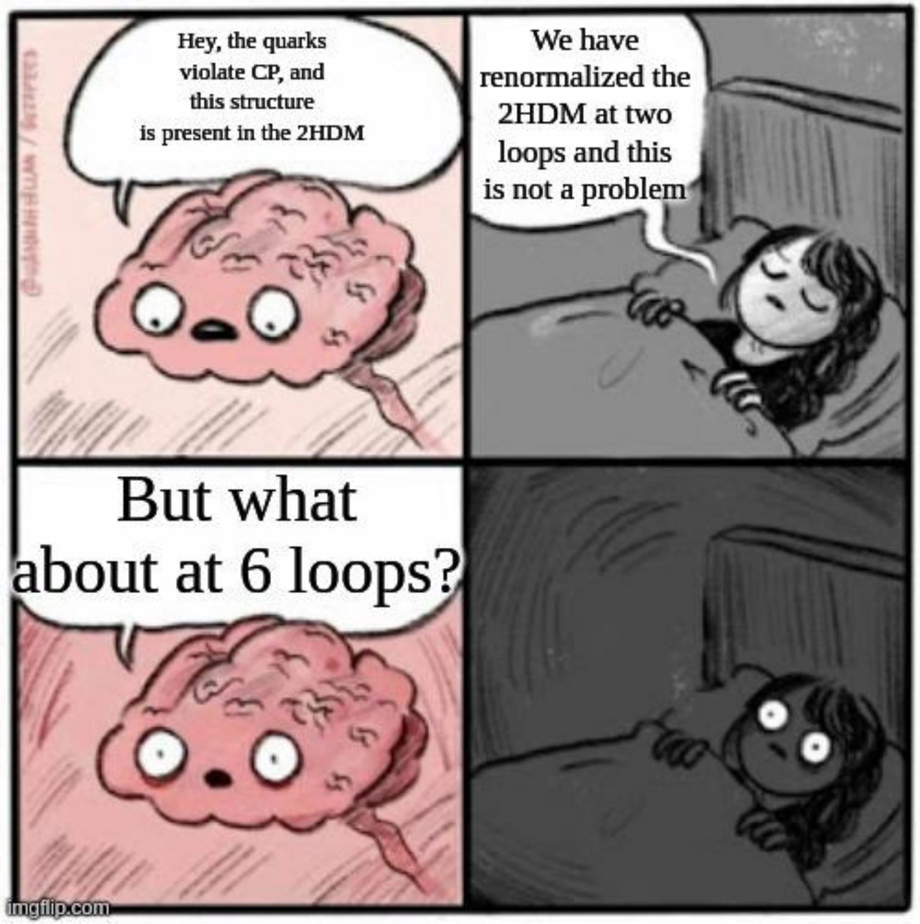
Can CP be conserved in the two-Higgs doublet model?



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2403.17052

Workshop on Multi-Higgs Models 2024

Motivation

- We got interested in this question after I came here in 2022.

Leaks of CP violation in the real two-Higgs doublet model

Duarte Fontes, Maximilian Löschner, Jorge C. Romão, João P. Silva

We discuss the Z_2 symmetric two Higgs doublet model with a real soft breaking term (real 2HDM). We explain in detail why it is not tenable to assume CP conservation in the scalar sector to keep the dimension two term real, while CP is violated by the dimension four Yukawa couplings. We propose the calculation of the infinite tadpole of the (would-be) pseudoscalar neutral scalar. We construct a simple toy model with the same flaws, where the unrenormalizable infinity is easier to calculate. We then turn our attention to the same tadpole in the real 2HDM. We spearhead this effort focusing on diagrams involving solely bare quantities. This involves hundreds of Feynman three-loop diagrams that could feed the CP violation from the quark into the scalar sector, and is only possible with state of the art automatic computation tools. Remarkably, some intermediate results agree when using three independent derivations, including the peculiar cancellation of the leading pole divergence due to a subtle interplay between masses and the Jarlskog invariant, which we calculate analytically. The calculation is not complete however, since the full two-loop renormalization of the real 2HDM is not yet available in the literature. Still, we argue convincingly that there is an irremovable infinity.

CP-leaks in the real two-Higgs doublet model

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Introduction

- CP violation discovered 1964
  - $K_S \to \pi\pi$ (CP even), $K_L \to \pi\pi\pi$ (CP odd) but also $K_L \to \pi\pi$ 0.3% of the time!

- CPV from the SM is completely described by the quark mixing matrix (Jarlskog invariant).

- 2HDM extends the SM scalar sector with another doublet which can have additional CPV, literature explore the complex 2HDM and real 2HDM. Most recent phenomenological studies focus on the real 2HDM because of EDM constraints.

Is the real 2HDM a theoretically consistent model since we know that the SM has CPV?

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The problem

- We know there is CP violation in the CKM matrix.
- CKM CPV can be transmitted to other operators via loop diagrams
- CKM phase is hard-breaking of CP, so no apparent reason why those generated imaginary parts shouldn't be divergent!
- The real 2HDM would not have the parameters to absorb this divergence: not theoretically consistent!

Is the real 2HDM a theoretically consistent model since we know that the SM has CPV?

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Softly broken Z_2 2HDM

- Softly broken Z_2 2HDM have one physical CP phase $\mathrm{Im}((m_{12}^{2\,*})^2\lambda_5)$:

$$V_{2HDM} = m_{11}^2\phi_1^\dagger\phi_1 + m_{22}^2\phi_2^\dagger\phi_2 - m_{12}^2\phi_1^\dagger\phi_2$$
$$+\frac{1}{2}\lambda_1(\phi_1^\dagger\phi_1)^2+\frac{1}{2}\lambda_2(\phi_2^\dagger\phi_2)^2+\lambda_3(\phi_1^\dagger\phi_1)\left(\phi_2^\dagger\phi_2\right) + \lambda_4(\phi_1^\dagger\phi_2)(\phi_2^\dagger\phi_1)$$
$$+\frac{1}{2}\lambda_5(\phi_1^\dagger\phi_2)^2+\mathrm{h.c.}$$

- Real 2HDM has both λ_5 and m_{12}^2 real on the same basis. The quark sector have CP violation from complex Yukawas:

$$\mathcal{L}_{Yuk} = -Y_d^{(1)}\bar{Q}_L\phi_1 d_R - Y_u^{(2)}\bar{Q}_L\tilde{\phi}_2 d_R + \mathrm{h.c.} \quad \text{(type II)}$$

This talk covers Type II only, for Type I see 2403.17052

Symmetries protecting CP

- The Yukawa sector is invariant under a global U(1) symmetry

$$\phi_1 \leftrightarrow e^{-i\theta}\phi_1 \qquad \phi_2 \leftrightarrow e^{i\theta}\phi_2$$

$$u_R \to e^{i\theta}u_R \quad d_R \to e^{-i\theta}d_R \quad \text{(type I)}$$
$$u_R \to e^{i\theta}u_R \quad d_R \to e^{i\theta}d_R \quad \text{(type II)}$$

- Imposing this symmetry forces $\lambda_5 = 0$
- The RGE for λ_5 and m_{12}^2 must transform under this symmetry:
  - RGE equation for λ_5 must be proportional to λ_5 !

