On flavor conserving and violating couplings in 2HDM and Beyond

Carlos E.M. Wagner
Physics Department, Enrico Fermi Institute, KICP, University of Chicago
HEP Division, Argonne National Laboratory

Workshop on Multi-Higgs Models
Instituto Superior Técnico, Lisbon, September 5, 2024

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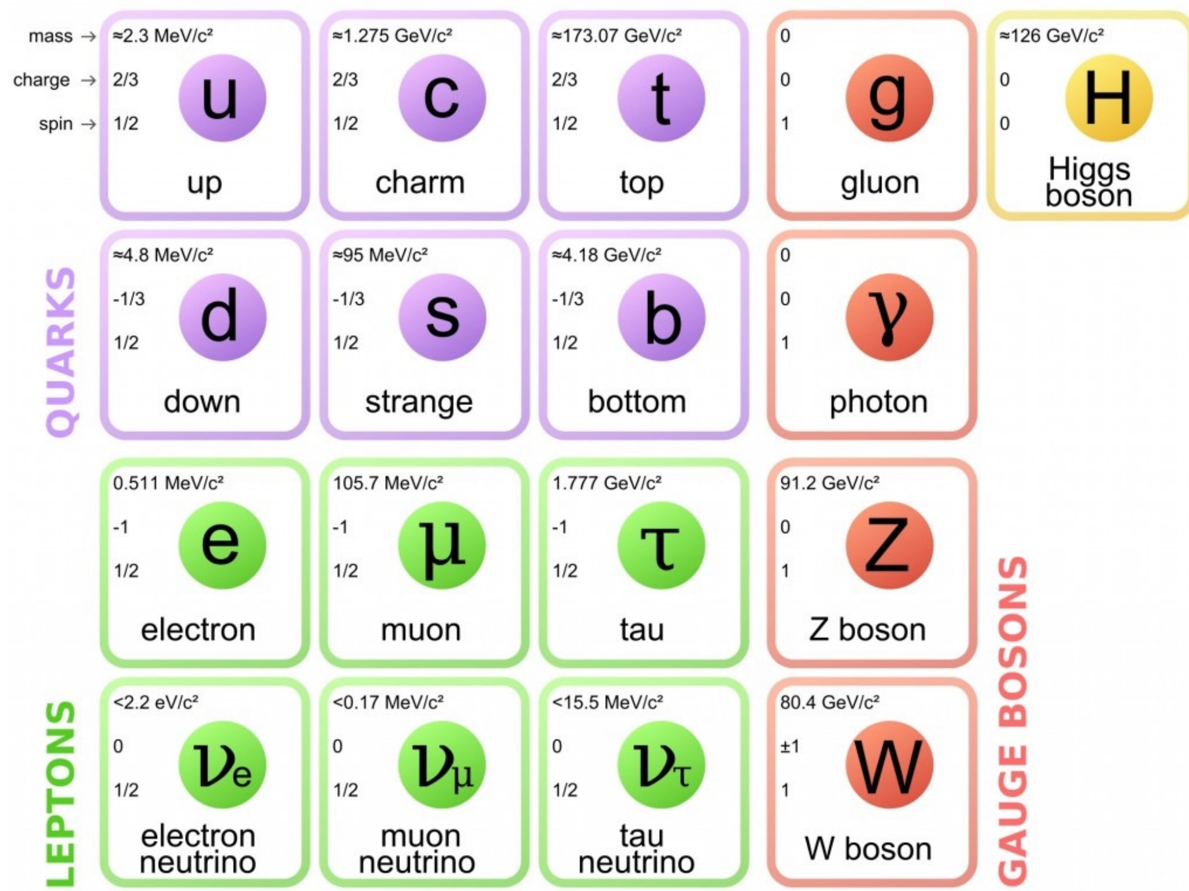
The Standard Model

Is an extremely successful Theory that describes interactions between the known elementary particles.

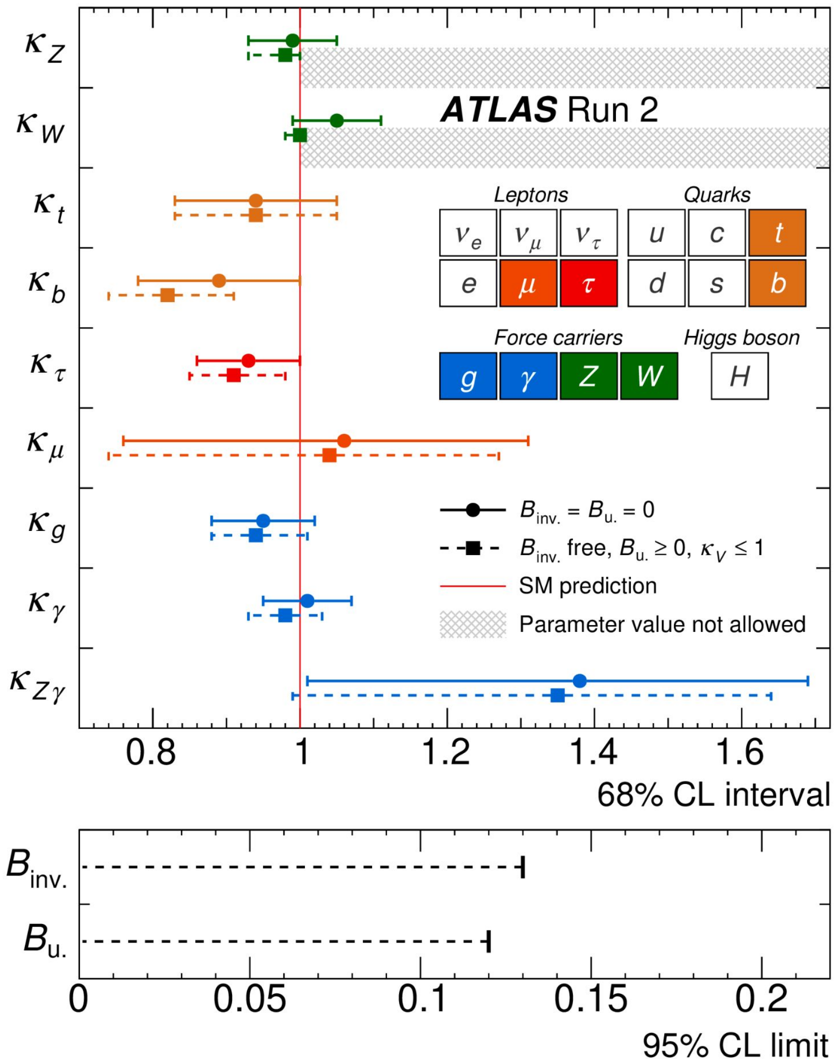
3 generations of fermions (mattter)

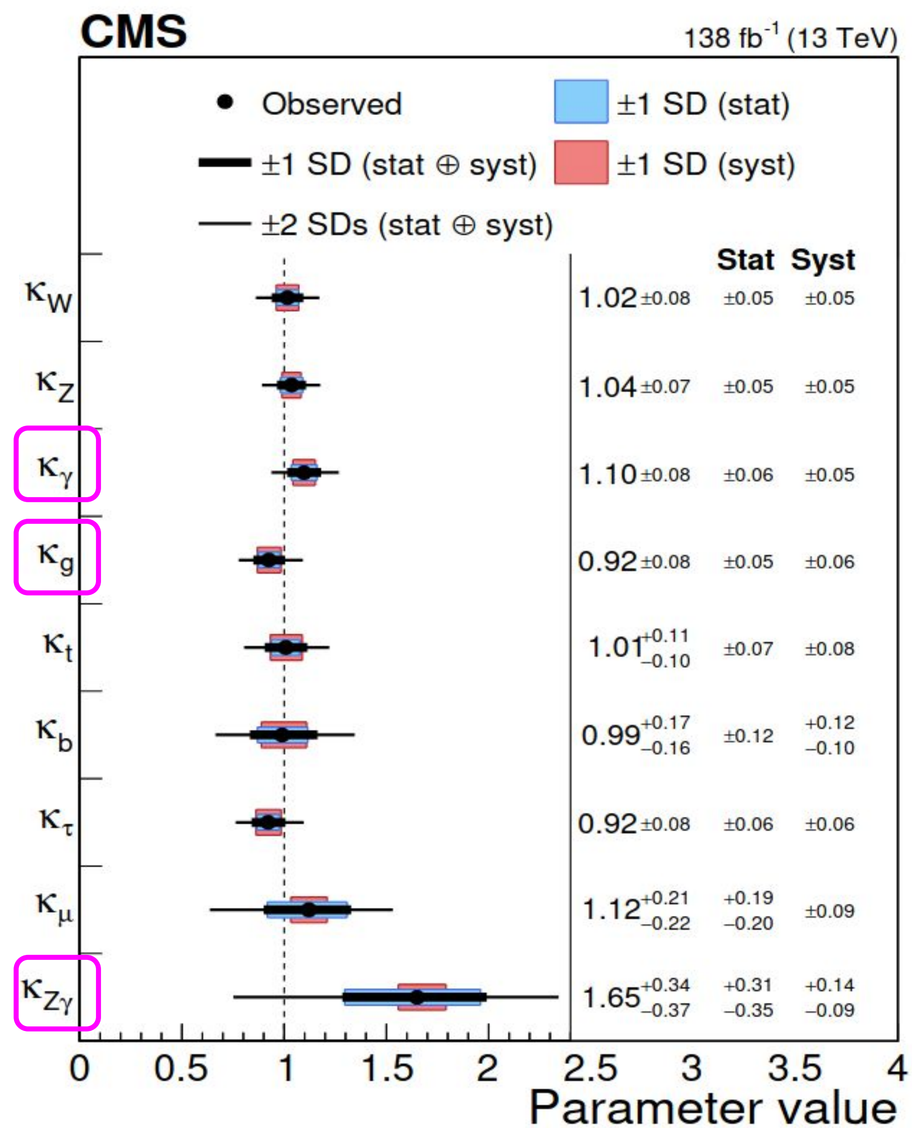
Gauge and Higgs Fields



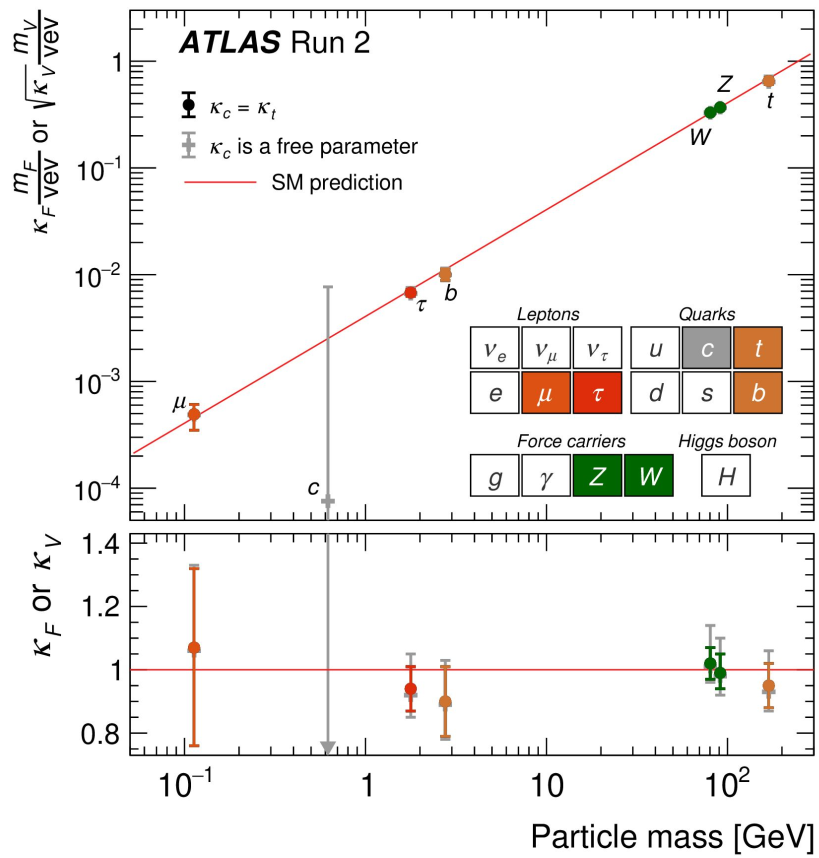
ATLAS and CMS Fit to Higgs Couplings

Departure from SM predictions of the order of few tens of percent allowed at this point.

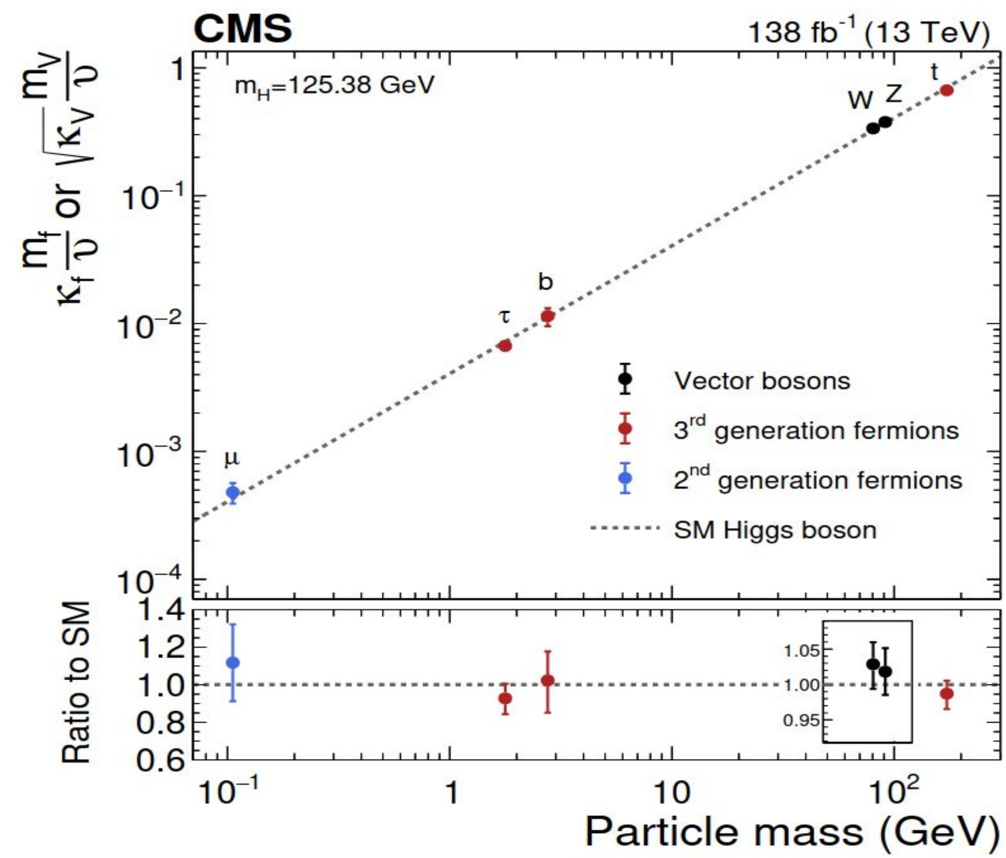




Correlation between masses and couplings consistent with the Standard Model expectations



H couplings vs particle mass



$$g_{hf\bar{f}} = \frac{m_f}{v}, \qquad g_{hVV} = \frac{m_V^2}{v}$$

Third generation coupling that are constrained at the 10 percent level,
will be constrained at the few percent level (including the muon) at the end of the LHC era

Why we should not be surprised

- There is a well known, amazing property of the SM as an effective field theory
- Take any sector with gauge invariant mass terms, which do not involve the Higgs v.e.v.

$$\mathcal{L} = -m_\phi^2 \phi^\dagger \phi + (M_\Psi \bar{\Psi} \Psi)$$

- The Appelquist-Carrazonne decoupling theorem says that as we push these gauge invariant masses up, the low energy effective theory will reduce to the Standard Model !
- The speed of decoupling depends on how these sector couple to the SM. In general, for a coupling κ, decoupling occurs when

$$\frac{k^2}{m_{\rm new}^2} \ll \frac{1}{v^2}$$

- Obviously decoupling doesn't occur if the masses are proportional to the v.e.v.
- These properties are behind the EFT program.

Simple Framework for analysis of coupling deviations
2HDM : General Potential

- General, renormalizable potential has seven quartic couplings, with three of them, given in the last line, which may be complex.

$$
\begin{aligned}
V = {} & m_{11}^2 \Phi_1^\dagger \Phi_1 + m_{22}^2 \Phi_2^\dagger \Phi_2 - (m_{12}^2 \Phi_1^\dagger \Phi_2 + h.c.) \\
& + \frac{\lambda_1}{2} (\Phi_1^\dagger \Phi_1)^2 + \frac{\lambda_2}{2} (\Phi_2^\dagger \Phi_2)^2 + \lambda_3 (\Phi_1^\dagger \Phi_1)(\Phi_2^\dagger \Phi_2) + \lambda_4 (\Phi_1^\dagger \Phi_2)(\Phi_2^\dagger \Phi_1) \\
& + \left[\frac{\lambda_5}{2} (\Phi_1^\dagger \Phi_2)^2 + \lambda_6 (\Phi_1^\dagger \Phi_1)(\Phi_1^\dagger \Phi_2) + \lambda_7 (\Phi_2^\dagger \Phi_2)(\Phi_1^\dagger \Phi_2) + h.c. \right] ,
\end{aligned}
$$

- In general, it is assumed that lambda 6 and 7 are zero, since this condition appears naturally in models with flavor conservation. However, this condition is basis dependent and it is not necessary.

