One-loop scalar mass calculations in 2HDMs

- **Tomás Pinto** (Faculty of Sciences of the University of Lisbon, Master in Physics)
- Supervisor: **Pedro M. Ferreira** (ISEL and CFTC, University of Lisbon)

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Ciências ULisboa

Objectives

- Compute the one-loop self-energy for the pseudoscalars in the 2HDM.
- Compare two similar cases: $U(1)$ and Z_3 symmetries.
- How do these extensions impact the mass of the physical pseudoscalar?

Outline

- 2HDM with a $U(1)$ global symmetry
- Spontaneous Symmetry Breaking
- A model-independent method to compute scalar self-energies
- One-loop pseudoscalar self-energy

2HDM scalar potential with a $U(1)$ global symmetry

$$U(1): \quad \Phi_1 \to U\Phi_1, \; \Phi_2 \to \Phi_2$$

$$m_{12} = \lambda_5 = \lambda_6 = \lambda_7 = 0$$

The most general renormalizable potential, with a $U(1)$ symmetry, is

$$V = m_{11}^2 \Phi_1^\dagger \Phi_1 + m_{22}^2 \Phi_2^\dagger \Phi_2 + \frac{\lambda_1}{2}\left(\Phi_1^\dagger \Phi_1\right)^2 + \frac{\lambda_2}{2}\left(\Phi_2^\dagger \Phi_2\right)^2 + \lambda_3 \left(\Phi_1^\dagger \Phi_1\right)\left(\Phi_2^\dagger \Phi_2\right) + \lambda_4 \left(\Phi_1^\dagger \Phi_2\right)\left(\Phi_2^\dagger \Phi_1\right)$$

where all parameters are real.

The scalar spectrum includes:

- 3 pseudo-Goldstone Bosons – 2 charged $G^\pm$ and 1 neutral (CP-odd) G^0
- 2 neutral (CP-even) scalars, h and H (one of which is the Higgs boson)
- 2 physical charged scalars, $H^\pm$
- 1 physical neutral pseudoscalar, A

It is well known that the scalar potential (and gauge sector) is the same for a 2HDM with a Z_3 symmetry instead (*accidental* $U(1)$ *symmetry*). However, this is not necessarily true for fermions.

Spontaneous Symmetry Breaking

When the doublets acquire non-zero VEVs, occurs the SSB of the global $U(1)$.
For normal minima, we find the following (tree-level) minimization conditions

$$m_{11}^2 + \frac{1}{2}\left[\lambda_1 v_1^2 + (\lambda_3 + \lambda_4)\, v_2^2\right] = 0$$

$$m_{22}^2 + \frac{1}{2}\left[\lambda_2 v_2^2 + (\lambda_3 + \lambda_4)\, v_1^2\right] = 0$$

Normal vacua

$$\langle \Phi_1 \rangle = \begin{pmatrix} 0 \\ \frac{v_1}{\sqrt{2}} \end{pmatrix}$$

$$\langle \Phi_2 \rangle = \begin{pmatrix} 0 \\ \frac{v_2}{\sqrt{2}} \end{pmatrix}$$

$$v_1, v_2 \in \mathbb{R}$$

The pseudoscalar mass matrix (in the minimum) is

$$\left[M^2_{\text{CP-odd}}\right]_{\min} = \begin{pmatrix} m_{11}^2 + \frac{1}{2}\left[\lambda_1 v_1^2 + (\lambda_3+\lambda_4)v_2^2\right] & 0 \\ 0 & m_{22}^2 + \frac{1}{2}\left[\lambda_2 v_2^2 + (\lambda_3+\lambda_4)v_1^2\right] \end{pmatrix} = \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}$$

which means that the physical pseudoscalar A is a Goldstone boson (Goldstone's theorem).

However, the theorem only applies to discrete symmetries... We will then calculate the pseudoscalar mass at one-loop level, which is when fermions start contributing.

A model-independent method to compute scalar self-energies

In a general renormalizable theory containing real scalars, two-component Weyl fermions and vector fields, the interaction Lagrangian terms are

$$\mathcal{L}_{\mathrm{S}} = -\frac{1}{6}\lambda^{ijk}R_iR_jR_k - \frac{1}{24}\lambda^{ijkl}R_iR_jR_kR_l$$

$$\mathcal{L}_{\mathrm{FV}} = g_I^{aJ}A_a^{\mu}\psi^{\dagger I}\overline{\sigma}_{\mu}\psi_J$$

$$\mathcal{L}_{\mathrm{SV}} = -g^{aij}A_a^{\mu}R_i\partial_{\mu}R_j - \frac{1}{4}g^{abij}A_a^{\mu}A_{\mu b}R_iR_j - \frac{1}{2}g^{abi}A_a^{\mu}A_{\mu b}R_i$$

$$\mathcal{L}_{\mathrm{SF}} = -\frac{1}{2}y^{IJk}\psi_I\psi_JR_k + \mathrm{c.c.}$$