CP is conserved at all orders in the U(1) 2HDM

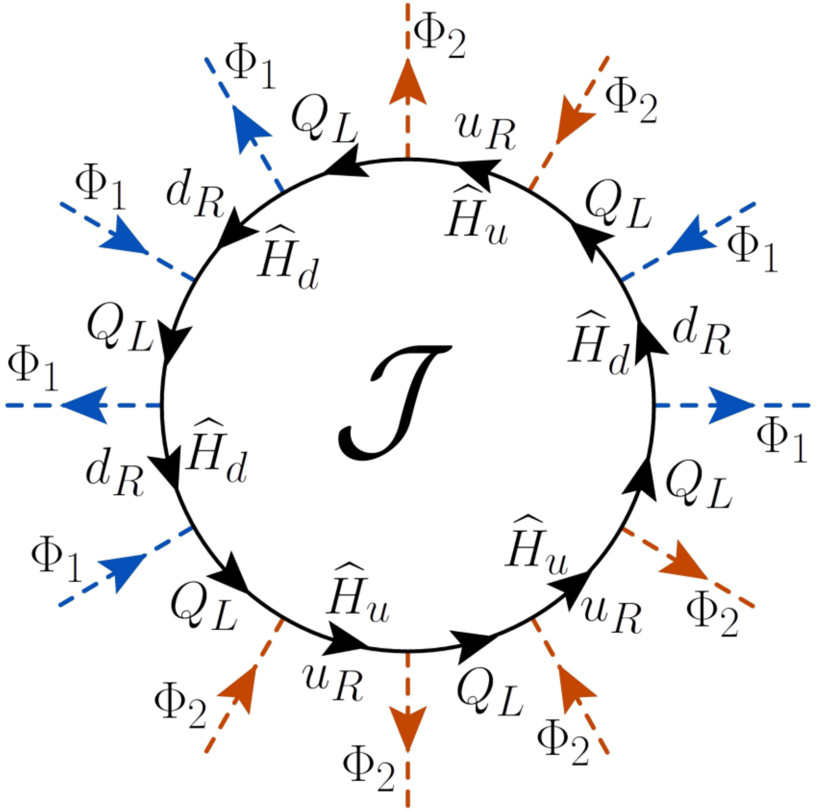
The primitive diagram

- Jarlskog invariant is proportional to 12 Yukawas

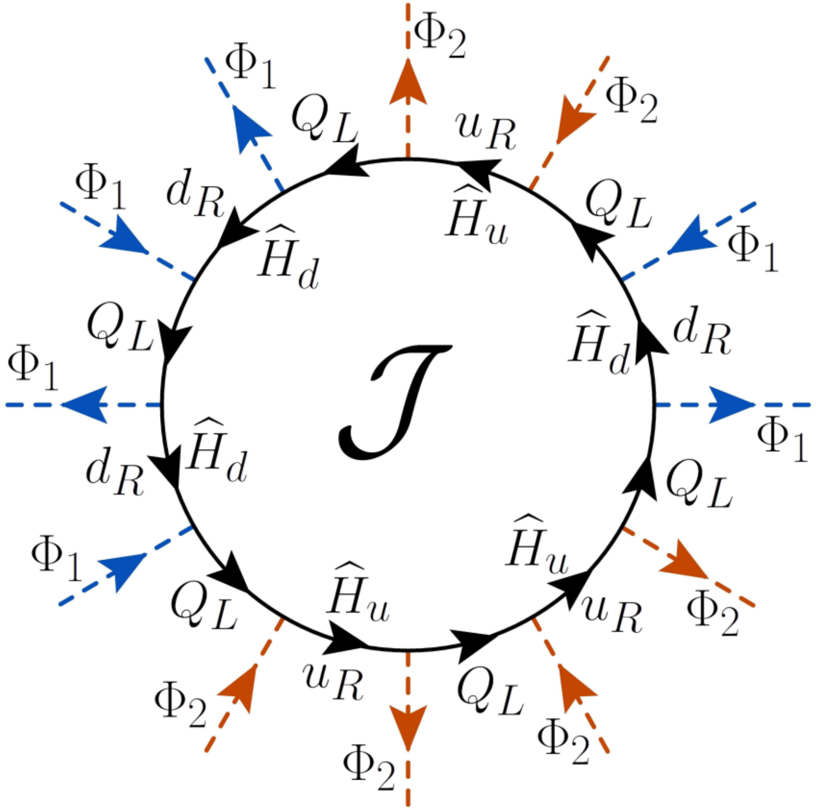
$\hat{H}_u = Y_u Y_u^\dagger \quad \hat{H}_d = Y_d Y_d^\dagger \qquad \boldsymbol{\mathcal{J}} = \mathrm{Tr}(\hat{H}_u \hat{H}_d \hat{H}_u{}^2 \hat{H}_d{}^2)$

- Any diagram that contribute to $\mathrm{Im}(\lambda_5)$ comes from the primitive $\boldsymbol{\mathcal{J}}$ and $\boldsymbol{\mathcal{J}}^*$ diagrams.

- Can we find relations between the diagrams that are preserved once we start closing legs? YES!



