Study of Potentials of Higgs Multiplets

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Why study Higgs multiplet extensions?

- T. D. Lee introduced extension of Higgs sector getting new source for CP violation.
- Supersymmetry requires at least two Higgs doublets.
- Dark matter models realized by Higgs extensions.
- Neutrino mixing models realized by Higgs extensions.
- Pragmatically, no reason for only one doublet.

Restrictions from electroweak sector

- From the kinetic terms of electroweak gauge bosons, tree level,

$$\rho \equiv \frac{m_W^2}{\cos^2(\theta_W) m_Z^2} = \frac{\sum_i \left(T_i(T_i+1) - T_{3i}^2 \right) |v_i|^2}{2 \sum_i T_{3i}^2 |v_i|^2} \approx 1 \ .$$

T_i, T_{3i} :	isospin, z component of multiplet i
v_i :	VEV

- Multiplets corresponding to $\rho = 1$:

singlets:	$T = T_3 = 0,$
doublets:	$T = 1/2, T_3 = \pm 1/2,$
septuplets:	$T = 3, T_3 = \pm 2,$
⋮	

- Restriction can be circumvented, for instance by combination of Higgs bosons, absence of VEV's.

- Example one doublet ($T = T_3 = 1/2$) and one quadruplet ($T = T_3 = 3/2$),

$$\rho = 1 - 6 \frac{v_{3/2}^2}{v_{1/2}^2 + 9 v_{3/2}^2}$$

Georgi, Machacek, NPB262 (1985)
Kannike JHEP01 (2024)

- Disregarding restrictions let us consider arbitrary multiplets.

Gauge redundancies

- In conventional method Higgs multiplets plagued with gauge redundancies.
- Guiding principle: Physics independent of gauge.
- Caveat: Gauge symmetries versus physical symmetries.

Picture: chatGPT4.o

S Elitzur *Impossibility of spontaneously breaking local symmetries* **PRD12** (1975)

Fröhlich, Morchio, Strocchi *Higgs phenomenon without symmetry breaking order parameter* **NPB190** (1981)

- Principal ideal: Construct gauge invariant model and keep gauge symmetry manifest.

- Simple example: Standard Model with one Higgs-boson doublet,

$$\varphi(x) = \begin{pmatrix} \phi^1(x) \\ \phi^2(x) \end{pmatrix}$$

- Define a real gauge-invariant bilinear field

$$K(x) = \varphi^\dagger \varphi$$

- Domain: $K \geqslant 0$

$$V_{SM} = -\mu^2 K + \lambda K^2$$

- Minimum: $\langle K \rangle = \frac{\mu^2}{2\lambda}$, electroweak symmetry unbroken for $\langle K \rangle = 0$, partially broken for $\langle K \rangle > 0$.

Higgs multiplets

isospin T	0	$1/2$	1	$3/2$	$\frac{m-1}{2}$
T_3	0	$-1/2, +1/2$	$-1, 0, +1$	$-3/2, \ldots, 3/2$	$-\frac{m-1}{2}, \ldots, +\frac{m-1}{2}$
multiplicity	1	2	3	4	m
multiplet $\varphi^{(m)}$	ϕ^1	$\begin{pmatrix}\phi^1\\ \phi^2\end{pmatrix}$	$\begin{pmatrix}\phi^1\\ \phi^2\\ \phi^3\end{pmatrix}$	$\begin{pmatrix}\phi^1\\ \vdots\\ \phi^4\end{pmatrix}$	$\begin{pmatrix}\phi^1\\ \vdots\\ \phi^m\end{pmatrix}$

Multiplets in bilinear representation

- Example: 2 triplets, $T = 1$, multiplicity $m = 3$

$$\varphi_1^{(3)} = \begin{pmatrix} \phi_1^1 \\ \phi_1^2 \\ \phi_1^3 \end{pmatrix}, \qquad \varphi_2^{(3)} = \begin{pmatrix} \phi_2^1 \\ \phi_2^2 \\ \phi_2^3 \end{pmatrix}$$