[Stephen P . Martin, 2002, DOI:10.1103/physrevd.65.116003]
[Stephen P . Martin, 2004, DOI:10.1103/PhysRevD.70.016005]

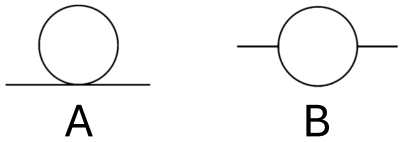
after expanding around the VEVs of the scalars. Here, all fields are physical eigenstates. The self-energy function is defined so that the pole masses are the solutions for complex

$$s_k = M_k^2 - i\Gamma_k M_k$$

of the equation

$$\mathrm{Det}\left[(m_i^2 - s_k^{(1)})\,\delta_{ij} + \frac{1}{16\pi^2}\Pi_{ij}^{(1)}(m_k^2)\right] = 0$$

One-loop self-energy matrix function for scalars



$$\Pi^{(1)}_{ij} = \frac{1}{2}\lambda^{ijkk}\mathrm{A}_S(m_k^2) + \frac{1}{2}\lambda^{ikl}\lambda^{jkl}\mathrm{B}_{SS}(m_k^2, m_l^2) + \mathrm{Re}[y^{KLi}y_{KLj}]\mathrm{B}_{FF}(m_K^2, m_L^2) + \mathrm{Re}[y^{KLi}y^{K'L'j}M_{KK'}M_{LL'}]\mathrm{B}_{\overline{FF}}(m_K^2, m_L^2)$$
$$+ g^{aik}g^{ajk}\mathrm{B}_{SV}(m_k^2, m_a^2) + \frac{1}{2}g^{aaij}\mathrm{A}_V(m_a^2) + \frac{1}{2}g^{abi}g^{abj}\mathrm{B}_{VV}(m_a^2, m_b^2)$$

Loop-integral functions

$$\mathrm{A}_S(x) = A(x),$$

$$\mathrm{B}_{SS}(x,y) = -B(x,y),$$

$$\mathrm{B}_{FF}(x,y) = (x+y-s)B(x,y) - A(x) - A(y),$$

$$\mathrm{B}_{\overline{FF}}(x,y) = 2B(x,y),$$

$$\mathrm{A}_V(x) = 4A(x) + 2x\,\delta_{\overline{\mathrm{MS}}} - \mathcal{L}_x[xA(x)],$$

$$\mathrm{B}_{SV}(x,y) = (2x - y + 2s)B(x,y) + A(x) - 2A(y) + \mathcal{L}_y[(x+y-s)A(y) - (x-s)^2 B(x,y)],$$

$$\mathrm{B}_{VV}(x,y) = -\frac{7}{2}B(x,y) + 2\,\delta_{\overline{\mathrm{MS}}} + \frac{1}{2}\mathcal{L}_x[xB(x,y)] + \frac{1}{2}\mathcal{L}_y[yB(x,y)] + \frac{1}{4}\mathcal{L}_x\mathcal{L}_y\{xA(y) + yA(x) + [2s(x+y) - x^2 - y^2 - s^2]B(x,y)\}.$$

$$\delta_{\overline{\mathrm{MS}}} \equiv \begin{cases} 1 & \text{for } \overline{\mathrm{MS}} \\ 0 & \text{for } \overline{\mathrm{DR}} \end{cases}$$

$$\mathcal{L}_x f(x) \equiv \frac{f(x) - f(\xi x)}{x}$$

Landau gauge: $\xi = 0$

Feynman gauge: $\xi = 1$

One-loop effective potential

$$V = V_0 + V_1$$

$$V_1 = \frac{1}{64\pi^2} \sum_\alpha n_\alpha \, m_\alpha^4(\varphi_i) \left[\log\left(\frac{m_\alpha^2(\varphi_i)}{\mu^2}\right) - \frac{3}{2}\right]$$

Degrees of freedom for each particle

$$n_\alpha = (-1)^{2s_\alpha} Q_\alpha C_\alpha (2s_\alpha + 1)$$

$m_\alpha^2(\varphi_i)$ are the field-dependent tree-level squared-masses, and the sum runs over all particles in the model.

Following Martin's procedure, we work in the Landau gauge and with the dimensional reduction (DRED) scheme.