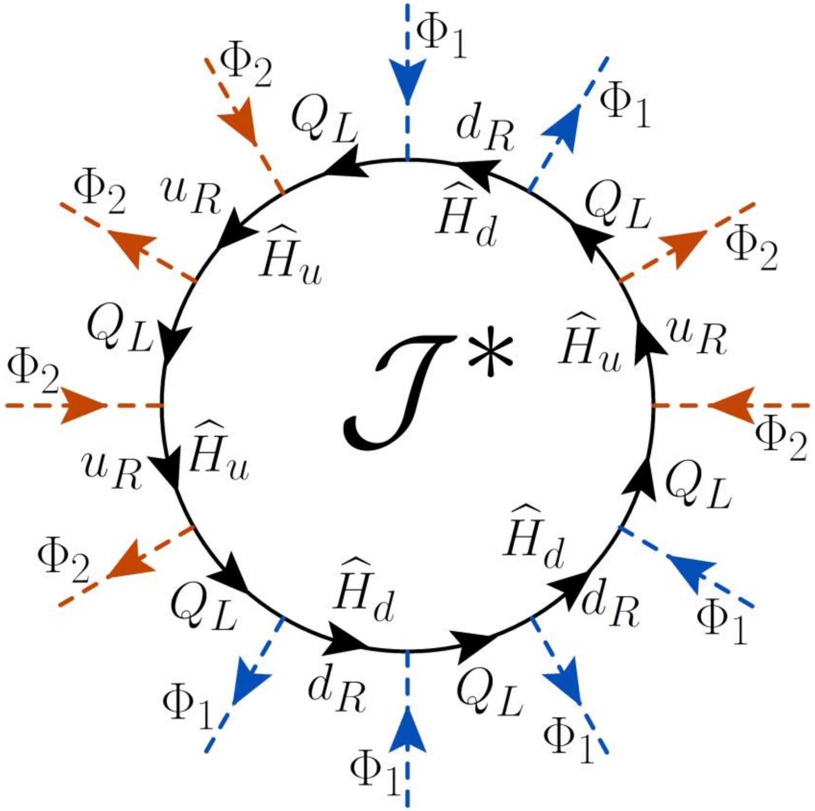
Symmetries of the primitive diagram



Generalized CP transformation

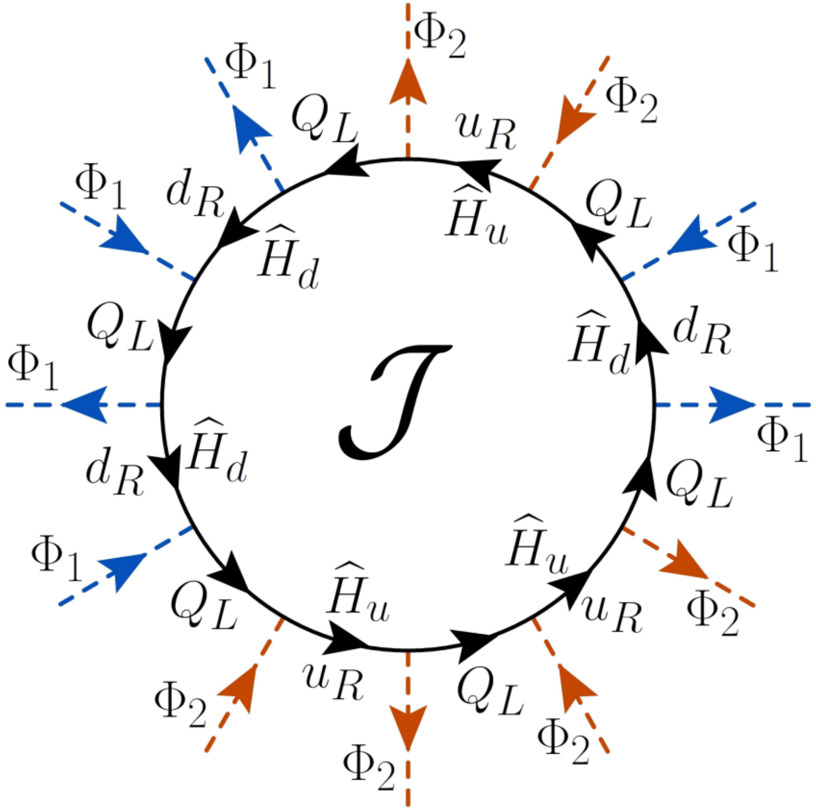
$\phi_1 \leftrightarrow \tilde{\phi}_2$ (type II)

λ_5 is invariant!

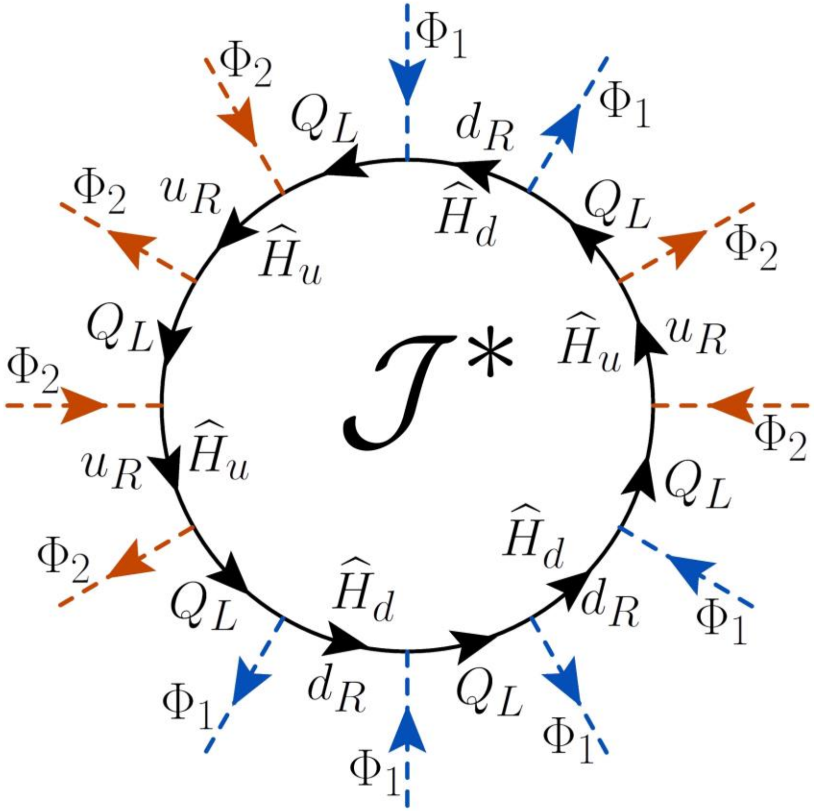


SU(2) structure preserved!