- We form

$$\psi^{(3)} = \begin{pmatrix} \varphi_1^{(3)\mathrm{T}} \\ \varphi_2^{(3)\mathrm{T}} \end{pmatrix} = \begin{pmatrix} \phi_1^1 & \phi_1^2 & \phi_1^3 \\ \phi_2^1 & \phi_2^2 & \phi_2^3 \end{pmatrix}$$

- Gauge-invariant matrix $\underline{K}^{(3)}$

$$\underline{K}^{(3)} = \psi^{(3)} \psi^{(3)^\dagger} = \begin{pmatrix} \varphi_1^{(3)\dagger} \varphi_1^{(3)} & \varphi_2^{(3)\dagger} \varphi_1^{(3)} \\ \varphi_1^{(3)\dagger} \varphi_2^{(3)} & \varphi_2^{(3)\dagger} \varphi_2^{(3)} \end{pmatrix}$$

- Write $\underline{K}^{(3)}$ in basis of unit and Pauli matrices,

$$\underline{K}^{(3)} = \frac{1}{2} K_\alpha^{(m)} \sigma_\alpha, \qquad \alpha \in \{0, \ldots, 3\} \,,$$

with $\sigma_0 = \mathbb{1}_2$.

- Bilinears explicitly

$$\begin{aligned} K_0^{(3)} &= \varphi_1^{(3)\dagger} \varphi_1^{(3)} + \varphi_2^{(3)\dagger} \varphi_2^{(3)} \\ K_1^{(3)} &= \varphi_1^{(3)\dagger} \varphi_2^{(3)} + \varphi_2^{(3)\dagger} \varphi_1^{(3)} \\ K_2^{(3)} &= i\varphi_2^{(3)\dagger} \varphi_1^{(3)} - i\varphi_1^{(3)\dagger} \varphi_2^{(3)} \\ K_3^{(3)} &= \varphi_1^{(3)\dagger} \varphi_1^{(3)} - \varphi_2^{(3)\dagger} \varphi_2^{(3)} \end{aligned}$$

- Generalization to n_m multiplets of multiplicity m: Form

$$\psi^{(m)} = \begin{pmatrix} \varphi_1^{(m)\mathrm{T}} \\ \vdots \\ \varphi_{n_m}^{(m)\mathrm{T}} \end{pmatrix} \in (n_m \times m)$$

- Build $\underline{K}^{(m)}$ and write in basis of generalized Pauli matrices,

$$\underline{K}^{(m)} = \psi^{(m)} \psi^{(m)\dagger} = \frac{1}{2} K_\alpha^{(m)} \lambda_\alpha, \qquad \alpha \in \{0, \ldots, n_m^2 - 1\} ,$$

with $\lambda_0 = \sqrt{\frac{2}{n_m}} \mathbb{1}_{n_m}$.

- We arrive at bilinears

$$K_\alpha^{(m)}$$

- One-to-one correspondence - except gauge redundancies.

Domain of multiplets

- Stability

  M Abud, G Sartori **PLB104** (1981)
  J Kim **NPB196** (1982)
  A Degee, I P Ivanov, V Keuss **JHEP02** (2013)
  O Nachtmann, MM **PRD92** (2015)

- Assignment of hypercharge Y to multiplets.
- Gell–Mann–Nishijima $Q = Y + T_3$
- Example: 2 triplets, $T_3 \in \{-1, 0, +1\}$ with $Y = 1$,

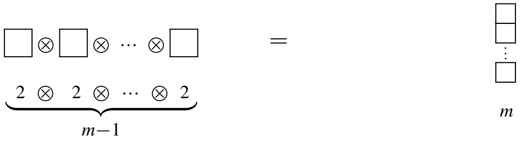
$$\varphi_i = \begin{pmatrix} \phi_i^{++} \\ \phi_i^{+} \\ \phi_i^{0} \end{pmatrix}, \qquad \psi^{(3)} = \begin{pmatrix} \phi_1^{++} & \phi_1^{+} & \phi_1^{0} \\ \phi_2^{++} & \phi_2^{+} & \phi_2^{0} \end{pmatrix}, \qquad \underline{K}^{(3)} = \psi^{(3)} \psi^{(3)\dagger}$$

