

# Vacuum (in)stability in 2HDMS vs N2HDM

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# Vacuum Stability in multi-Higgs models

- Higgs mechanism in SM relies on stable vacuum, guaranteed at tree-level.
- Higher order corrections can spoil the stability, top quark mass plays a crucial role.
- With extended scalar sectors, the vacuum stability, unlike SM is challenged at tree-level.
- In case of non-supersymmetric extensions of the SM scalar sector, there can be potential charge and CP-breaking minima as well as a second wrong EW vacuum(panic vacuum).

- If there is no stationary point deeper than the EW vacuum, then EW vacuum is *absolutely stable*.
- If there are deeper minima, but the transition time into those is larger than age of universe, EW vacuum is *metastable*, else *unstable*.
- Vacuum structure of 2HDM and its real scalar singlet extension, namely N2HDM, has been studied. [Phys.Lett.B 603 \(2004\) 219-229](#), [JHEP 09 \(2019\) 006](#)
- In 2HDM, any stationary point that is charge or CP breaking is necessarily a saddle point that lies above the normal EW minimum.  
[Phys.Lett.B 603 \(2004\) 219-229](#)
- N2HDM (real scalar extension of 2HDM), due to addition of an extra scalar degree of freedom, shows quite different vacuum phenomena.  
[JHEP 09 \(2019\) 006](#)

# Going beyond N2HDM with 2HDM+complex singlet(2HDMS)

- The objective is to study the vacuum instabilities in 2HDMS.
- A detailed comparison with N2HDM.
  - 1 Intrinsic difference between the vacuum structure of the two models.
  - 2 How much of that difference stands the *experimental test*?
  - 3 Can we probe the different vacuum structures of the two models?

## The models

The part of the scalar potential involving the singlet  $S$  in N2HDM with  $Z_2$  symmetry on the scalar  $S$ .

$$V_S = \frac{1}{2}m_S^2 S^2 + \frac{1}{8}\lambda_6 S^4 + \frac{1}{2}\lambda_7 |\Phi_1|^2 S^2 + \frac{1}{2}\lambda_8 |\Phi_2|^2 S^2$$

and in 2HDMS with complex singlet  $S + iP$ ,  $Z_2$  symmetry is imposed on additional complex singlet. With the assumption of real co-efficients, we arrive at the following.

$$V'_S = \frac{1}{2}m_S^2 S^2 + \frac{1}{2}m_{S'}^2 P^2 + \frac{1}{8}\lambda_6 S^4 + \frac{1}{8}\lambda_9 P^4 + \frac{1}{8}\lambda_{10} S^2 P^2 \\ + \frac{1}{2}(\lambda_7 |\Phi_1|^2 + \lambda_8 |\Phi_2|^2) S^2 + \frac{1}{2}(\lambda_{11} |\Phi_1|^2 + \lambda_{12} |\Phi_2|^2) P^2$$

## Possible Vacua: N2HDM

$$\mathcal{N}_s \rightarrow \langle \Phi_1 \rangle_0 = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v_1 \end{pmatrix}, \quad \langle \Phi_2 \rangle_0 = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v_2 \end{pmatrix}, \quad \langle S \rangle_0 = v_s$$

$$\mathcal{CB} \rightarrow \langle \Phi_1 \rangle_0 = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ c_1 \end{pmatrix}, \quad \langle \Phi_2 \rangle_0 = \frac{1}{\sqrt{2}} \begin{pmatrix} c_2 \\ c_3 \end{pmatrix}, \quad \langle S \rangle_0 = 0$$

$$\mathcal{CB}_s \rightarrow \langle \Phi_1 \rangle_0 = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ c_1 \end{pmatrix}, \quad \langle \Phi_2 \rangle_0 = \frac{1}{\sqrt{2}} \begin{pmatrix} c_2 \\ c_3 \end{pmatrix}, \quad \langle S \rangle_0 = s$$

- Similarly  $\mathcal{CP}$  and  $\mathcal{CP}_s$  can also exist in N2HDM.

