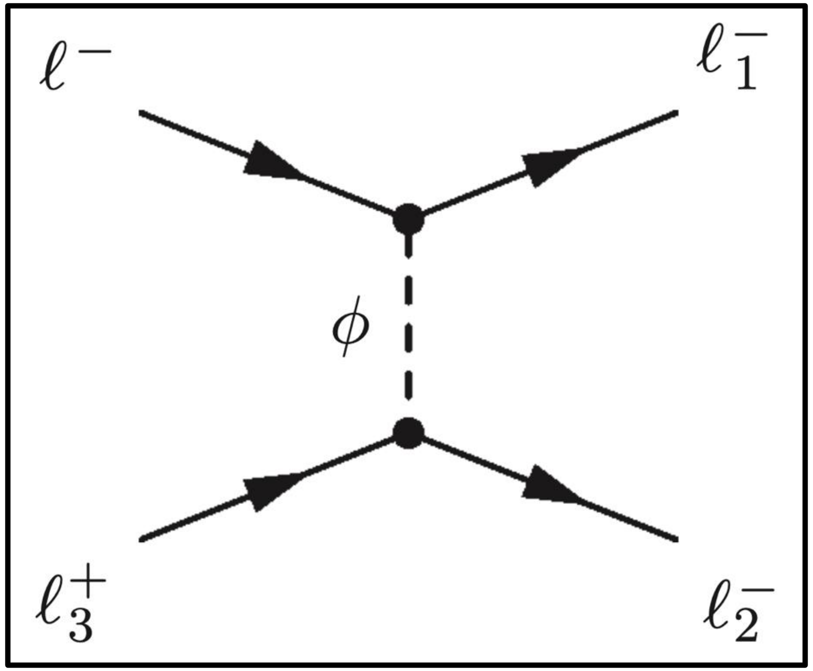
$$\tau^- \to \mu^- + \mu^+ + \mu^-$$

(Other 3 lepton final states all have very small BR's)

