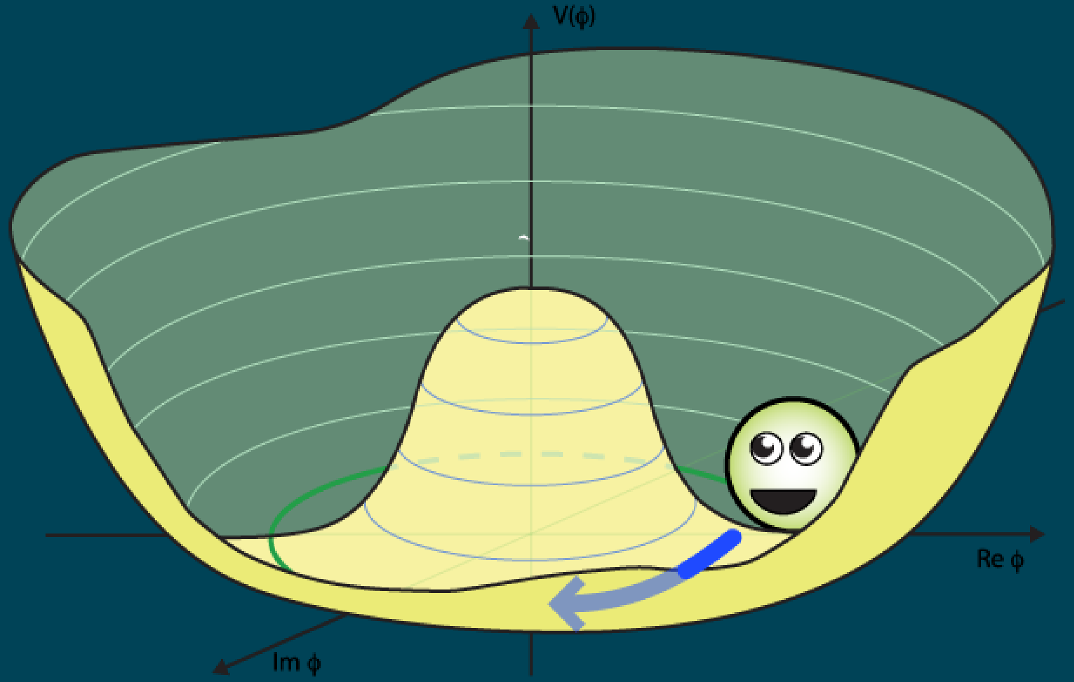
Western Norway University of Applied Sciences

Tree-level metastability bounds of the 2HDM in terms of physical parameters

Talk given at workshop on Multi-Higgs models, september 2024



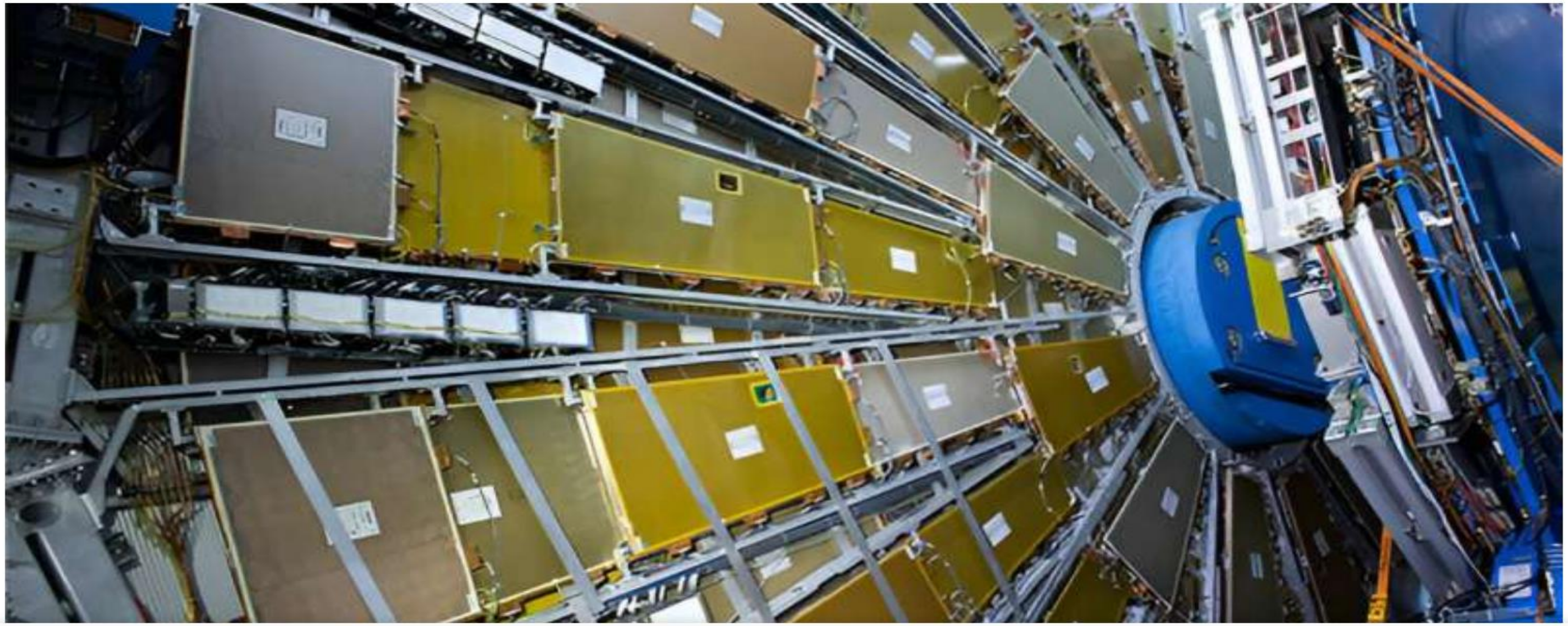
Odd Magne Øgreid

AUGUST 22, 2024

Editors' notes

Deviations in particle interactions point to the existence of new bosons

by Wits University



The 2HDM potential

$$V = V_2 + V_4$$

$$V_2 = -\frac{1}{2}\left\{m_{11}^2\Phi_1^\dagger\Phi_1 + m_{22}^2\Phi_2^\dagger\Phi_2 + \left[\underline{m_{12}^2}\Phi_1^\dagger\Phi_2 + \text{h.c.}\right]\right\}$$

$$V_4 = \frac{\lambda_1}{2}(\Phi_1^\dagger\Phi_1)^2 + \frac{\lambda_2}{2}(\Phi_2^\dagger\Phi_2)^2 + \lambda_3(\Phi_1^\dagger\Phi_1)(\Phi_2^\dagger\Phi_2) + \lambda_4(\Phi_1^\dagger\Phi_2)(\Phi_2^\dagger\Phi_1)$$
$$+\frac{1}{2}\left[\underline{\lambda_5}(\Phi_1^\dagger\Phi_2)^2 + \text{h.c.}\right] + \left\{\left[\underline{\lambda_6}(\Phi_1^\dagger\Phi_1) + \underline{\lambda_7}(\Phi_2^\dagger\Phi_2)\right](\Phi_1^\dagger\Phi_2) + \text{h.c.}\right\}$$

- 14 parameters (reducible to 11)
- 4 **complex** parameters

The physical parameter set $\mathcal{P}$ and counting of parameters.

- Potential has initially 14 parameters
- Exploit the freedom to change basis and reduce to 11 independent parameters.
- Traditional approach:
  Work out masses and couplings expressed in terms of the initial 14 (or 11) parameters of the potential (exchange some for VEVs).
- Alternative approach:
  Work the other way around. Pick a set of 11 independent physical masses and couplings (all invariants) and express the initial 14 parameters in terms of these
- We now choose our set of 11 independent parameters to consist of:
  - Four squared masses
  - Three gauge couplings
  - Four scalar couplings

$$\mathcal{P} \equiv \{M_{H\pm}^2, M_1^2, M_2^2, M_3^2, e_1, e_2, e_3, q_1, q_2, q_3, q\}$$

- Observables from the potential (invariants) expressible through these.
- Trilinear and quadrilinear scalar couplings expressible through these.

$$e_i \equiv \frac{2}{g^2}\text{Coefficient}(\mathcal{L}, H_i W^- W^+)$$
$$q_i \equiv \text{Coefficient}(V, H_i H^- H^+)$$
$$q \equiv \text{Coefficient}(V, H^- H^- H^+ H^+).$$

Satisfying: $v^2 = e_1^2 + e_2^2 + e_3^2$

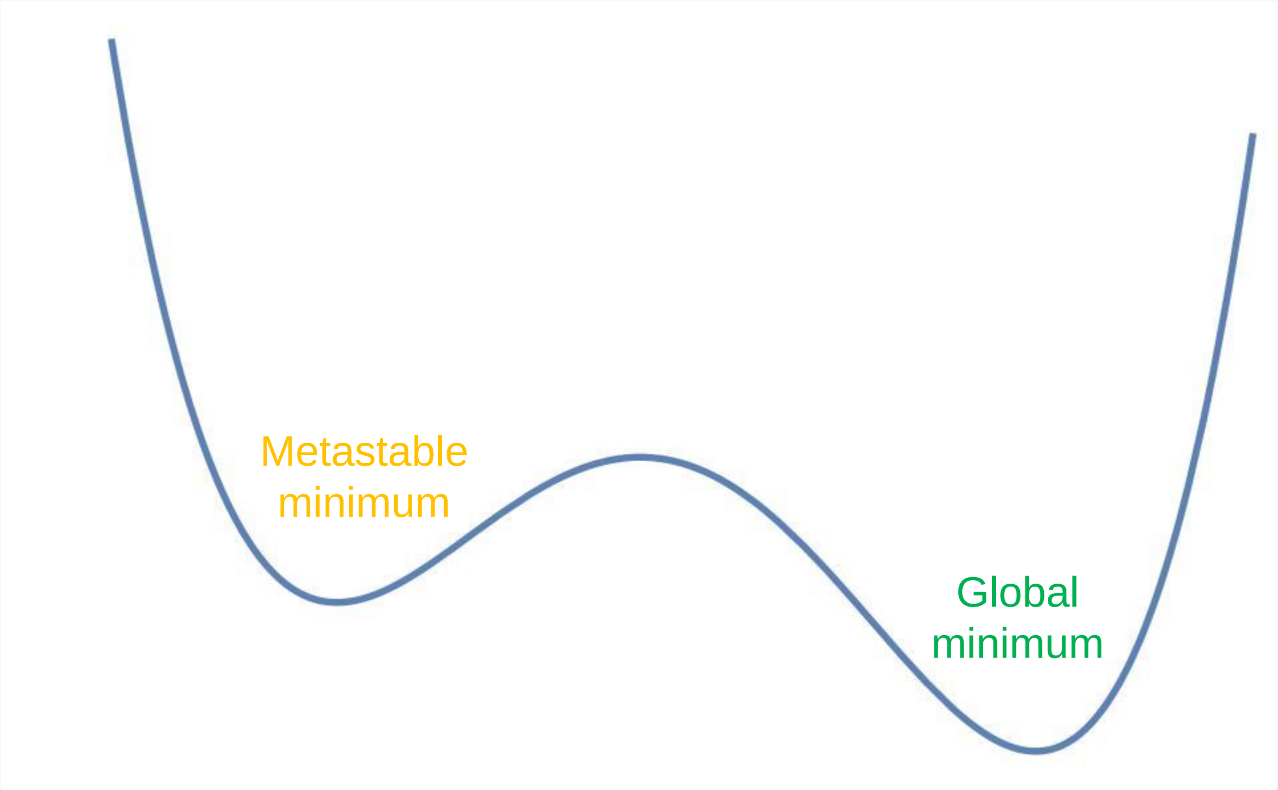
Description of translation process:
Ogreid: PoS CORFU2017 (2018) 065

Remaining scalar couplings expressible in terms of $\mathcal{P}$:
Grzadkowski, Haber, Ogreid & Osland: JHEP 12 (2018) 056

Symmetries of potential (exact, spontaneously broken or softly broken all described in terms of $\mathcal{P}$:
Ferreira, Grzadkowski, Ogreid & Osland: JHEP 02 (2021) 196
Ferreira, Grzadkowski, Ogreid & Osland: JHEP 01 (2023) 143

Further applications?
- Tree-level unitarity constraints
- Boundedness from below (BFB) constraints
- Vacuum metastability constraints

Vacuum metastability in the 2HDM:



- 2HDM potentials allow for more than one minimum.
- How to discriminate the metastable from the global one?
- Boundedness from below important.