- We will therefore concentrate on the general 2HDM, with all quartic couplings different from zero. As it is well known, an important parameter in these models is

$$
\tan\beta = \frac{v_2}{v_1}
$$

Higgs Basis

- An interesting basis for the phenomenological analyses of these models is the Higgs basis

$$H_1 = \Phi_1 \cos\beta + \Phi_2 \sin\beta$$
$$H_2 = \Phi_1 \sin\beta - \Phi_2 \cos\beta$$

$$H_1 = \begin{pmatrix} G^+ \\ \frac{1}{\sqrt{2}}(v + \phi_1^0 + iG^0) \end{pmatrix}, \quad H_2 = \begin{pmatrix} H^+ \\ \frac{1}{\sqrt{2}}(\phi_2^0 + ia^0) \end{pmatrix}$$

- The field ϕ_1^0 is therefore associated with the field direction that acquires a vacuum expectation value and acts as a SM-like Higgs

- The behavior of the neutral mass eigenstates depend on the projection on the fields in this basis.

- Typically, it is the lightest neutral Higgs boson that behaves like the SM-like Higgs. The case in which one can identify the state ϕ_1^0 with the mass eigenstate is called alignment.

- In the alignment limit the tree-level couplings agree with the SM ones. Large departures from the alignment limit are heavily restricted by LHC measurements.

$$h = \sin(\beta - \alpha)\phi_1^0 + \cos(\beta - \alpha)\phi_2^0$$

Quartic Couplings in the Higgs basis

Similar notation as in the generic basis, but changing lambdas by Z's

$$V \supset \frac{Z_1}{2}(H_1^\dagger H_1)^2 + \frac{Z_2}{2}(H_2^\dagger H_2)^2 + Z_3(H_1^\dagger H_1)(H_2^\dagger H_2) + Z_4(H_1^\dagger H_2)(H_2^\dagger H_1)$$
$$+ \left[\frac{Z_5}{2}(H_1^\dagger H_2)^2 + Z_6(H_1^\dagger H_1)H_1^\dagger H_2 + Z_7(H_2^\dagger H_2)H_1^\dagger H_2 + h.c.\right]$$

Observe that since only H1 acquires vacuum expectation value in this basis, the mixing between the Higgs states of both doublets can only occur via Z6

Mass Matrix in the Higgs Basis

- The neutral Higgs mass matrix takes a particularly simple form in the Higgs basis (Zi are the quartic couplings in this basis)

$$\mathcal{M}^2 = v^2 \begin{pmatrix} Z_1 & Z_6^R & -Z_6^I \\ Z_6^R & \frac{M_{H^\pm}^2}{v^2} + \frac{1}{2}(Z_4 + Z_5^R) & -\frac{1}{2}Z_5^I \\ -Z_6^I & -\frac{1}{2}Z_5^I & \frac{M_{H^\pm}^2}{v^2} + \frac{1}{2}(Z_4 - Z_5^R) \end{pmatrix}$$

- Two things are obvious from here. First, in the CP-conserving case, the condition of alignment, $Z_6 \ll 1$ implying small mixing between the lightest and heavier eigenstates is given by

$$\cos(\beta - \alpha) = -\frac{Z_6 v^2}{m_H^2 - m_h^2} \qquad \text{Decoupling}: \quad Z_6 v^2 \ll m_H^2$$

- Second, while in the alignment limit the real part of Z_5 contributes to the splitting of the two heavier mass eigenstates, its imaginary part contributes to the splitting and their mixing.

$$M_{h_3,h_2}^2 = M_{H^\pm}^2 + \frac{1}{2}(Z_4 \pm |Z_5|)v^2 \,.$$

$$m_h^2 = Z_1 v^2, \qquad m_h = 125\ \text{GeV}$$

Amazing Properties of the SM Higgs sector

- The interactions with fermions present an amazing story. We start with a completely arbitrary 3x3 Yukawa matrix interactions, where this three is related to generations

$$y_{ij}\bar{\psi}^i_L H \psi^j_R + h.c.$$

- Now, when you give the Higgs a v.e.v. this becomes a mass matrix that you must diagonalize when going to the physical states.

- But, due to the fact that mass and Yukawa matrices are proportional to each other, the interactions become flavor diagonal

$$y_{hnm} = \frac{m_f}{v}\delta_{nm}$$

- In general, there are no tree-level Flavor Changing Neutral Currents ! No tree-level CP violation. All these effects occur at the loop-level, via the charged weak interactions, and are proportional to CKM matrix elements.

- I don't need to tell you how amazing this is ! Moreover, all available data is consistent with these predictions.

Mimicking the SM behavior

- In 2HDM, one can mimic the SM behavior by just allowing the fermions with a giving charge (up quarks, down quarks, charge leptons and neutrinos) to couple to only one of the Higgs fields.

- This leads to the so-called type I to IV 2HDM, depending on which couplings are allowed.

	Up-type	Down-type	Lepton
Type-I	Φ_1	Φ_1	Φ_1
Type-II	Φ_1	Φ_2	Φ_2
Type-LS	Φ_1	Φ_1	Φ_2
Type-F	Φ_1	Φ_2	Φ_1

- In type I, all fermions couple to the same Higgs. In type II, down quarks and charge leptons couple to one of the Higgs boson doublets and up quarks and neutrinos to the other. This is the scheme allowed at tree-level in SUSY theories.

- Let me emphasize that at the loop level in SUSY theories couplings to the other Higgs boson doublet appear.

Couplings in low energy supersymmetry (tree level) : Type II 2HDM

Modifying the top and bottom couplings in two Higgs Doublet Models

$$\kappa_t = \sin(\beta - \alpha) + \cot\beta\cos(\beta - \alpha)$$ (Fermion Fields that couple to Φ_2)

$$\kappa_b = \sin(\beta - \alpha) - \tan\beta\cos(\beta - \alpha)$$ (Fermion Fields that couple to Φ_1)

$$\kappa_V = \sin(\beta - \alpha) \simeq 1$$

Alignment : $\cos(\beta - \alpha) = 0$

$$\tan\beta = \frac{v_u}{v_d}$$

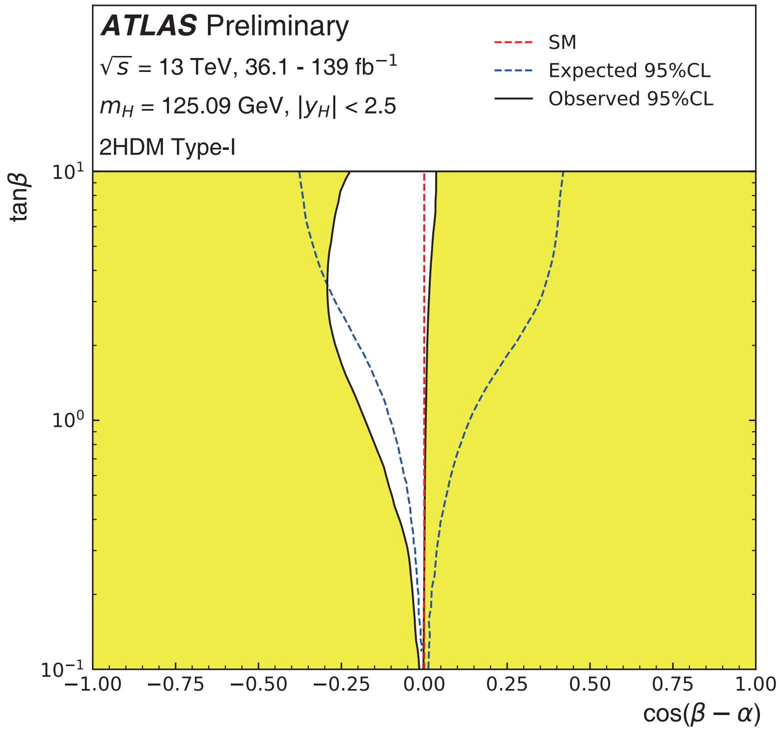
$$h = \sin(\beta - \alpha) H_1^0 + \cos(\beta - \alpha) H_2^0$$

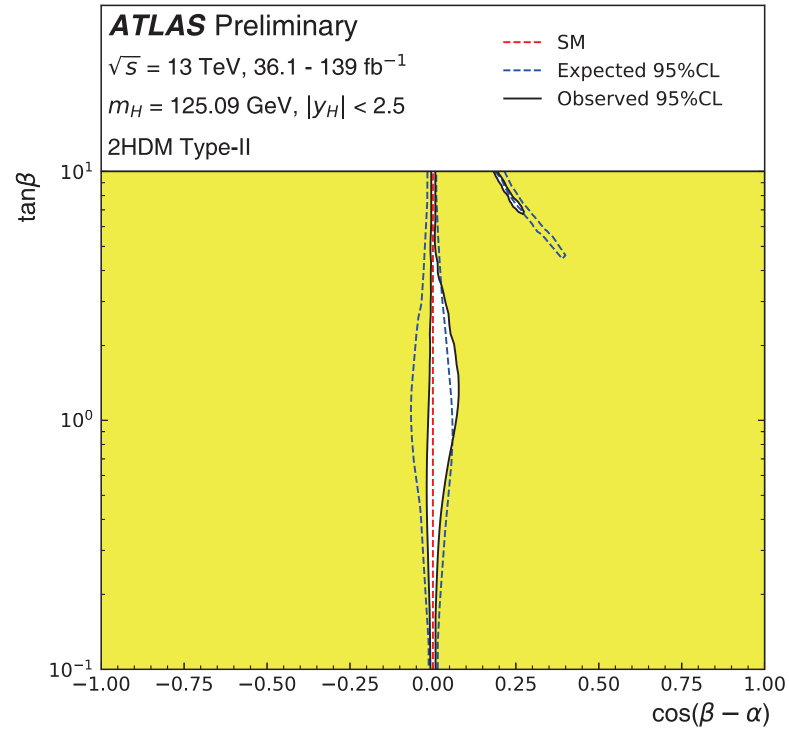
$$H = \cos(\beta - \alpha) H_1^0 - \sin(\beta - \alpha) H_2^0$$

(Neutral Higgs bosons in the Higgs basis)

We will keep in mind that the LHC favors and SM-like Higgs boson

LHC constraints on Higgs alignment in the 2HDM





Regions excluded by fits to the measured rates of the productions and decay of the Higgs boson (assumed to be h of the 2HDM). Contours at 95% CL. ATLAS-CONF-2021-053

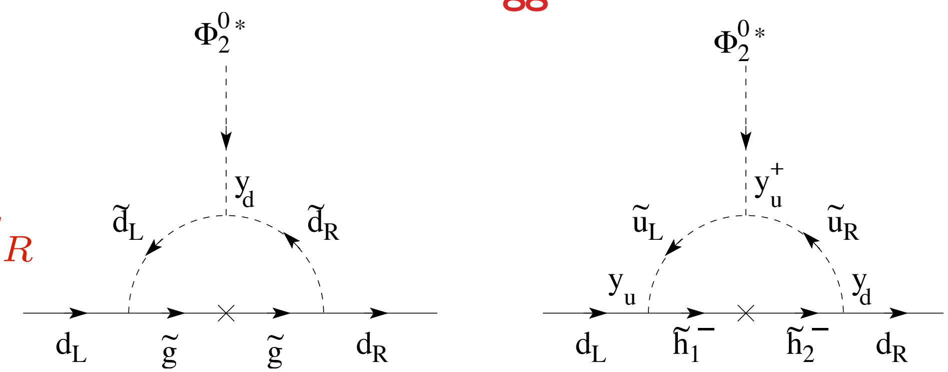
SUSY at Loop Level

Hempfling '93
Hall, Rattazzi, Sarid'93
Carena, Olechowski, Pokorski, C.W.'93

Radiative Corrections to Flavor Conserving Higgs Couplings

- Couplings of down and up quark fermions to both Higgs fields arise after radiative corrections.

$$\mathcal{L} = \bar{d}_L (h_d H_1^0 + \Delta h_d H_2^0) d_R$$



- The radiatively induced coupling depends on ratios of supersymmetry breaking parameters

$$m_b = h_b v_1 \left(1 + \frac{\Delta h_b}{h_b} \tan\beta \right) \qquad \boxed{\tan\beta = \frac{v_2}{v_1}}$$

$$\frac{\Delta_b}{\tan\beta} = \frac{\Delta h_b}{h_b} \simeq \frac{2\alpha_s}{3\pi} \frac{\mu M_{\tilde{g}}}{\max(m_{\tilde{b}_i}^2, M_{\tilde{g}}^2)} + \frac{h_t^2}{16\pi^2} \frac{\mu A_t}{\max(m_{\tilde{t}_i}^2, \mu^2)}$$

$$X_t = A_t - \mu/\tan\beta \simeq A_t \qquad \Delta_b = (E_g + E_t h_t^2) \tan\beta$$

Resummation : Carena, Garcia, Nierste, C.W.'00

Generic case

- Although it is important to consider models that mimic the SM suppression of flavor violation, one should also analyze a more generic case, since it is what quite generally appears at low energies.

- So, let's write the coupling modifications in 2HDM for the case in which each type of fermions couple to both Higgs

$$\mathcal{L} \supset -(y_\alpha^{ij} \bar{F}_L \Phi_\alpha f_R + h.c.)$$

- The fermion mass matrix will then be given by

$$M^{ij} = (y_1^{ij} \cos\beta + y_2^{ij} \sin\beta) v$$

- We shall denote with a bar the Yukawas in the physical basis where the mass is diagonal. Hence

$$M_d^{ii} = (\bar{y}_1^{ij} \cos\beta + \bar{y}_2^{ij} \sin\beta) v$$

- Therefore, for $i \neq j$ $\bar{y}_1^{ij} \cos\beta = -\bar{y}_2^{ij} \sin\beta$

Arbitrary Yukawas : $\mathcal{L} \supset -(y_\alpha^{ij} \bar{F}_L \Phi_\alpha f_R + h.c.)$ N. Coyle, D. Rocha, C.W. ' 24

General expression for neutral Higgs couplings

Mass term coming mainly from coupling to Φ_1

$$\mathcal{L}_{h_1^0} = -\frac{m_i}{v}\left[\sin(\beta-\alpha) - \frac{\cos(\beta-\alpha)}{(1+\Delta_i)}\left(\tan\beta - \frac{\Delta_i}{\tan\beta}\right)\right] h_1^0 \bar{f}_i f_i$$
$$+ \left[\left(\frac{\mathrm{Re}(\bar{y}_2^{ij})}{\cos\beta\sqrt{2}}\cos(\beta-\alpha)(1-\delta^{ij}) + i\frac{\mathrm{Im}(\bar{y}_2^{ij})}{\cos\beta\sqrt{2}}\cos(\beta-\alpha)\right) h_1^0 \bar{f}_L^i f_R^j + h.c.\right]$$

Mass term coming mainly from coupling to Φ_2

$$= -\frac{m_i}{v}\left[\sin(\beta-\alpha) + \frac{\cos(\beta-\alpha)}{(1+\tilde{\Delta}_i)}\left(\frac{1}{\tan\beta} - \tilde{\Delta}_i\tan\beta\right)\right] h_1^0 \bar{f}_i f_i$$
$$- \left[\left(\frac{\mathrm{Re}(\bar{y}_1^{ij})}{\sin\beta\sqrt{2}}\cos(\beta-\alpha)(1-\delta^{ij}) + i\frac{\mathrm{Im}(\bar{y}_1^{ij})}{\sin\beta\sqrt{2}}\cos(\beta-\alpha)\right) h_1^0 \bar{f}_L^i f_R^j + h.c.\right]$$

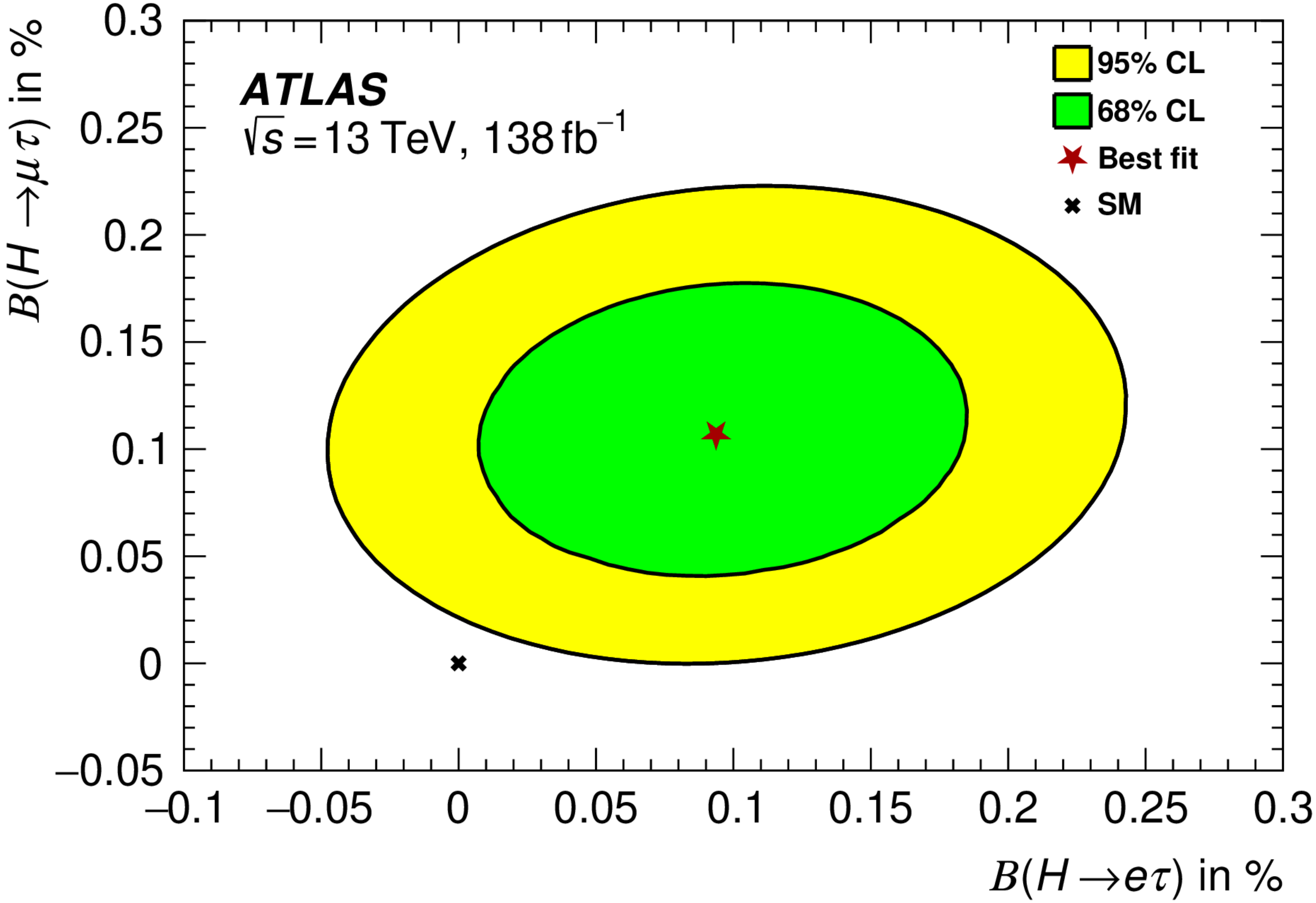
$$M_d = U_L M U_R^\dagger \qquad \bar{y}_i = U_L y_i U_R^\dagger$$

$$\Delta_i = \frac{\mathrm{Re}(\bar{y}_2^{ii})}{\mathrm{Re}(\bar{y}_1^{ii})}\tan\beta \qquad \tilde{\Delta}_i = \frac{1}{\Delta_i}$$