## Possible Vacua: 2HDMS

$$\mathcal{N}_s \rightarrow \langle \Phi_1 \rangle_0 = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v_1 \end{pmatrix}, \quad \langle \Phi_2 \rangle_0 = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v_2 \end{pmatrix}, \quad \langle S \rangle_0 = v_s, \quad \langle P \rangle_0 = 0$$

$$\mathcal{N}_{sp} \rightarrow \langle \Phi_1 \rangle_0 = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v_1 \end{pmatrix}, \quad \langle \Phi_2 \rangle_0 = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v_2 \end{pmatrix}, \quad \langle S \rangle_0 = v_s, \quad \langle P \rangle_0 = v_p$$

$$\mathcal{CB} \rightarrow \langle \Phi_1 \rangle_0 = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ c_1 \end{pmatrix}, \quad \langle \Phi_2 \rangle_0 = \frac{1}{\sqrt{2}} \begin{pmatrix} c_2 \\ c_3 \end{pmatrix}, \quad \langle S \rangle_0 = 0, \quad \langle P \rangle_0 = 0$$

$$\mathcal{CB}_s \rightarrow \langle \Phi_1 \rangle_0 = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ c_1 \end{pmatrix}, \quad \langle \Phi_2 \rangle_0 = \frac{1}{\sqrt{2}} \begin{pmatrix} c_2 \\ c_3 \end{pmatrix}, \quad \langle S \rangle_0 = s, \quad \langle P \rangle_0 = 0$$

$$\mathcal{CB}_p \rightarrow \langle \Phi_1 \rangle_0 = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ c_1 \end{pmatrix}, \quad \langle \Phi_2 \rangle_0 = \frac{1}{\sqrt{2}} \begin{pmatrix} c_2 \\ c_3 \end{pmatrix}, \quad \langle S \rangle_0 = 0, \quad \langle P \rangle_0 = p$$

$$\mathcal{CB}_{sp} \rightarrow \langle \Phi_1 \rangle_0 = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ c_1 \end{pmatrix}, \quad \langle \Phi_2 \rangle_0 = \frac{1}{\sqrt{2}} \begin{pmatrix} c_2 \\ c_3 \end{pmatrix}, \quad \langle S \rangle_0 = s, \quad \langle P \rangle_0 = p$$

- There can be also  $\mathcal{CP}$ ,  $\mathcal{CP}_s$ ,  $\mathcal{CP}_p$  and  $\mathcal{CP}_{sp}$  in 2HDMS.

- In total, there are four extra charge and CP-breaking vacua in 2HDMS.
- There are also “wrong” (*panic*) neutral vacua present in both N2HDM and 2HDMS. Here too, naturally the number of potentially dangerous vacua is lot more in 2HDMS.
- Therefore, the stability of the parameter points naturally deteriorates in 2HDMS compared to N2HDM.



## Comparison in terms of extra model parameters : $\mathcal{N}_{sp}$ in 2HDM vs $\mathcal{N}_s$ in N2HDM

A *stable* BP of N2HDM in mass basis:

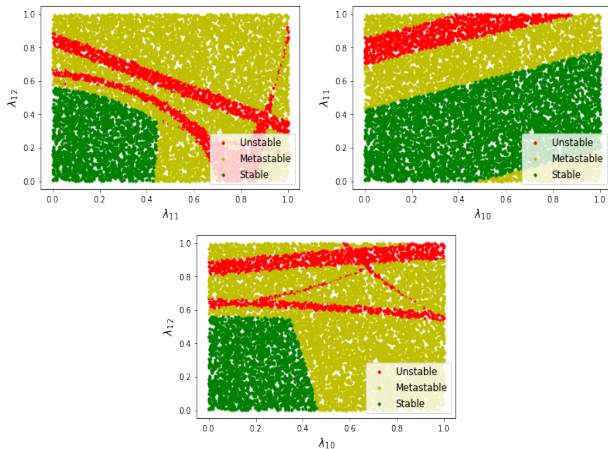
$m_{h_1}$	$m_{h_2}$	$m_{h_3}$	$m_A = m_H^\pm$	$m_{12}^2$	$\tan \beta$	$v_s$	$\{\alpha_1, \alpha_2, \alpha_3\}$
95	125	601	621	9529.17	1.37	468.1	$\{-0.49, 0.31, -0.09\}$

In the interaction basis:

$\lambda_1$	$\lambda_2$	$\lambda_3$	$\lambda_4 = \lambda_5$	$\lambda_6$	$\lambda_7$	$\lambda_8$	$m_{12}^2$	$\tan \beta$	$v_s$
1.43	0.24	12.02	-6.05	2.97	2.11	-0.41	9529.17	1.37	468.1

The benchmark accommodates 95 GeV excess with its observed  $\mu_{\gamma\gamma}^{\text{combined}}$ .