«Evading death by vacuum»

Vacuum metastability in the 2HDM:

Has been investigated in a series of papers:

«Evading ☠ by vacuum», (softly broken U(1))
Augusto Barroso, Pedro Ferreira, Igor Ivanov, Rui Santos, João Silva, EPJC 73 (2013)

«Metastability bounds on the two Higgs doublet model», (softly broken Z_2)
Augusto Barroso, Pedro Ferreira, Igor Ivanov, Rui Santos, JHEP 45 (2013)

«Tree-level metastability bounds for the most general two Higgs doublet model»,
Igor Ivanov, João Silva, PRD 92 (2015)

- In all papers, a discriminant *D* is presented, whose properties help determine metastability bounds.
- BUT – the discriminant *D* differs from paper to paper, and *D* for softly broken U(1) and Z_2 are not special cases of the *D* presented for the general 2HDM. They are related.

Vacuum metastability in the 2HDM:

«Tree-level metastability bounds for the most general two Higgs doublet model»,
Igor Ivanov, João Silva, PRD 92 (2015)

- Process described for the general case can be expressed in terms of the parameters of $\mathcal{P}$

After translating process to physical parameters, we can specialize to specific models with symmetries by using the results from
Ferreira, Grzadkowski, Ogreid & Osland: JHEP 02 (2021) 196
Ferreira, Grzadkowski, Ogreid & Osland: JHEP 01 (2023) 143

- Models with at least a Z_2 invariant V_4 ($\lambda_6 = \lambda_7 = 0$) simplify the process considerably, including IDM, C2HDM, $U(1)$, CP2, r_0, CP3, $SO(3)$.
- In the formalism using only physical parameters, we are guaranteed to be in a vacuum provided all squared masses $\{M_{H^\pm}^2, M_1^2, M_2^2, M_3^2\}$ are positive. Always assume this.
- The masses, together with the remaining parameters $\{e_1, e_2, e_3, q_1, q_2, q_3, q\}$ will specify the shape of the potential which could have another minimum.
- In that other minimum physical parameters would be different.

The process of Ivanov & Silva:

1. Identify a parameter ζ . Turns out that $\zeta = \frac{M_{H^\pm}^2}{v^2}$ always.
2. Calculate a discriminant D
   a) If $D > 0$ we are in the global minimum.
   b) If $D < 0$ we must continue
3. Calculate the eigenvalues of a matrix Λ_E . If some eigenvalues are complex, discard point (not BFB). If all are real, use projection operator to identify time-like eigenvalue Λ_0 . Remains three non-time-like eigenvalues: Λ_k
   a) If $\Lambda_0 < \Lambda_k$ for some value of k , potential is not BFB. Discard point.
   b) If $\Lambda_0 > \Lambda_k$ for all values of k , we continue:
      i. If $\zeta > \Lambda_0$ we are in a global minimum
      ii. If $\zeta < \Lambda_0$ we are in the metastable minimum

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Open questions:
What about *D*=0? (happens iff one eigenvalue equals ζ)
What about $\Lambda_0 = \max(\Lambda_k)$
What about $\zeta = \Lambda_0$ (implies *D*=0)

The discriminant *D* for the general 2HDM

$$\Lambda_E = \frac{1}{2}\begin{pmatrix} \frac{1}{2}(\lambda_1+\lambda_2)+\lambda_3 & \mathrm{Re}(\lambda_6+\lambda_7) & -\mathrm{Im}(\lambda_6+\lambda_7) & \frac{1}{2}(\lambda_1-\lambda_2) \\ -\mathrm{Re}(\lambda_6+\lambda_7) & -\lambda_4-\mathrm{Re}\lambda_5 & \mathrm{Im}\lambda_5 & -\mathrm{Re}(\lambda_6-\lambda_7) \\ \mathrm{Im}(\lambda_6+\lambda_7) & \mathrm{Im}\lambda_5 & -\lambda_4+\mathrm{Re}\lambda_5 & \mathrm{Im}(\lambda_6-\lambda_7) \\ -\frac{1}{2}(\lambda_1-\lambda_2) & -\mathrm{Re}(\lambda_6-\lambda_7) & \mathrm{Im}(\lambda_6-\lambda_7) & -\frac{1}{2}(\lambda_1+\lambda_2)+\lambda_3 \end{pmatrix}$$

$$D = -\det(\Lambda_E - \zeta), \quad \zeta = \frac{M_{H^\pm}^2}{v^2}$$

The discriminant D in terms of physical parameters

$$D = -\frac{1}{4v^{10}}\left[M_2^2 M_3^2 (v^2 q_1 - 2e_1 M_{H\pm}^2)^2 + M_1^2 M_3^2 (v^2 q_2 - 2e_2 M_{H\pm}^2)^2 + M_1^2 M_2^2 (v^2 q_3 - 2e_3 M_{H\pm}^2)^2\right] + \frac{M_1^2 M_2^2 M_3^2 q}{2v^6}$$

- Surprisingly(?) simple.
- Linear in q and quadratic in $M_{H\pm}^2$
- Linear also in $M_1^2,\ M_2^2,\ M_3^2$
- If $D > 0$, our minimum is the global minimum. Requires q to be positive, ($q = \frac{\lambda_2}{2}$ in HB).
- If $D < 0$, we cannot conclude yet. Further investigation is necessary. (q is not “too large”)

The discriminant *D* in terms of physical parameters

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$$+\frac{M_1^2M_2^2M_3^2q}{2v^6}$$

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- If *D* > 0, our minimum is the global minimum. Requires *q* to be positive, ($q = \frac{\lambda_2}{2}$ in HB).
- If *D* < 0, we cannot conclude yet. Further investigation is necessary. (*q* is not “too large”)
- If *D* = 0, two degenerate minima? Global minimum? Flat direction? Metastable minimum? Unbounded from below?

The discriminant D for a 2HDM with spontaneous CP violation

- If CP is spontaneously broken, we know that there exist two different minima of the same depth.
- We then know that in terms of physical parameters (Grzadkowski, Ogreid, Osland Phys. Rev. D 94, 115002 (2016))

$$\frac{M_{H^\pm}^2}{v^2} = \frac{(e_1 q_1 M_2^2 M_3^2 + e_2 q_2 M_3^2 M_1^2 + e_3 q_3 M_1^2 M_2^2 - M_1^2 M_2^2 M_3^2)}{2(e_1^2 M_2^2 M_3^2 + e_2^2 M_3^2 M_1^2 + e_3^2 M_1^2 M_2^2)},$$

$$q = \frac{(e_2 q_3 - e_3 q_2)^2 M_1^2 + (e_3 q_1 - e_1 q_3)^2 M_2^2 + (e_1 q_2 - e_2 q_1)^2 M_3^2 + M_1^2 M_2^2 M_3^2}{2(e_1^2 M_2^2 M_3^2 + e_2^2 M_3^2 M_1^2 + e_3^2 M_1^2 M_2^2)}$$

- Using this, we find that $D = 0$ (as expected?).
- Are there other situations when $D = 0$ other then spontaneous CP violation? YES!

The discriminant *D* in terms of physical parameters

- Let us parametrize in terms of parameters which measures deviation from SCPV

$$\Delta m_+ \equiv \frac{M_{H^\pm}^2}{v^2} - \frac{(e_1 q_1 M_2^2 M_3^2 + e_2 q_2 M_3^2 M_1^2 + e_3 q_3 M_1^2 M_2^2 - M_1^2 M_2^2 M_3^2)}{2(e_1^2 M_2^2 M_3^2 + e_2^2 M_3^2 M_1^2 + e_3^2 M_1^2 M_2^2)},$$

$$\Delta q \equiv q - \frac{(e_2 q_3 - e_3 q_2)^2 M_1^2 + (e_3 q_1 - e_1 q_3)^2 M_2^2 + (e_1 q_2 - e_2 q_1)^2 M_3^2 + M_1^2 M_2^2 M_3^2}{2(e_1^2 M_2^2 M_3^2 + e_2^2 M_3^2 M_1^2 + e_3^2 M_1^2 M_2^2)}$$

- SCPV iff $\Delta m_+ = \Delta q = 0$
- Discriminant takes on a simple form with these parameters:

$$D = \frac{1}{2v^6}\left[(2(\Delta m_+) + \Delta q) M_1^2 M_2^2 M_3^2 - 2(\Delta m_+)^2 (e_1^2 M_2^2 M_3^2 + e_2^2 M_3^2 M_1^2 + e_3^2 M_1^2 M_2^2)\right]$$

- Can vanish also when we do not have SCPV

What about $D < 0$?

- Follow the prescription of Ivanov & Silva.
- We need to find all four eigenvalues of the matrix Λ_E .

  Λ_0 : time-like eigenvalue
  Λ_k : non-time-like eigenvalues (k=1,2,3)

- For the general 2HDM, this involves solving the general quartic equation.
- Simplifies for models with $\lambda_6 = \lambda_7 = 0$ (Z_2 symmetry in V_4)
- If some eigenvalues are complex, we are not in the global minimum (discard point)
- If all eigenvalues are real, use projection operator to determine the time-like Λ_0
- If $\Lambda_0 < \Lambda_k$ for some k = 1,2,3, potential is not bounded from below (discard point)
- If $\Lambda_0 > \Lambda_k$ for all k = 1,2,3, and: $\zeta > \Lambda_0$, we are in the global minimum.
  $\zeta < \Lambda_0$, we are in the metastable minimum.

$$\zeta = \frac{M^2_{H^\pm}}{v^2}$$

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$$\zeta = \frac{M_{H^\pm}^2}{v^2}$$

$\Lambda_0 = \max(\Lambda_k)$?

$\zeta = \Lambda_0$?