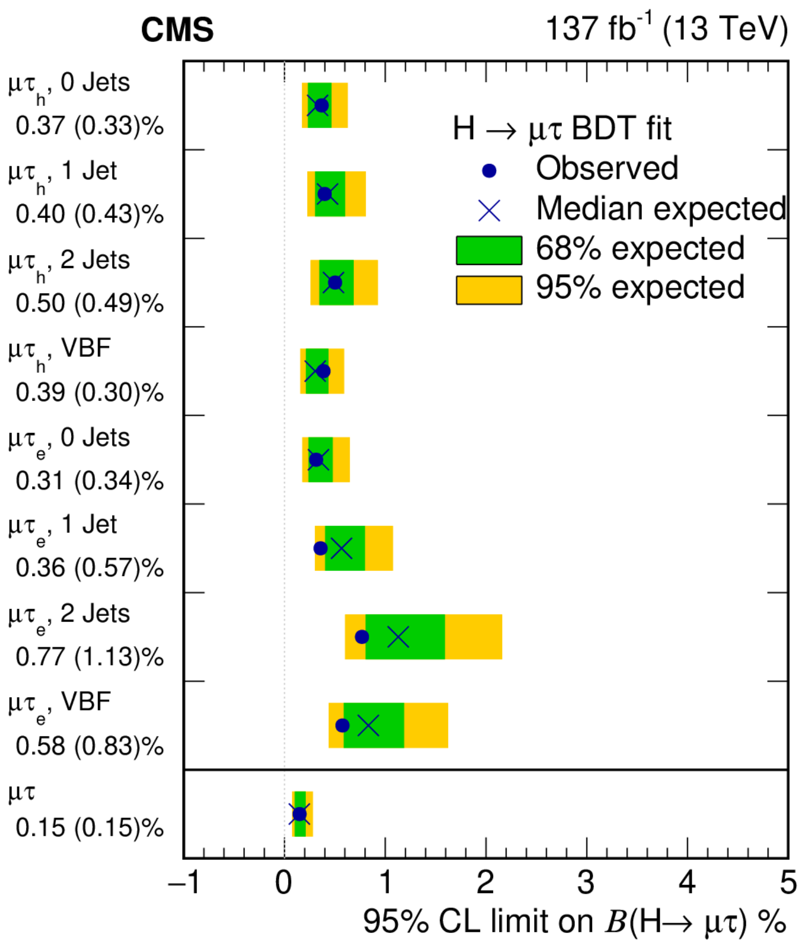
Higgs FCNC demands flavor as well as Higgs misalignment !

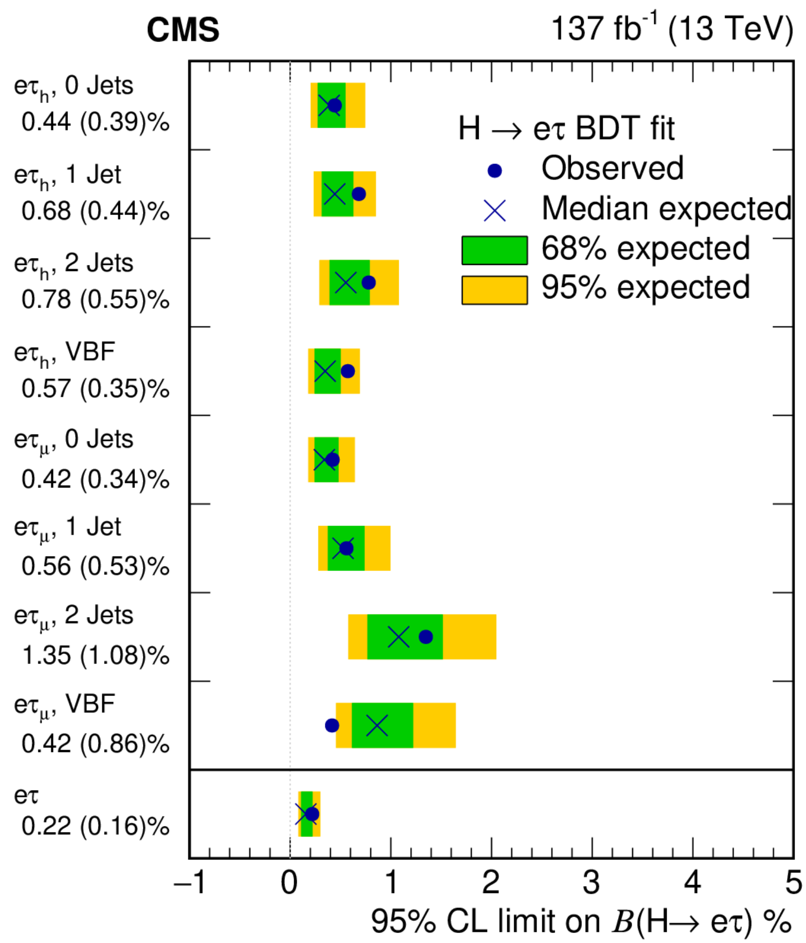
$$\bar{y}_1 v_1 + \bar{y}_2 v_2 = \mathrm{Diag}(m) \rightarrow \bar{y}_1\cos\beta + \bar{y}_2\sin\beta = \mathrm{Diag}(m/v)$$

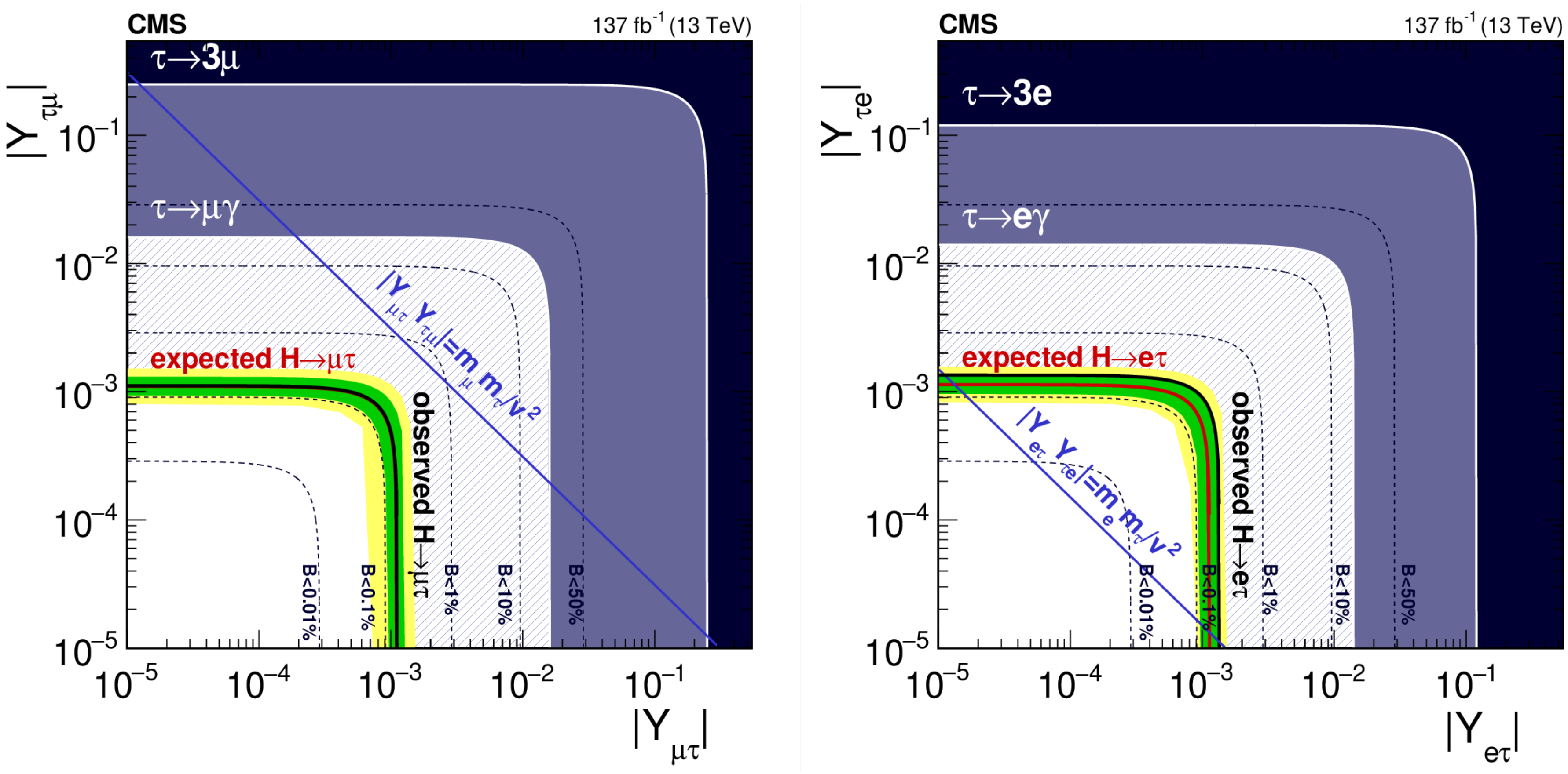
Possible flavor violation in Higgs decays



No hint from CMS, though : $BR(H \to \tau\mu, e) < 0.15\%$


CMS
137 fb-1 (13 TeV)
μτh, 0 Jets
0.37 (0.33)%
μτh, 1 Jet
0.40 (0.43)%
μτh, 2 Jets
0.50 (0.49)%
μτh, VBF
0.39 (0.30)%
μτe, 0 Jets
0.31 (0.34)%
μτe, 1 Jet
0.36 (0.57)%
μτe, 2 Jets
0.77 (1.13)%
μτe, VBF
0.58 (0.83)%
μτ
0.15 (0.15)%
H → μτ BDT fit
Observed
Median expected
68% expected
95% expected
95% CL limit on B(H→ μτ) %


CMS
137 fb-1 (13 TeV)
eτh, 0 Jets
0.44 (0.39)%
eτh, 1 Jet
0.68 (0.44)%
eτh, 2 Jets
0.78 (0.55)%
eτh, VBF
0.57 (0.35)%
eτμ, 0 Jets
0.42 (0.34)%
eτμ, 1 Jet
0.56 (0.53)%
eτμ, 2 Jets
1.35 (1.08)%
eτμ, VBF
0.42 (0.86)%
eτ
0.22 (0.16)%
H → eτ BDT fit
Observed
Median expected
68% expected
95% expected
95% CL limit on B(H→ eτ) %


CMS
137 fb^-1 (13 TeV)
τ→3μ
τ→μγ
expected H→μτ
observed H→μτ
|Y_μτ Y_τμ|=m_μ m_τ/v²
B<0.01%
B<0.1%
B<1%
B<10%
B<50%
|Y_τμ|
|Y_μτ|
CMS
137 fb^-1 (13 TeV)
τ→3e
τ→eγ
expected H→eτ
observed H→eτ
|Y_eτ Y_τe|=m_e m_τ/v²
B<0.01%
B<0.1%
B<1%
B<10%
B<50%
|Y_τe|
|Y_eτ|

Couplings in the Higgs basis

- Let me emphasize that the Higgs basis is a convenient mathematical construction, and that the couplings can be derived by taking the limit of tanβ = 0 of the above expressions.

- It is simple to show that in this case the deviation of diagonal couplings as well as the flavor violating couplings are governed by the diagonal and off diagonal components of the Higgs that does not acquire vev (the Yukawa matrix to the Higgs that acquire vev is obviously diagonal in this case) (see Howie Haber's talk)

- Although in principle the Yukawa couplings to the second Higgs look arbitrary and not related to fermion masses, they must have a structure in the construction of the mass matrix in the original basis where both Higgs bosons acquire a vev. (otherwise the off-diagonal elements will look dangerously large in the non-decoupling limit).

$$\mathcal{L} \supset -(y_\alpha^{ij} \bar{Q}_L H_\alpha f_R + h.c.)$$

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Non-SM Higgs Coupling

$$\begin{aligned}\mathcal{L}_{h_2^0} &= -\frac{m_i}{v}\delta^{ij}\left[\cos(\beta-\alpha) + \left(\frac{\tan\beta}{1+\Delta_i} - \frac{\Delta_i}{\tan\beta(1+\Delta_i)}\right)\sin(\beta-\alpha)\right] h_2^0 \bar{f}_i f_i \\ &+ \left[\left(\frac{\mathrm{Re}(\bar{y}_2^{ij})}{\sqrt{2}\cos\beta}(1-\delta^{ij})\sin(\beta-\alpha) + i\frac{\mathrm{Im}(\bar{y}_2^{ij})}{\sqrt{2}\cos\beta}\sin(\beta-\alpha)\right) h_2^0 \bar{f}_L^i f_R^j + h.c.\right]\end{aligned}$$

H_1-coupling

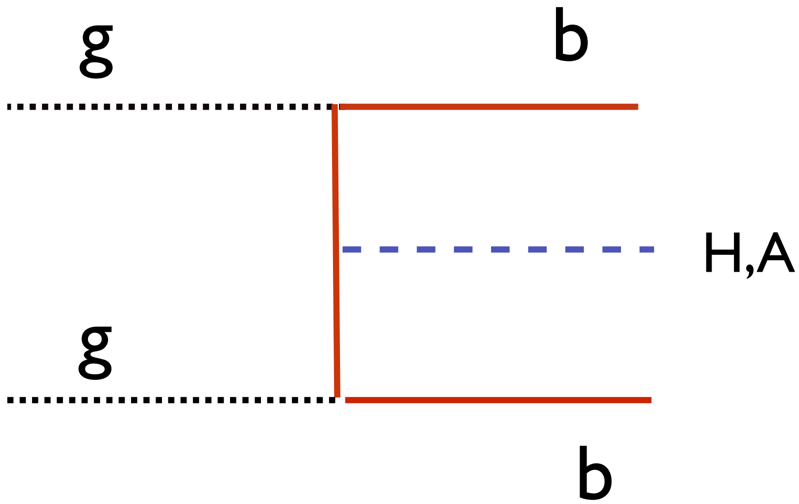
$$\begin{aligned}\mathcal{L}_{h_2^0} &= -\frac{m_i}{v}\delta^{ij}\left[\cos(\beta-\alpha) - \left(\frac{1}{\tan\beta(1+\tilde{\Delta}_i)} - \frac{\tilde{\Delta}_i\tan\beta}{(1+\tilde{\Delta}_i)}\right)\sin(\beta-\alpha)\right] h_2^0 \bar{f}_i f_i \\ &- \left[\left(\frac{\mathrm{Re}(\bar{y}_1^{ij})}{\sqrt{2}\sin\beta}(1-\delta^{ij})\sin(\beta-\alpha) + i\frac{\mathrm{Im}(\bar{y}_1^{ij})}{\sqrt{2}\sin\beta}\sin(\beta-\alpha)\right) h_2^0 \bar{f}_L^i f_R^j + h.c.\right]\end{aligned}$$

H_2-coupling

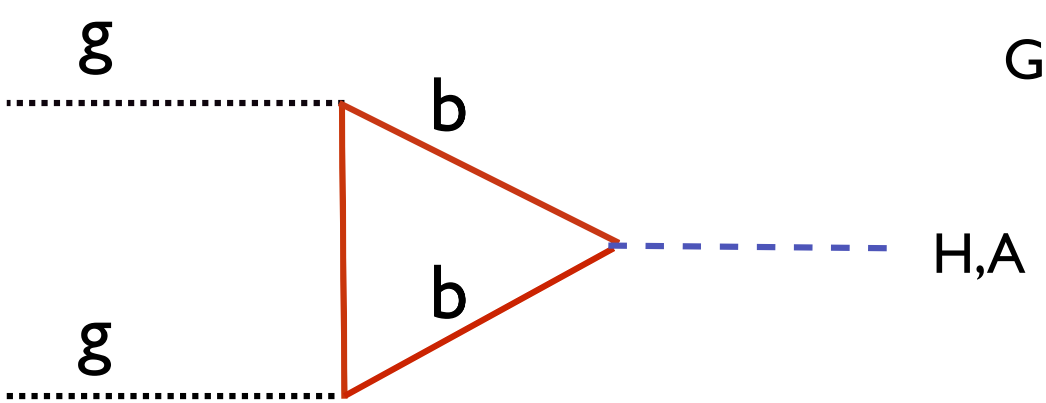
Higgs alignment, of course, does not ensure flavor alignment in the non-standard Higgs sector