One-loop minimization conditions

$$\frac{\partial V}{\partial v_i} = \frac{\partial V_0}{\partial v_i} + \frac{\partial V_1}{\partial v_i}$$

$$\frac{\partial V_1}{\partial v_i} = \frac{1}{32\pi^2} \sum_\alpha n_\alpha m_\alpha^2 \frac{\partial m_\alpha^2}{\partial v_i} \left[\log\left(\frac{m_\alpha^2}{\mu^2}\right) - 1\right]$$

$$m_{11}^2 + \frac{1}{2}\left[\lambda_1 v_1^2 + (\lambda_3 + \lambda_4)\, v_2^2\right] + \frac{1}{v_1}\frac{\partial V_1}{\partial v_1} = 0$$

$$m_{22}^2 + \frac{1}{2}\left[\lambda_2 v_2^2 + (\lambda_3 + \lambda_4)\, v_1^2\right] + \frac{1}{v_2}\frac{\partial V_1}{\partial v_2} = 0$$

One-loop pseudoscalar self-energy in the $U(1)$ model

Substituting the one-loop minimization conditions in the tree-level pseudoscalar matrix gives

$$\left[M_0^2\right]_{\text{CP-odd}} = \begin{pmatrix} -\frac{1}{v_1}\frac{\partial V_1}{\partial v_1} & 0 \\ 0 & -\frac{1}{v_2}\frac{\partial V_1}{\partial v_2} \end{pmatrix}$$

$$\left[M^2\right]_{ij} = \left[M_0^2\right]_{ij} + \frac{1}{16\pi^2}\left[\Pi^{(1)}\right]_{ij}$$

$$\left[\Pi^{(1)}\right]_{ij} = \left[\Pi_S^{(1)}\right]_{ij} + \left[\Pi_{GB}^{(1)}\right]_{ij} + \left[\Pi_F^{(1)}\right]_{ij}$$

Scalar contributions

Since the tree-level mass matrix is diagonal, then the off-diagonal entries of the self-energy matrix need to be zero:

$$\begin{aligned} \left[\Pi_S^{(1)}\right]_{A,G^0} &= \frac{\lambda_4}{2}\left\{2s_\beta c_\beta\left[A(m_{G^\pm}^2) - A(m_{H^\pm}^2)\right] + v_1 v_2 \lambda_4 B(m_{H^\pm}^2, m_{G^\pm}^2)\right\} \\ &= \frac{2s_\beta^2 c_\beta^2}{v_1 v_2}\left\{A(m_{G^\pm}^2) - A(m_{H^\pm}^2) + \left[m_{G^\pm}^2 - m_{H^\pm}^2\right] B(m_{H^\pm}^2, m_{G^\pm}^2)\right\}\left[m_{G^\pm}^2 - m_{H^\pm}^2\right] \\ &= 0 \end{aligned}$$

The diagonal entries of the self-energy matrix are:

$$\left[\Pi_S^{(1)}\right]_{G^0,G^0} = \frac{1}{2}\left\{\lambda_{34}A(m_A^2) + 3\lambda_1 A(m_{G^0}^2) + \left[\lambda_{34}c_\alpha^2 + \lambda_1 s_\alpha^2\right]A(m_H^2) + \left[\lambda_1 c_\alpha^2 + \lambda_{34}s_\alpha^2\right]A(m_h^2)\right\}$$
$$+ \left[\lambda_3 c_\beta^2 + \lambda_1 s_\beta^2\right]A(m_{G^\pm}^2) + \left[\lambda_1 c_\beta^2 + \lambda_3 s_\beta^2\right]A(m_{H^\pm}^2) - \frac{v_2^2\lambda_4^2}{2}B(m_{G^\pm}^2, m_{H^\pm}^2)$$
$$- \left[v_2\lambda_{34}c_\alpha + v_1\lambda_1 s_\alpha\right]^2 B(m_{G^0}^2, m_H^2) - \left[v_1\lambda_1 c_\alpha - v_2\lambda_{34}s_\alpha\right]^2 B(m_{G^0}^2, m_h^2)$$

$$\left[\Pi_S^{(1)}\right]_{A,A} = \frac{1}{2}\left\{3\lambda_2 A(m_A^2) + \lambda_{34}A(m_{G^0}^2) + \left[\lambda_2 c_\alpha^2 + \lambda_{34}s_\alpha^2\right]A(m_H^2) + \left[\lambda_{34}c_\alpha^2 + \lambda_2 s_\alpha^2\right]A(m_h^2)\right\}$$
$$+ \left[\lambda_2 c_\beta^2 + \lambda_3 s_\beta^2\right]A(m_{G^\pm}^2) + \left[\lambda_3 c_\beta^2 + \lambda_2 s_\beta^2\right]A(m_{H^\pm}^2) - \frac{v_1^2\lambda_4^2}{2}B(m_{G^\pm}^2, m_{H^\pm}^2)$$
$$- \left[v_2\lambda_2 c_\alpha + v_1\lambda_{34}s_\alpha\right]^2 B(m_A^2, m_H^2) - \left[v_1\lambda_{34}c_\alpha - v_2\lambda_2 s_\alpha\right]^2 B(m_A^2, m_h^2)$$