Unbroken Z_2 symmetry – The IDM

- The Z_2 invariant potential:

$$V(\Phi_1,\Phi_2) = -\frac{1}{2}\left\{m_{11}^2\Phi_1^\dagger\Phi_1 + m_{22}^2\Phi_2^\dagger\Phi_2\right\} + \frac{\lambda_1}{2}(\Phi_1^\dagger\Phi_1)^2 + \frac{\lambda_2}{2}(\Phi_2^\dagger\Phi_2)^2 + \lambda_3(\Phi_1^\dagger\Phi_1)(\Phi_2^\dagger\Phi_2) + \lambda_4(\Phi_1^\dagger\Phi_2)(\Phi_2^\dagger\Phi_1) + \frac{\lambda_5}{2}\left[(\Phi_1^\dagger\Phi_2)^2 + \text{h.c.}\right].$$

- Seven free parameters
- The Z_2 invariant vacuum:

$$\langle\Phi_1\rangle = \frac{1}{\sqrt{2}}\begin{pmatrix}0\\ v\end{pmatrix}, \quad \langle\Phi_2\rangle = \begin{pmatrix}0\\ 0\end{pmatrix}.$$

- Alternative description in terms of physical parameters (Ferreira, Grzadkowski, Ogreid & Osland: JHEP 02 (2021) 196)

$$e_2 = q_2 = e_3 = q_3 = 0, \quad (e_1 = v = 246\ \text{GeV})$$

Metastability analysis for the IDM continued

- Discriminant given by: $D = \frac{M_2^2 M_3^2 (2v^2 M_1^2 q - (2M_{H\pm}^2 - vq_1)^2)}{4v^8}$
- If $2v^2 M_1^2 q - (2M_{H\pm}^2 - vq_1)^2 > 0$ we are in the global minimum
- If $2v^2 M_1^2 q - (2M_{H\pm}^2 - vq_1)^2 = 0$?
- If $2v^2 M_1^2 q - (2M_{H\pm}^2 - vq_1)^2 < 0$ we must continue (calculate eigenvalues of Λ_E)
- For all eigenvalues to be real, we must have $q > 0$. $0 < q < \frac{(2M_{H\pm}^2 - vq_1)^2}{2v^2 M_1^2}$
- Eigenvalues:

$$\begin{aligned}
\Lambda_1 &= \frac{M_{H\pm}^2 - M_2^2}{v^2} \\
\Lambda_2 &= \frac{M_{H\pm}^2 - M_3^2}{v^2} \\
\Lambda_3 &= \frac{q_1 - \sqrt{2M_1^2 q}}{2v} \\
\Lambda_4 &= \frac{q_1 + \sqrt{2M_1^2 q}}{2v} \equiv \Lambda_0 \text{ (time-like)}
\end{aligned}$$

$$\begin{aligned}
(\hat{P}^1)_{00} &= 0 \\
(\hat{P}^2)_{00} &= 0 \\
(\hat{P}^3)_{00} &= -\frac{(\sqrt{2q}v - \sqrt{M_1^2})^2}{4v\sqrt{2M_1^2 q}} \\
(\hat{P}^4)_{00} &= \frac{(\sqrt{2q}v + \sqrt{M_1^2})^2}{4v\sqrt{2M_1^2 q}}
\end{aligned}$$

Metastability analysis for the IDM continued

- If $\Lambda_0 < \Lambda_k$ for some *k* = 1,2,3, potential is not bounded from below (discard point)
- That happens whenever

$$\frac{q_1 + \sqrt{2M_1^2 q}}{2v} < \frac{M_{H^\pm}^2 - \min(M_2^2, M_3^2)}{v^2}$$

$$\Downarrow$$

$$vq_1 + v\sqrt{2M_1^2 q} - 2M_{H^\pm}^2 + 2\min(M_2^2, M_3^2) < 0$$

- We must have $\Lambda_0 > \Lambda_k$ for all *k* = 1,2,3,

$$\Downarrow$$

$$vq_1 + v\sqrt{2M_1^2 q} - 2M_{H^\pm}^2 + 2\min(M_2^2, M_3^2) > 0$$

Metastability analysis for the IDM continued

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⇓

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⇓

$$vq_1 + v\sqrt{2M_1^2 q} - 2M_{H^\pm}^2 + 2\min(M_2^2, M_3^2) > 0$$

- What happens if $vq_1 + v\sqrt{2M_1^2 q} - 2M_{H^\pm}^2 + 2\min(M_2^2, M_3^2) = 0$?

Metastability analysis for the IDM continued

- If $vq_1 + v\sqrt{2M_1^2 q} - 2M_{H^\pm}^2 + 2\min(M_2^2, M_3^2) > 0$

  and: $\zeta > \Lambda_0$, we are in the global minimum.
  $\zeta < \Lambda_0$, we are in the metastable minimum.
  $\zeta = \Lambda_0$, two degenerate minima(?)

$$\zeta = \frac{M_{H^\pm}^2}{v^2}$$

- Global minimum if: $\frac{M_{H^\pm}^2}{v^2} > \frac{q_1 + \sqrt{2M_1^2 q}}{2v}$
- Metastable minimum if: $\frac{M_{H^\pm}^2}{v^2} < \frac{q_1 + \sqrt{2M_1^2 q}}{2v}$
- Degenerate minima (?) if: $\frac{M_{H^\pm}^2}{v^2} = \frac{q_1 + \sqrt{2M_1^2 q}}{2v}$

Metastability analysis for the IDM continued

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- Degenerate minima (?) if: $$\frac{M_{H^\pm}^2}{v^2} = \frac{q_1 + \sqrt{2M_1^2 q}}{2v}$$

Metastability analysis for the IDM continued

- If $vq_1 + v\sqrt{2M_1^2 q} - 2M_{H^\pm}^2 + 2\min(M_2^2, M_3^2) > 0$

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- Global minimum if: $\frac{M_{H^\pm}^2}{v^2} > \frac{q_1 + \sqrt{2M_1^2 q}}{2v}$ $\quad vq_1 + v\sqrt{2M_1^2 q} - 2M_{H^\pm}^2 < 0$
- Metastable minimum if: $\frac{M_{H^\pm}^2}{v^2} < \frac{q_1 + \sqrt{2M_1^2 q}}{2v}$ $\quad vq_1 + v\sqrt{2M_1^2 q} - 2M_{H^\pm}^2 > 0$
- Degenerate minima (?) if: $\frac{M_{H^\pm}^2}{v^2} = \frac{q_1 + \sqrt{2M_1^2 q}}{2v}$ $\quad vq_1 + v\sqrt{2M_1^2 q} - 2M_{H^\pm}^2 = 0$

Visualization for IDM

- Already have

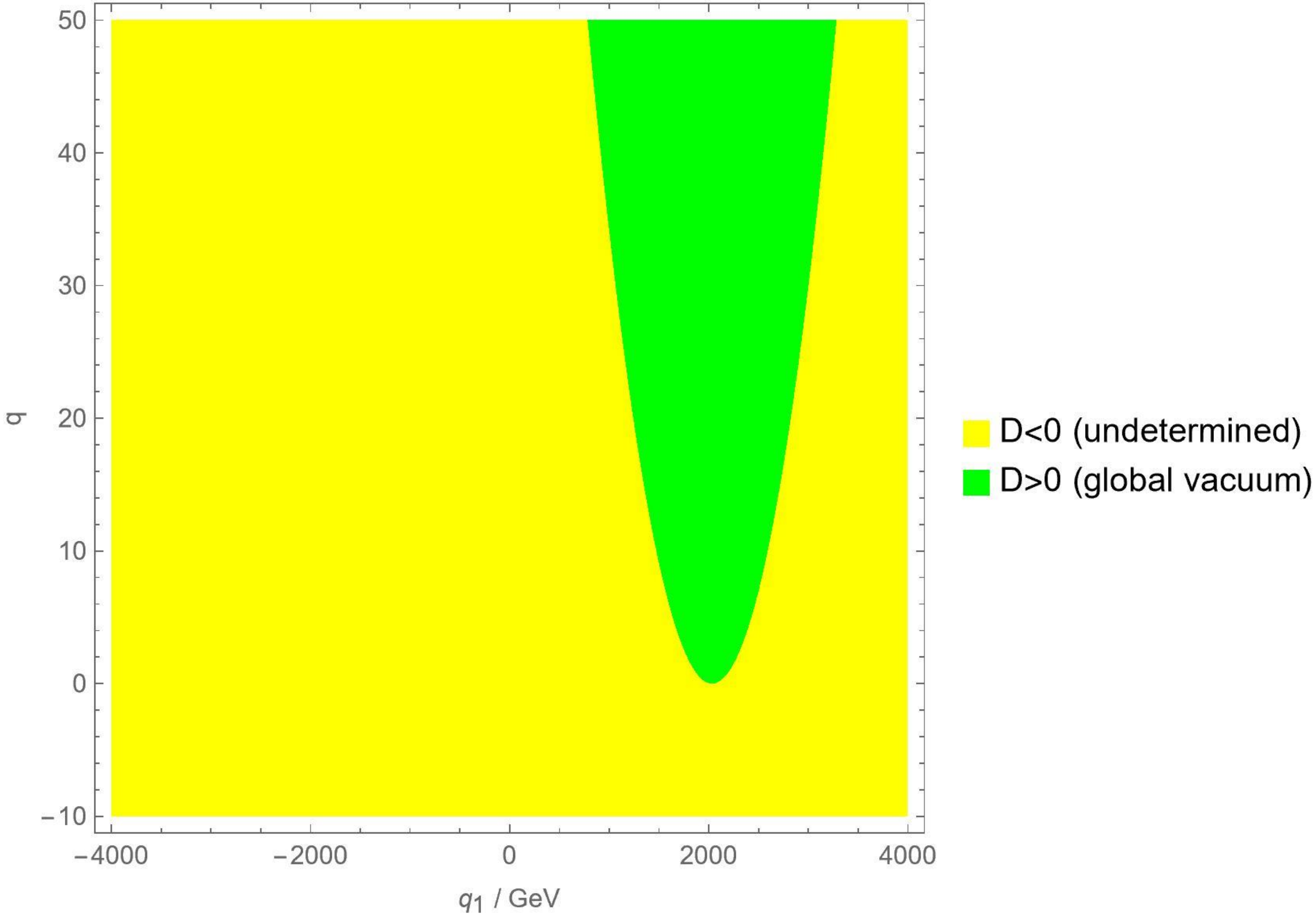
$$e_2 = q_2 = e_3 = q_3 = 0, \quad (e_1 = v = 246 \text{ GeV})$$

- Pick set of numerical values

$$\begin{aligned} M_1 &= 125 \text{ GeV} \\ M_2 &= 200 \text{ GeV} \\ M_3 &= 300 \text{ GeV} \\ M_{H^\pm} &= 500 \text{ GeV} \end{aligned}$$