- $0 \leqslant \mathrm{rank}(\psi^{(m)}) \leqslant m$ gives rank of $\underline{K}^{(m)}$.
- Neutral non-trivial vacuum corresponds to rank one.
- Simple restrictions on domain, example

$$K_0^{(3)} \geqslant 0 \,, \qquad \mathrm{tr}^2(\underline{K}^{(3)}) - \mathrm{tr}((\underline{K}^{(3)})^2) = 0$$

Doublet representation

- What about mixed multiplet terms in the potential?
- Totally symmetric irrep. of tensor product of $m-1$ doublets $SU(2)$



- Example: Higgs-boson triplet with hypercharge $Y = 1$,

$$\varphi^{(3)} = \begin{pmatrix} \phi^{++} \\ \phi^{+} \\ \phi^{0} \end{pmatrix}, \qquad \Delta_{11} = \phi^{++}, \quad \Delta_{12} = \Delta_{21} = \frac{1}{\sqrt{2}}\phi^{+}, \quad \Delta_{22} = \phi^{(0)}$$

that is, with SU(2) indices

$$(\Delta_{ij}) = \begin{pmatrix} \phi^{++} & \frac{1}{\sqrt{2}}\phi^{+} \\ \frac{1}{\sqrt{2}}\phi^{+} & \phi^{(0)} \end{pmatrix}.$$

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multiplet $\varphi^{(m)}$	ϕ^1	$\begin{pmatrix} \phi^1 \\ \phi^2 \end{pmatrix}$	$\begin{pmatrix} \phi^1 \\ \phi^2 \\ \phi^3 \end{pmatrix}$	$\begin{pmatrix} \phi^1 \\ \vdots \\ \phi^4 \end{pmatrix}$	$\begin{pmatrix} \phi^1 \\ \vdots \\ \phi^m \end{pmatrix}$
doublet rep.		Δ_i	Δ_{ij}	Δ_{ijk}	$\Delta_{i_1 \ldots i_{m-1}}$

Multi-Higgs potential

- Conjugated multiplets with upper indices

$$\Delta^{*i_1,\ldots,i_{m-1}} \equiv (\Delta_{j_1,\ldots j_{m-1}})^* \qquad \text{with } i_k, j_k \in \{1,2\}\ .$$

We can also lower single indices,

$$\epsilon_{ij}\Delta^{*\ldots j \ldots} \quad \text{with } \epsilon = i\sigma_2 = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}\ .$$

- $SU(2)$ invariance corresponds to contracted indices.

S Keppeler *Birdtracks for SU(N)* arXiv 1707.07280
Kannike JHEP01 (2024)

- Examples

$$\Delta^{*ij}\Delta_i\Delta_j, \qquad \Delta_{ij}\Delta^{*i}\Delta^{*j}, \qquad \Delta_j\Delta_k\Delta^{*m}\epsilon_{mi}\Delta^{*ijk}$$

- We have to take care of $U_Y(1)$.

Symmetries

- Symmetry transformation for bilinears

$$K_0^{(m)}(x) \to K_0'^{(m)}(x) = K_0^{(m)}(x'), \qquad K_a^{(m)}(x) \to K_a'^{(m)}(x) = X_{ab}^{(m)} K_b^{(m)}(x'),$$
$$a, b \in \{1, \ldots, n_m^2 - 1\}, \qquad X^{(m)} = (X_{ab}^{(m)}) \in O(n_m^2 - 1)\ .$$

- Symmetry transformation for doublet representation

$$\Delta_a^{(m)}(x) \to U_{ab}^{(m)} \Delta_b^{(m)}(x')\ , \qquad \Delta_c^{(m)*}(x) \to \Delta_d^{(m)*}(x') U_{dc}^{(m)\dagger}.$$

- Spontaneous symmetry, consider $\langle K_\mu^{(m)} \rangle$, $\langle \Delta_a^{(m)} \rangle$.