Non-Standard Higgs Production

QCD: S. Dawson, C.B. Jackson, L. Reina, D.Wackeroth, hep-ph/0603112



Associated Production

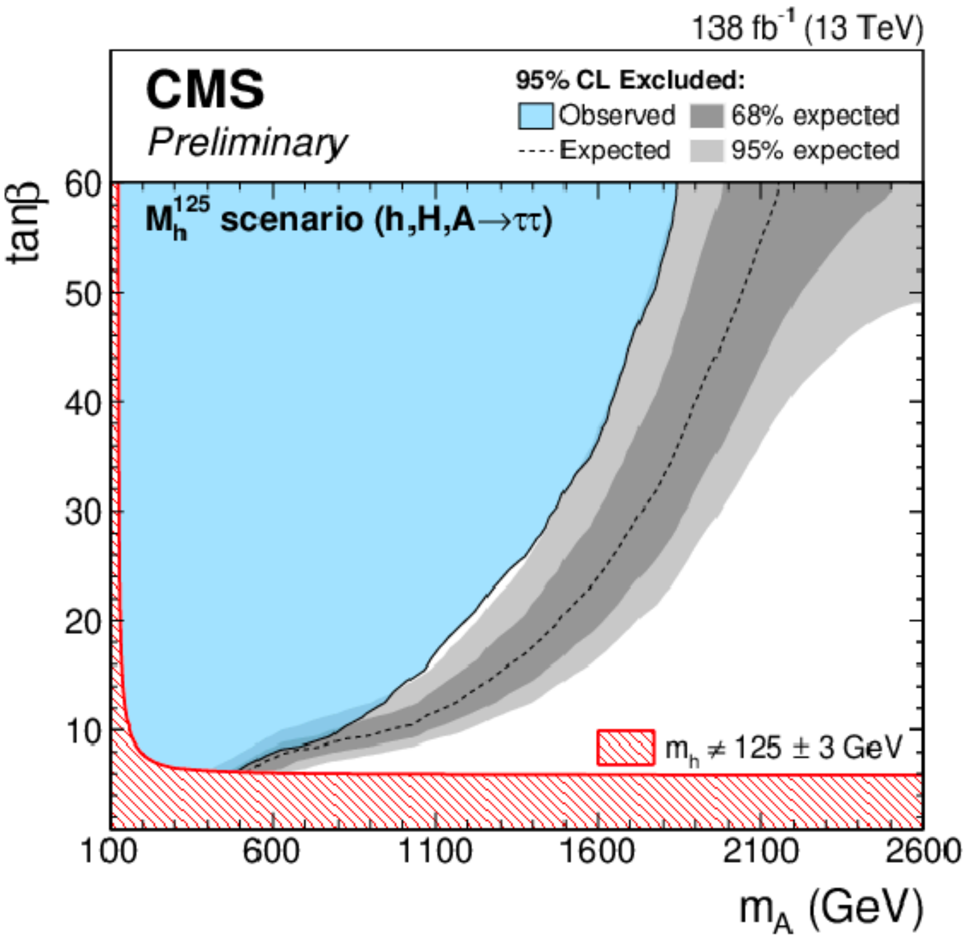


Gluon Fusion

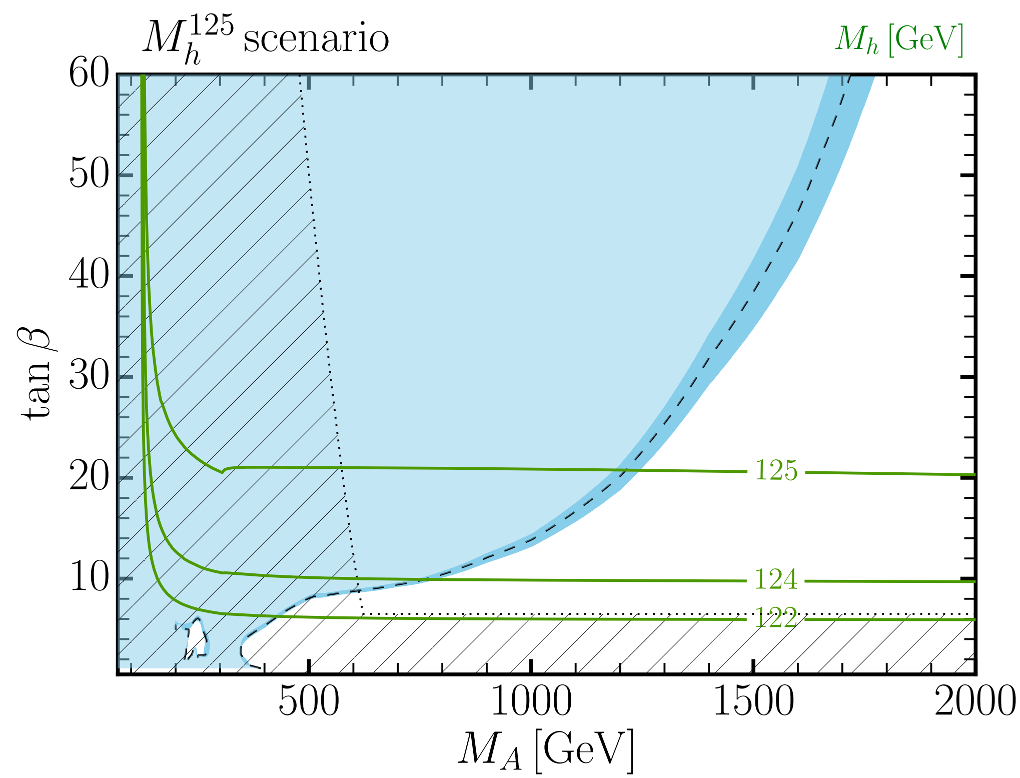
$$g_{Abb} \simeq g_{Hbb} \simeq \frac{m_b \tan\beta}{(1+\Delta_b)v}, \qquad g_{A\tau\tau} \simeq g_{H\tau\tau} \simeq \frac{m_\tau \tan\beta}{v}$$

Complementarity of Direct and Indirect Bounds

Bahl, Fuchs, Hahn, Heinemeyer, Liebler, Patel, Slavich, Stefaniak, Weiglein, C.W. arXiv:1808.07542



Dashed area, constrained by precision measurements.
Low values of the Higgsino Mass assumed in this Figure.



Interesting but not compelling excess appears at CMS.
No similar excess appears at ATLAS.

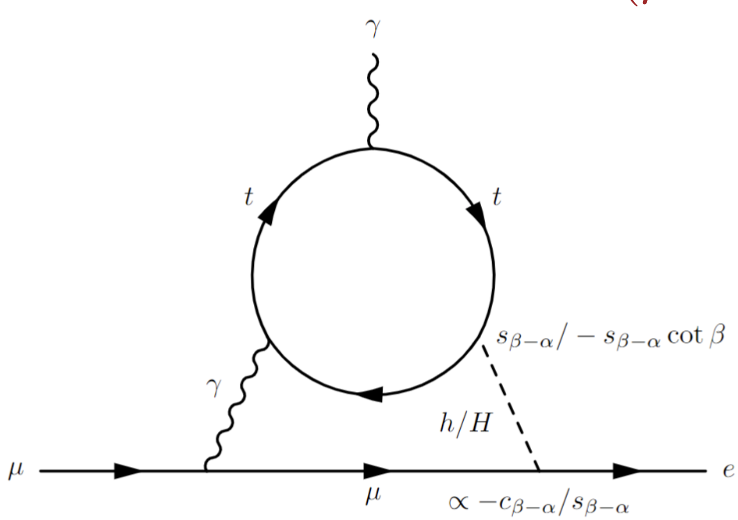
Higgs Flavor violation

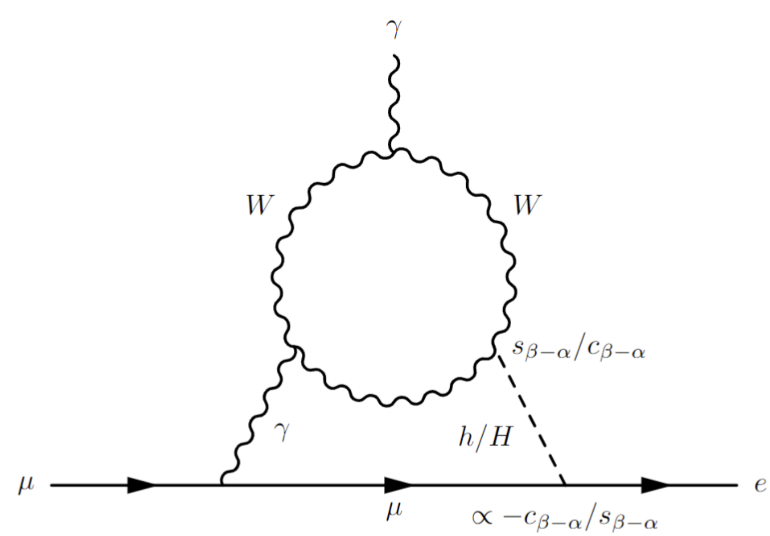
Induces flavor violating processes which do not involve the Higgs directly

One example is the radiative decay of heavy leptons into lighter ones

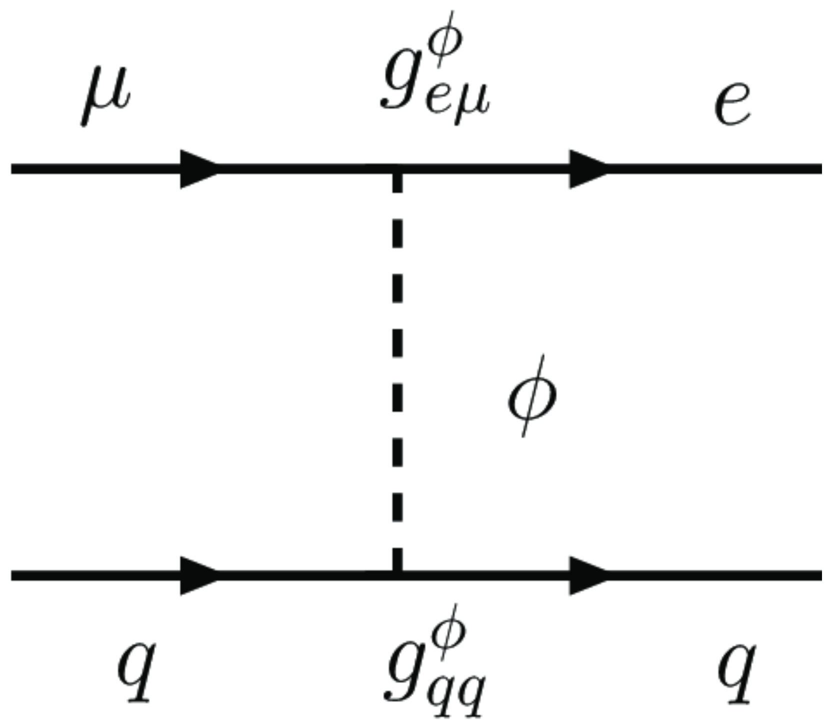
Here I assume that the top and leptons have dominant couplings like in type II scenarios

$$BR(\mu \to e\gamma) < 4.2 \times 10^{-13}$$





μ to e Conversion



Less relevant interference

Harnik, Kopp, Zupan, arXiv:1209.1937

Flavor Conserving and Violating Processes

- There can be interesting cancellations between the flavor violating contributions of light and heavy Higgs bosons.
- The large hierarchy between the different generations can be explained in different ways.
- Generically, if we assume the dominant Yukawa to lead to the generation of the tau mass and the other to lead to the generation of the muon and electron masses, the off-diagonal elements are proportional to, for instance,

$$\bar{y}_{l_i l_j} \propto \frac{\sqrt{m_i m_j}}{v} \qquad \text{or} \qquad \bar{y}_{l_i l_j} \propto \frac{\mathrm{Min}(\mathrm{m_i}, \mathrm{m_j})}{v}$$

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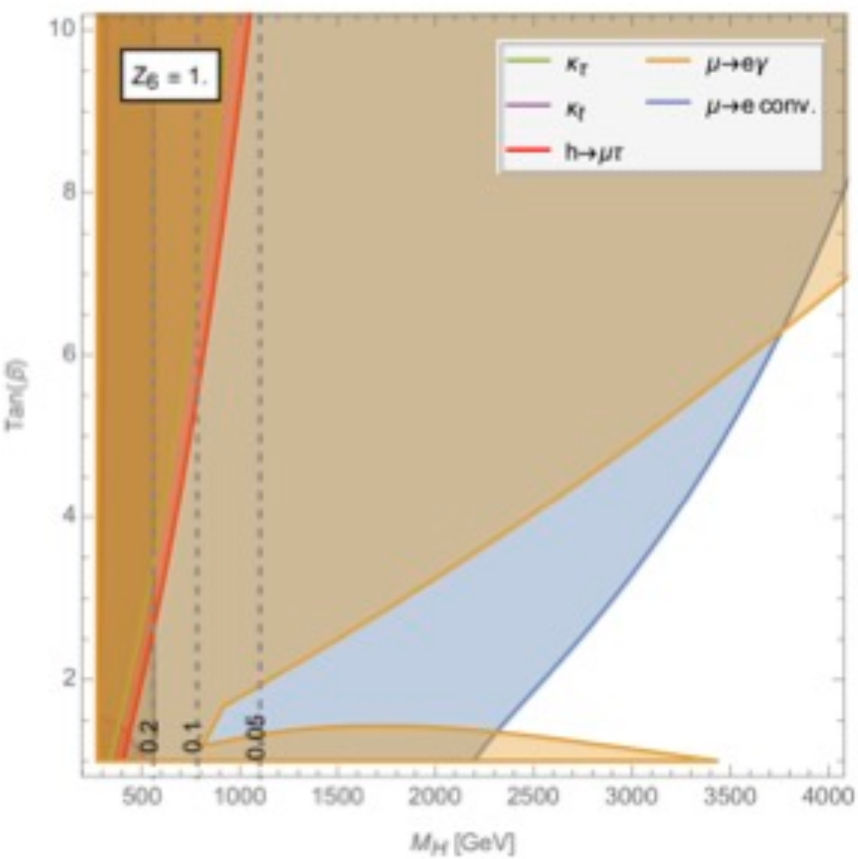
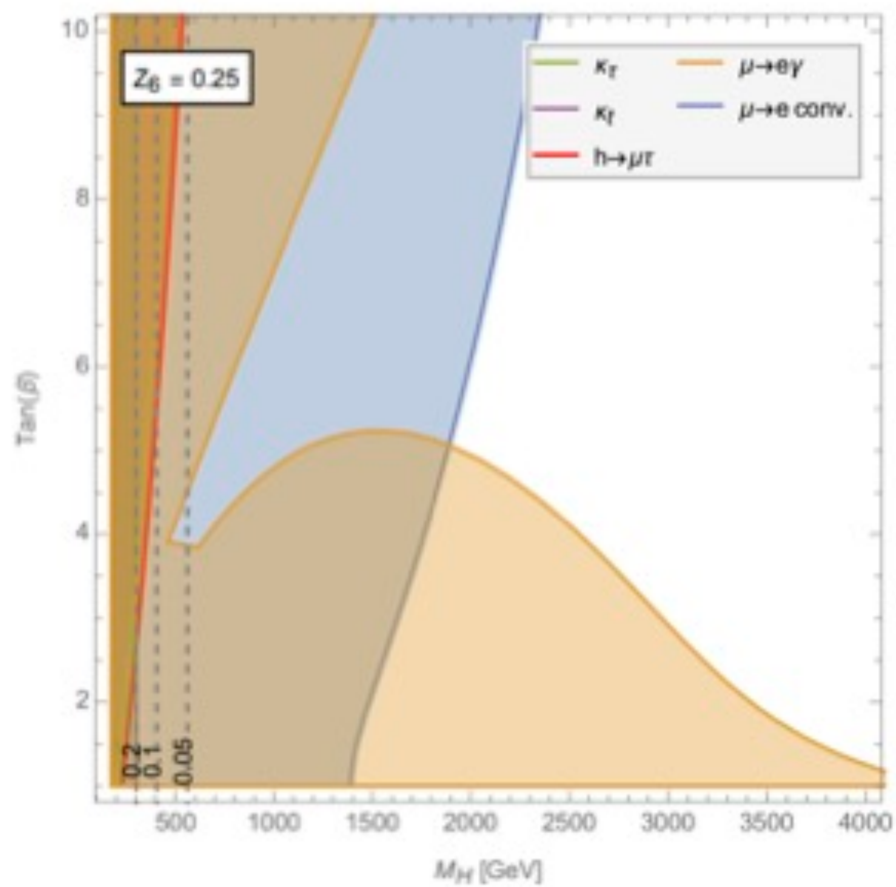
Case in which $\bar{y}_{l_i l_j} \propto \frac{\sqrt{m_i m_j}}{v}$