Symmetries of the primitive diagram



Close external legs on both sides and match diagrams

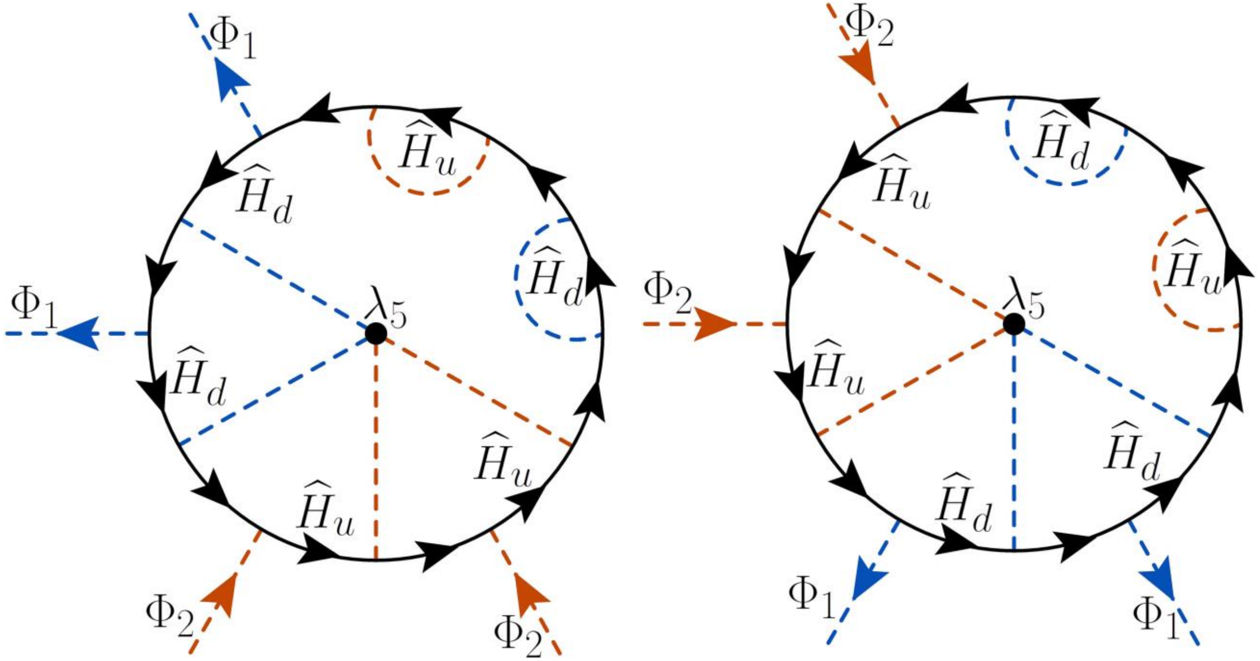


Divergent contribution does not distinguish internal ϕ_1 and ϕ_2!

λ_5 at 6 loops

Type II – 6 loops

- For each diagram, there exists a second diagram with identical topology and momentum structure but in which $u_R \leftrightarrow d_R$
- 3k diagrams proportional to $\mathcal{J}$
- Always pairwise diagrams as λ_5 is invariant under this symmetry! (Note that λ_1 and λ_2 are not, important later!)
- This means that we have the divergent contribution for λ_5 to be **REAL**!



No divergent leaks at 6 loops

No leaks of CP in the 2HDM at 6 loops

- The contribution for both type I and II (X and Y also) is real, but for different reasons!
- Type I – Also cancel, but from the topology of the diagrams, see more in 2403.17052
- Type II – Generalized CP symmetry protects the CP violation at 6 loop. The rest of the Lagrangian does not respect this symmetry, do we have leaks at 7 loop?
- Complete the sequence:

$$0, 0, 0, 0, 0, 0, 0, ?$$

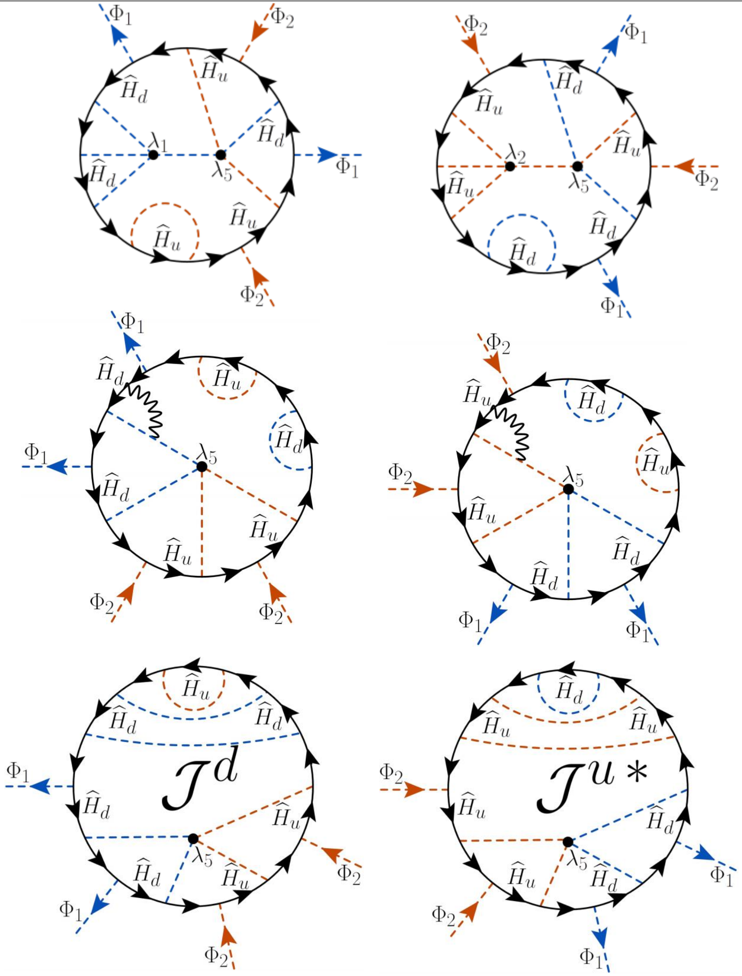
λ_5 at 7 loops

Type II – Call the plumber! CP is leaking

GCP transformation we have $\lambda_1 \leftrightarrow \lambda_2$. This breaks the cancelation of imaginary part if $\lambda_1 \neq \lambda_2$

The Hypercharge interaction differentiate u and d. This breaks the cancelation of imaginary proportional to the difference of Hypercharges.

Inclusion of one additional Yukawa breaks the symmetry between u and d. The relation now relates different objects and no cancelation of imaginary divergent piece!



Type I also leaks at 7 loops, but for **different** reasons!

Is the Real 2HDM inconsistent?

- If the RG evolution is indeed nonzero, even if extremely small, the real 2HDM would not be a theoretically consistent model! One should start to question if it makes sense to restrict the CP violation to zero when the model does not respect this decision.

- There could be additional symmetries which we did not spot canceling the 7-loop diagrams. Only brute force checking, or another formalism can tackle this.

$$\frac{d\,\mathrm{Im}(\lambda_5)}{d\ln\mu} = \frac{\lambda_5 \mathrm{Im}(\mathcal{J})}{(16\pi^2)^7} \begin{cases} \left[a^{\lambda}(\lambda_1 - \lambda_2) + a^{g'} g'^2 + a^{y}(y_t^2 - y_b^2 + \ldots)\right] & \text{(type II)} \\ \left[b^{\lambda_3}\lambda_3 + b^{\lambda_4}\lambda_4 + b^{g'} g'^2 + b^{g} g^2\right] & \text{(type I)} \end{cases}$$