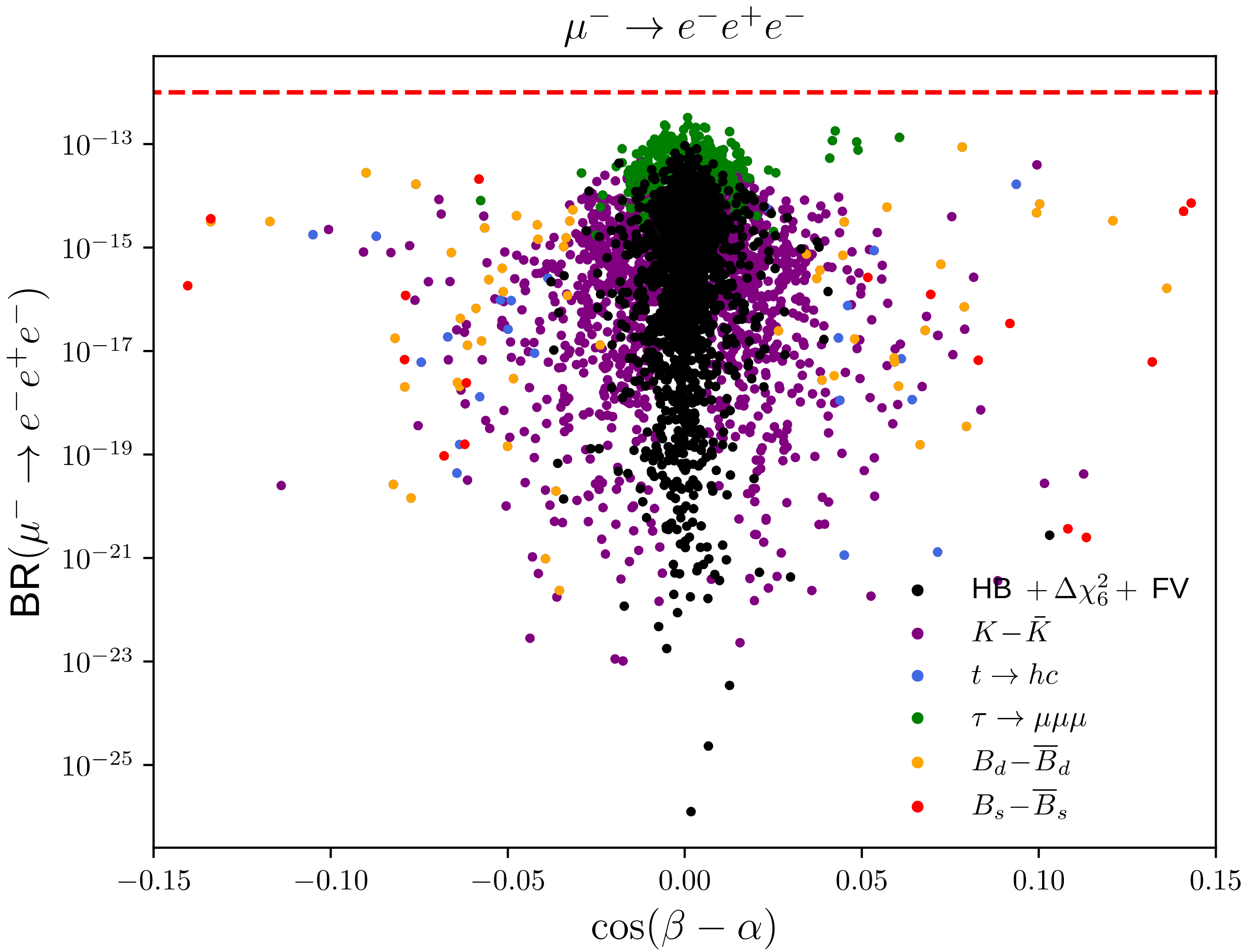




$$\mu^- \to e^- + e^+ + e^-$$



ℓ^-
ℓ_1^-
ϕ
ℓ_3^+
ℓ_2^-



$\mu^- \to e^- e^+ e^-$
$\mathrm{BR}(\mu^- \to e^- e^+ e^-)$
$\cos(\beta - \alpha)$
10^{-13}
10^{-15}
10^{-17}
10^{-19}
10^{-21}
10^{-23}
10^{-25}
-0.15
-0.10
-0.05
0.00
0.05
0.10
0.15
HB $+\Delta\chi^2_6+$ FV
$K-\bar{K}$
$t \to hc$
$\tau \to \mu\mu\mu$
$B_d-\overline{B}_d$
$B_s-\overline{B}_s$

Higgs-mediated Neutral meson mixing

Higgs mediated contributions to neutral meson mixing ($B_{d,s} - \bar{B}_{d,s}$, $K - \bar{K}$ and $D - \bar{D}$ mixing) arise in our model. Integrating out the three neutral Higgs bosons, we obtain the following dimension six effective Lagrangian describing B_s meson mixing

$$\mathscr{L}_{\text{eff}} = C_2\left(\bar{b}_R s_L\right)^2 + \tilde{C}_2\left(\bar{b}_L s_R\right)^2 + C_4\left(\bar{b}_R s_L\right)\left(\bar{b}_L s_R\right) + \text{ h.c.}$$

with Wilson coefficients,

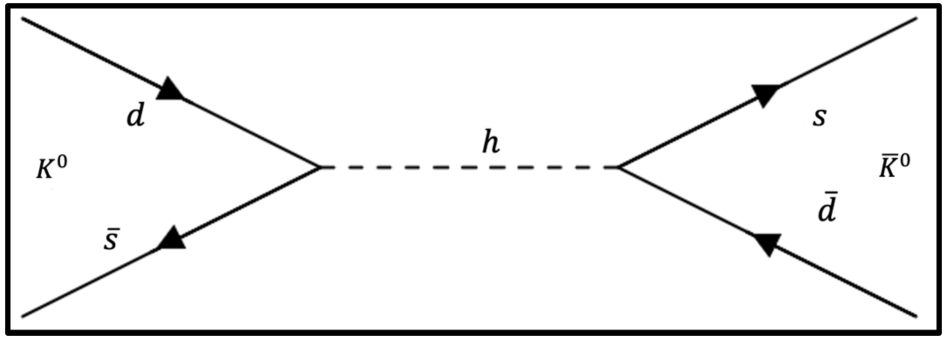
$$C_2 = \frac{\left(\rho_{32}^D\right)^2}{4}\left(\frac{\sin^2(\beta-\alpha)}{m_H^2} + \frac{\cos^2(\beta-\alpha)}{m_h^2} - \frac{1}{m_A^2}\right),$$