$BR(\mu \to e\gamma) < 4.2 \times 10^{-13}$ $k_\tau < 0.2$ $BR(h \to \tau\mu) < 0.002$

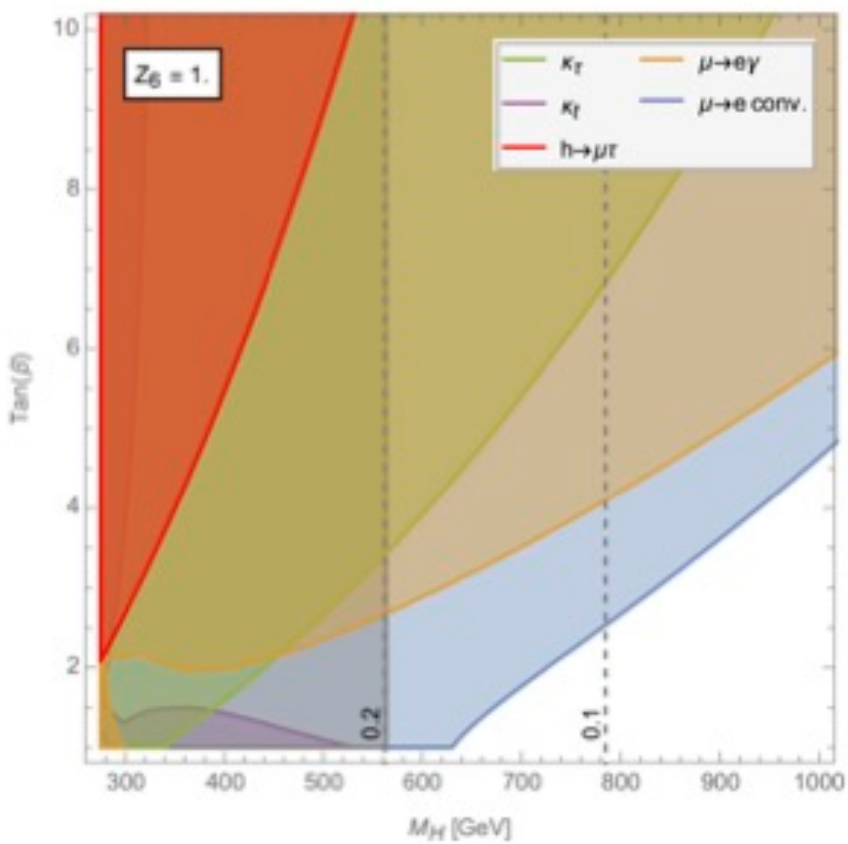


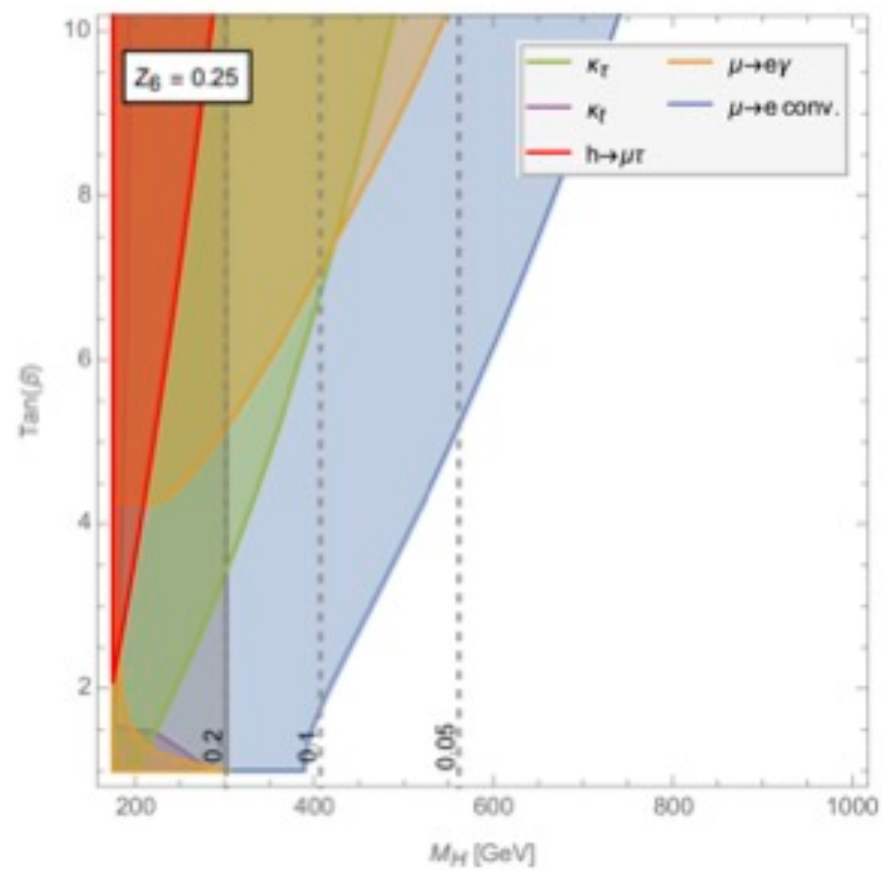


Visible interference between light and heavy Higgs contributions

N. Coyle, D. Rocha, C.W. ' 24

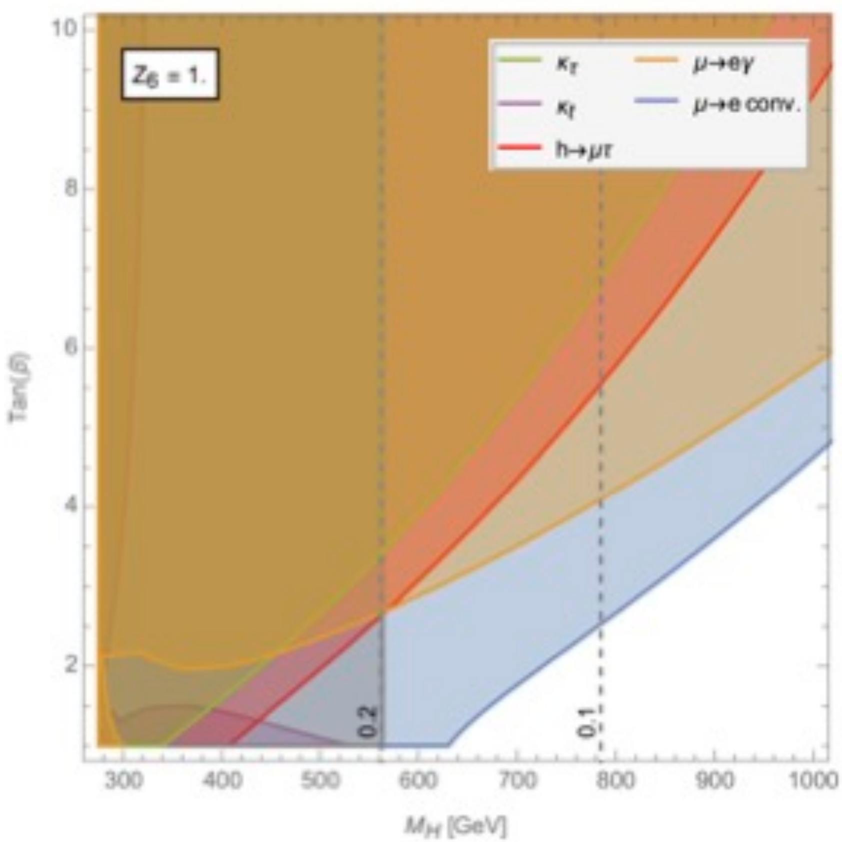
Case in which $\bar{y}_{l_i l_j} \propto \frac{\mathrm{Min(m_i, m_j)}}{v}$

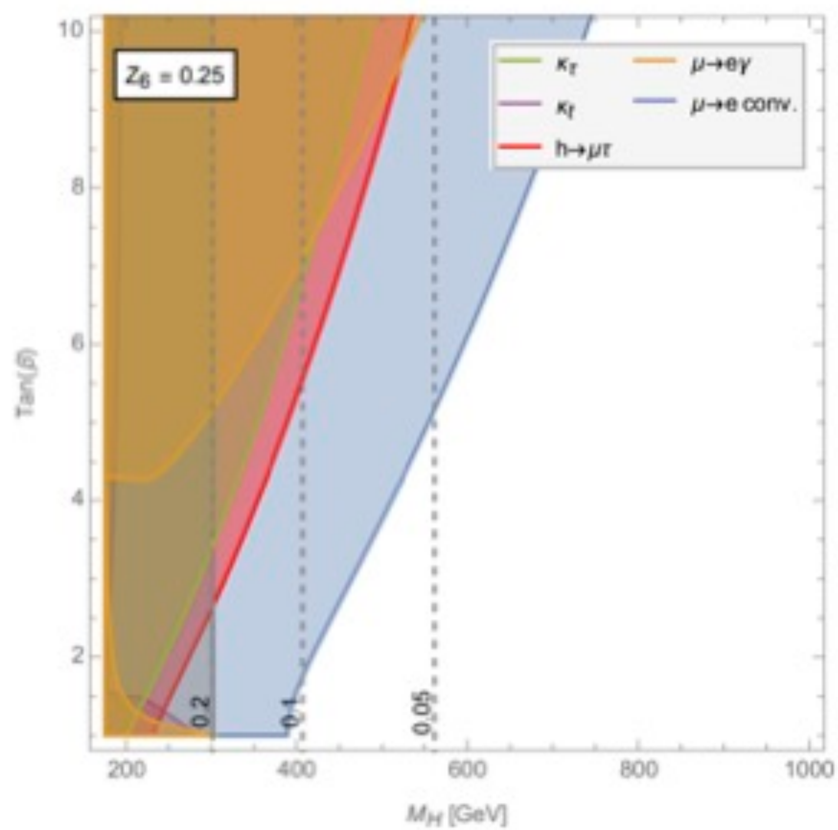




N. Coyle, D. Rocha, C.W. ‘ 24

Case in which $\bar{y}_{\mu\tau} \propto \frac{\sqrt{m_\tau m_\mu}}{v}, \quad \bar{y}_{\mu e} \propto \frac{m_e}{v}$

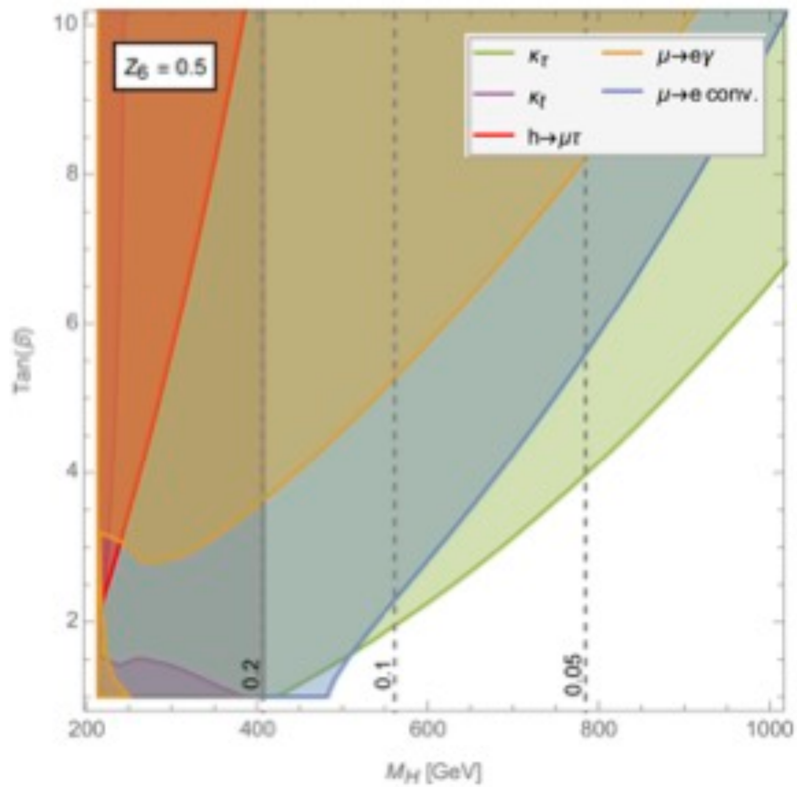




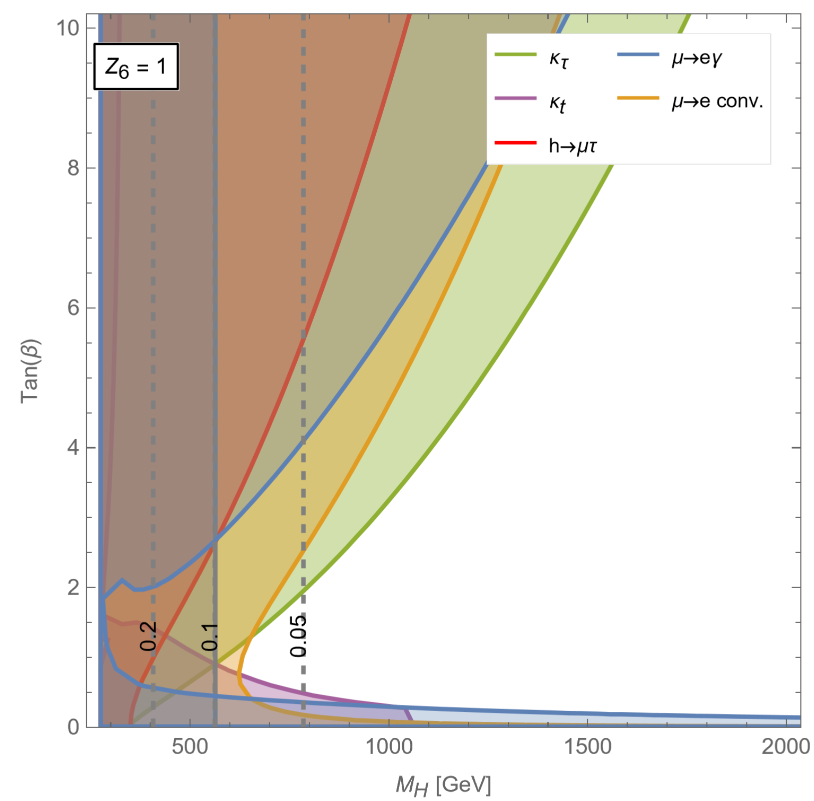
N. Coyle, D. Rocha, C.W. ' 24

Influence of Diagonal Couplings

$$\bar{y}_{l_i l_j} \propto \frac{\mathrm{Min(m_i, m_j)}}{v}$$



$$\bar{y}_{\mu\tau} \propto \frac{\sqrt{m_\tau m_\mu}}{v}, \qquad \bar{y}_{\mu e} \propto \frac{m_e}{v}$$



For Diagonal values $\bar{y}_2^{ii} = 0$ (impact of $\Delta_i = 0$).

Alignment Condition

$$Z_6 = [\lambda_2 s_\beta^2 - \lambda_1 c_\beta^2 + (\lambda_3 + \lambda_4 + \lambda_5) c_{2\beta}] s_{\frac{2\beta}{2}} + \lambda_7 s_\beta s_{3\beta}$$

$$Z_6 = \cos(\beta - \alpha) = 0$$

Possible alignment solutions :

1. At large tanβ, and if λ7 is small, generated at the loop level, as in the MSSM,

$$t_\beta \lambda_7 = \lambda_2 - (\lambda_3 + \lambda_4 + \lambda_5) = \frac{m_h^2}{v^2} - (\lambda_3 + \lambda_4 + \lambda_5)$$

$$\text{MSSM}: \lambda_3 + \lambda_4 + \lambda_5 = -\frac{M_Z^2}{v^2}$$

2. For small tanβ, the term in square brackets must be cancelled. This could happen if

$$\lambda_1 = \lambda_2 = \lambda_3 + \lambda_4 + \lambda_5$$

This can be due to a symmetry relation, that we will explore.

