Diluting quark flavor hierarchies using dihedral symmetry: a multi-Higgs example

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♣ Sources : arXiv:1908.03961 (PRD) and arXiv:2107.03756 (EPJC)

♣ Collaborators : M. Levy and A. Srivastava

Towards a world without miracles

- Approximate Reality:

$$m_u \approx 0\,, \qquad m_d \approx 0\,, \qquad V_{\text{CKM}} \approx \begin{pmatrix} \cos\theta_C & -\sin\theta_C & 0 \\ \sin\theta_C & \cos\theta_C & 0 \\ 0 & 0 & 1 \end{pmatrix}.$$

- The zeros are unrelated accidents in the SM.
- Rule of Attraction:

$$\text{Attractiveness of a theory} \propto \frac{1}{(\text{No. of miracles it needs to work})^n}$$

  The value of 'n' depends on personal taste!
- We will use D_4 flavor symmetry to reduce the no. of miracles (hopefully!).

" True beauty comes from symmetry, not chance,
As those move easiest who have learned to dance. "

D_4 basics

"Symmetry is what we see at a glance; based on the fact that there is no reason for any difference..."

- D_4 has five irreducible representations: $\mathbf{1}_{++}$, $\mathbf{1}_{+-}$, $\mathbf{1}_{-+}$, $\mathbf{1}_{--}$ and $\mathbf{2}$
- We pick a basis such that the generators in the $\mathbf{2}$ representation are given by

$$a = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}, \qquad b = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}. \tag{1}$$

  a is of order 4 and b is of order 2.

- In this basis, the relevant tensor products are obtained as

$$\begin{aligned} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}_{\mathbf{2}} \otimes \begin{bmatrix} y_1 \\ y_2 \end{bmatrix}_{\mathbf{2}} &= [x_1 y_1 + x_2 y_2]_{\mathbf{1}_{++}} \oplus [x_1 y_2 - x_2 y_1]_{\mathbf{1}_{--}} \\ &\quad \oplus [x_1 y_2 + x_2 y_1]_{\mathbf{1}_{-+}} \oplus [x_1 y_1 - x_2 y_2]_{\mathbf{1}_{+-}}, \quad (2a) \\ \mathbf{1}_{rs} \otimes \mathbf{1}_{r's'} &= \mathbf{1}_{r''s''}, \quad (2b) \end{aligned}$$

  where $r'' = r \cdot r'$ and $s'' = s \cdot s'$.

D_4 symmetric 2HDM

Quarks transform in the following way under D_4 :

$$\mathbf{2} \; : \; \begin{bmatrix} Q_1 \\ Q_2 \end{bmatrix}, \; \begin{bmatrix} p_{1R} \\ p_{2R} \end{bmatrix}, \; \begin{bmatrix} n_{1R} \\ n_{2R} \end{bmatrix},$$

$$\mathbf{1}_{++} \; : \; Q_3 \, , \qquad \mathbf{1}_{--} \; : \; p_{3R} \, , \qquad \mathbf{1}_{-+} \; : \; n_{3R} \, ,$$

where

$$Q_k = \begin{pmatrix} p_{kL} \\ n_{kL} \end{pmatrix} \qquad k = 1, 2, 3 \, .$$

We also have two $SU(2)_L$ scalar doublets transforming under D_4 as follows:

$$\begin{bmatrix} \phi_1 \\ \phi_2 \end{bmatrix} \; : \qquad \mathbf{2}$$

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Yukawa Lagrangian that follows from the symmetry:

$$\begin{aligned} -\mathscr{L}_Y \;=\; & A_u \left(\bar{Q}_1 \tilde{\phi}_2 - \bar{Q}_2 \tilde{\phi}_1 \right) p_{3R} + B_u \, \bar{Q}_3 \left(\tilde{\phi}_1 p_{1R} + \tilde{\phi}_2 p_{2R} \right) \\ & + A_d \left(\bar{Q}_1 \phi_2 + \bar{Q}_2 \phi_1 \right) n_{3R} + B_d \, \bar{Q}_3 \left(\phi_1 n_{1R} + \phi_2 n_{2R} \right) + \text{h.c.} \, , \end{aligned}$$

Just as many Yukawa parameters as there are nonzero masses. Only extra parameter available to fix the quark mixing is $\tan\beta = \langle \phi_2 \rangle / \langle \phi_1 \rangle$.