$$\left[M^2\right]_{G^0,G^0} = \left[M_0^2\right]_{G^0,G^0} + \frac{1}{16\pi^2}\left[\Pi^{(1)}\right]_{G^0,G^0}$$
$$= -\frac{1}{32\pi^2 v_1}\sum_\alpha n_\alpha m_\alpha^2 \frac{\partial m_\alpha^2}{\partial v_1}\left(\log\left(\frac{m_\alpha^2}{\mu^2}\right) - 1\right) + \frac{1}{16\pi^2}\left[\Pi^{(1)}\right]_{G^0,G^0} = 0$$

Gauge bosons contributions

$$\mathcal{L} \supset (D^\mu \Phi_1)^\dagger (D_\mu \Phi_1) + (D^\mu \Phi_2)^\dagger (D_\mu \Phi_2)$$

$$\Pi_{ij}^{(1)} = \frac{1}{2}\lambda^{ijkk} A_S(m_k^2) + \frac{1}{2}\lambda^{ikl}\lambda^{jkl} B_{SS}(m_k^2, m_l^2) + \mathrm{Re}[y^{KLi} y_{KLj}] B_{FF}(m_K^2, m_L^2) + \mathrm{Re}[y^{KLi} y^{K'L'j} M_{KK'} M_{LL'}] B_{\overline{FF}}(m_K^2, m_L^2)$$

$$+ g^{aik} g^{ajk} B_{SV}(m_k^2, m_a^2) + \frac{1}{2} g^{aaij} A_V(m_a^2) + \frac{1}{2} g^{abi} g^{abj} B_{VV}(m_a^2, m_b^2)$$

$$\left[\Pi_{GB}^{(1)}\right]_{A,G^0} = \frac{e^2 s_\alpha c_\alpha}{4 s_{\theta_W}^2 c_{\theta_W}^2} \left[B_{SV}(m_H^2, m_Z^2) - B_{SV}(m_h^2, m_Z^2)\right] + \frac{e^2 s_\beta c_\beta}{8 s_{\theta_W}^2 c_{\theta_W}^2} \left[B_{SV}(m_{G^\pm}^2, m_{W^\pm}^2) - B_{SV}(m_{H^\pm}^2, m_{W^\pm}^2)\right] = 0$$

$$\left[\Pi_{GB}^{(1)}\right]_{A,A} = \frac{e^2}{2 s_{\theta_W}^2} \left[c_\beta^2 B_{SV}(m_{G^\pm}^2, m_{W^\pm}^2) + s_\beta^2 B_{SV}(m_{H^\pm}^2, m_{W^\pm}^2)\right] + \frac{e^2}{4 s_{\theta_W}^2 c_{\theta_W}^2} \left[s_\alpha^2 B_{SV}(m_h^2, m_Z^2) - A_V(m_Z^2)\right] + \frac{e^2 c_\alpha^2}{64 s_{\theta_W}^2 c_{\theta_W}^2} B_{SV}(m_H^2, m_Z^2)$$

$$-\frac{1}{32\pi^2 v_1}\left[6\frac{\partial m_{W^\pm}^2}{\partial v_1} A(m_{W^\pm}^2) + 3\frac{\partial m_Z^2}{\partial v_1} A(m_Z^2) + 3\frac{\partial m_\gamma^2}{\partial v_1} A(m_\gamma^2)\right] + \left[\Pi_{GB}^{(1)}\right]_{G^0,G^0} = 0$$

$$-\frac{1}{32\pi^2 v_2}\left[6\frac{\partial m_{W^\pm}^2}{\partial v_2} A(m_{W^\pm}^2) + 3\frac{\partial m_Z^2}{\partial v_2} A(m_Z^2) + 3\frac{\partial m_\gamma^2}{\partial v_2} A(m_\gamma^2)\right] + \left[\Pi_{GB}^{(1)}\right]_{A,A} = 0$$

Fermion contributions in a Type I model

$$\Pi_{ij}^{(1)} = \frac{1}{2}\lambda^{ijkk}A_S(m_k^2) + \frac{1}{2}\lambda^{ikl}\lambda^{jkl}B_{SS}(m_k^2, m_l^2) + \mathrm{Re}[y^{KLi}y_{KLj}]B_{FF}(m_K^2, m_L^2) + \mathrm{Re}[y^{KLi}y^{K'L'j}M_{KK'}M_{LL'}]B_{\overline{FF}}(m_K^2, m_L^2)$$

$$+ g^{aik}g^{ajk}B_{SV}(m_k^2, m_a^2) + \frac{1}{2}g^{aaij}A_V(m_a^2) + \frac{1}{2}g^{abi}g^{abj}B_{VV}(m_a^2, m_b^2)$$