- Remaining parameters: q_1 and q
- Scan over q_1-q-space

Visualization for IDM



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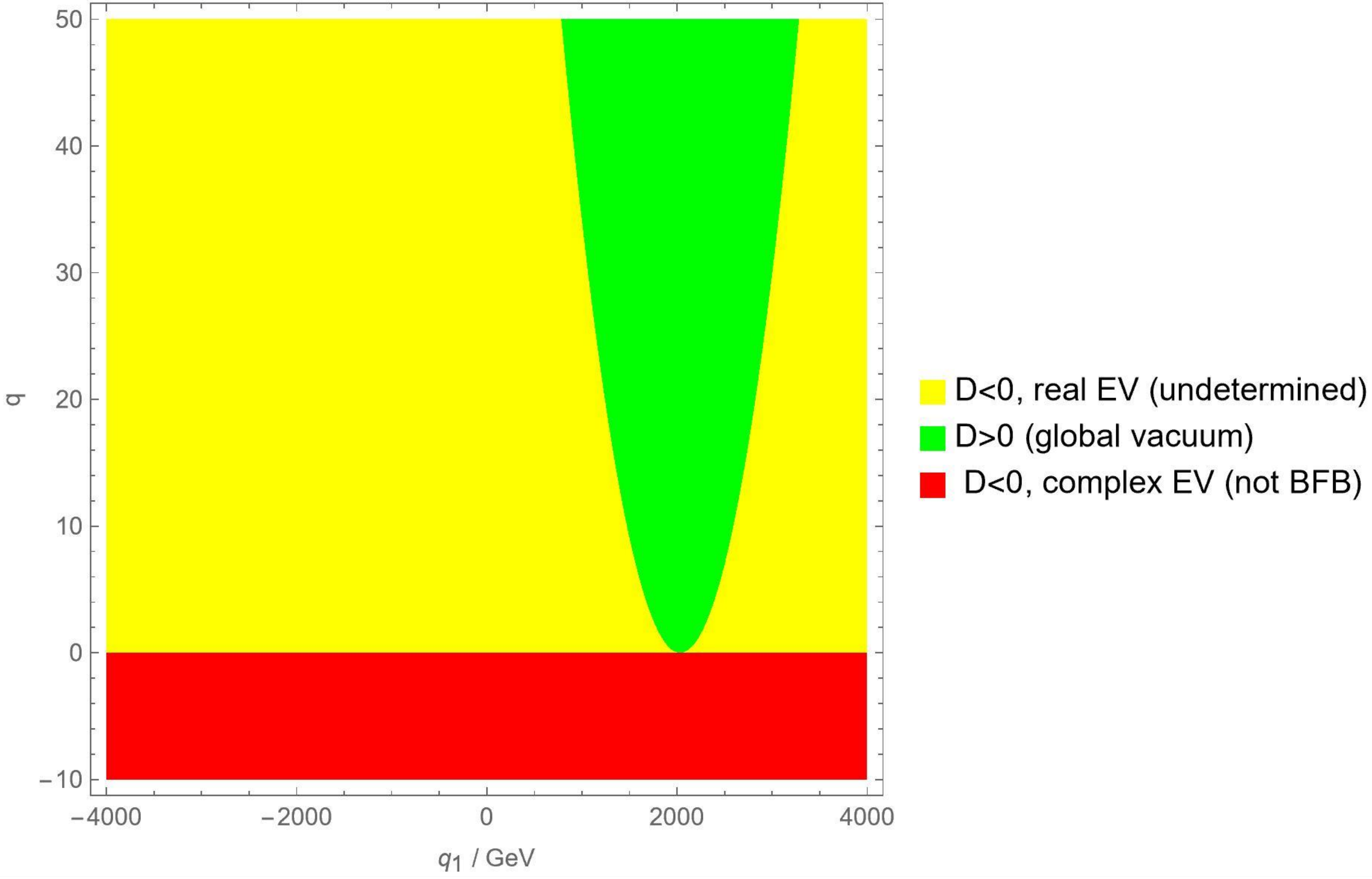

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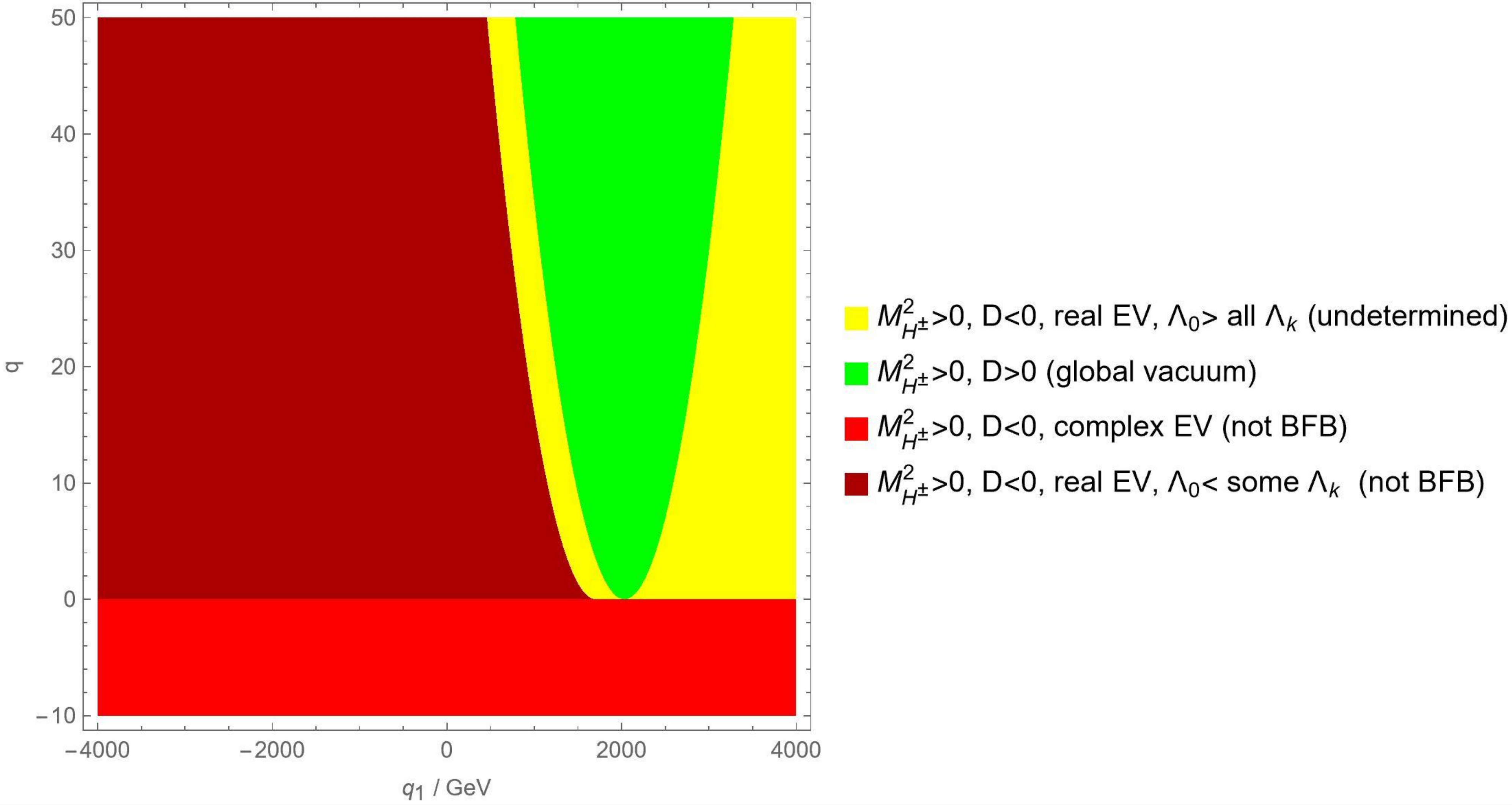

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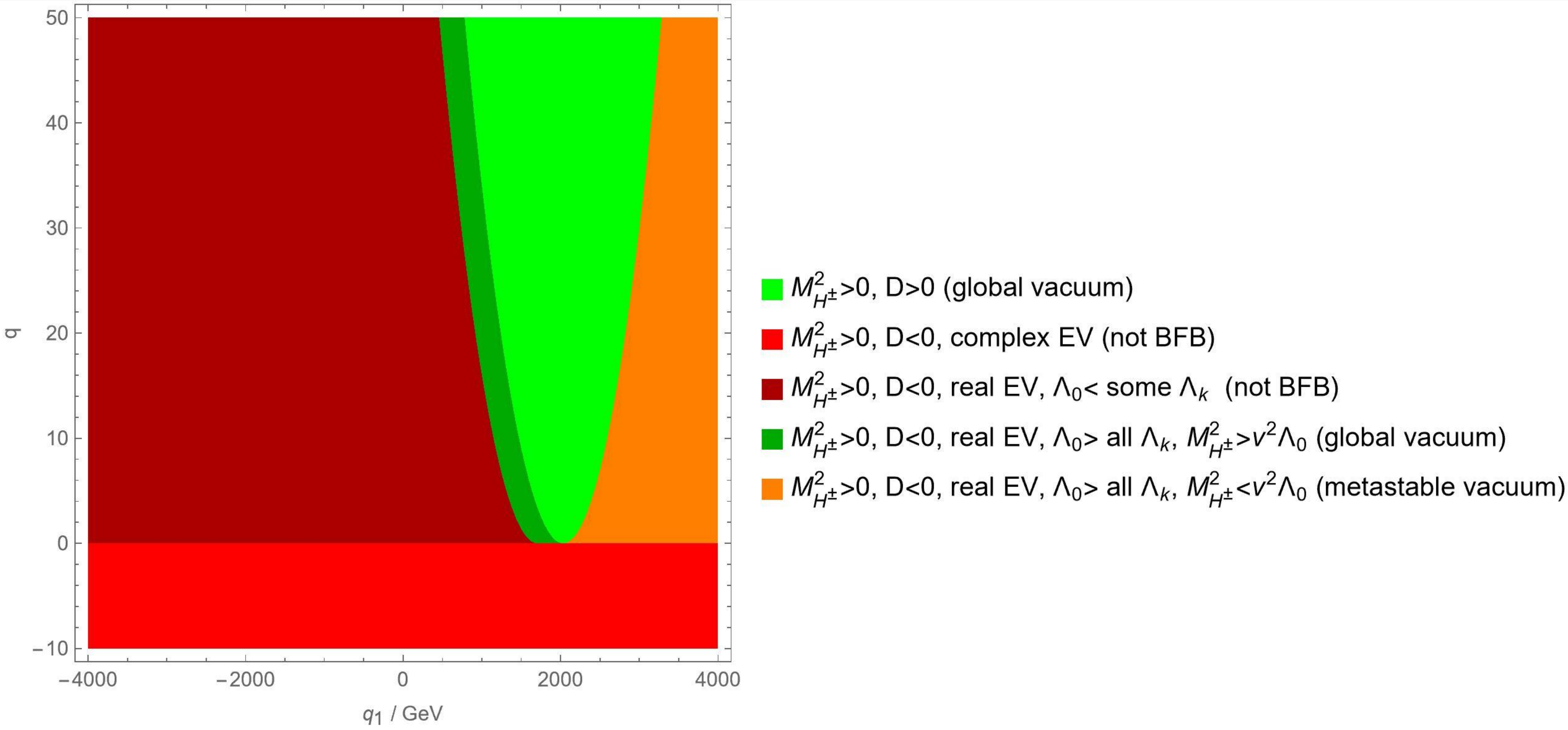
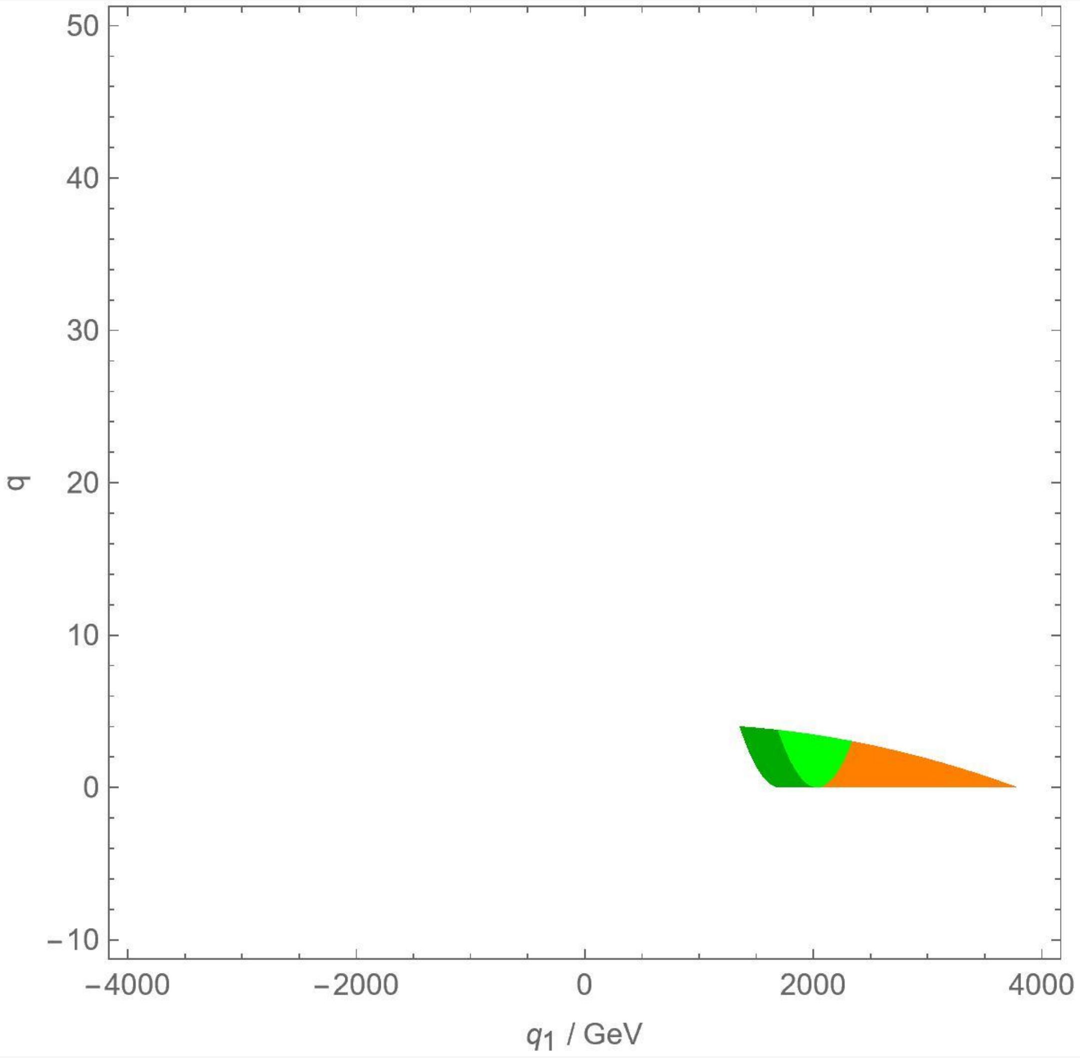