# Fate in 2HDMS with additional free parameters



Numerical analysis done with **EVADE**. Bounded-from-below condition applied beforehand.

## Will experimental observations change the picture?

- CAUTION!! The extra parameters of 2HDMS are not really 'free'.
- Imposing the observed scalar masses and signal strengths already constrains the free parameters of 2HDMS, thereby alleviating the difference between the two models.
- For this analysis we will consider mass basis and physical observables.

## Comparison between N2HDM and 2HDMS with couplings ( $h_i f \bar{f}$ and $h_i VV$ ) in similar range

- We want to check the differences in the vacuum structure in N2HDM and 2HDMS after all the physical observables are kept at similar ranges in both cases.
- Since all the couplings of physical scalars to fermions and gauge bosons are functions of mixing matrix elements, we demand all the elements of the  $3 \times 3$  subspace of the  $4 \times 4$  mixing matrix elements of 2HDMS are within  $\lesssim 15\%$  of the matrix elements of the  $3 \times 3$  mixing matrix of N2HDM.

$$R = \begin{pmatrix} c_{\alpha_1} c_{\alpha_2} & s_{\alpha_1} c_{\alpha_2} & s_{\alpha_2} \\ -s_{\alpha_1} c_{\alpha_3} - c_{\alpha_1} s_{\alpha_2} s_{\alpha_3} & c_{\alpha_1} c_{\alpha_3} - s_{\alpha_1} s_{\alpha_2} s_{\alpha_3} & c_{\alpha_2} s_{\alpha_3} \\ s_{\alpha_1} s_{\alpha_3} - c_{\alpha_1} s_{\alpha_2} c_{\alpha_3} & -c_{\alpha_1} s_{\alpha_3} - s_{\alpha_1} s_{\alpha_2} c_{\alpha_3} & c_{\alpha_2} c_{\alpha_3} \end{pmatrix},$$

$$\begin{aligned}
R'_{11} &= c_{\alpha_1} c_{\alpha_2} c_{\alpha_4} \\
R'_{12} &= c_{\alpha_2} c_{\alpha_4} s_{\alpha_1} \\
R'_{13} &= c_{\alpha_4} s_{\alpha_2} \\
R'_{14} &= s_{\alpha_4} \\
R'_{21} &= c_{\alpha_5} (-c_{\alpha_3} s_{\alpha_1} - c_{\alpha_1} s_{\alpha_2} s_{\alpha_3}) - c_{\alpha_1} c_{\alpha_2} s_{\alpha_4} s_{\alpha_5} \\
R'_{22} &= c_{\alpha_5} (c_{\alpha_1} c_{\alpha_3} - s_{\alpha_1} s_{\alpha_2} s_{\alpha_3}) - c_{\alpha_2} s_{\alpha_1} s_{\alpha_4} s_{\alpha_5} \\
R'_{23} &= c_{\alpha_2} c_{\alpha_5} s_{\alpha_3} - s_{\alpha_2} s_{\alpha_4} s_{\alpha_5} \\
R'_{24} &= c_{\alpha_4} s_{\alpha_5} \\
R'_{31} &= c_{\alpha_6} (-c_{\alpha_1} c_{\alpha_3} s_{\alpha_2} + s_{\alpha_1} s_{\alpha_3}) + (-c_{\alpha_1} c_{\alpha_2} c_{\alpha_5} s_{\alpha_4} - (-c_{\alpha_3} s_{\alpha_1} - c_{\alpha_1} s_{\alpha_2} s_{\alpha_3}) s_{\alpha_5}) s_{\alpha_6} \\
R'_{32} &= c_{\alpha_6} (-s_{\alpha_1} c_{\alpha_3} s_{\alpha_2} - c_{\alpha_1} s_{\alpha_3}) + (-s_{\alpha_1} c_{\alpha_2} c_{\alpha_5} s_{\alpha_4} - (-c_{\alpha_3} c_{\alpha_1} - s_{\alpha_1} s_{\alpha_2} s_{\alpha_3}) s_{\alpha_5}) s_{\alpha_6} \\
R'_{33} &= c_{\alpha_2} c_{\alpha_3} c_{\alpha_6} + (-c_{\alpha_5} s_{\alpha_2} s_{\alpha_4} - c_{\alpha_2} s_{\alpha_3} s_{\alpha_5}) s_{\alpha_6} \\
R'_{34} &= c_{\alpha_4} c_{\alpha_5} s_{\alpha_6} \\
R'_{41} &= c_{\alpha_6} (-c_{\alpha_1} c_{\alpha_2} c_{\alpha_5} s_{\alpha_4} - (-c_{\alpha_3} s_{\alpha_1} - c_{\alpha_1} s_{\alpha_2} s_{\alpha_3}) s_{\alpha_5}) - (-c_{\alpha_1} s_{\alpha_2} c_{\alpha_3} + s_{\alpha_1} s_{\alpha_3}) s_{\alpha_6} \\
R'_{42} &= c_{\alpha_6} (-s_{\alpha_1} c_{\alpha_2} c_{\alpha_5} s_{\alpha_4} - (c_{\alpha_3} c_{\alpha_1} - s_{\alpha_1} s_{\alpha_2} s_{\alpha_3}) s_{\alpha_5}) - (-s_{\alpha_1} s_{\alpha_2} c_{\alpha_3} - c_{\alpha_1} s_{\alpha_3}) s_{\alpha_6} \\
R'_{43} &= c_{\alpha_6} (-c_{\alpha_5} s_{\alpha_2} s_{\alpha_4} - c_{\alpha_2} s_{\alpha_3} s_{\alpha_5}) - c_{\alpha_2} c_{\alpha_3} s_{\alpha_6} \\
R'_{44} &= c_{\alpha_4} c_{\alpha_5} c_{\alpha_6}
\end{aligned}$$