$$\left[\Pi_F^{(1)}\right]_{G^0,A} = 0$$

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fermions do not couple to the first doublet

$$\left[\Pi_F^{(1)}\right]_{A,A} = \sum_{K=d,s,b}\left[\left(\Gamma_{KK}^2\right)^2 B_{FF}(m_K^2, m_K^2) - \frac{v_2^2}{2}\left(\Gamma_{KK}^2\right)^4 B_{\overline{FF}}(m_K^2, m_K^2)\right] + \sum_{K=u,c,t}\left[\left(\Delta_{KK}^2\right)^2 B_{FF}(m_K^2, m_K^2) - \frac{v_2^2}{2}\left(\Delta_{KK}^2\right)^4 B_{\overline{FF}}(m_K^2, m_K^2)\right]$$

$$-\frac{1}{32\pi^2 v_2}\sum_{\alpha=u,c,t,d,s,b}(-12)\frac{\partial m_\alpha^2}{\partial v_2}A(m_\alpha^2) + \left[\Pi_F^{(1)}\right]_{A,A} = 0$$

For a Type II model with a $U(1)$ symmetry we obtained the same result.

Fermion contributions in a Type I model

$$\Pi_{ij}^{(1)}=\frac{1}{2}\lambda^{ijkk}A_S(m_k^2)+\frac{1}{2}\lambda^{ikl}\lambda^{jkl}B_{SS}(m_k^2,m_l^2)+\mathrm{Re}[y^{KLi}y_{KLj}]B_{FF}(m_K^2,m_L^2)+\mathrm{Re}[y^{KLi}y^{K'L'j}M_{KK'}M_{LL'}]B_{\overline{FF}}(m_K^2,m_L^2)$$
$$+g^{aik}g^{ajk}B_{SV}(m_k^2,m_a^2)+\frac{1}{2}g^{aaij}A_V(m_a^2)+\frac{1}{2}g^{abi}g^{abj}B_{VV}(m_a^2,m_b^2)$$

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$$+\sum_{K=u,c,t}\left[\left(\Delta_{KK}^2\right)^2B_{FF}(m_K^2,m_K^2)-\frac{v_2^2}{2}\left(\Delta_{KK}^2\right)^4B_{\overline{FF}}(m_K^2,m_K^2)\right]$$

Collecting the previous results, we find what was to be expected…

$$M^2=M_0^2+\frac{1}{16\pi^2}\left(\Pi_S^{(1)}+\Pi_{GB}^{(1)}+\Pi_F^{(1)}\right)=\begin{pmatrix}0&0\\0&0\end{pmatrix}$$

two pseudoscalar Goldstone bosons at one-loop level.

For a Type II mo

Z_3 model (with 3 quark generations)

$$\omega = e^{2\pi i/3}$$

$$\Phi_2 \to \omega^2 \Phi_2$$

$$Q_{L1} \to \omega^2 Q_{L1}$$

$$Q_{L2} \to \omega Q_{L2}$$

$$n_{R3} \to \omega n_{R3}$$

$$p_{R3} \to \omega p_{R3}$$

$$\mathcal{L}_{\mathrm{Y,\ quarks}} = -\left[\Gamma_a\right]_{ij} \overline{Q_{L}}_i \Phi_a n_{Rj} - \left[\Delta_a\right]_{ij} \overline{Q_{L}}_i \tilde{\Phi}_a p_{Rj} + \mathrm{H.c.}$$

The model features a particular Z_3 symmetry, for which the Yukawa matrices are

$$\Gamma_1, \Delta_1 \sim \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & \times \\ \times & \times & 0 \end{pmatrix} \qquad \Gamma_2 \sim \begin{pmatrix} \times & \times & 0 \\ 0 & 0 & 0 \\ 0 & 0 & \otimes \end{pmatrix} \qquad \Delta_2 \sim \begin{pmatrix} 0 & 0 & \otimes \\ \otimes & \otimes & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

[P. M. Ferreira, João P. Silva, 2011, DOI:10.1103/PhysRevD.83.065026]
[P. M. Ferreira, L. Lavoura, João P. Silva, 2011, DOI:10.1016/j.physletb.2011.08.071]

The highlighted matrix entries only "survive" a Z_3 transformation because they are proportional to $\omega^3 = 1$. By putting them to zero, we would expect to recover the continuous $U(1)$ case.