Thank You

Extra material

General 2HDM

- Add a second Higgs doublet to the SM:
  - Immediately screw up flavor and CP! Need model building to avoid experimentally-excluded levels of flavor and CPV

$$V_{2HDM} = m_{11}^2\phi_1^\dagger\phi_1 + m_{22}^2\phi_2^\dagger\phi_2 - m_{12}^2\phi_1^\dagger\phi_2$$
$$+\frac{1}{2}\lambda_1(\phi_1^\dagger\phi_1)^2+\frac{1}{2}\lambda_2(\phi_2^\dagger\phi_2)^2+\lambda_3(\phi_1^\dagger\phi_1)(\phi_2^\dagger\phi_2) + \lambda_4(\phi_1^\dagger\phi_2)(\phi_2^\dagger\phi_1)$$
$$+\frac{1}{2}\lambda_5(\phi_1^\dagger\phi_2)^2+\lambda_6(\phi_1^\dagger\phi_1)(\phi_1^\dagger\phi_2) + \lambda_7(\phi_2^\dagger\phi_2)(\phi_1^\dagger\phi_2) + \text{h.c.}$$

$$\mathcal{L}_{Yuk} = -Y_d^{(1)}\bar{Q}_L\phi_1 d_R - Y_u^{(1)}\bar{Q}_L\tilde{\phi}_1 d_R$$
$$-Y_d^{(2)}\bar{Q}_L\phi_2 d_R - Y_u^{(2)}\bar{Q}_L\tilde{\phi}_2 d_R + \text{h.c.}$$

Natural flavor conservation

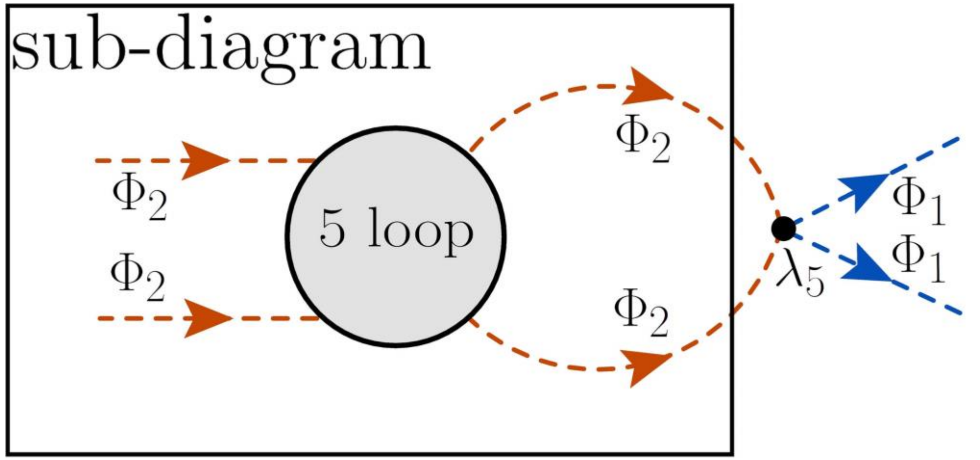
- Sidestep the FCNC problem by imposing Natural Flavor Conservation
- Easy to impose using a Z_2 symmetry:

$$\phi_1 \to -\phi_1 \qquad \phi_2 \to \phi_2$$
$$u_R \to u_R \quad d_R \to d_R \quad \text{(type I)}$$
$$u_R \to u_R \quad d_R \to -d_R \quad \text{(type II)}$$

- Z_2 forces $\lambda_6 = 0, \lambda_7 = 0$ and $m_{12}^2 = 0$
- Z_2 2HDM has no decoupling limit → needs soft breaking ($m_{12}^2 \neq 0$)

Type I – 6 loops

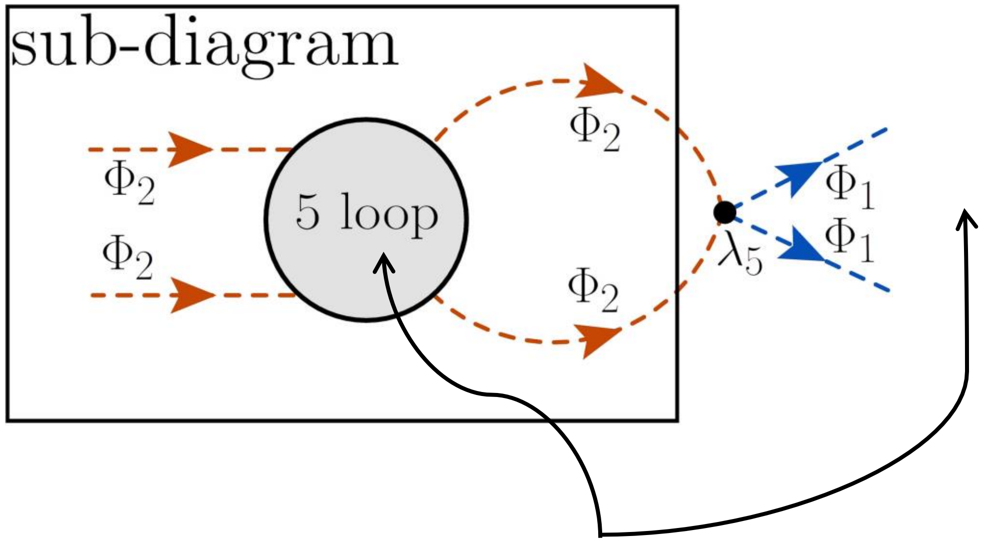
- Type I is special, the only way to create 2 ϕ_1 are with the λ_5 insertion.
- Cut the λ_5 internal propagators → separate the diagram into a 5 loop sub-diagram
- The constant contribution of the 5 loop diagram is Hermitian! The divergent contribution of the 6 loop diagram is then also Hermitian!
- The matching is harder to spot
- 10k diagrams proportional to $\mathcal{J}$



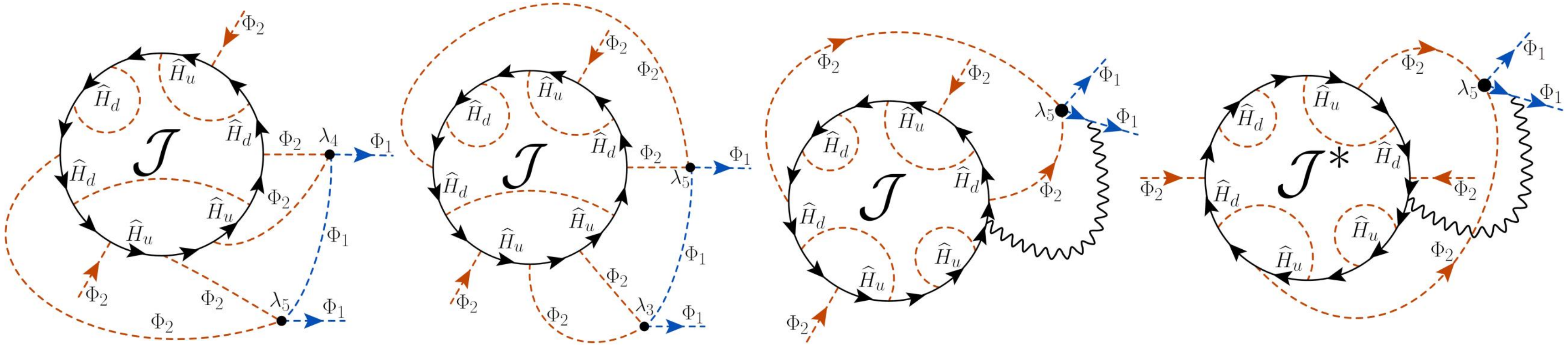
No divergent leaks at 6 loops

Type I – Call the plumber! CP is leaking

- Diagrams which renormalize λ_2 or that renormalize the external line cannot generate an imaginary divergent contribution.
- Gauge interactions and diagrams containing λ_3 or λ_4 can destroy the sub-diagram structure! No clear cancelation!