Visualization for IDM – also unitarity constraints



$M^2_{H^\pm}>0$, D>0 (global vacuum)

$M^2_{H^\pm}>0$, D<0, real EV, $\Lambda_0>$ all Λ_k, $M^2_{H^\pm}>v^2\Lambda_0$ (global vacuum)

$M^2_{H^\pm}>0$, D<0, real EV, $\Lambda_0>$ all Λ_k, $M^2_{H^\pm}<v^2\Lambda_0$ (metastable vacuum)

Softly broken Z_2 with complex m_{12}^2 (C2HDM)

- The C2HDM potential

$$V(\Phi_1,\Phi_2) = -\frac{1}{2}\left\{m_{11}^2\Phi_1^\dagger\Phi_1 + m_{22}^2\Phi_2^\dagger\Phi_2 + \left[m_{12}^2\Phi_1^\dagger\Phi_2 + \text{h.c.}\right]\right\}$$
$$+\frac{\lambda_1}{2}(\Phi_1^\dagger\Phi_1)^2 + \frac{\lambda_2}{2}(\Phi_2^\dagger\Phi_2)^2 + \lambda_3(\Phi_1^\dagger\Phi_1)(\Phi_2^\dagger\Phi_2)$$
$$+\lambda_4(\Phi_1^\dagger\Phi_2)(\Phi_2^\dagger\Phi_1) + \frac{1}{2}\left[\lambda_5(\Phi_1^\dagger\Phi_2)^2 + \text{h.c.}\right].$$

- Vacuum:

$$\langle\Phi_1\rangle = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 \\ v_1 \end{pmatrix}, \quad \langle\Phi_2\rangle = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 \\ v_2 \end{pmatrix}.$$

- 9 free parameters
- Popular model since FCNC are constrained and CP is broken.

C2HDM can alternatively be described by physical parameters

$$q = d_{010} - \frac{1}{2}d_{012} - d_{101} - \frac{4\,\mathrm{Im}J_{11} + \mathrm{Im}J_2 + 2\mathrm{Im}J_{30}}{2\mathrm{Im}J_1},$$

$$\begin{aligned} M_{H^\pm}^2 &= v^2\{2(d_{010}d_{012} - d_{010}d_{101} - d_{022} + d_{200})(\mathrm{Im}J_1)^2 \\ &\quad + [4(2d_{101} - d_{010})\mathrm{Im}J_{11} + (d_{012} - 2d_{010} + 3d_{101})\mathrm{Im}J_2 + 2(d_{101} - d_{012})\mathrm{Im}J_{30}]\mathrm{Im}J_1 \\ &\quad + (2\,\mathrm{Im}J_{11} + \mathrm{Im}J_2)(4\,\mathrm{Im}J_{11} + \mathrm{Im}J_2 + 2\,\mathrm{Im}J_{30})\}/\{2\mathrm{Im}J_1[2(d_{012} + d_{101} - d_{010})\mathrm{Im}J_1 \\ &\quad + 4\,\mathrm{Im}J_{11} + \mathrm{Im}J_2 + 2\,\mathrm{Im}J_{30}]\} \end{aligned}$$

Ferreira, Grzadkowski, Ogreid & Osland: JHEP 01 (2023) 143

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$$\begin{aligned} \mathrm{Im}J_1 &= \frac{1}{v^5} \sum_{i,j,k} \epsilon_{ijk} M_i^2 e_i e_k q_j \\ \mathrm{Im}J_{11} &= \frac{1}{v^7} \sum_{i,j,k} \epsilon_{ijk} M_i^2 M_j^2 e_i e_k q_j \\ \mathrm{Im}J_2 &= \frac{2e_1e_2e_3}{v^9}(M_1^2 - M_2^2)(M_2^2 - M_3^2)(M_3^2 - M_1^2) \\ \mathrm{Im}J_{30} &= \frac{1}{v^5} \sum_{i,j,k} \epsilon_{ijk} q_i M_i^2 e_j q_k \end{aligned}$$

$$d_{ijk} = \frac{q_1^i M_1^{2j} e_1^k + q_2^i M_2^{2j} e_2^k + q_3^i M_3^{2j} e_3^k}{v^{i+2j+k}}$$

Metastability for C2HDM

- Plug expressions for q and $M_{H^\pm}^2$ into expression for D - no obvious simplification
- Remember now that $M_{H^\pm}^2$ is function of $\mathcal{P}' \equiv \{M_1^2, M_2^2, M_3^2, e_1, e_2, e_3, q_1, q_2, q_3\}$
- Ensure that $M_{H^\pm}^2$ is positive (we start from a minimum), analyze sign of D
- Eigenvalues: Characteristic equation factorizes into $(\Lambda^2 + b_1\Lambda + c_1)(\Lambda^2 + b_2\Lambda + c_2) = 0$ where b_1, c_1, b_2, c_2 are analytical expressions of the parameters in $\mathcal{P}'$
- One pair of eigenvalues may be complex – discard such points
- When eigenvalues are real, follow prescription of Ivanov&Silva by comparing eigenvalues and compare to charged mass

Visualization for C2HDM

- Pick set of numerical values

$$
\begin{aligned}
M_1 &= 125 \text{ GeV} \\
M_2 &= 200 \text{ GeV} \\
M_3 &= 300 \text{ GeV} \\
v &= 246 \text{ GeV} \\
e_2 &= 5 \text{ GeV} \\
e_3 &= 10 \text{ GeV} \\
q_1 &= 90 \text{ GeV}
\end{aligned}
$$

- Not too far from AL $e_2 = e_3 = 0$
- Remaining parameters: q_2 and q_3
- Scan over q_2 - q_3 – space
- Ensure positivity of $M_{H^\pm}^2$

Visualization for C2HDM

- Pick set of numerical values

$$
\begin{aligned}
M_1 &= 125 \text{ GeV} \\
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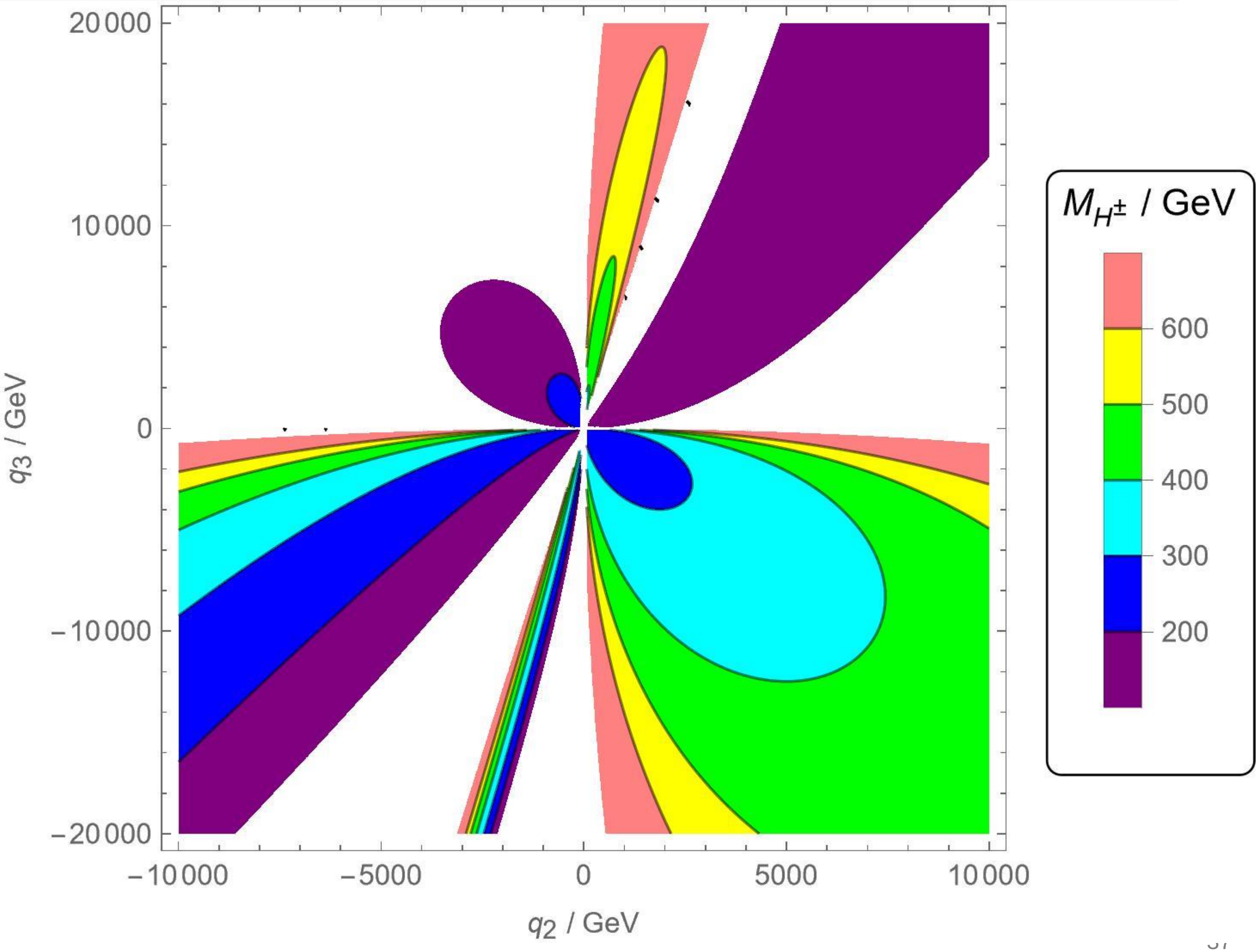
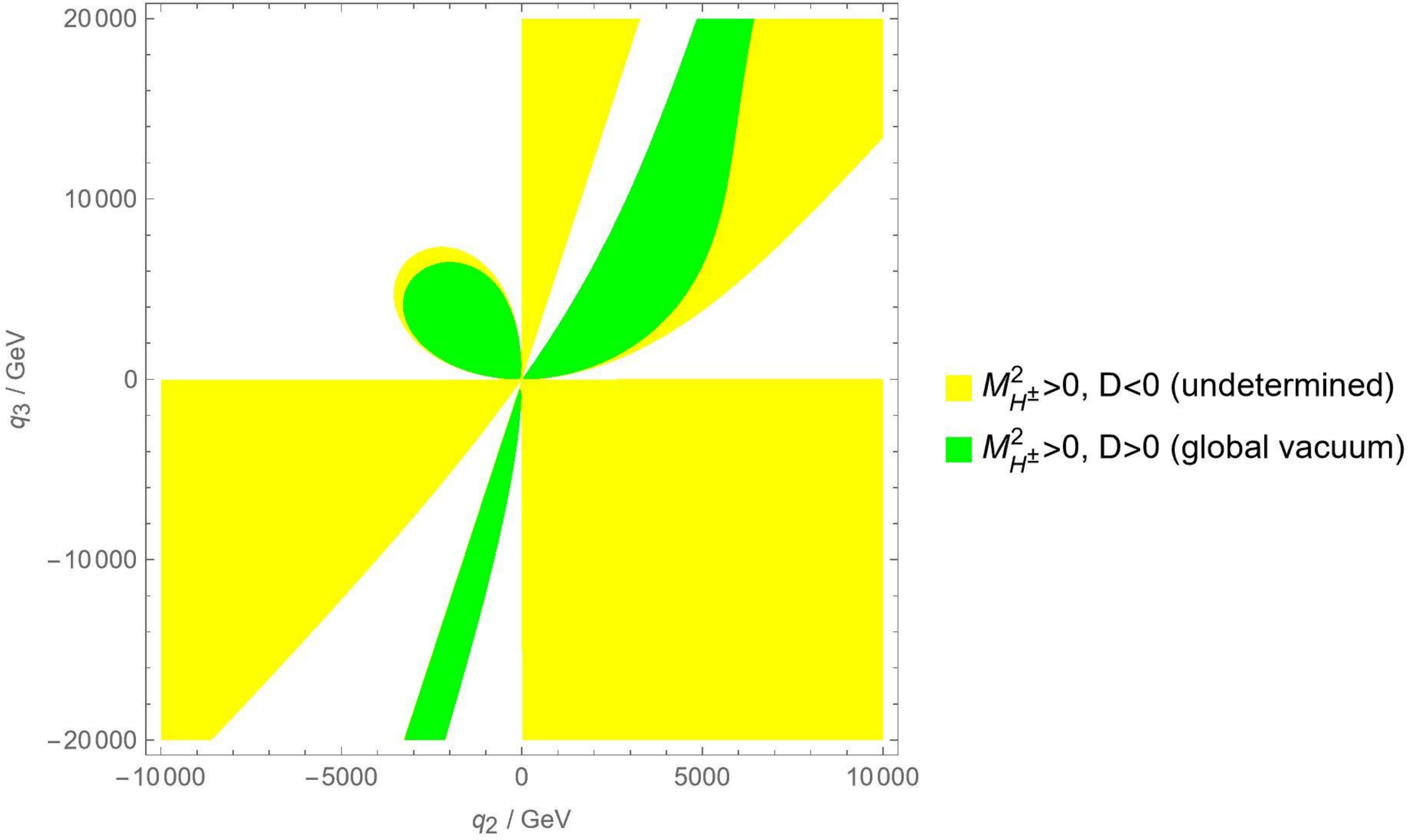
v &= 246 \text{ GeV} \\
e_2 &= 5 \text{ GeV} \\
e_3 &= 10 \text{ GeV} \\
q_1 &= 90 \text{ GeV}
\end{aligned}
$$