Z_3 model (with 3 quark generations)

$$\omega = e^{2\pi i/3}$$

$$\Phi_2 \to \omega^2 \Phi_2$$

$$Q_{L1} \to \omega^2 Q_{L1}$$

$$Q_{L2} \to \omega Q_{L2}$$

$$n_{R3} \to \omega n_{R3}$$

$$p_{R3} \to \omega p_{R3}$$

$$\mathcal{L}_{\text{Y, quarks}} = -[\Gamma_a]_{ij}\, \overline{Q_{Li}} \Phi_a n_{Rj} - [\Delta_a]_{ij}\, \overline{Q_{Li}} \tilde{\Phi}_a p_{Rj} + \text{H.c.}$$

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$$\Gamma_1, \Delta_1 \sim \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & \times \\ \times & \times & 0 \end{pmatrix} \qquad \Gamma_2 \sim \begin{pmatrix} \times & \times & 0 \\ 0 & 0 & 0 \\ 0 & 0 & \otimes \end{pmatrix} \qquad \Delta_2 \sim \begin{pmatrix} 0 & 0 & \otimes \\ \otimes & \otimes & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

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The Yukawa sector is then diagonalized numerically and the fermion contributions to the scalar self-energy are computed using the same method.

Summary

- We used Martin's results to compute the one-loop masses of the pseudoscalars in a 2HDM with a $U(1)$ symmetry.
- The Goldstone's theorem was verified at one-loop level.
- The fermion contributions will dictate the mass of the pseudoscalar in the Z_3 model.

Thank you for listening!

At two-loop level...

A. Diagrams with only scalar propagators

The two-loop Feynman diagrams that involve only scalar propagator lines yield the following contribution to the self-energy:

$$\Pi^{(2)}_{ij} = \frac{1}{4}\lambda^{ijkl}\lambda^{kmn}\lambda^{lmn}W_{SSSS}(m_k^2,m_l^2,m_m^2,m_n^2) + \frac{1}{4}\lambda^{ijkl}\lambda^{klmm}X_{SSS}(m_k^2,m_l^2,m_m^2) + \frac{1}{2}\lambda^{ikl}\lambda^{jkm}\lambda^{lmnn}Y_{SSSS}(m_k^2,m_l^2,m_m^2,m_n^2)$$

$$+\frac{1}{4}\lambda^{ikl}\lambda^{jmn}\lambda^{klmn}Z_{SSSS}(m_k^2,m_l^2,m_m^2,m_n^2) + \frac{1}{6}\lambda^{iklm}\lambda^{jklm}S_{SSS}(m_k^2,m_l^2,m_m^2) + \frac{1}{2}(\lambda^{ikl}\lambda^{jkmn}$$

$$+\lambda^{jkl}\lambda^{ikmn})\lambda^{lmn}U$$

$$+\frac{1}{2}\lambda^{ikm}\lambda^{jln}\lambda^{klp}\lambda$$

Here the loop integral fun

In several of these functi
limits, one obtains

B. Diagrams with scalar and fermion propagators

In this subsection, I present the results for Feynman diagrams that involve both scalar and fermion propagators, but no vector propagators.

The contributions from diagrams with the topology W in Fig. 2, in which propagators 1, 2 are scalars and propagators 3, 4 are fermions, are

$$\Pi^{(2)}_{ij} = \frac{1}{2}\lambda^{ijkl}\mathrm{Re}[y^{MNk}y^{M'N'l}M_{MM'}M_{NN'}]W_{SS\overline{FF}}(m_k^2,m_l^2,m_M^2,m_N^2) + \frac{1}{2}\lambda^{ijkl}y^{MNk}y_{MNl}W_{SSFF}(m_k^2,m_l^2,m_M^2,m_N^2),$$

where

$$W_{SS\overline{FF}}(x,$$

$$W_{SSFF}(x,$$

$$W_{SSFF}(x,$$

The contributions from diagram

C. Diagrams with scalar and one vector propagators

The contributions from two-loop Feynman diagrams with scalar propagators, exactly one vector propagator, and no fermion propagators are

$$\Pi^{(2)}_{ij} = \frac{1}{2}\lambda^{ijkl}g^{akm}g^{alm}W_{SSSV}(m_k^2,m_l^2,m_m^2,m_a^2) + \frac{1}{4}\lambda^{ijkl}g^{aakl}X_{SSV}(m_k^2,m_l^2,m_a^2) + \frac{1}{2}\lambda^{ikl}\lambda^{jkm}g^{aalm}Y_{SSSV}(m_k^2,m_l^2,m_m^2,m_a^2)$$

$$+\frac{1}{2}g^{aik}g^{ajl}\lambda^{klmm}Y_{VSSS}(m_a^2,m_k^2,m_l^2,m_m^2) + \frac{1}{2}(g^{aik}\lambda^{jkmn} + g^{ajk}\lambda^{ikmn})g^{amn}U_{SVSS}(m_k^2,m_a^2,m_m^2,m_n^2)$$

$$+\lambda^{ikl}\lambda^{jkm}g^{aln}g^{amn}V_{SSSSV}(m_k^2,m_l^2,m_m^2,m_n^2,m_a^2) + \frac{1}{2}g^{aik}g^{ajl}\lambda^{kmn}\lambda^{lmn}V_{VSSSS}(m_a^2,m_k^2,m_l^2,m_m^2,m_n^2)$$

$$+\frac{1}{2}(g^{aik}\lambda^{jkl} + g^{ajk}\lambda^{ikl})g^{amn}\lambda^{lmn}V_{SVSSS}(m_k^2,m_a^2,m_l^2,m_m^2,m_n^2)$$