Example potential

- Example potential: 2 doublets, 3 triplets:

$$V_{\text{example}} = m_2^2 K_0^{(2)} + \sqrt{\frac{3}{2}} m_3^2 K_0^{(3)} + \lambda_2 (K_0^{(2)})^2 + \frac{3}{2} \lambda_3 (K_0^{(3)})^2 + \lambda_{23} K_1^{(2)} K_1^{(3)} \ .$$

- (Standard) CP transformation

$$\psi^{(m)}(x) \rightarrow \psi'^{(m)}(x) = \psi^{(m)^*}(x'), \qquad \text{with } x = (t, \boldsymbol{x}), \quad x' = (t, -\boldsymbol{x}) \ .$$

- Bilinears transform as

$$K_0^{(m)}(x) \rightarrow K_0^{(m)}(x'), \qquad K_a^{(m)}(x) \rightarrow C_{ab}^{(m)} K_b^{(m)}(x') \ .$$

with

$$C^{(2)} = \mathrm{diag}(1, -1, 1), \qquad C^{(3)} = \mathrm{diag}(1, 1, -1, -1, -1, 1, 1) \ .$$

- Potential invariant under (standard) CP transformations.

I. F. Ginzburg, M. Krawczyk, PRD 72 (2005)
I. P. Ivanov and C. C. Nishi, PRD 82 (2010)
MM, O. Nachtmann, JHEP 11 151 (2011)
V. Keus, S.F. King, S. Moretti, JHEP 1401 (2014)
B. Grzadkowski, MM, J. Wudka, JHEP 1111 (2011)
P. M. Ferreira, H. E. Haber, MM, O. Nachtmann, J. P. Silva, Int.J.Mod.Phys. A26 (2011)

Conclusions

- Multi-Higgs bosons attractive.
- Explicit gauge redundancies obscure physics.
- Description in terms of gauge-invariant bilinears.
- Doublet representation of arbitrary multiplets.
- Symmetry studies transparent.

Thank you for your attention!

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Change of basis

- Consider the following unitary mixing of the doublets

$$\begin{pmatrix} \varphi_1(x)^{\mathrm{T}} \\ \vdots \\ \varphi_n(x)^{\mathrm{T}} \end{pmatrix} \rightarrow U \begin{pmatrix} \varphi_1(x)^{\mathrm{T}} \\ \vdots \\ \varphi_n(x)^{\mathrm{T}} \end{pmatrix}$$

- Bilinears transform as

$$\psi \rightarrow U\psi, \qquad \underline{K} = \psi\psi^\dagger \rightarrow U\underline{K}U^\dagger$$

hence,

$$K_0 = \mathrm{tr}\,(\underline{K}\lambda_0) \rightarrow \mathrm{tr}\,\left(U\underline{K}U^\dagger\lambda_0\right) = \mathrm{tr}\,\left(U\underline{K}\lambda_0U^\dagger\right) = \mathrm{tr}\,(\underline{K}\lambda_0) = K_0,$$

$$K_a = \mathrm{tr}\,(\underline{K}\lambda_a) \rightarrow \mathrm{tr}\,\left(U\underline{K}U^\dagger\lambda_a\right) = \mathrm{tr}\,\left(\underbrace{U^\dagger\lambda_aU}_{\equiv R_{ab}\lambda_b}\underline{K}\right) = R_{ab}K_b$$

- Change of basis correspond to proper rotations $R = (R_{ab}) \in SO(n^2-1)$.

- Under a change of basis

$$K_0 \to K_0, \quad \boldsymbol{K} \to R\boldsymbol{K}$$

The potential

$$V = \xi_0 K_0 + \boldsymbol{\xi}^{\mathrm{T}} \boldsymbol{K} + \eta_{00} {K_0}^2 + 2K_0\, \boldsymbol{\eta}^{\mathrm{T}} \boldsymbol{K} + \boldsymbol{K}^{\mathrm{T}} E \boldsymbol{K}$$

changes to

$$V' = \xi_0 K_0 + \boldsymbol{\xi}^{\mathrm{T}} R\boldsymbol{K} + \eta_{00} {K_0}^2 + 2K_0\, \boldsymbol{\eta}^{\mathrm{T}} R\boldsymbol{K} + \boldsymbol{K}^{\mathrm{T}} R^{\mathrm{T}} E R\boldsymbol{K}$$

- Potential invariant, $V = V'$, iff

$$\boldsymbol{\xi} = R\, \boldsymbol{\xi}, \quad \boldsymbol{\eta} = R\, \boldsymbol{\eta}, \quad E = R\, E\, R^{\mathrm{T}}.$$