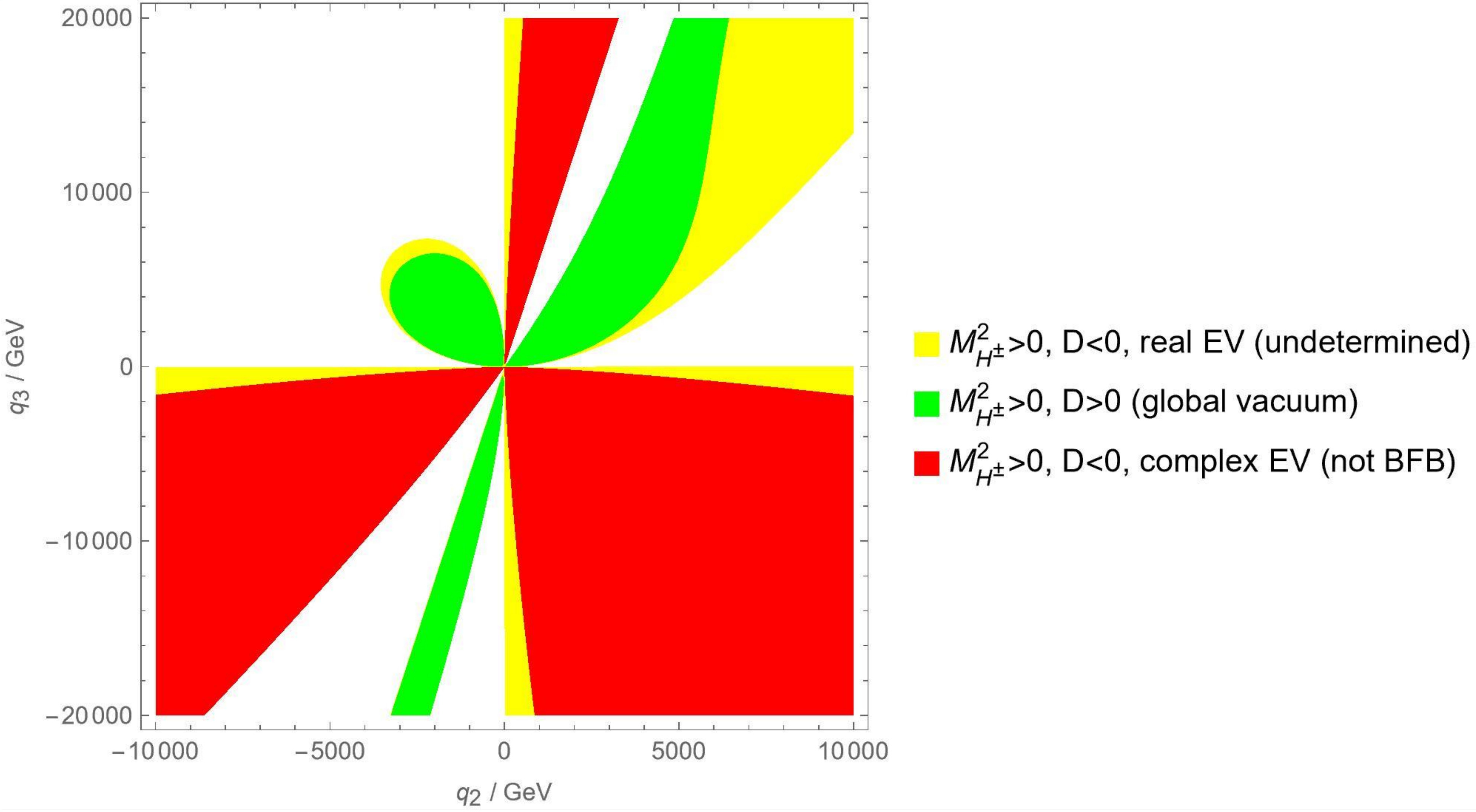
- Not too far from AL $e_2 = e_3 = 0$
- Remaining parameters: q_2 and q_3
- Scan over q_2 - q_3 – space
- Ensure positivity of $M^2_{H^\pm}$