Loops on the external legs or inside preserve the sub diagram structure

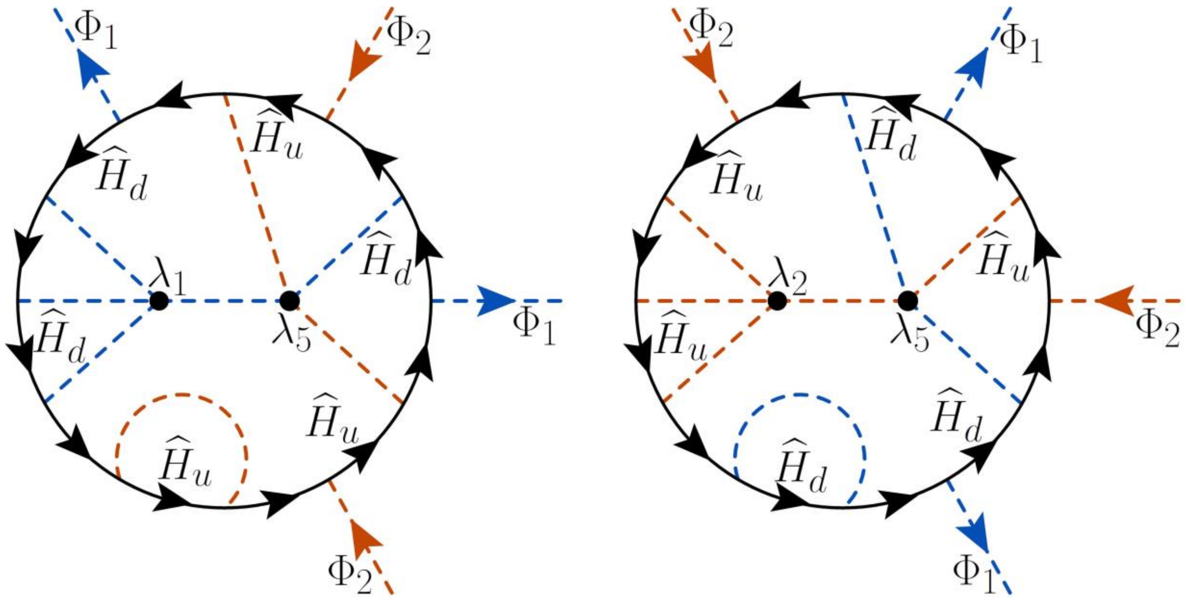


Type II – Call the plumber! CP is leaking

- Under the GCP transformation we have $\lambda_1 \leftrightarrow \lambda_2$. This breaks the cancelation of imaginary part if $\lambda_1 \neq \lambda_2$!

- We expect an imaginary divergent contribution to λ_5 at 7 loops proportional to

$$\lambda_5(\lambda_1 - \lambda_2)\,\mathrm{Im}(\mathcal{J})$$

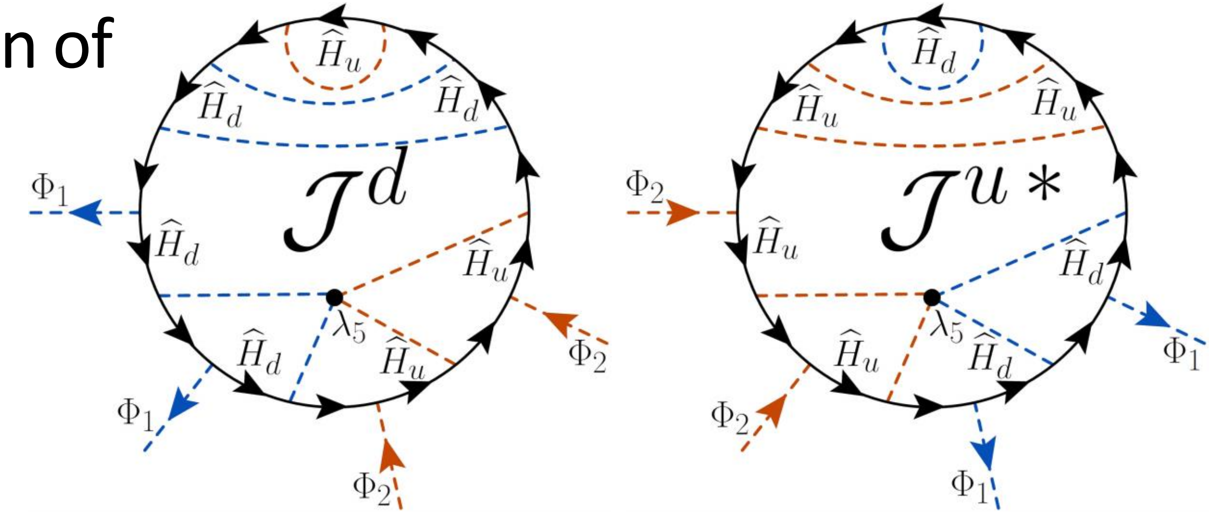


Type II – Call the plumber! CP is leaking

- Inclusion of one additional Yukawa breaks the symmetry between u and d. The relation now relates different objects and no cancelation of imaginary divergent piece!