Mass matrices and rotations

The mass matrices that follow from the Yukawa Lagrangian:

$$M_u = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 & 0 & A_u v_2 \\ 0 & 0 & -A_u v_1 \\ B_u v_1 & B_u v_2 & 0 \end{pmatrix}, \qquad M_d = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 & 0 & A_d v_2 \\ 0 & 0 & A_d v_1 \\ B_d v_1 & B_d v_2 & 0 \end{pmatrix}, \tag{3}$$

where $\langle \phi_k \rangle = v_k/\sqrt{2}$.

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where $\langle \phi_k \rangle = v_k/\sqrt{2}$. The diagonal mass matrices can be obtained via the following biunitary transformations:

$$D_u = V_L \cdot M_u \cdot V_R^\dagger = \mathrm{diag}(m_u, m_c, m_t)\,, \tag{4a}$$

$$D_d = U_L \cdot M_d \cdot U_R^\dagger = \mathrm{diag}(m_d, m_s, m_b)\,. \tag{4b}$$

The matrices, V and U relate the quark fields in the gauge basis to those in the mass basis as follows:

$$u_L = V_L\, p_L\,, \qquad u_R = V_R\, p_R\,, \tag{5a}$$

$$d_L = U_L\, n_L\,, \qquad d_R = U_R\, n_R\,, \tag{5b}$$

The CKM matrix is then given by

$$V_{\mathrm{CKM}} = V_L \cdot U_L^\dagger\,. \tag{6}$$

Quark sector at the zeroth order

The matrices, V_L and U_L can be obtained by diagonalizing $M_u M_u^\dagger$ and $M_d M_d^\dagger$ respectively:

$$M_u M_u^\dagger = \frac{1}{2}\begin{pmatrix} A_u^2 v_2^2 & -A_u^2 v_1 v_2 & 0 \\ -A_u^2 v_1 v_2 & A_u^2 v_1^2 & 0 \\ 0 & 0 & B_u^2 v^2 \end{pmatrix}, \qquad M_d M_d^\dagger = \frac{1}{2}\begin{pmatrix} A_d^2 v_2^2 & A_d^2 v_1 v_2 & 0 \\ A_d^2 v_1 v_2 & A_d^2 v_1^2 & 0 \\ 0 & 0 & B_d^2 v^2 \end{pmatrix}, \tag{7}$$

To diagonalize the above matrices, introduce

$$U_\beta = \begin{pmatrix} \cos\beta & \sin\beta & 0 \\ -\sin\beta & \cos\beta & 0 \\ 0 & 0 & 1 \end{pmatrix}, \tag{8}$$

and check that

$$D_u^2 = U_\beta \cdot (M_u M_u^\dagger) \cdot U_\beta^\dagger = \text{diag}\left(0, A_u^2 v^2/2, B_u^2 v^2/2\right), \tag{9a}$$

$$D_d^2 = U_\beta^\dagger \cdot (M_d M_d^\dagger) \cdot U_\beta = \text{diag}\left(0, A_d^2 v^2/2, B_d^2 v^2/2\right). \tag{9b}$$

Quark sector at the zeroth order

- Thus, we can identify the masses of the physical quarks as

$$m_{u,d}^2 = 0\,, \qquad m_{c,s}^2 = \frac{1}{2}A_{u,d}^2 v^2\,, \qquad m_{t,b}^2 = \frac{1}{2}B_{u,d}^2 v^2\,. \tag{10}$$

- Also, comparing with the definitions in Eq. (4), we can conclude

$$V_L = U_\beta\,, \qquad U_L = U_\beta^\dagger\,. \tag{11}$$

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- Using Eq. (6) we can now easily calculate the CKM matrix as follows:

$$V_{\text{CKM}} = U_\beta \cdot U_\beta = \begin{pmatrix} \cos 2\beta & \sin 2\beta & 0 \\ -\sin 2\beta & \cos 2\beta & 0 \\ 0 & 0 & 1 \end{pmatrix}. \tag{12}$$