$$C_{h;VV} = \cos \beta R'_{i1}^{(r)} + \sin \beta R'_{i2}^{(r)}, \quad C_{h;t\bar{t}} = \frac{R'_{i2}^{(r)}}{\sin \beta}, \quad C_{h;b\bar{b}} = C_{h;\tau\bar{\tau}} = \frac{R'_{i1}^{(r)}}{\cos \beta}$$

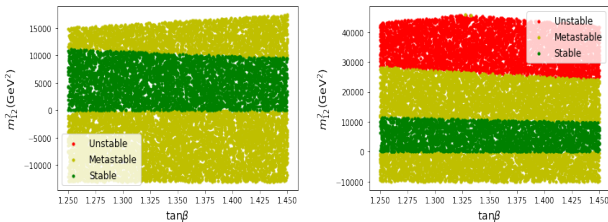


Figure: BP1 in N2HDM and 2HDMS :  $\Delta C/C \lesssim 15\%$ .  $\rightarrow \alpha_4 \approx \alpha_5 \approx \alpha_6 \lesssim 0.2$ .

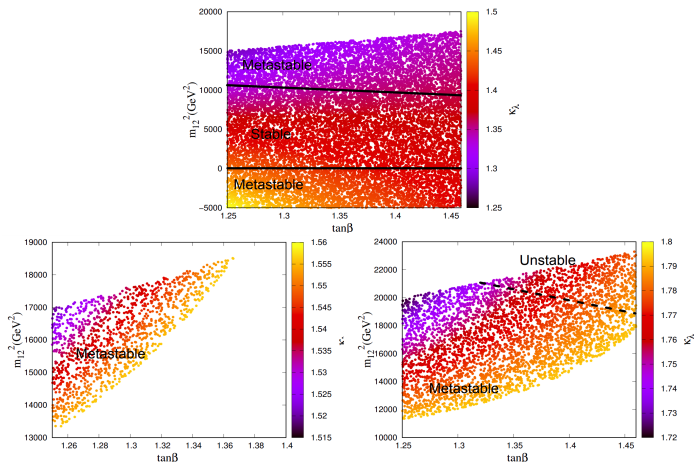
- The upper limit on  $m_{12}^2$  from BFB.
- The lower limit on  $m_{12}^2$  from perturbativity.

## Probing vacuum structure with tri-linear couplings

- We calculate the tri-linear coupling of the 125 GeV Higgs at tree-level in both models.
- Tri-linear coupling measurement can in addition put constraint on the still allowed parameter space of N2HDM and 2HDMS.
- In particular  $m_{12}^2$ ,  $\tan \beta$  and correspondingly all the  $\lambda$ 's become strongly constrained.