Backup slides

Tree-level scalar masses

For neutral minima, we can separate the full 8×8 scalar mass matrix into two 4×4 matrices for charged and neutral scalars. The matrix of neutral scalars has the form

$$[M_N^2]_{ij} = \begin{pmatrix} [M_h^2] & [M_I^2] \\ [M_I^2]^T & [M_A^2] \end{pmatrix} \qquad [M_I^2] = \begin{pmatrix} A_I & B_I \\ C_I & D_I \end{pmatrix}$$

If all couplings in the potential are real the off-diagonal blocks are zero and thus, there is no mixing between CP-odd and CP-even scalars.

$$A_I = \frac{1}{2} v_2 \left[\mathrm{Im}(\lambda_5) v_2 + 2\,\mathrm{Im}(\lambda_6) v_1 \right],$$

$$B_I = \mathrm{Im}(m_{12}^2) - \mathrm{Im}(\lambda_5) v_1 v_2 - \frac{3}{2} \mathrm{Im}(\lambda_6) v_1^2 - \frac{1}{2} \mathrm{Im}(\lambda_7) v_2^2,$$

$$C_I = -\mathrm{Im}(m_{12}^2) + \mathrm{Im}(\lambda_5) v_1 v_2 + \frac{1}{2} \mathrm{Im}(\lambda_6) v_1^2 - \frac{3}{2} \mathrm{Im}(\lambda_7) v_2^2,$$

$$D_I = -\frac{1}{2} v_1 \left[\mathrm{Im}(\lambda_5) v_1 + 2\,\mathrm{Im}(\lambda_7) v_2 \right].$$

[G.C. Branco, P.M. Ferreira, L. Lavoura, M.N. Rebelo, Marc Sher, J.P. Silva, 2012, DOI:10.1016/j.physrep.2012.02.002]

$U(1)$ model with normal vacua:

- no mixing between CP-odd and CP-even scalars;
- the pseudoscalar A is a (physical) Goldstone boson;
- no mixing in the pseudoscalars mass matrix.

Expansions for small s

$$B(x,y) = \frac{A(y)-A(x)}{x-y} + \frac{s}{2(x-y)^3}[x^2-y^2+2xy\ln(y/x)] + \frac{s^2}{6(x-y)^5}[(x-y)(x^2+y^2+10xy)+6xy(x+y)\ln(y/x)] + \cdots$$

$$B(x,x) = -\overline{\ln}\,x + \frac{s}{6x} + \frac{s^2}{60x^2} + \cdots$$

Since the Goldstone bosons have zero physical mass, we evaluate the expressions at zero external momentum ($s = 0$) which significantly simplifies the calculations (e.g. B_{SV} functions become zero in the Landau gauge).

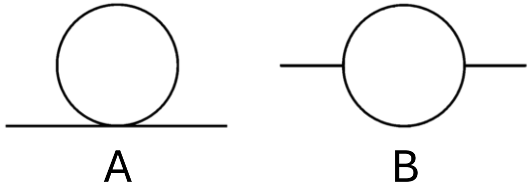
The squared-masses are diagonal– the fields are squared-mass eigenstates – but the masses of the fermions are not necessarily. It is only required that the latter have a block diagonal form, with non-zero entries for states with the same squared-mass.

One-loop self-energy integrals

At one-loop level, the topologies contributing to the self-energy function are

$$\mathbf{A}(x) = C\int d^dk \frac{1}{[k^2+x]}$$

$$\mathbf{B}(x,y) = C\int d^dk \frac{1}{[k^2+x][(k-p)^2+y]}$$



[Stephen P . Martin, 2003, DOI:10.1103/PhysRevD.68.075002]

Modified integrals with the divergent parts subtracted, at one-loop order (Passarino-Veltman functions):

$$A(x) = \lim_{\epsilon\to 0}[\mathbf{A}(x) + x/\epsilon] = x(\overline{\ln}\, x - 1)$$

$$\overline{\ln}\, X \equiv \ln(X/Q^2)$$

$$B(x,y) = \lim_{\epsilon\to 0}[\mathbf{B}(x,y) - 1/\epsilon]$$

$$= -\int_0^1 dt\, \overline{\ln}[tx + (1-t)y - t(1-t)s]$$

The loop integrals are functions of squared-masses and a common external momentum invariant s (although the argument is omitted).