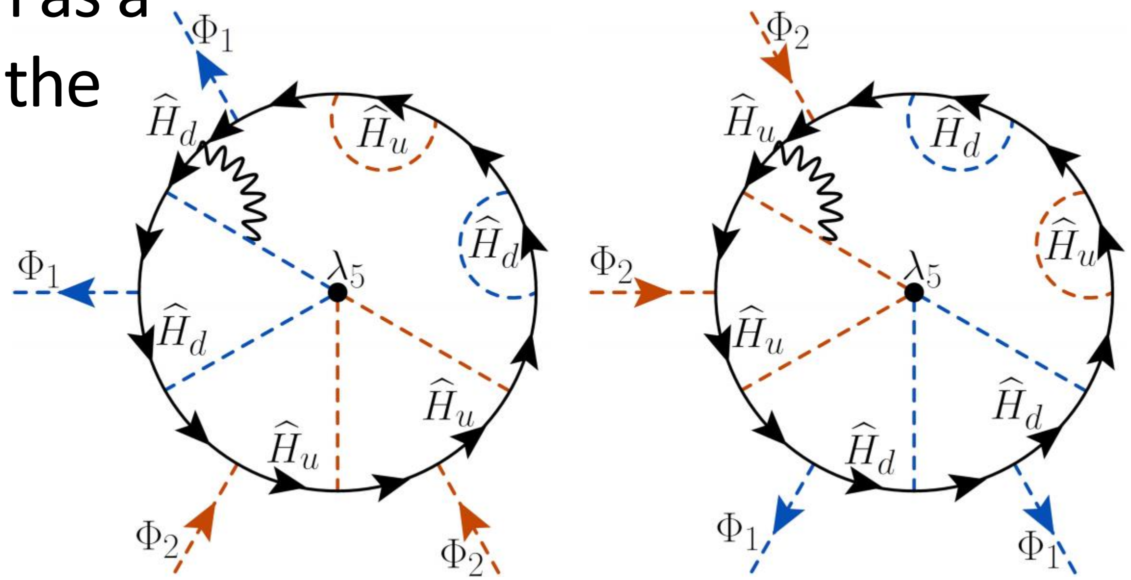
- We expect then an imaginary divergent contribution proportional to

$$\lambda_5 \mathrm{Im}\left(\mathcal{J}^u - \mathcal{J}^d\right) = \left(y_t^2 - y_b^2 + \cdots\right) \lambda_5 \mathrm{Im}(\mathcal{J})$$



Type II – Call the plumber! CP is leaking

- The Hypercharge interaction differentiate between u and d. This breaks the cancelation as a function of the difference of Hypercharge of the quarks!

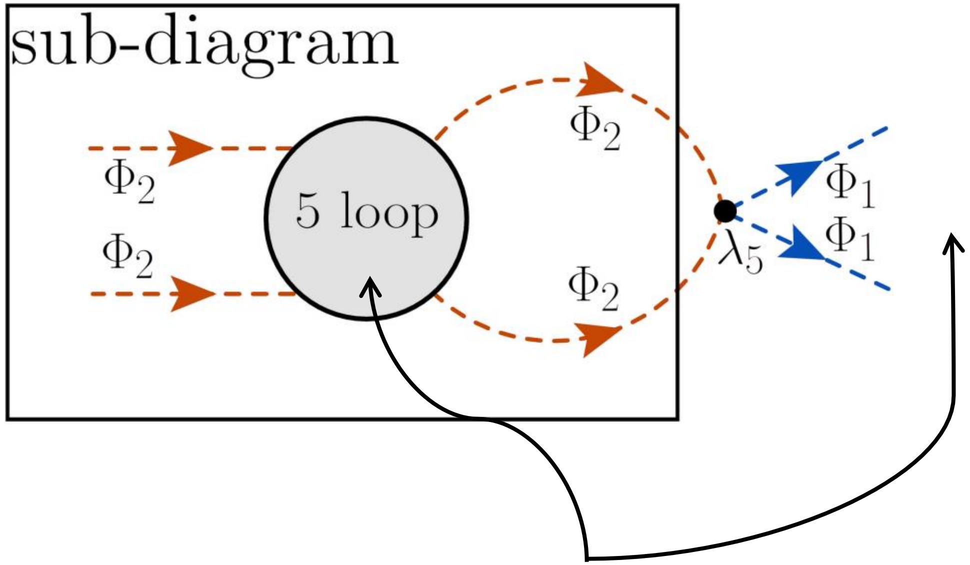


- We expect then an imaginary divergent contribution proportional to

$$\lambda_5 g'^2 \mathrm{Im}(\mathcal{J})$$

Type I – Call the plumber! CP is leaking

- Diagrams which renormalize λ_2 or that renormalize the external line cannot generate an imaginary divergent contribution

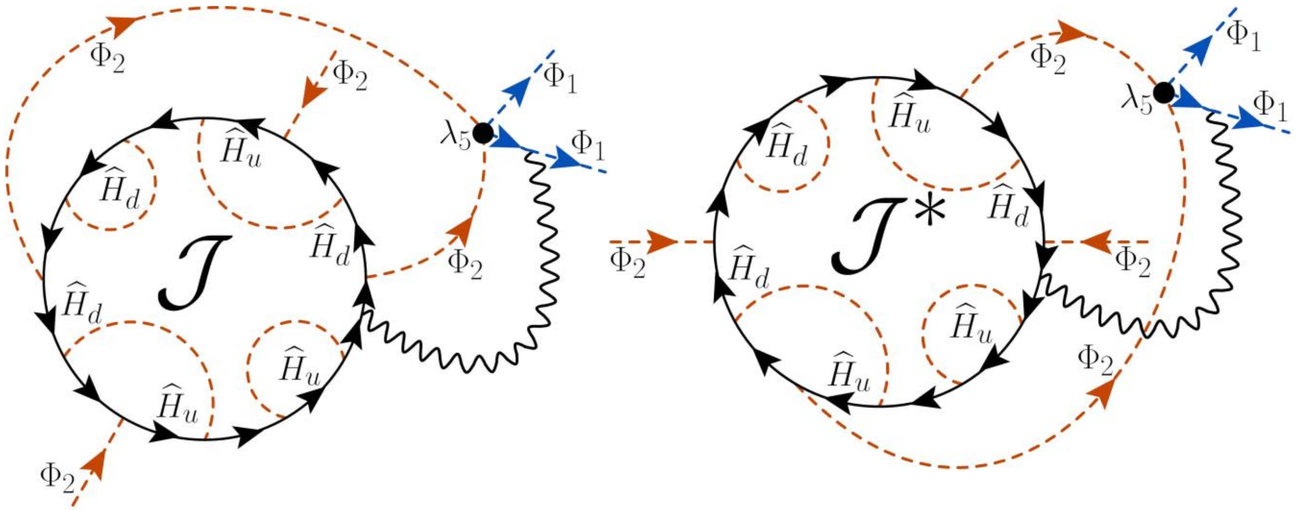


- However, not all diagrams have this topology!