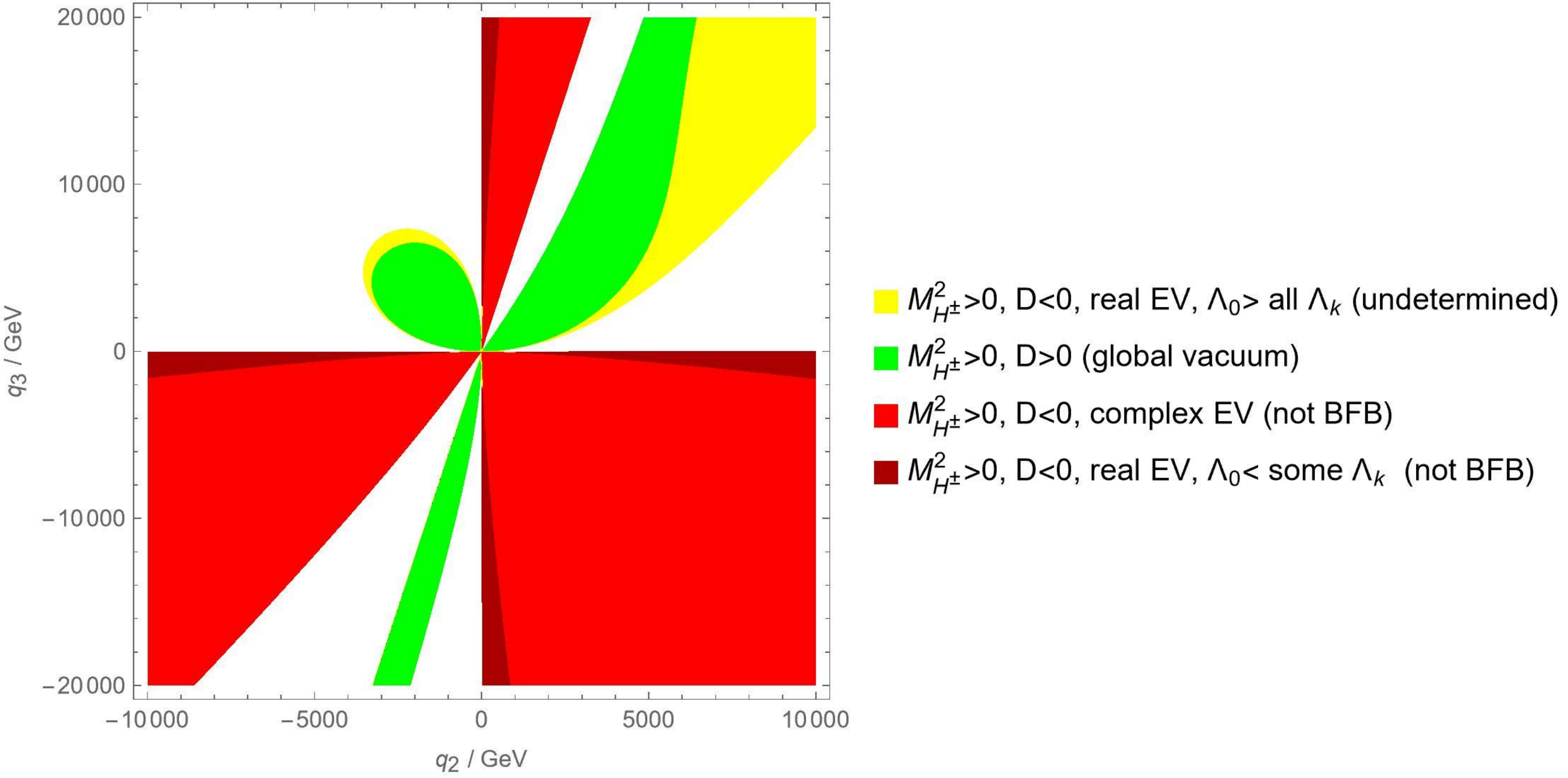
Visualization for C2HDM



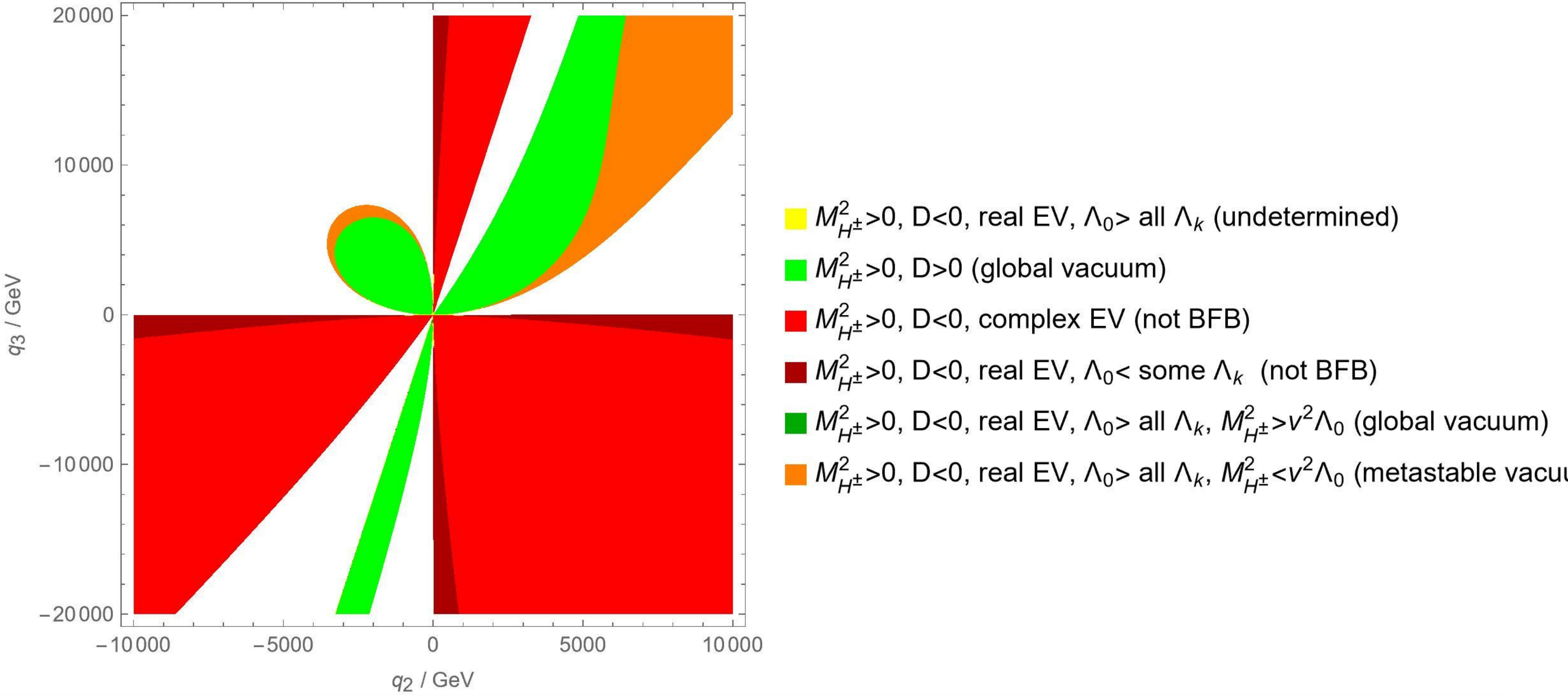
Visualization for C2HDM



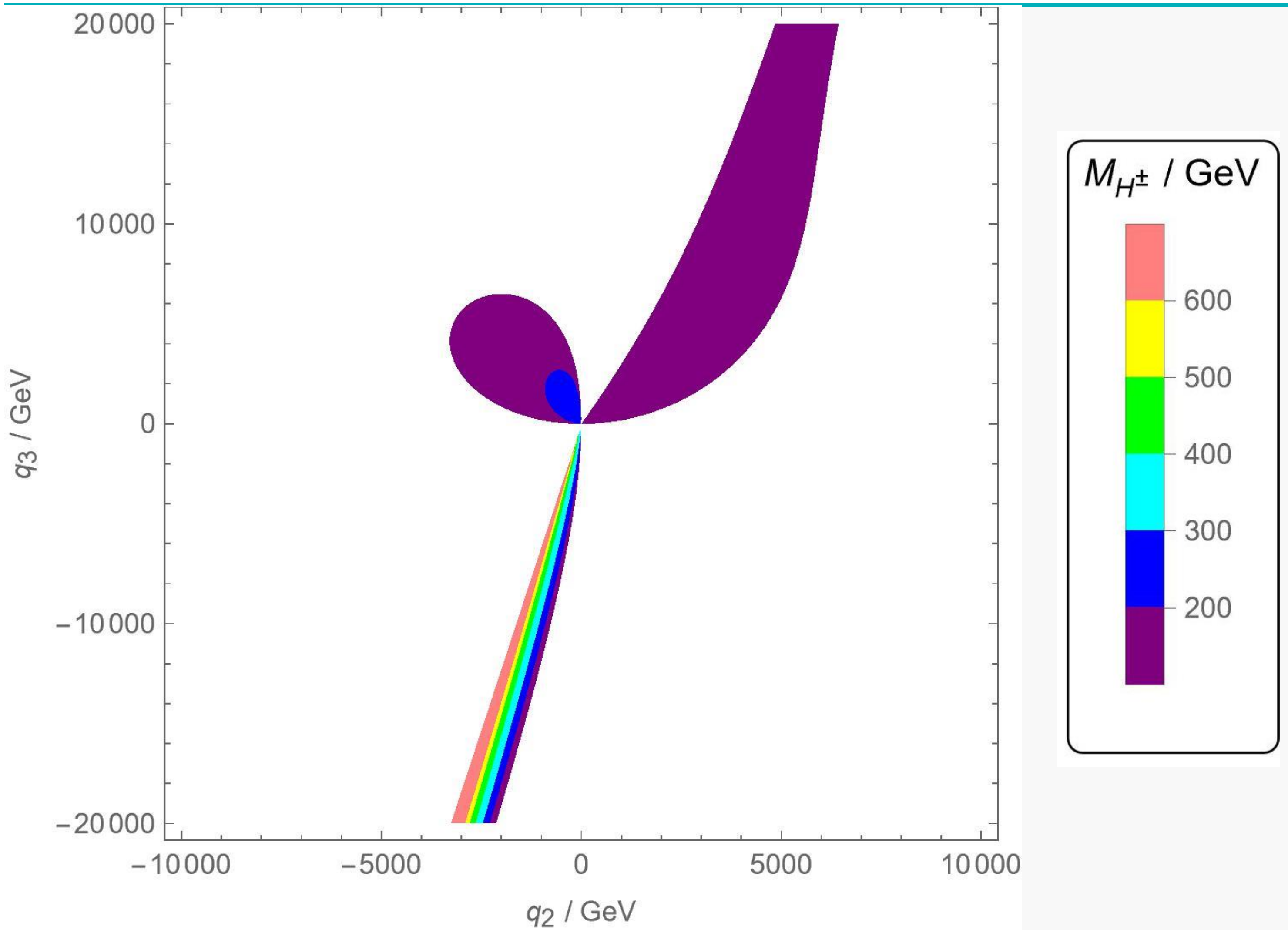
Visualization for C2HDM



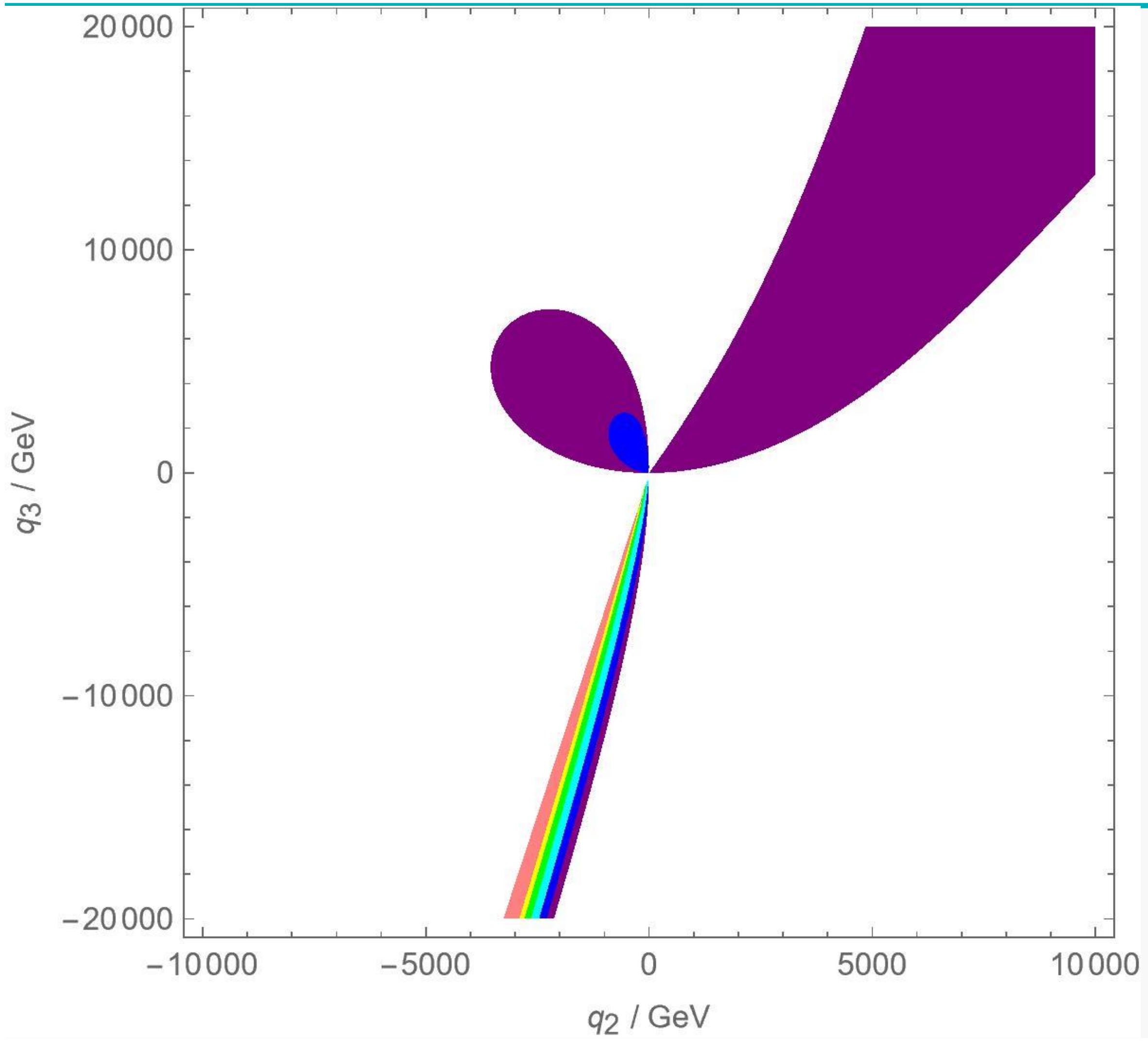
Visualization for C2HDM

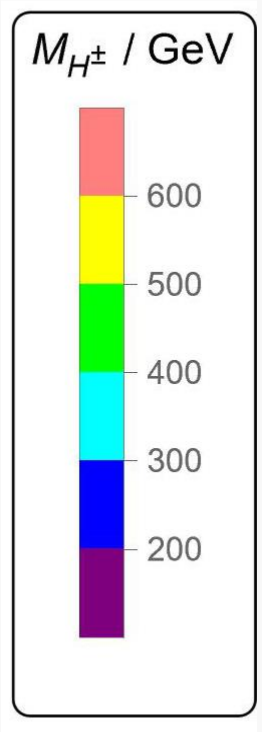


Visualization for C2HDM – charged mass in stable region

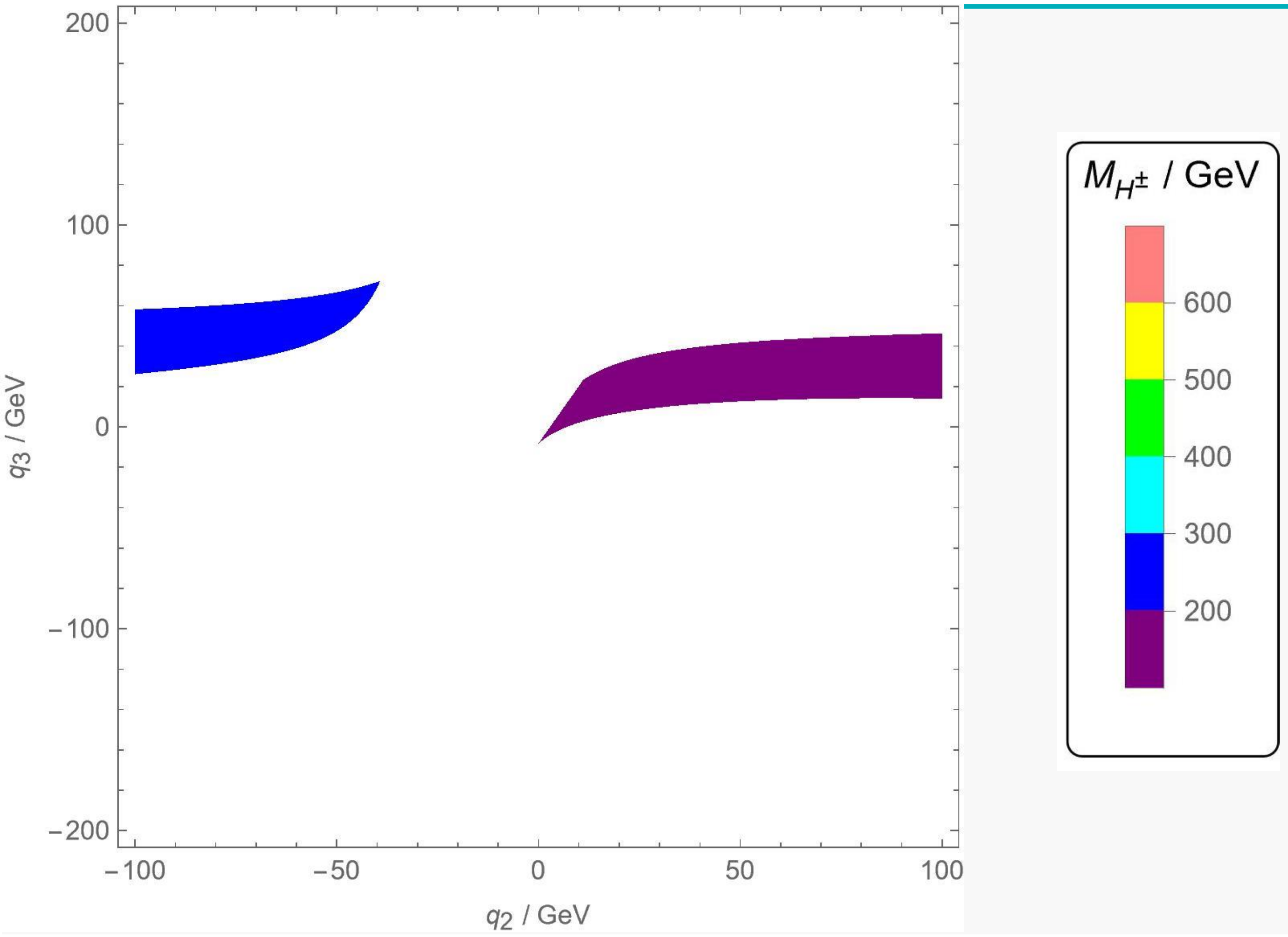


Visualization for C2HDM – charged mass in metastable/stable regions





Visualization for C2HDM – unitarity constraints added



General 2HDM

- Already have discriminant *D*

- Eigenvalues found from characteristic equation

$$\Lambda^4 + b\Lambda^3 + c\Lambda^2 + d\Lambda + e = 0$$

- All coefficients expressed in terms of masses and couplings

$$\begin{aligned}
b &= -2m_+ + d_{010} - d_{012} - d_{101}, \\
c &= m_+^2 + m_+(-d_{010} + d_{012} + 2d_{101}) - \frac{d_{012}}{2}q \\
&\quad - d_{010}d_{012} - d_{010}d_{101} + \frac{d_{010}^2}{2} - \frac{d_{020}}{2} + d_{022} + \frac{d_{101}^2}{4} + d_{111}, \\
d &= -d_{101}m_+^2 + d_{012}qm_+ + m_+\left(d_{010}d_{101} - \frac{d_{101}^2}{2} - d_{111}\right) + q\left(\frac{d_{022}}{2} - \frac{d_{010}d_{012}}{2}\right) \\
&\quad - \frac{1}{2}d_{010}^2 d_{101} + \frac{1}{4}d_{010}d_{101}^2 + d_{010}d_{111} + \frac{d_{012}d_{200}}{4} + \frac{d_{020}d_{101}}{2} - \frac{d_{101}d_{111}}{2} - d_{121}, \\
e &= -\frac{d_{012}}{2}qm_+^2 + \frac{d_{101}^2}{4}m_+^2 + qm_+\left(\frac{d_{010}d_{012}}{2} - \frac{d_{022}}{2}\right) \\
&\quad + m_+\left(-\frac{1}{4}d_{010}d_{101}^2 - \frac{d_{012}d_{200}}{4} + \frac{d_{101}d_{111}}{2}\right) \\