3. Alternative, for sizable tanβ, and very small λ7, there could be an accidental cancellation.

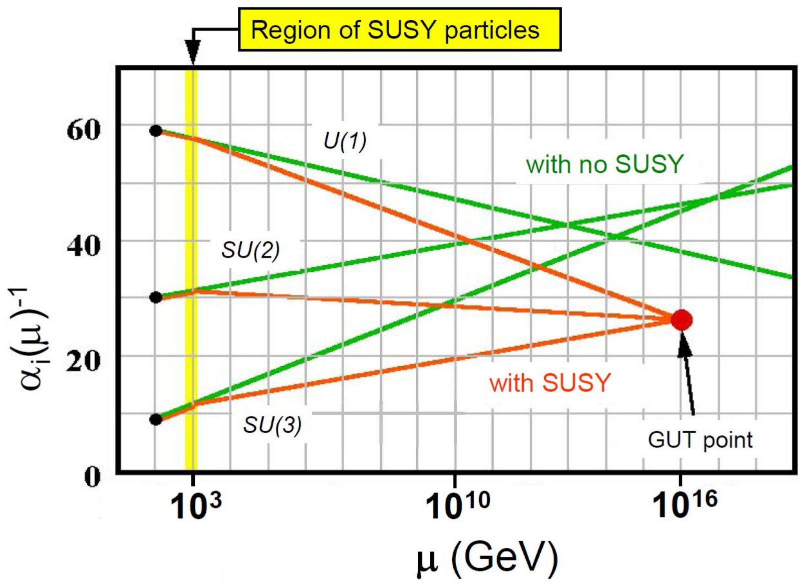
For instance, at large values of tanβ, this can happen whenever

$$\lambda_2 = \frac{m_h^2}{v^2} = \lambda_3 + \lambda_4 + \lambda_5$$

This mechanism is at work in the NMSSM, where $\Delta\lambda_4 = \lambda^2$

A well motivated example : Supersymmetry

Unification



SUSY Algebra

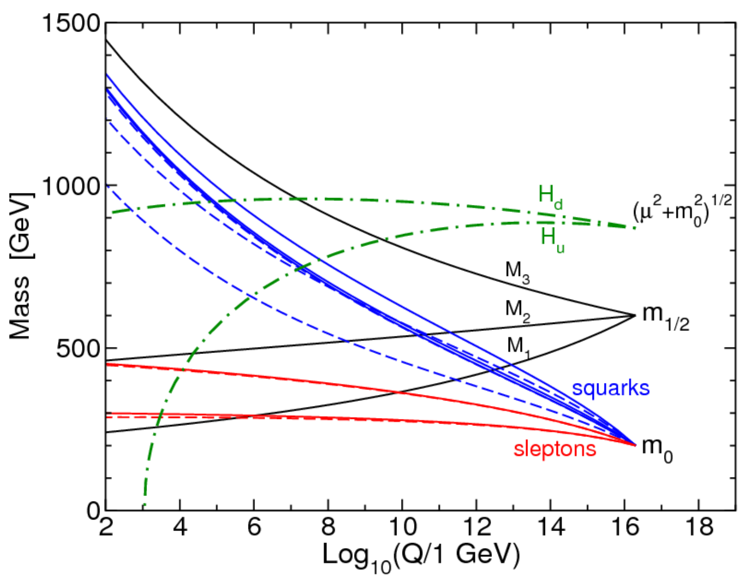
$$\{Q_\alpha, \bar{Q}_{\dot{\alpha}}\} = 2\sigma^{\mu}_{\alpha\dot{\alpha}} P_\mu$$

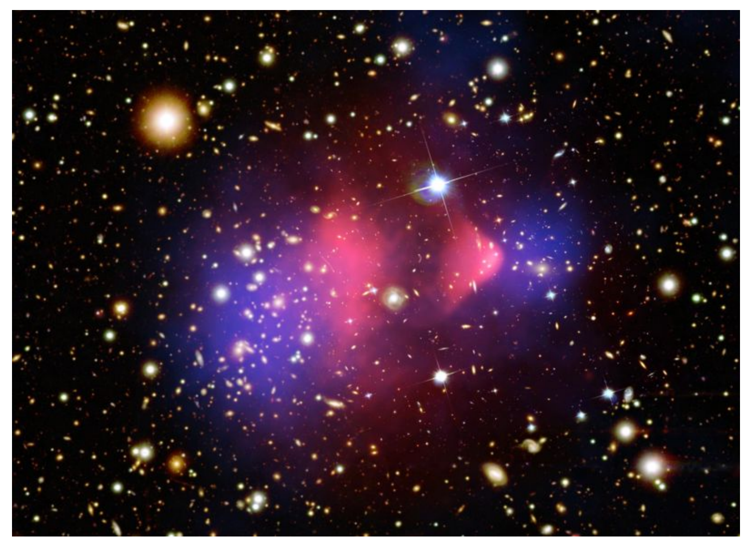
$$[Q_\alpha, P_\mu] = [\bar{Q}_{\dot{\alpha}}, P_\mu] = 0$$

Quantum Gravity ?

Ultraviolet Insensitivity

Electroweak Symmetry Breaking





If R-Parity is Conserved the Lightest SUSY particle is a good Dark Matter candidate

Stop Searches : MSSM Guidance ?

Lightest SM-like Higgs mass strongly depends on:

* CP-odd Higgs mass m_A

* tan beta $= \frac{v_u}{v_d}$

*the top quark mass

* the stop masses and mixing

$$M^2_{\tilde{t}} = \begin{pmatrix} m_Q^2 + m_t^2 + D_L & m_t X_t \\ m_t X_t & m_U^2 + m_t^2 + D_R \end{pmatrix}$$

M_h depends logarithmically on the averaged stop mass scale M_{SUSY} and has a quadratic and quartic dep. on the stop mixing parameter X_t. [and on sbottom/stau sectors for large tan beta]

For moderate to large values of tan beta and large non-standard Higgs masses

$$m_h^2 \cong M_Z^2 \cos^2 2\beta + \frac{3}{4\pi^2}\frac{m_t^4}{v^2}\left[\frac{1}{2}\tilde{X}_t + t + \frac{1}{16\pi^2}\left(\frac{3}{2}\frac{m_t^2}{v^2} - 32\pi\alpha_3\right)\left(\tilde{X}_t t + t^2\right)\right]$$

$$t = \log(M_{SUSY}^2 / m_t^2) \qquad \tilde{X}_t = \frac{2X_t^2}{M_{SUSY}^2}\left(1 - \frac{X_t^2}{12 M_{SUSY}^2}\right)$$

$X_t = A_t - \mu/\tan\beta \rightarrow$ LR stop mixing

Carena, Espinosa, Quiros, C.W.'95,96

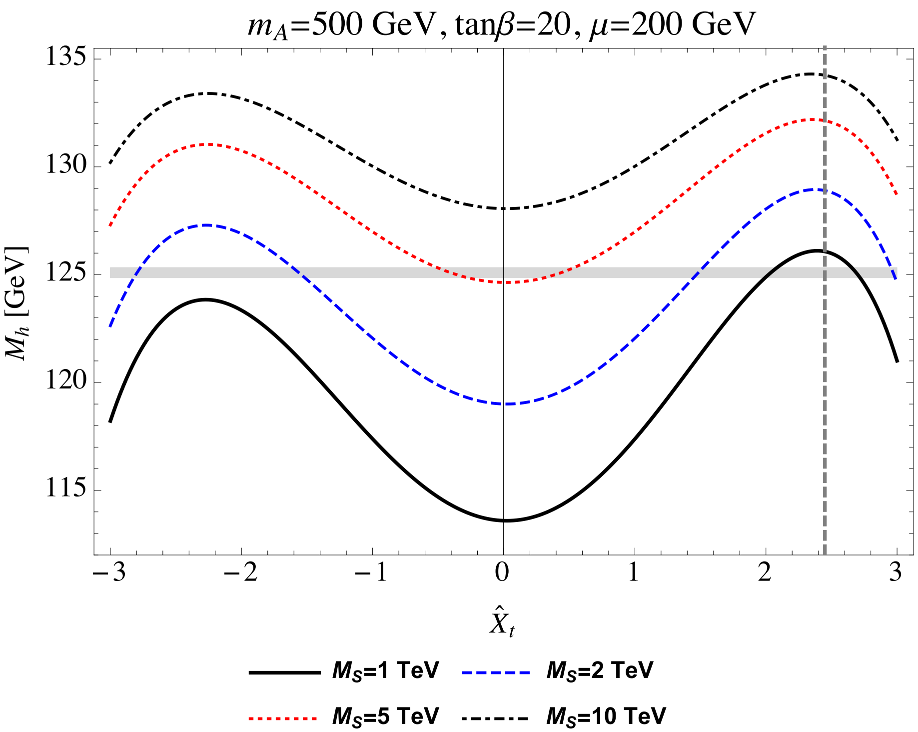
Analytic expression valid for $M_{SUSY} \sim m_Q \sim m_U$

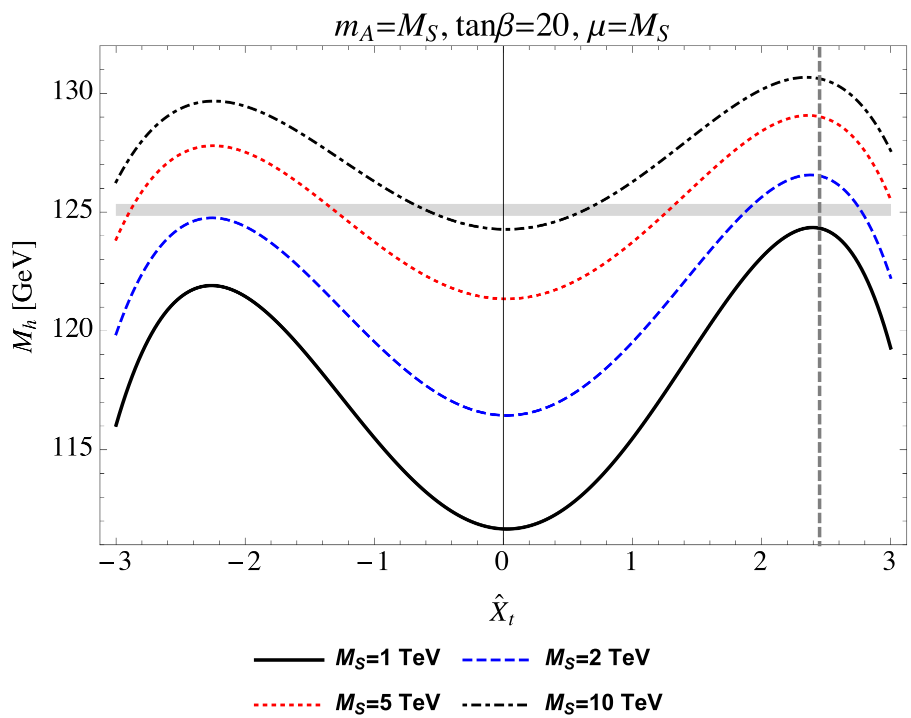
MSSM Guidance:
Stop Masses above about 1 TeV lead to the right Higgs Masss

P. Slavich, S. Heinemeyer et al, arXiv:2012.15629

P. Draper, G. Lee, C.W.'13, Bagnaschi et al' 14, Vega and Villadoro '14, Bahl et al'17

G. Lee, C.W. arXiv:1508.00576

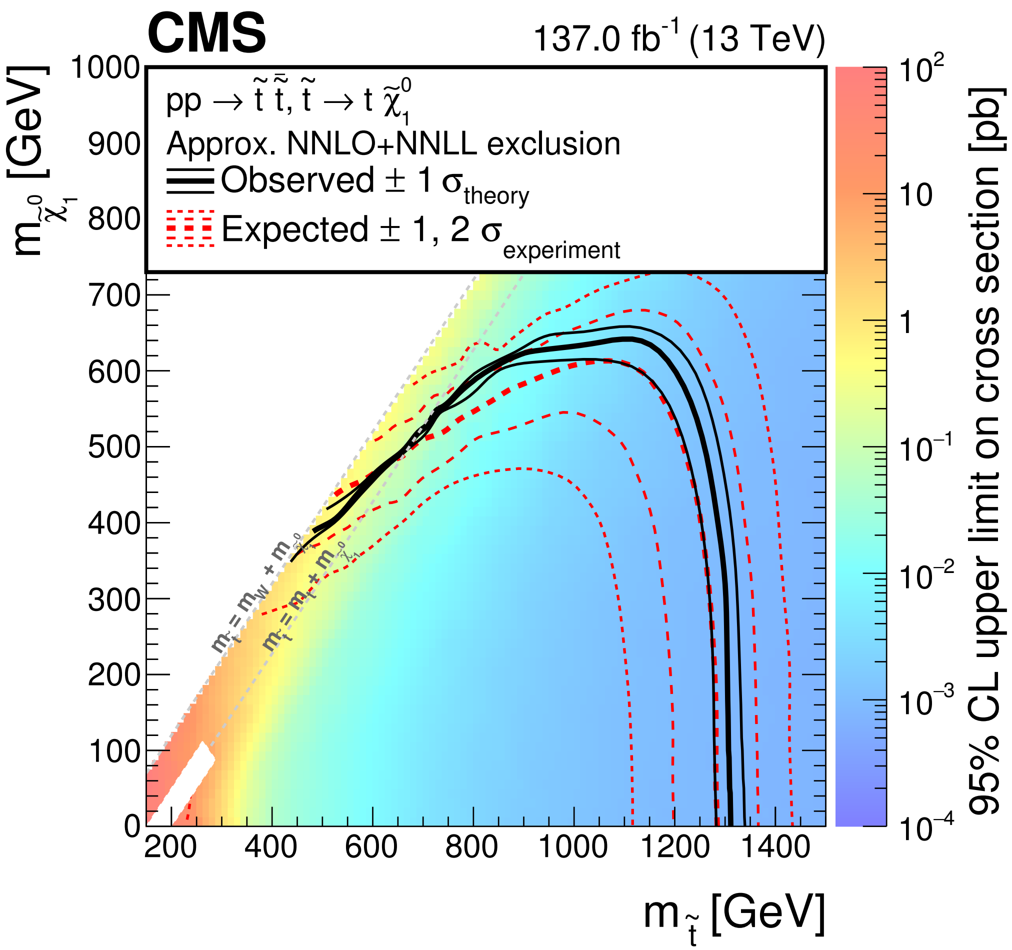


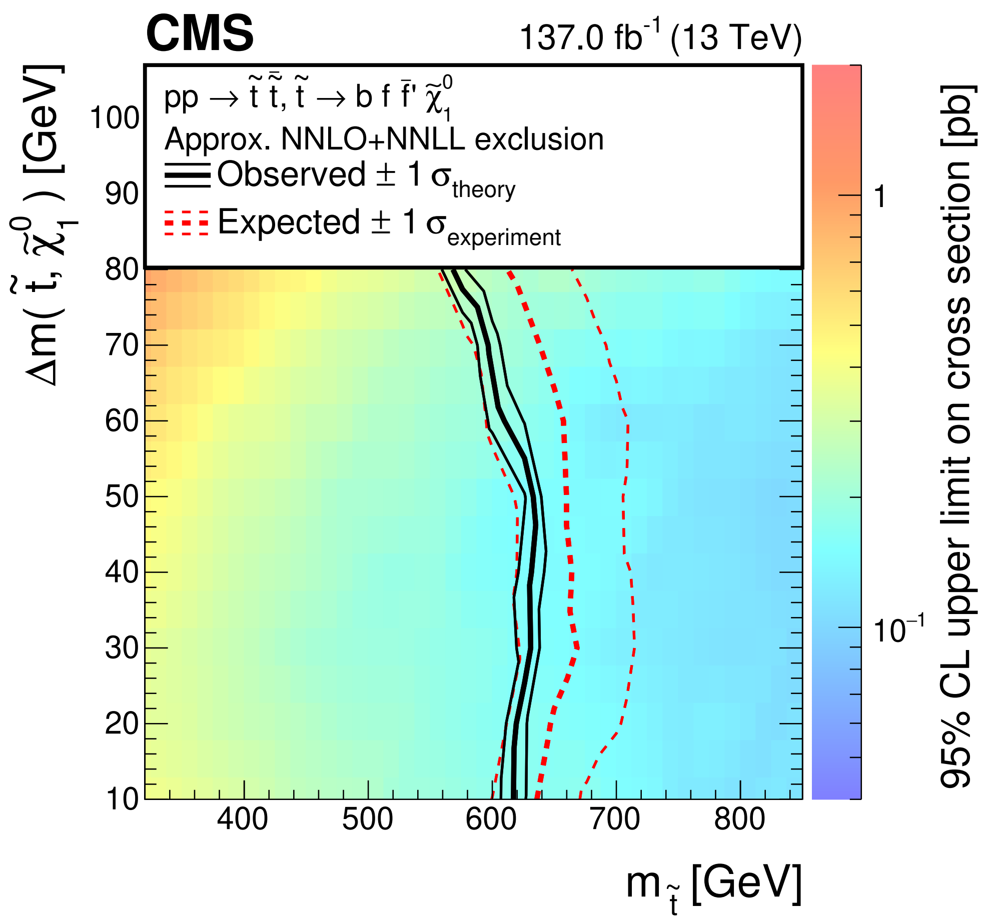


Necessary stop masses increase for lower values of tanβ, larger values of μ smaller values of the CP-odd Higgs mass or lower stop mixing values.

Lighter stops demand large splittings between left- and right-handed stop masses

Stop Searches





Combining all searches, in the simplest decay scenarios, it is hard to avoid the constraints of 700 GeV for sbottoms and 600 GeV for stops. Islands in one search are covered by other searches.

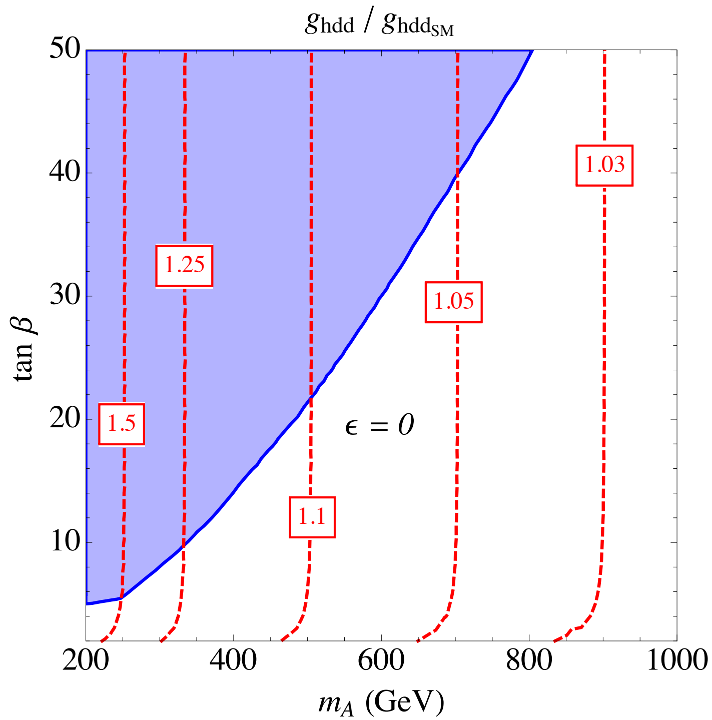
We are starting to explore the mass region suggested by the Higgs mass determination !