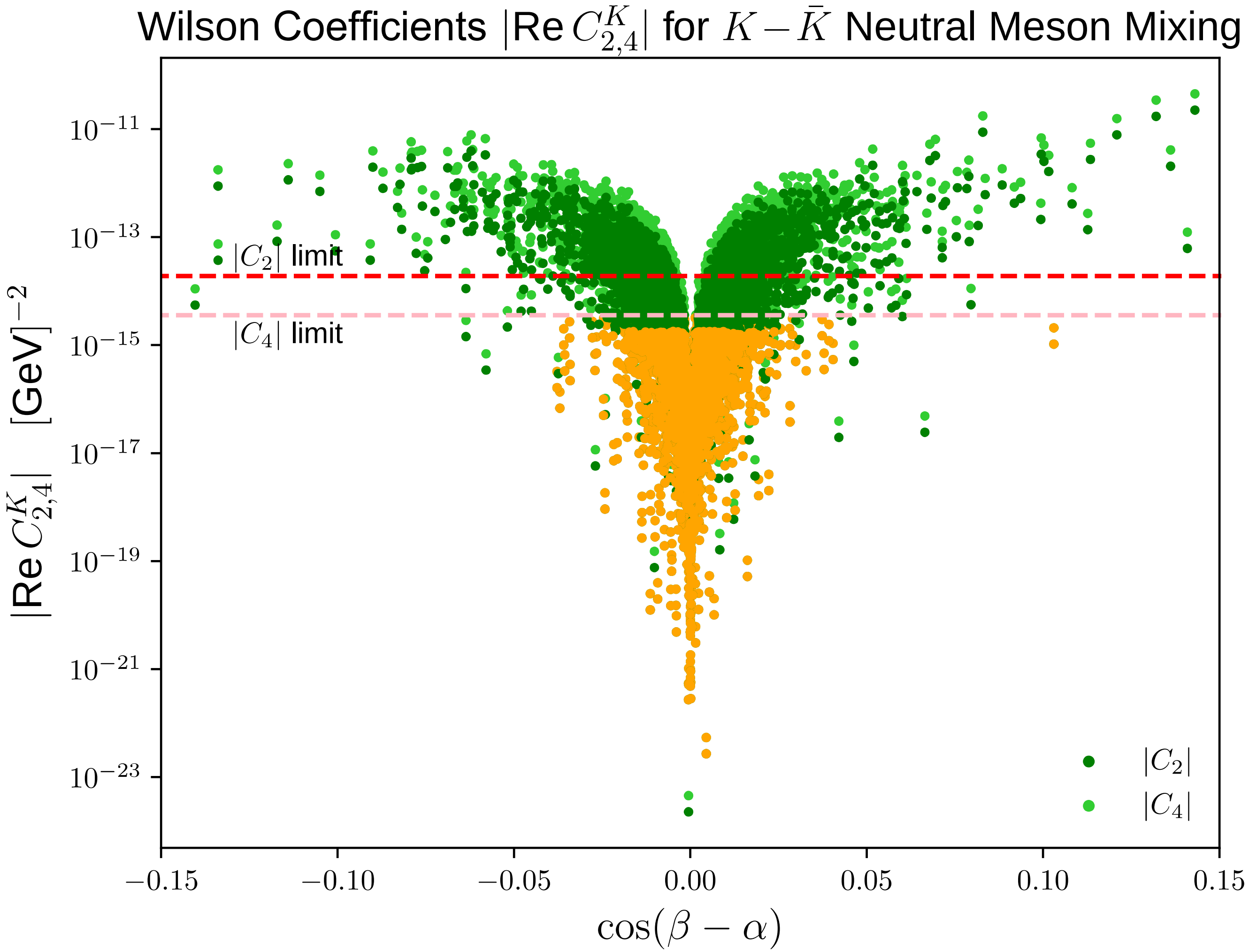
$$\tilde{C}_2 = \frac{\left(\rho_{23}^{D*}\right)^2}{4}\left(\frac{\sin^2(\beta-\alpha)}{m_H^2} + \frac{\cos^2(\beta-\alpha)}{m_h^2} - \frac{1}{m_A^2}\right),$$