&\quad + q\left(-\frac{1}{4}d_{010}^2 d_{012} + \frac{d_{010}d_{022}}{2} + \frac{d_{012}d_{020}}{4} - \frac{d_{032}}{2}\right) \\
&\quad + \frac{1}{4}d_{010}d_{012}d_{200} + \frac{1}{8}d_{010}^2 d_{202} - \frac{d_{010}d_{212}}{2} - \frac{d_{012}d_{210}}{4} \\
&\quad - \frac{d_{020}d_{202}}{8} - \frac{d_{022}d_{200}}{4} + \frac{3d_{222}}{4}.
\end{aligned}$$

$$m_+ \equiv \frac{M_{H^\pm}^2}{v^2}$$

General 2HDM in the alignment limit

- Pick set of numerical values

$$\begin{aligned} M_1 &= 125 \text{ GeV} \\ M_2 &= 95 \text{ GeV} \\ M_3 &= 152 \text{ GeV} \\ M_{H^\pm} &= 200 \text{ GeV} \\ q &= 3 \\ v &= 246 \text{ GeV} \\ e_2 &= 0 \\ e_3 &= 0 \\ q_1 &= 90 \text{ GeV} \end{aligned}$$

- Exact AL $e_2 = e_3 = 0$
- Remaining parameters: q_2 and q_3
- Scan over q_2 - q_3 – space

General case in the alignment limit

- Pick set of numerical values

$$
\begin{aligned}
M_1 &= 125\ \text{GeV} \\
M_2 &= 95\ \text{GeV} \\
M_3 &= 152\ \text{GeV} \\
M_{H^\pm} &= 200\ \text{GeV} \\
q &= 3 \\
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e_2 &= 0 \\
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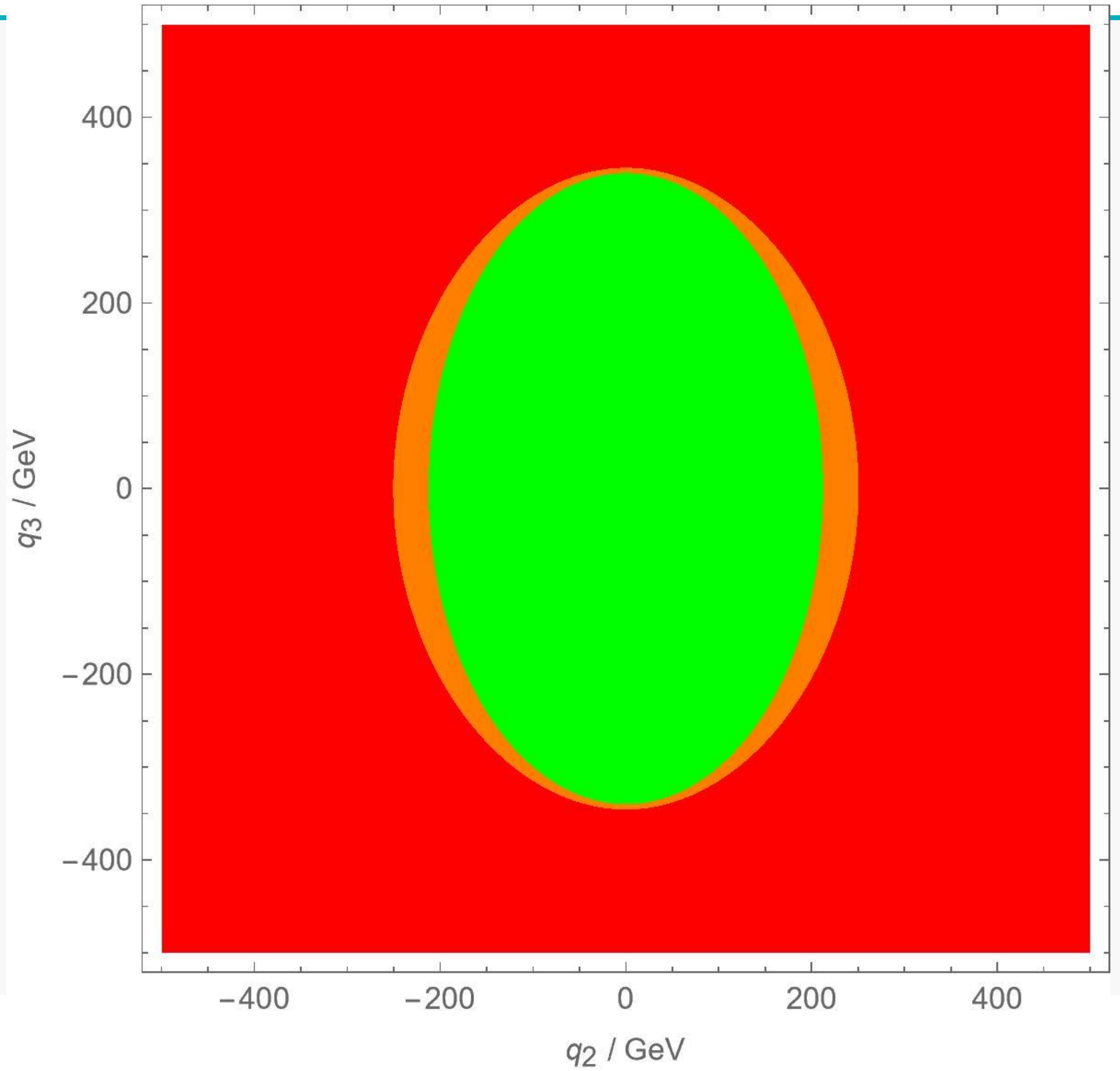
\end{aligned}
$$

- Exact AL $e_2 = e_3 = 0$
- Remaining parameters: q_2 and q_3
- Scan over q_2 - q_3 – space



Summary!

- Tree-level metastability bounds for 2HDM can be described in terms of physical parameters.
- General 2HDM as well as models with imposed symmetries.
- Well suited for scans over the physical parameters of the model.
- Also other constraints in terms of physical parameters can be imposed.
- Experimental constraints will be given in terms of physical parameters.