$$\kappa_\lambda = 1.4$$

## N2HDM



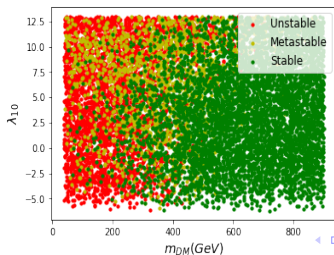
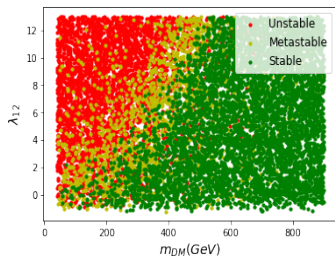
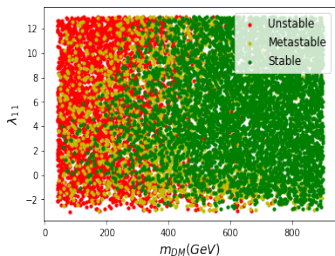
2HDMS:  $\Delta\kappa_\lambda/\kappa_\lambda \lesssim 10\%$  with  $\alpha \approx 0.05$  (left),  $\lesssim 30\%$  with  $\alpha \approx 0.1$  (right).



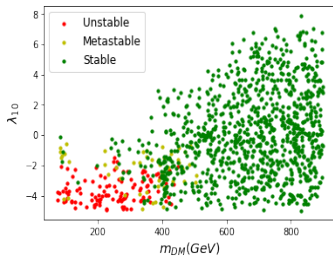
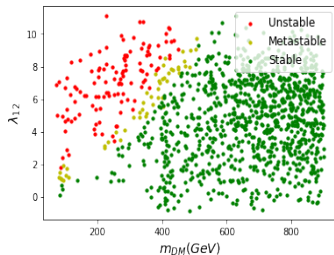
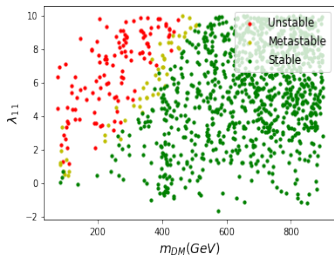
## Comparison between $\mathcal{N}_S$ vacuum of N2HDM and 2HDMS

- This phase of 2HDMS is the DM phase (**Dark 2HDMS**), unlike N2HDM.
- Here no mixing between the additional scalar sector of 2HDMS with the scalar sector of N2HDM.
- All the couplings (including tri-linear couplings) are same in both models at tree-level.
- Benchmark of N2HDM will map onto 2HDMS.
- Dark sector couplings are completely decoupled from the visible sector phenomenology, can be varied freely, other than impacts on invisible branchings of the scalars.
- This scenario changes at loop level.

# Impact of dark sector parameters on vacuum stability of 2HDMS



# After DM constraints are applied



## Conclusion

- There are additional charge and CP-breaking as well as “panic” neutral minima in 2HDMS compared to N2HDM.
- The stability criterion depends strongly on the extra free parameters of 2HDMS.
- In  $\mathcal{N}_{sp}$ -type vacuum, there can still be some difference in the vacuum stability of N2HDM and 2HDMS parameter points even if they lead to the similar masses and fermion and gauge boson couplings of scalars.
- Tri-linear coupling measurement can directly probe the scalar self-couplings and can thereby probe vacuum stability further.
- Loop effects can affect the outcomes of our analysis.
- In case of  $\mathcal{N}_s$ -type vacuum of 2HDMS, dark sector phenomenology is closely related to vacuum stability and corresponding difference between N2HDM and 2HDMS.

the singlet potential  $V_S$  is,

$$\begin{aligned} V_S = & m_S^2 S^* S + \left( \frac{m_S'^2}{2} S^2 + h.c. \right) \\ & + \left( \frac{\lambda_1''}{24} S^4 + h.c. \right) + \left( \frac{\lambda_2''}{6} (S^2 S^* S) + h.c. \right) + \frac{\lambda_3''}{4} (S^* S)^2 \\ & + S^* S [\lambda_1' \Phi_1^\dagger \Phi_1 + \lambda_2' \Phi_2^\dagger \Phi_2] + [S^2 (\lambda_4' \Phi_1^\dagger \Phi_1 + \lambda_5' \Phi_2^\dagger \Phi_2) + h.c.]. \quad (1) \end{aligned}$$

# Free parameters in both models

	Interaction basis	Mass-basis
N2HDM	$\lambda_{1,..8}, v_s, m_{12}^2, \tan \beta, v$	$m_{h_{1,..3}}, m_A, m_{H^\pm}, \alpha_{1,..3}, v_s, m_{12}^2, \tan \beta, v$
2HDMS	$\lambda_{1,..12}, v_s, v_p, m_{12}^2, \tan \beta, v$	$m_{h_{1,..4}}, m_A, m_{H^\pm}, \alpha_{1,..6}, v_s, v_p, m_{12}^2, \tan \beta, v$