Quark sector at the zeroth order

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- One can now identify the Cabibbo mixing angle as

$$\sin\theta_C = \sin 2\beta \approx 0.22\,. \tag{13}$$

Note on the scalar sector

$$V = -\mu_1^2\left(\phi_1^\dagger\phi_1\right) - \mu_2^2\left(\phi_2^\dagger\phi_2\right) - \mu_{12}^2\left(\phi_1^\dagger\phi_2 + \phi_2^\dagger\phi_1\right) + \lambda_1\left(\phi_1^\dagger\phi_1 + \phi_2^\dagger\phi_2\right)^2 + \lambda_2\left(\phi_1^\dagger\phi_2 - \phi_2^\dagger\phi_1\right)^2 + \lambda_3\left(\phi_1^\dagger\phi_2 + \phi_2^\dagger\phi_1\right)^2 + \lambda_4\left(\phi_1^\dagger\phi_1 - \phi_2^\dagger\phi_2\right)^2 . \tag{14}$$

- The D_4 symmetry needs to be softly broken in the bilinear terms so that general values of $\tan\beta$ can be allowed.
- The physical scalar masses (assuming alignment limit) are given by

$$m_h^2 = 2(\lambda_1 + \lambda_3)v^2 , \tag{15}$$

$$m_H^2 = \frac{2\mu_{12}^2}{\sin 2\beta} + 2(\lambda_4 - \lambda_3)v^2 , \tag{16}$$

$$m_+^2 = \frac{2\mu_{12}^2}{\sin 2\beta} - 2\lambda_3 v^2 , \tag{17}$$

$$m_A^2 = \frac{2\mu_{12}^2}{\sin 2\beta} - 2(\lambda_2 + \lambda_3)v^2 . \tag{18}$$

- The soft-breaking parameter μ_{12}^2 helps the nonstandard scalars to become heavy and thus can get decoupled from the EW scale.

FCNCs? Sure!

$$
\begin{aligned}
\mathscr{L}_Y^{\text{CP even}} \quad = \quad & -\frac{h}{v}\left(\bar{u}\, D_u\, u + \bar{d}\, D_d\, d\right) \\
& -\frac{H}{v}\left[\bar{u}\left(N_u P_R + N_u^\dagger P_L\right) u + \bar{d}\left(N_d P_R + N_d^\dagger P_L\right) d\right], \qquad (19)
\end{aligned}
$$

$$
N_u = -\begin{pmatrix} 0 & m_c & 0 \\ 0 & 0 & 0 \\ m_t & 0 & 0 \end{pmatrix}, \quad N_d = -\begin{pmatrix} 0 & m_s & 0 \\ 0 & 0 & 0 \\ m_b & 0 & 0 \end{pmatrix}. \qquad (20)
$$

Appropriately suppressed scalar mediated FCNCs are a prediction! Bound on nonstandard mass scale from $\Delta F = 2$ processes $M > \mathcal{O}\,(3\ \text{TeV})$.

How to get closer to the reality?

- How to generate correct nonzero values of the small parameters?
- This $D4$-2HDM might be a constituent part of a more elaborate theoretical framework.

“ Whoever thinks a faultless piece to see,
Thinks what ne’er was, nor is, nor e’er shall be, ”

Going beyond 2HDM

- $$\mathbf{2} : \begin{bmatrix} Q_{1L} \\ Q_{2L} \end{bmatrix} , \begin{bmatrix} \phi_1 \\ \phi_2 \end{bmatrix} , \tag{21}$$
  $$\mathbf{1}_{++} : n_{1R} , \quad \mathbf{1}_{--} : n_{2R} , n_{3R} , \phi_u ,$$
  $$\mathbf{1}_{-+} : p_{2R} , p_{3R} , \phi_d , \quad \mathbf{1}_{+-} : Q_{3L} , p_{1R} . \tag{22}$$

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- The gauge and D_4 invariant Yukawa Lagrangians in the up and down quark sectors are then given by

$$\begin{aligned} -\mathscr{L}_u &= A_u(\bar{Q}_{1L}\tilde{\phi}_1 - \bar{Q}_{2L}\tilde{\phi}_2)p_{1R} + B_u(\bar{Q}_{1L}\tilde{\phi}_2 + \bar{Q}_{2L}\tilde{\phi}_1)p_{2R} + C_u(\bar{Q}_{1L}\tilde{\phi}_2 + \bar{Q}_{2L}\tilde{\phi}_1)p_{3R} \\ &\quad + X_u\bar{Q}_{3L}\tilde{\phi}_u p_{2R} + Y_u\bar{Q}_{3L}\tilde{\phi}_u p_{3R} , \qquad (23a) \\ -\mathscr{L}_d &= A_d(\bar{Q}_{1L}\phi_1 + \bar{Q}_{2L}\phi_2)n_{1R} + B_d(\bar{Q}_{1L}\phi_2 - \bar{Q}_{2L}\phi_1)n_{2R} + C_d(\bar{Q}_{1L}\phi_2 - \bar{Q}_{2L}\phi_1)n_{3R} \\ &\quad + X_d\bar{Q}_{3L}\phi_d n_{2R} + Y_d\bar{Q}_{3L}\phi_d n_{3R} . \qquad (23b) \end{aligned}$$