Loops on the external legs or inside preserve the sub diagram structure

Type I – Call the plumber! CP is leaking

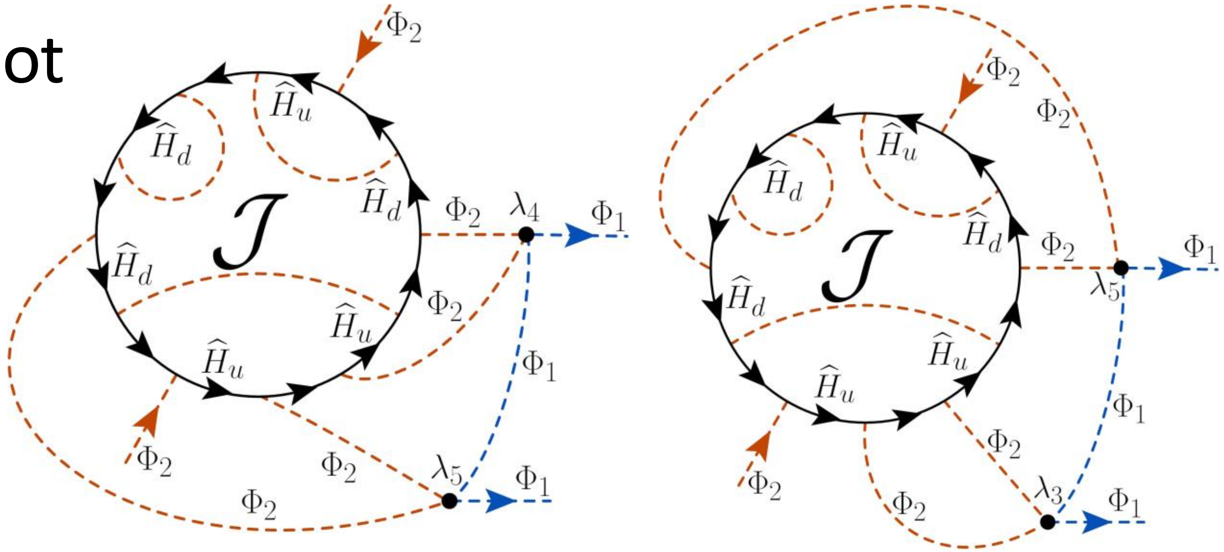
- Gauge interactions can destroy the sub-diagram structure. Becomes harder to analyze!



- In the fermion line, u and d are indistinguishable. The gauge interactions should not be able to tell the primitive diagrams apart. However, no clear path to proof this

Type I – Call the plumber! CP is leaking

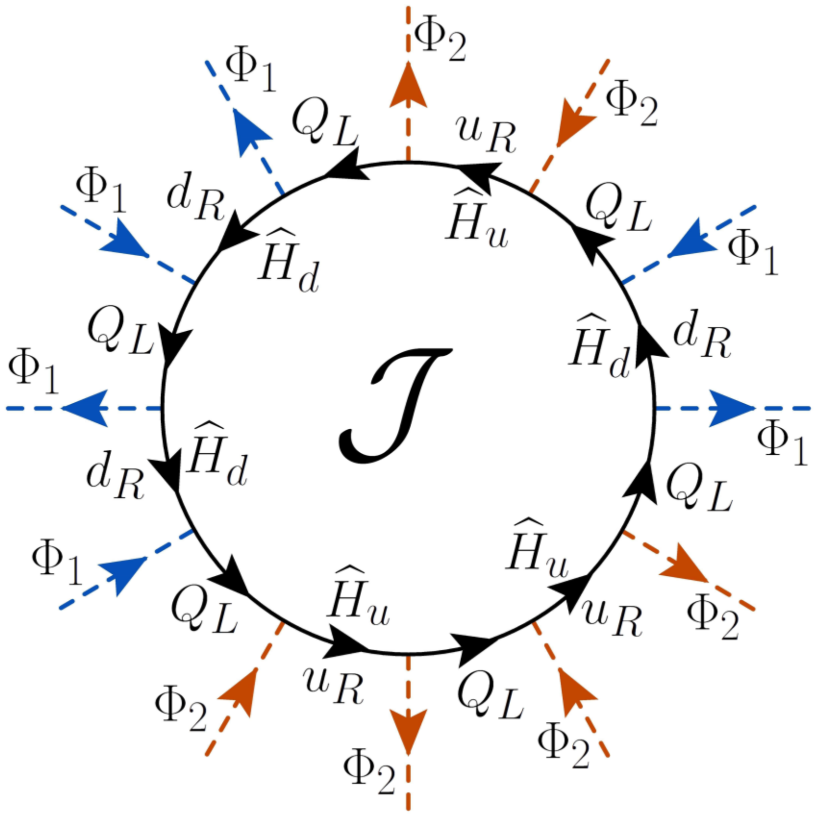
- Diagrams containing λ_3 or λ_4 also do not cancel, this structure is impossible to construct from $\mathcal{J}^*$!



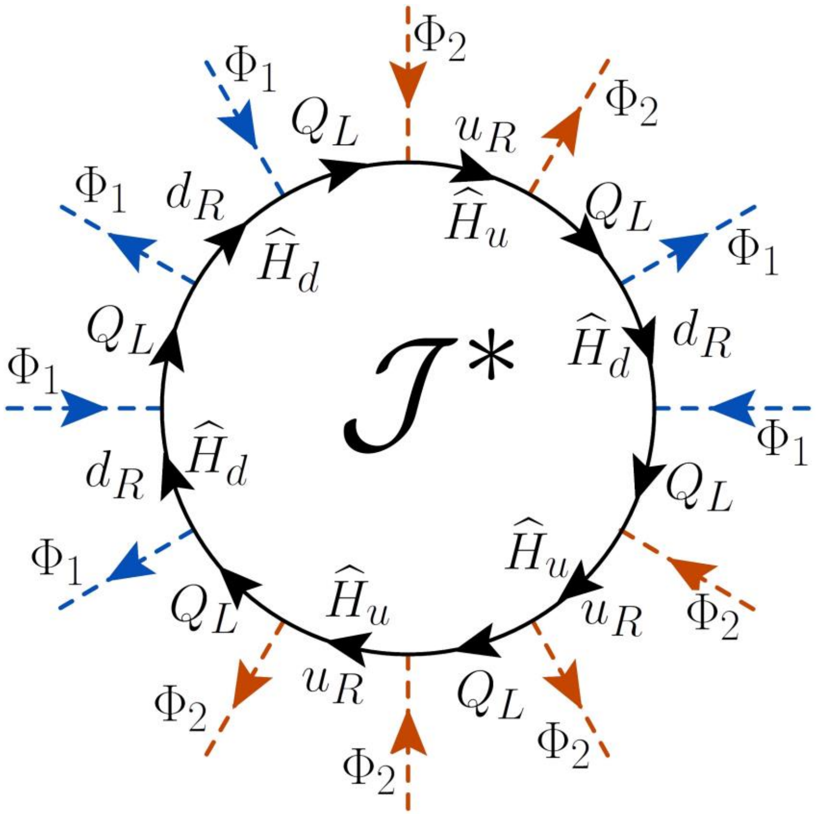
- We expect an imaginary divergent contribution proportional to λ_3 and λ_4

- No clear relation between the coefficients of these terms!

Reversal of Fermion flow



Reversing the fermion flow



$$\mathcal{L}_{Yuk} = -Y_d^{(1)}\bar{Q}_L\phi_2 d_R - Y_u^{(2)}\bar{Q}_L\tilde{\phi}_2 d_R + \text{h.c.} \quad \text{(type I)}$$