Down Couplings in the MSSM for low values of μ

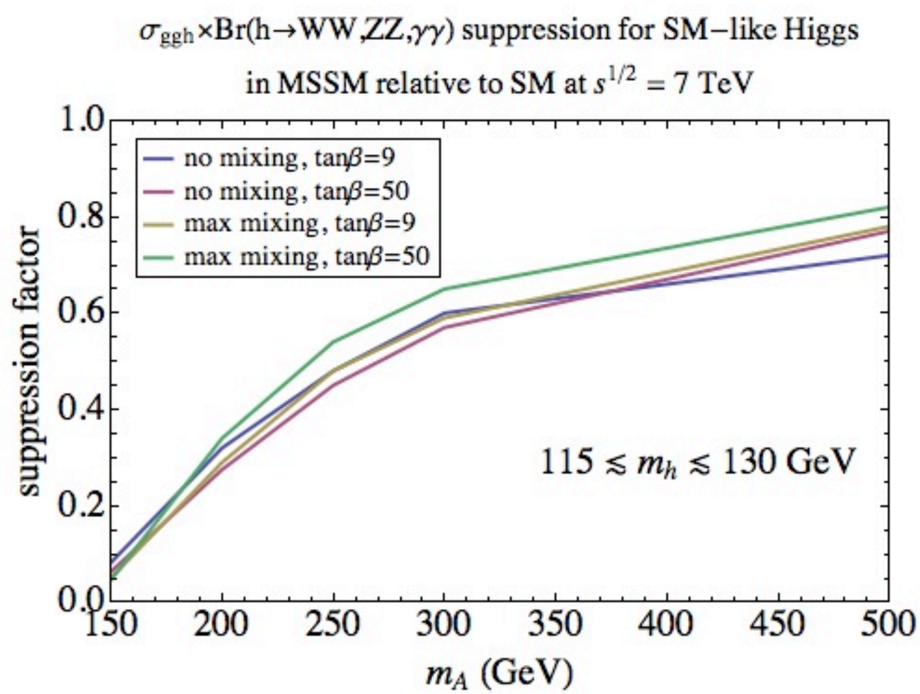
- Higgs Decay into bottom quarks is the dominant one
- A modification of the bottom quark coupling affects all other decays

$$t_\beta \, c_{\beta-\alpha} \simeq \frac{-1}{m_H^2 - m_h^2}\left[m_h^2 + m_Z^2 + \frac{3m_t^4}{4\pi^2 v^2 M_S^2}\left\{A_t \mu t_\beta \left(1 - \frac{A_t^2}{6M_S^2}\right) - \mu^2\left(1 - \frac{A_t^2}{2M_S^2}\right)\right\}\right]$$

Carena, Haber, Low, Shah, C.W. '14



Carena, Low, Shah, C.W.'13



Enhancement of bottom quark and tau couplings independent of $\tan\beta$

Carena, Haber, Low, Shah, C.W.'15

Naturalness and Alignment in the (N)MSSM

see also Kang, Li, Li,Liu, Shu'13, Agashe,Cui,Franceschini'13

- It is well known that in the NMSSM there are new contributions to the lightest CP-even Higgs mass,

$$W = \lambda S H_u H_d + \frac{\kappa}{3} S^3$$

$$m_h^2 \simeq \lambda^2 \frac{v^2}{2} \sin^2 2\beta + M_Z^2 \cos^2 2\beta + \Delta_{\tilde{t}}$$

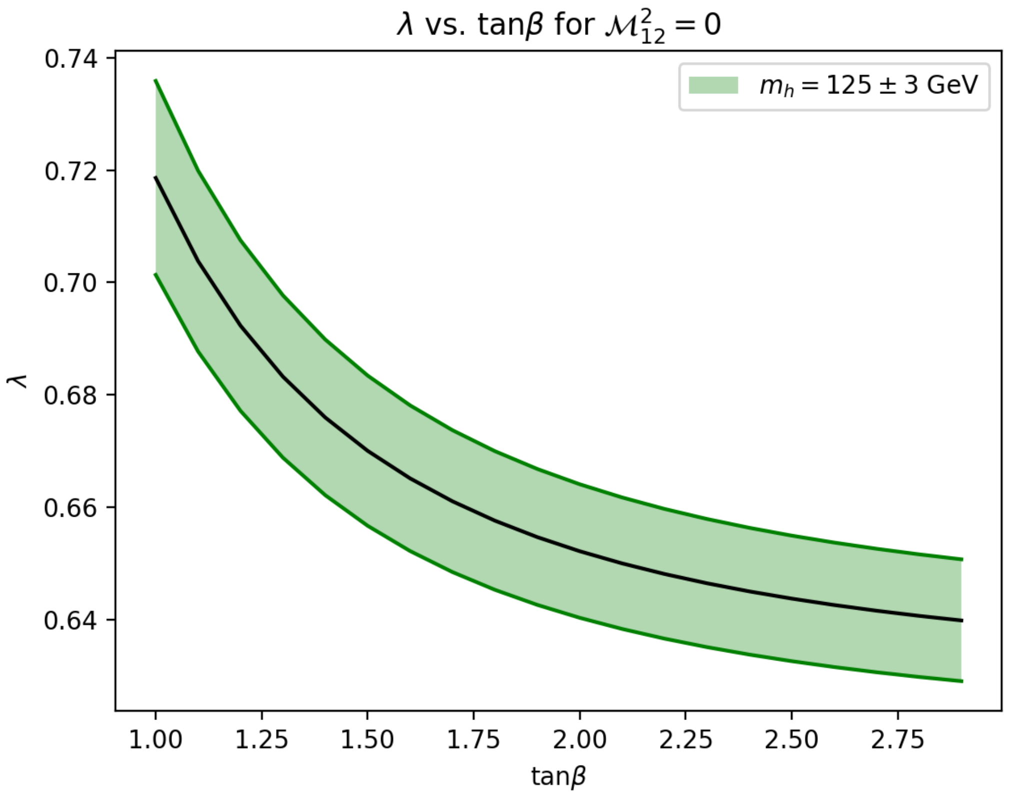
- It is perhaps less known that it leads to sizable corrections to the mixing between the MSSM like CP-even states. In the Higgs basis, (correction to $\Delta\lambda_4 = \lambda^2$)

$$M_S^2(1,2) \simeq \frac{1}{\tan\beta}\left(m_h^2 - M_Z^2 \cos 2\beta - \lambda^2 v^2 \sin^2\beta + \delta_{\tilde{t}}\right) \equiv Z_6 v^2$$

- The values of lambda end up in a very narrow range, between 0.65 and 0.7 for all values of tan(beta), that are the values that lead to naturalness with perturbativity up to the GUT scale

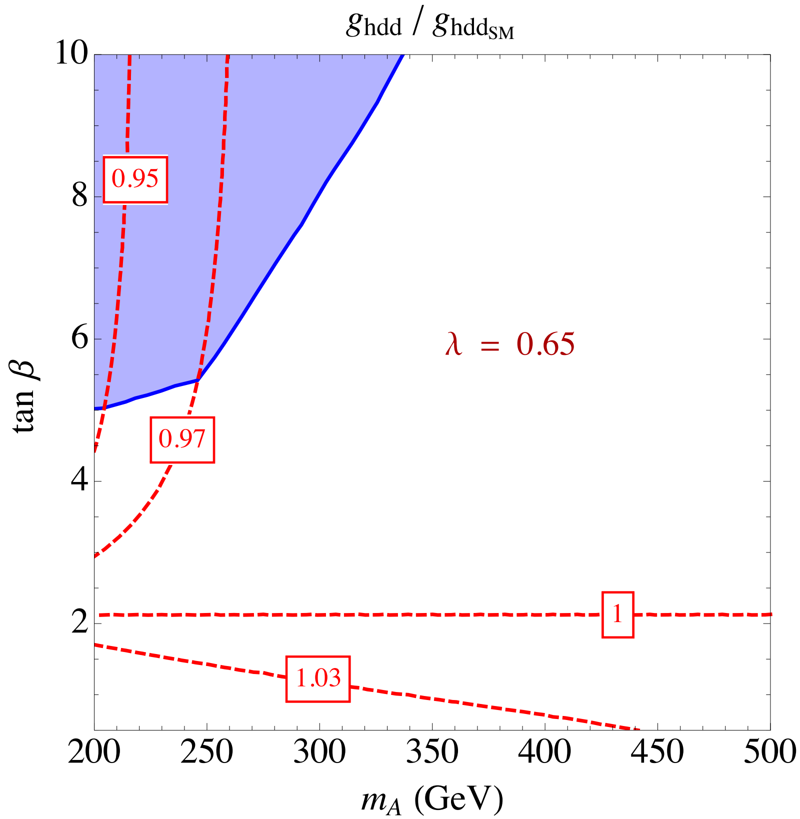
$$\lambda^2 = \frac{m_h^2 - M_Z^2 \cos 2\beta}{v^2 \sin^2\beta}$$

NMSSM : λ vs tanβ



Alignment in the NMSSM (heavy or Aligned singlets)

Carena, Low, Shah, C.W.'13



It is clear from this plot that the NMSSM does an amazing job in aligning the MSSM-like CP-even sector, provided $\lambda \sim 0.65$

Very relevant phenomenological properties

This range of couplings, and the subsequent alignment, may appear as emergent properties in a theory with strong interactions at high energies

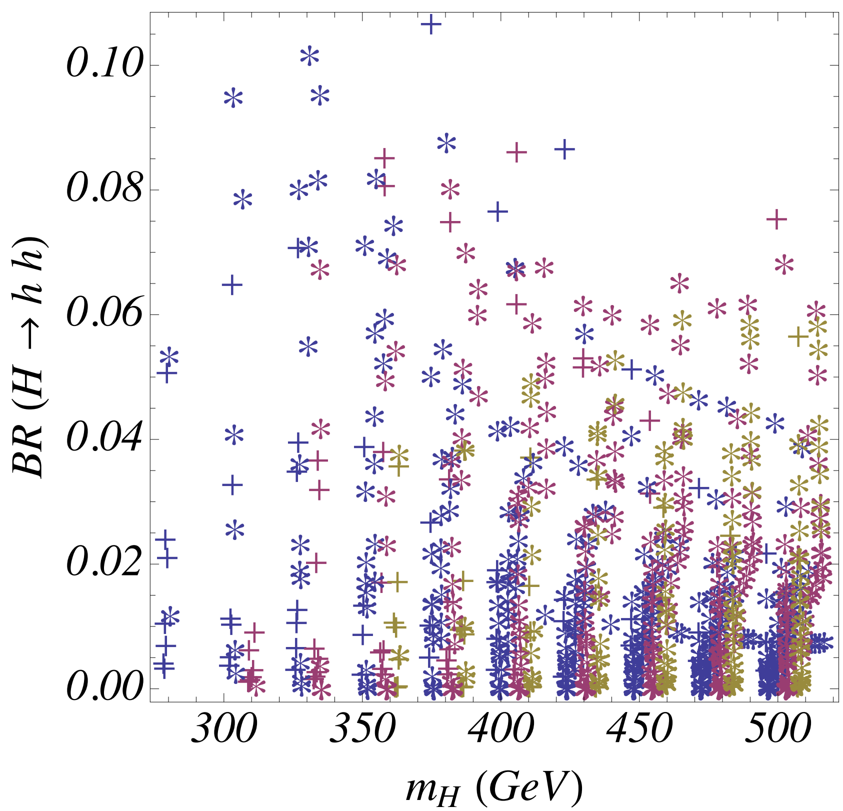
N. Coyle, C.W. arXiv:1912.01036

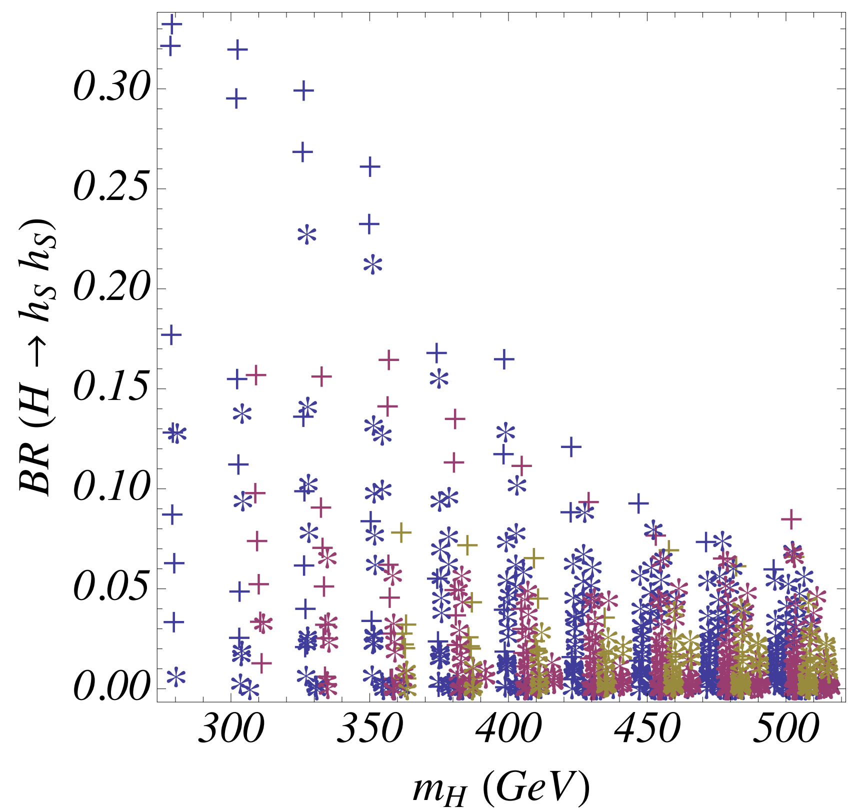
Decays into pairs of SM-like Higgs bosons suppressed by alignment

Carena, Haber, Low, Shah, C.W.'15

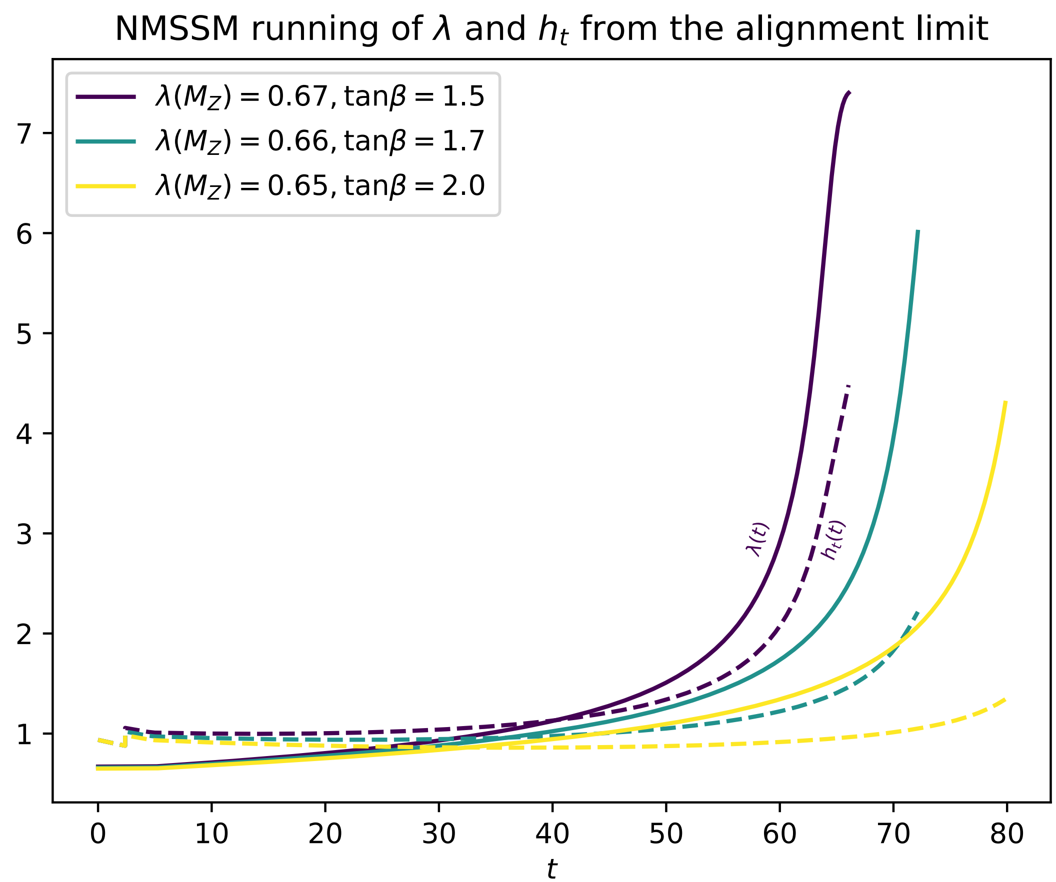
Crosses : H1 singlet like
Asterix : H2 singlet like