Yukawa sector in terms of Weyl fermions

$$\Psi_D = \begin{pmatrix} \xi_\alpha \\ \chi^{\dagger\dot\alpha} \end{pmatrix}$$

$$\overline{\Psi}_D = \Psi_D^\dagger \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} = \begin{pmatrix} \chi^\alpha & \xi^\dagger_{\dot\alpha} \end{pmatrix}$$

[Stephen P. Martin, *A Supersymmetry Primer*, arXiv:hep-ph/9709356v7]

$$\overline{\Psi}_i P_L \Psi_j = \chi_i \xi_j$$
$$\overline{\Psi}_i P_R \Psi_j = \xi_i^\dagger \chi_j^\dagger$$
$$\overline{\Psi}_i \gamma^\mu P_L \Psi_j = \xi_i^\dagger \overline{\sigma}^\mu \xi_j$$
$$\overline{\Psi}_i \gamma^\mu P_R \Psi_j = \chi_i \sigma^\mu \chi_j^\dagger$$

$$\begin{aligned}
\mathcal{L}_{\text{Y, quarks}} &= -\left[\Gamma_a\right]_{ij} \overline{Q_L}_i \Phi_a n_{Rj} - \left[\Delta_a\right]_{ij} \overline{Q_L}_i \tilde{\Phi}_a p_{Rj} + \text{H.c.} \\
&= -\frac{1}{\sqrt{2}} \left[\Gamma_a\right]_{ij} \left[\overline{p_L}_i n_{Rj} \varphi_a^+ + \overline{n_L}_i n_{Rj} (v_a + \varphi_a^0)\right] - \frac{1}{\sqrt{2}} \left[\Delta_a\right]_{ij} \left[\overline{p_L}_i p_{Rj} (v_a + \varphi_a^{0*}) - \overline{n_L}_i p_{Rj} \varphi_a^-\right] + \text{H.c.} \\
&= -\overline{p_L} \Gamma_a n_R \frac{\varphi_a^+}{\sqrt{2}} - \overline{n_L} \left(M_n + \Gamma_a \frac{\varphi_a^0}{\sqrt{2}}\right) n_R - \overline{p_L} \left(M_p + \Delta_a \frac{\varphi_a^{0*}}{\sqrt{2}}\right) p_R + \overline{n_L} \Delta_a p_R \frac{\varphi_a^-}{\sqrt{2}} + \text{H.c.} \\
&= -\overline{u_L} U_L^{p\dagger} \Gamma_a U_R^{n\dagger} d_R \frac{\varphi_a^+}{\sqrt{2}} - \overline{d_L} \left(M_d + U_L^{n\dagger} \Gamma_a U_R^n \frac{\varphi_a^0}{\sqrt{2}}\right) d_R - \overline{u_L} \left(M_u + U_L^{p\dagger} \Delta_a U_R^p \frac{\varphi_a^{0*}}{\sqrt{2}}\right) u_R \\
&\quad + \overline{d_L} U_L^{n\dagger} \Delta_a U_R^p u_R \frac{\varphi_a^-}{\sqrt{2}} + \text{H.c.} \\
&= -\xi_i^{u\dagger} \chi_j^{d\dagger} \left[U_L^{p\dagger} \Gamma_a U_R^{n\dagger}\right]_{ij} \frac{\varphi_a^+}{\sqrt{2}} - \xi_i^{d\dagger} \chi_j^{d\dagger} \left(\left[M_d\right]_{ij} + \left[U_L^{n\dagger} \Gamma_a U_R^n\right]_{ij} \frac{\varphi_a^0}{\sqrt{2}}\right) \\
&\quad - \xi_i^{u\dagger} \chi_j^{u\dagger} \left(\left[M_u\right]_{ij} + \left[U_L^{p\dagger} \Delta_a U_R^p\right]_{ij} \frac{\varphi_a^{0*}}{\sqrt{2}}\right) + \xi_i^{d\dagger} \chi_j^{u\dagger} \left[U_L^{n\dagger} \Delta_a U_R^p\right]_{ij} \frac{\varphi_a^-}{\sqrt{2}} + \text{H.c.}
\end{aligned}$$

$$M_n = \frac{1}{\sqrt{2}} \sum_{a=1}^{2} v_a \Gamma_a$$

$$M_p = \frac{1}{\sqrt{2}} \sum_{a=1}^{2} v_a \Delta_a$$

$$M_d \equiv U_L^{n\dagger} M_n U_R^n = \text{diag}(m_d, m_s, m_b)$$
$$M_u \equiv U_L^{p\dagger} M_n U_R^p = \text{diag}(m_u, m_c, m_t)$$