$$C_4 = \frac{\left(\rho_{32}^D\right)\left(\rho_{23}^{D*}\right)}{2}\left(\frac{\sin^2(\beta-\alpha)}{m_H^2} + \frac{\cos^2(\beta-\alpha)}{m_h^2} + \frac{1}{m_A^2}\right),$$

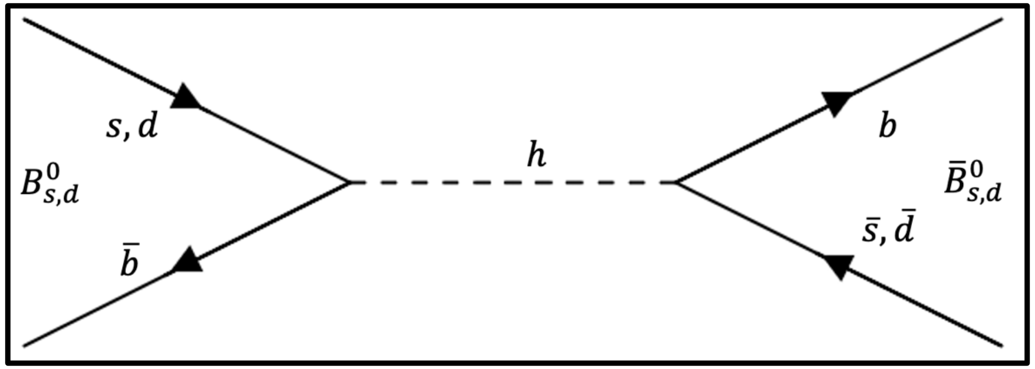
and corresponding Wilson coefficients for B_d, K, and D mixing.