Going beyond 2HDM

- $$\mathbf{2}: \begin{bmatrix} Q_{1L} \\ Q_{2L} \end{bmatrix}, \begin{bmatrix} \phi_1 \\ \phi_2 \end{bmatrix}, \tag{21}$$
$$\mathbf{1}_{++}: n_{1R}, \quad \mathbf{1}_{--}: n_{2R}, n_{3R}, \phi_u,$$
$$\mathbf{1}_{-+}: p_{2R}, p_{3R}, \phi_d, \quad \mathbf{1}_{+-}: Q_{3L}, p_{1R}. \tag{22}$$

- The gauge and D_4 invariant Yukawa Lagrangians in the up and down quark sectors are then given by

$$-\mathscr{L}_u = A_u(\bar{Q}_{1L}\tilde{\phi}_1 - \bar{Q}_{2L}\tilde{\phi}_2)p_{1R} + B_u(\bar{Q}_{1L}\tilde{\phi}_2 + \bar{Q}_{2L}\tilde{\phi}_1)p_{2R} + C_u(\bar{Q}_{1L}\tilde{\phi}_2 + \bar{Q}_{2L}\tilde{\phi}_1)p_{3R} + X_u\bar{Q}_{3L}\tilde{\phi}_u p_{2R} + Y_u\bar{Q}_{3L}\tilde{\phi}_u p_{3R}, \tag{23a}$$
$$-\mathscr{L}_d = A_d(\bar{Q}_{1L}\phi_1 + \bar{Q}_{2L}\phi_2)n_{1R} + B_d(\bar{Q}_{1L}\phi_2 - \bar{Q}_{2L}\phi_1)n_{2R} + C_d(\bar{Q}_{1L}\phi_2 - \bar{Q}_{2L}\phi_1)n_{3R} + X_d\bar{Q}_{3L}\phi_d n_{2R} + Y_d\bar{Q}_{3L}\phi_d n_{3R}. \tag{23b}$$

- VEV hierarchy: $v_{1,2} \ll v_{u,d}$
- The third generation of quarks will get their masses from v_u and v_d in the up and down sector respectively while $v_{12} = \sqrt{v_1^2 + v_2^2}$ is responsible for the first two generations of quarks.

Highlights

-
$$V_{\rm CKM} \approx \begin{pmatrix} \cos 2\beta & -\sin 2\beta & -\theta_d \sin 2\beta \\ \sin 2\beta & \cos 2\beta & \theta_d \cos 2\beta - \theta_u \\ \theta_u \sin 2\beta & \theta_d + \theta_u \cos 2\beta & 1 \end{pmatrix} . \tag{24}$$

- First generation: $m_u^2 = A_u^2 v_{12}^2 \,, \quad m_d^2 = A_d^2 v_{12}^2 \,.$
- $v_{12} \approx \mathcal{O}\left(1\ GeV\right)$, $v_{u,d} \approx \mathcal{O}\left(100\ GeV\right)$, implying $\theta_{u,d} \approx \mathcal{O}\left(\frac{v_{12}}{v_{u,d}}\right) \approx \mathcal{O}\left(\lambda^2\right) \Rightarrow$ Naturally leads to the Wolfenstein parametrization of $V_{\rm CKM}$
- $m_{t,b}^2 \approx (Y_{u,d}^2 + X_{u,d}^2) v_{u,d}^2$
- $m_{c,s}^2 \approx \frac{(B_{u,d} Y_{u,d} - C_{u,d} X_{u,d})^2}{(Y_{u,d}^2 + X_{u,d}^2)} v_{12}^2.$

Conclusions

People who believe in hidden patterns in quark masses and mixings

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THANK YOU !