## Stability Criteria

In both N2HDM and 2HDMS

Where

$$\mathcal{V}_{CB} - \mathcal{V}_{Ns} = \left(\frac{m_{H^\pm}^2}{2v^2}\right) N_s [(v'_2 c_1 - v'_1 c_3)^2 + v_1'^2 c_2^2] - s^2 m_{S1}^2 > 0 \quad (2)$$

with

$$m_{S1}^2 = m_S^2 + \lambda_7 c_1^2 + \lambda_8 (c_2^2 + c_3^2) \quad (3)$$

In 2HDMS in addition

$$\mathcal{V}_{CBp} - \mathcal{V}_{Ns} = \left(\frac{m_{H^\pm}^2}{2v^2}\right) N_s [(v'_2 c_1 - v'_1 c_3)^2 + v_1'^2 c_2^2] + c_5^2 m_p^2 - s^2 m_{S1}^2 > 0 \quad (4)$$

$m_p^2 = m_{S'}^2 + \frac{1}{4} \lambda_{10} s^2 + \frac{1}{2} (\lambda_{11} v_1'^2 + \lambda_{12} v_2'^2)$  is the pseudoscalar DM mass and  
 $m_{S1}^2 = m_S^2 + \lambda_7 c_1^2 + \lambda_8 (c_2^2 + c_3^2) + \frac{1}{4} \lambda_{10} c_5^2$

For  $N_{sp}$ -type vacuum of 2HDMS the stability criteria are in addition,

$$\mathcal{V}_{CB} - \mathcal{V}_{N_{sp}} = \left(\frac{m_{H^\pm}^2}{2v^2}\right)_{N_{sp}} [(v_2' c_1 - v_1' c_3)^2 + v_1'^2 c_2^2] - s^2 m_{S1}^2 - p^2 m_{S2}^2 > 0 \quad (5)$$

Where

$$m_{S1}^2 = m_S^2 + \lambda_7 c_1^2 + \lambda_8 (c_2^2 + c_3^2) \quad (6)$$

$$m_{S2}^2 = m_S'^2 + \lambda_{11} c_1^2 + \lambda_{12} (c_2^2 + c_3^2) \quad (7)$$

$$\mathcal{V}_{CBs} - \mathcal{V}_{N_{sp}} = \left(\frac{m_{H^\pm}^2}{2v^2}\right)_{N_{sp}} [(v_2' c_1 - v_1' c_3)^2 + v_1'^2 c_2^2] - p^2 m_{S2}^2 > 0 \quad (8)$$

Where

$$m_{S2}^2 = m_S'^2 + \lambda_{11} c_1^2 + \lambda_{12} (c_2^2 + c_3^2) + \frac{1}{4} \lambda_{10} c_4^2 \quad (9)$$



$$\mathcal{V}_{CBp} - \mathcal{V}_{N_{sp}} = \left(\frac{m_{H^\pm}^2}{2v^2}\right) N_{sp} [(v'_2 c_1 - v'_1 c_3)^2 + v_1'^2 c_2^2] - s^2 m_{S1}^2 > 0 \quad (10)$$

Where

$$m_{S1}^2 = m_S^2 + \lambda_7 c_1^2 + \lambda_8 (c_2^2 + c_3^2) + \frac{1}{4} \lambda_{10} c_5^2 \quad (11)$$

One *metastable* BP of N2HDM in mass basis:

$m_{h_1}$	$m_{h_2}$	$m_{h_3}$	$m_A = m_H^\pm$	$m_{12}^2$	$\tan \beta$	$v_s$	$\{\alpha_1, \alpha_2, \alpha_3\}$
95	125	607.8	628.0	-13222.9	1.48	286.1	$\{-0.45, 0.86, -0.09\}$

In the interaction basis:

$\lambda_1$	$\lambda_2$	$\lambda_3$	$\lambda_4 = \lambda_5$	$\lambda_6$	$\lambda_7$	$\lambda_8$	$m_{12}^2$	$\tan \beta$	$v_s$
12.44	0.58	12.84	-6.99	3.99	6.35	-0.30	-13222.9	1.48	286.1

# In 2HDMS