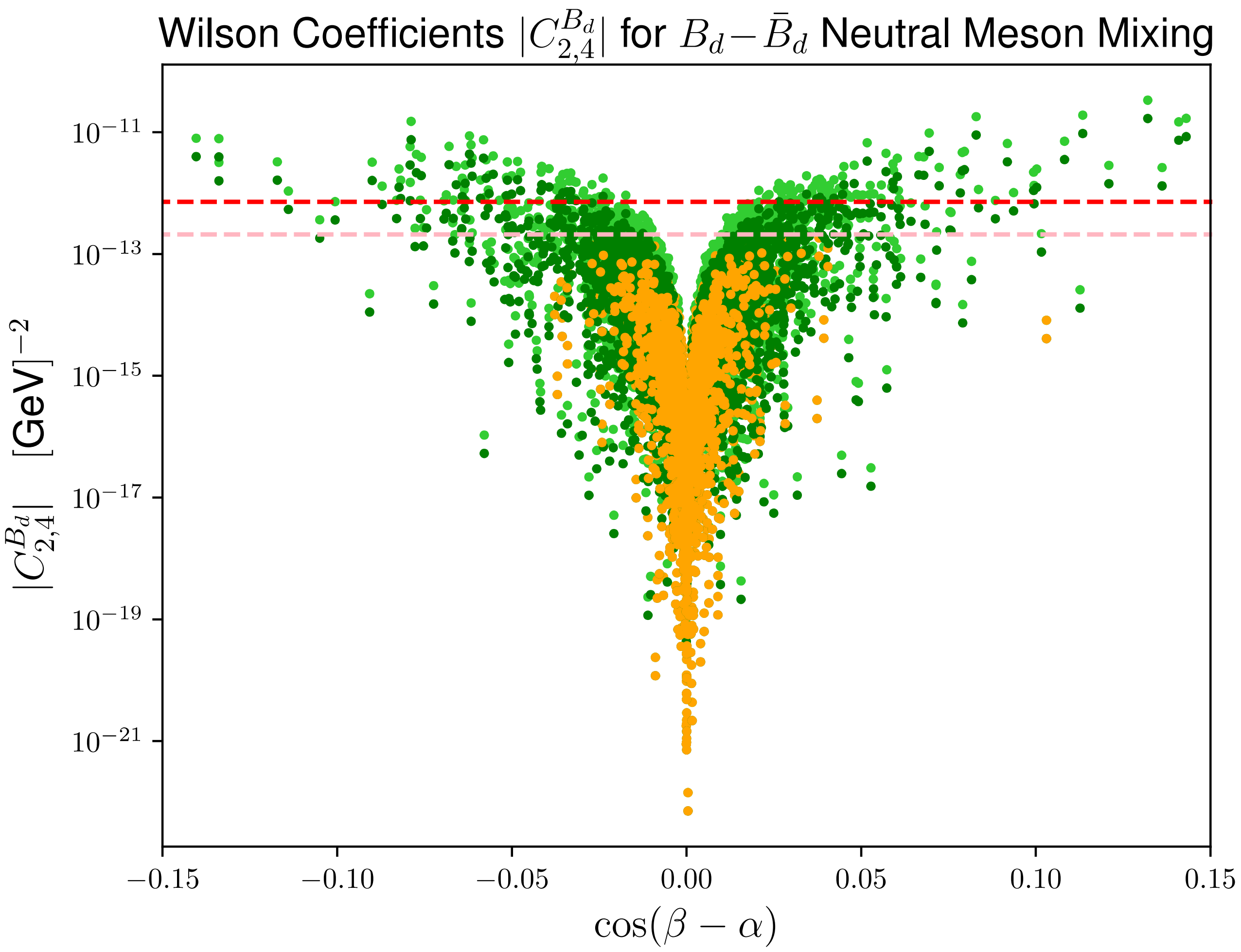
Kaon Mixing $K^0 - \overline{K}^0$



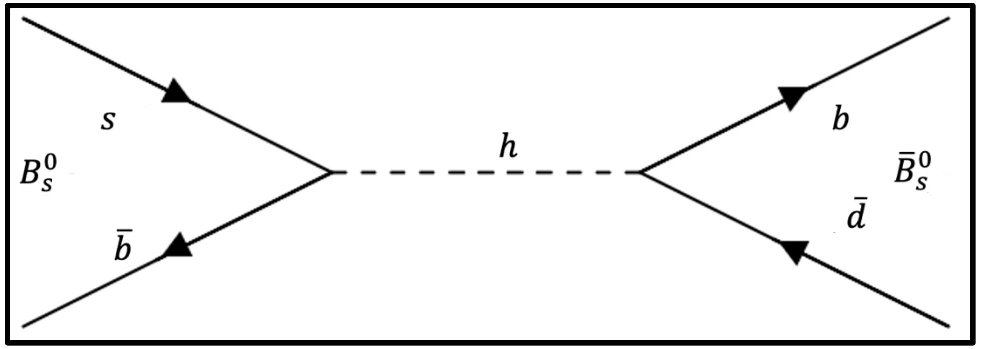


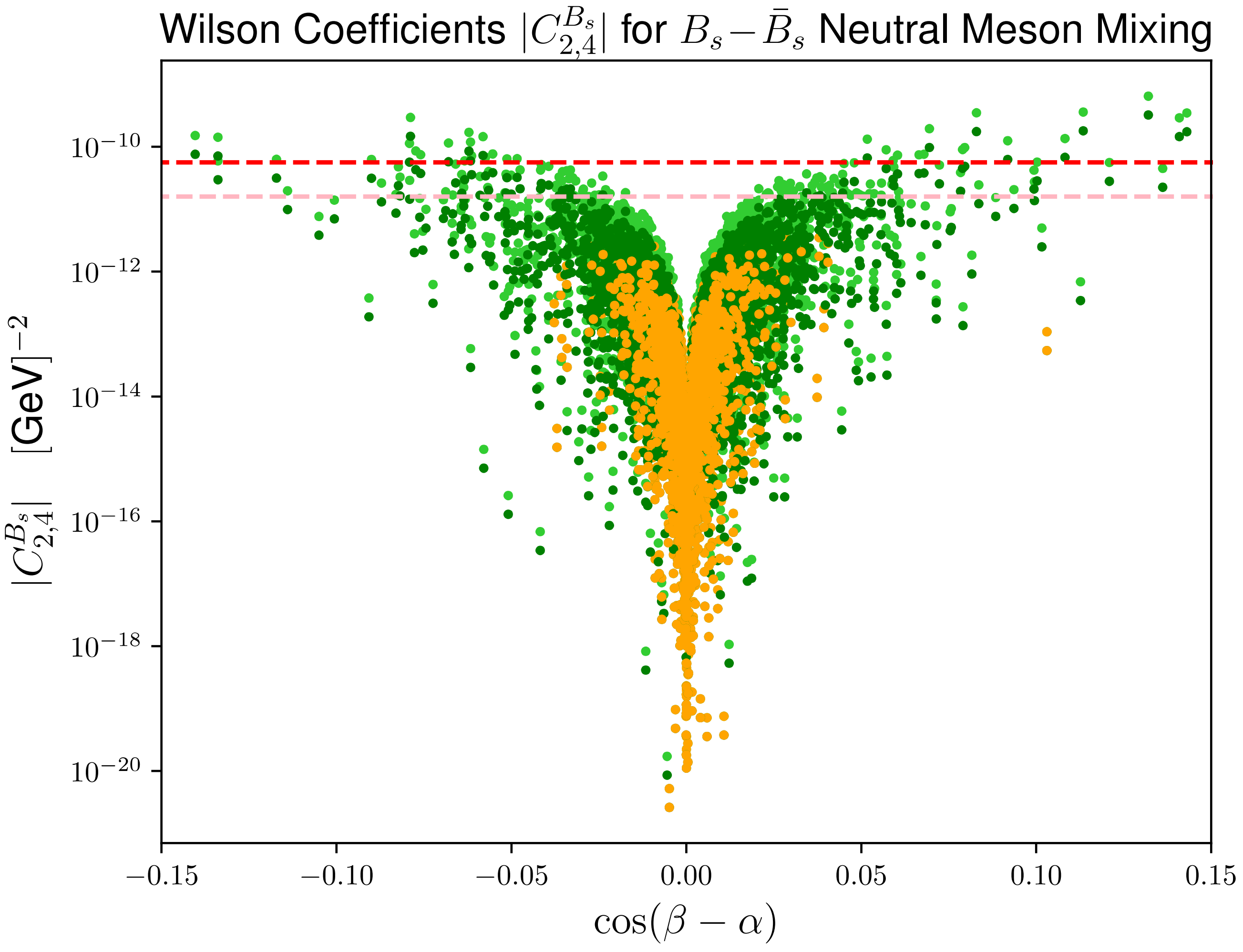
Neutral B Meson Mixing: $B_d^0 - \bar{B}_d^0$





Neutral Strange B Meson Mixing: $B_s^0 - \bar{B}_s^0$





Conclusions and Future work

- The viability of the Cheng-Sher ansatz for off-diagonal neutral Higgs–fermion Yukawa couplings should be examined...
  - in a formalism where the unphysical parameter $\tan\beta$ never appears.
  - by making use of the most recent analysis of the CKM parameters based on the Fritzsch textures for the up and down quark mass matrices.

- Phenomenological implications of (the less suppressed) flavor off-diagonal decays of the heavy Higgs scalars should be investigated.

- Extend the analysis to allow for CP-violating phases in the ρ-type Yukawa matrices and scalar potential.

- The Fritzsch and Cheng-Sher textures are not RG-stable. Thus, it would be useful to construct UV completions of the 2HDM that could provide an (approximate) explanation for the Yukawa matrix textures used here.