Blue : $\tan\beta = 2$
Red : $\tan\beta = 2.5$
Yellow: $\tan\beta = 3$





Running of Couplings. Landau Poles at High Energies



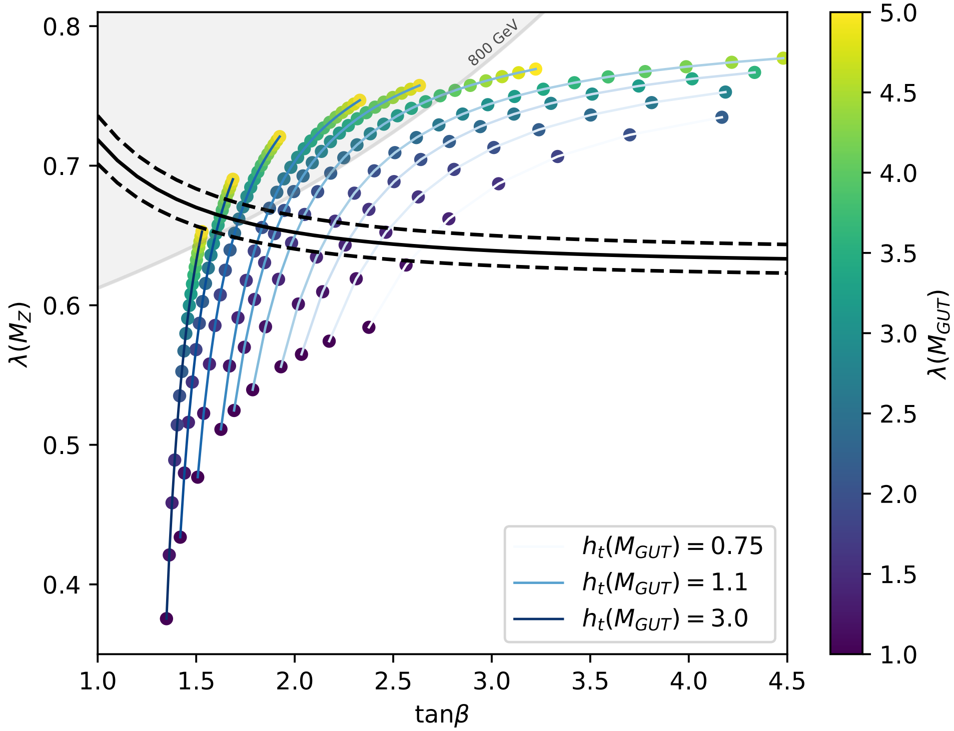
$$t = \ln\left(\frac{Q^2}{M_Z^2}\right)$$

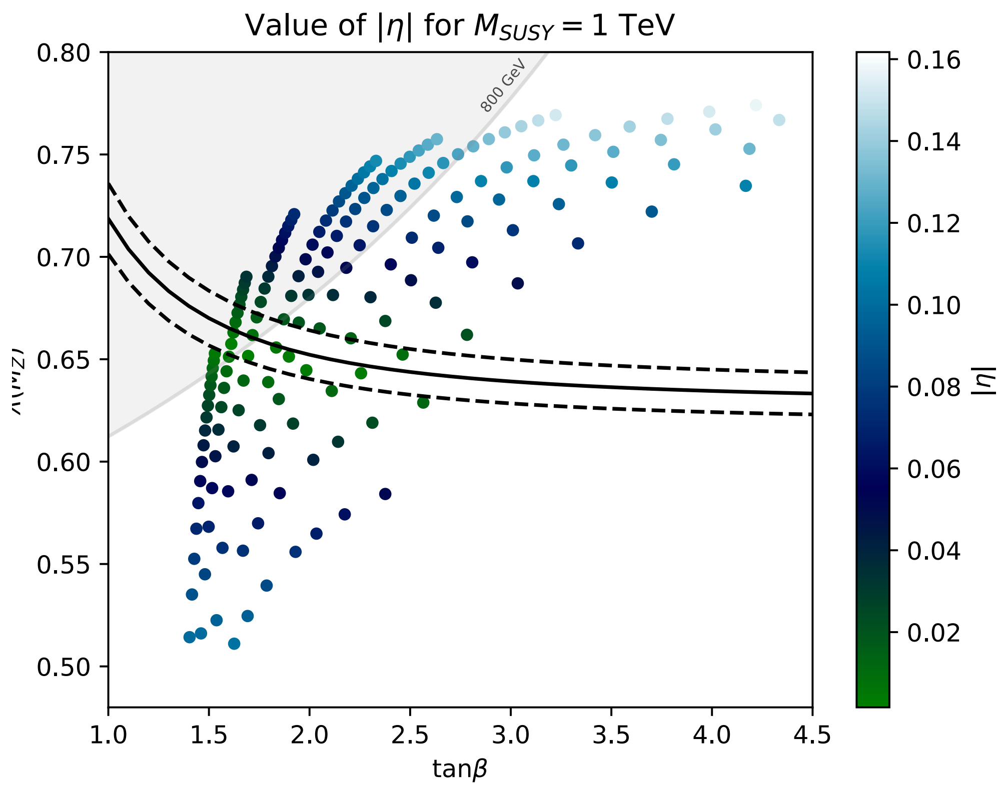
$$Q = M_{\rm GUT}, \quad t \sim 66$$

Range of values for Higgs alignment seems to suggest the appearance of a strongly interacting sector (Fat Higgs) at energies close to the GUT scale.

N. Coyle, C.W. arXiv:1912.01036

Higgs Alignment and the coupling λ





$$
\begin{aligned}
g_{hb\bar{b}} &= g^{\rm SM}_{hb\bar{b}}\,(1-\eta) \\
g_{ht\bar{t}} &= g^{\rm SM}_{ht\bar{t}}\left(1+\frac{\eta}{\tan^2\beta}\right) \\
g_{hVV} &= g^{\rm SM}_{hVV}\left(1-\frac{\eta^2}{2\tan^2\beta}\right)
\end{aligned}
$$

$$\eta = \cos(\beta-\alpha)\tan\beta$$

N. Coyle, C.W. arXiv:1912.01036

Comments

- Flavor or Higgs alignments are not guaranteed. Therefore, beyond the standard Higgs searches, there is a strong motivation to perform the following searches :

- Flavor violating decays of the Standard Higgs boson : modified diagonal couplings come usually together with flavor violating couplings. So, the simple kappa framework is not enough, for more than technical reasons $h \to \mu\tau, h \to \mu e, h \to e\tau, \mathrm{etc}$

- Flavor violating decays of non-standard Higgs bosons. They are unsuppressed $H \to tc, H \to \mu\tau, H \to \mu e, H \to e\tau, \mathrm{etc}$

- bs transitions are also of interest, although constrained by other processes

- Searches for heavy Higgs bosons decaying to other scalar states, non-necessarily SM Higgs bosons $H \to hX, H \to XY, etc.$

- I am aware that there are LHC groups working on these subjects. I would encourage more people to join these efforts.

Conclusions

- Precision Higgs measurement show a good agreement of all couplings with respect to the SM expectations
- Two Higgs Doublet Models and singlet extensions provide a good effective field theory to the study of LHC data
- Higgs Flavor violating couplings may lead to the first hints of physics BSM.
- Light non-standard Higgs bosons demand alignment in field space of the mass eigenstates with the directions acquiring vev's.
- We discussed a few ways in which alignment may be obtained.
- Higgs physics remains as the most vibrant field of particle physics, one in which many surprises may lay ahead, with profound implications for our understanding of Nature.

Backup Slides

Entanglement Suppression and Alignment

Two States System

- Let's take two distinguishable qubits, A and B, each of them with its own basis of vectors

$$|1>_I, |2>_I, \qquad I = A, B$$

- We can define a quantum state

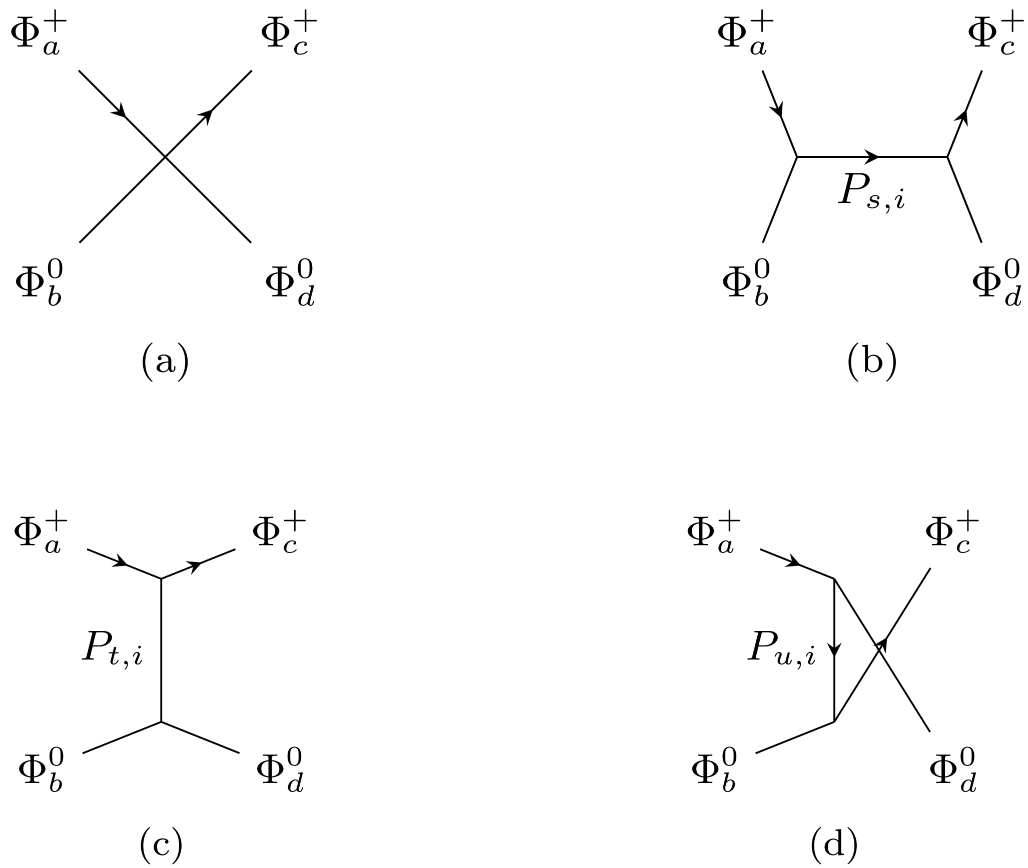
$$|\psi>= \sum_{i,j=1}^{2} c_{ij} |i>_A |j>_B$$

- Entanglement suppression will occur when we can write this as the product of a state in A times one in B. Mathematically, this occur whenever the so-called concurrence

$$\Delta = c_{11}c_{22} - c_{12}c_{21} = 0$$

Scattering Amplitudes

Let's apply these ideas to the case of two Higgs doublets, with spin states up (charged) and down (neutral). Let's start with a product state and demand that the final state is not entangled, namely we want to end up in another product state.



Carena, Low, C.W., Xiao, arXiv:2307.08112

Scattering Process

Considering the S Matrix for the scattering process of two distinguishable states, for which we will choose the neutral and charged components of the Higgs doublets in the Higgs basis

Carena, Low, C.W., Xiao, arXiv:2307.08112

$$S = 1 + i\,T$$

$$\langle \Phi_c \Phi_d | \, iT \, | \Phi_a \Phi_b \rangle = i(2\pi)^4 \delta^{(4)}(p_a + p_b - p_c - p_c) M_{ab,cd}$$

Incoming states :

$$|\Phi_a \Phi_b\rangle = (\kappa|1\rangle + \epsilon|2\rangle) \otimes (\gamma|1\rangle + \delta|2\rangle)$$

Outgoing states:

$$|\Phi_c \Phi_d\rangle = (\delta_{ac}\delta_{bd} + iM_{ab,cd})|\Phi_a\rangle \otimes |\Phi_b\rangle$$

Entanglement Suppression (at linear order in Mij,kl) :

$$M_{11,11} + M_{22,22} = M_{12,12} + M_{21,21} \;,$$
$$M_{11,22} = M_{12,21} = M_{21,12} = M_{22,11} = 0 \;,$$
$$M_{11,12} = M_{21,22} \;, \quad M_{11,21} = M_{12,22} \;.$$

Concurrence

$$|\Phi_c\Phi_d\rangle = (\delta_{ac}\delta_{bd} + iM_{ab,cd})|\Phi_a\rangle \otimes |\Phi_b\rangle = c_{ij}|ij\rangle$$

$$c_{11} = (1 + iM_{11,11})\,\kappa\gamma + iM_{12,11}\ \kappa\delta + iM_{21,11}\ \epsilon\gamma + iM_{22,11}\ \epsilon\delta$$

$$c_{12} = iM_{11,12}\ \kappa\gamma + (1 + iM_{12,12})\,\kappa\delta + iM_{21,12}\ \epsilon\gamma + iM_{22,12}\ \epsilon\delta$$

$$c_{21} = iM_{11,21}\ \kappa\gamma + iM_{12,21}\ \kappa\delta + (1 + iM_{21,21})\,\epsilon\gamma + iM_{22,21}\ \epsilon\delta$$

$$c_{22} = iM_{11,22}\ \kappa\gamma + iM_{12,22}\ \kappa\delta + iM_{21,22}\ \epsilon\gamma + (1 + iM_{22,22})\,\epsilon\delta$$

The concurrence is therefore given by

$$\begin{aligned}\Delta(|\Phi_c\Phi_d\rangle) = {} & i\kappa\epsilon\gamma\delta(M_{11,11} - M_{12,12} - M_{21,21} + M_{22,22}) \\ & + i\kappa\epsilon(\gamma^2 - \delta^2)(M_{21,22} - M_{11,12}) + i(\kappa^2 - \epsilon^2)\gamma\delta(M_{12,22} - M_{11,21}) \\ & - iM_{12,21}\ \kappa^2\delta^2 - iM_{21,12}\ \epsilon^2\gamma^2 + iM_{11,22}\ \kappa^2\gamma^2 + iM_{22,11}\ \epsilon^2\delta^2 + O((M_{ab,cd})^2)\end{aligned}$$

Amplitudes in the Higgs Basis

We shall perform the calculation in the Higgs basis: such U(2) rotation - no mixing between Φ^0 and Φ^+ - corresponds to a single-qubit operation and does not change the entanglement power of the S-Matrix

$$iM_{ab,cd} = iM^0_{ab,cd} - \frac{v^2}{2} \sum_i \sum_{r=s,t,u} M_i^r{}_{ab,cd}\, P_{r,i} \;,$$

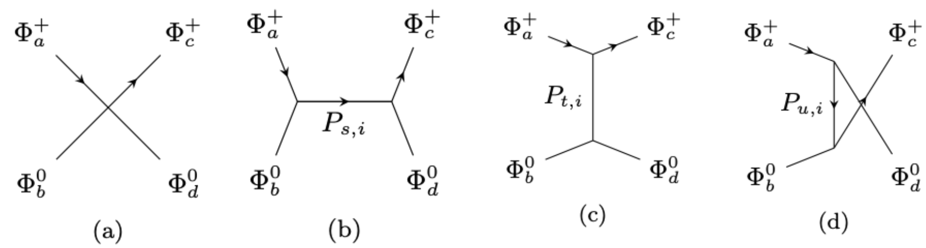
$$M^0_{ab,cd} = \begin{pmatrix} Z_1 & Z_6 & Z_6 & Z_5 \\ Z_6 & Z_3 & Z_4 & Z_7 \\ Z_6 & Z_4 & Z_3 & Z_7 \\ Z_5 & Z_7 & Z_7 & Z_2 \end{pmatrix} \;,$$

At tree level, in the symmetric phase, the amplitudes receive contributions from the quartic couplings. In the broken phase, however, receives contribution from diagrams that involve the interchange of standard and non-standard Higgs bosons.

Charged mediators, however, lead to entanglement in the broken phase and suppression of entanglement demands equality of the masses of the charged Higgs and Goldstone modes.

Carena, Low, C.W., Xiao, arXiv:2307.08112

Higgs Exchange Amplitudes



$$M^s_{i\;ab,cd} = M_{abi} M^*_{cdi} \;, \quad M^u_{i\;ab,cd} = M_{adi} M^*_{cbi}$$

$$M^t_{i\;ab,cd} = \sum_{j,k} \mathcal{R}_{ij} M_{ajc} (\mathcal{R}_{ik} M_{dkb,0})^* + \text{h.c.} \quad ,$$

rotation matrix in the neutral sector

$$P_{t,i} = i/(t - m^2_{0,i}) \text{ and } P_{r,i} = i/(r - m^2_{+,i}), \qquad r = s, u.$$

$$m_{0,i} = \{m_h, m_H, 0, m_A\} \text{ and } m_{+,i} = \{m_{H^\pm}, 0\}$$

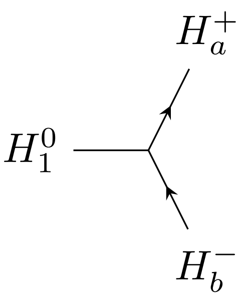
Higgs Masses and Trilinear Couplings in the Broken Phase

$$m_+^2 = \begin{pmatrix} 0 & 0 \\ 0 & Y_2 + Z_3 v^2/2 \end{pmatrix},$$

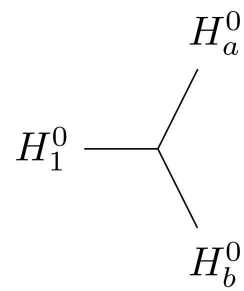
$$m_{\text{even}}^2 = \begin{pmatrix} Z_1 v^2 & Z_6 v^2 \\ Z_6 v^2 & Y_2 + (Z_3 + Z_4 + Z_5) v^2/2 \end{pmatrix}$$

$$m_{\text{odd}}^2 = \begin{pmatrix} 0 & 0 \\ 0 & Y_2 + (Z_3 + Z_4 - Z_5) v^2/2 \end{pmatrix}.$$

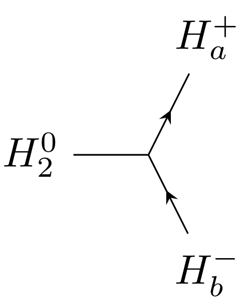
$Y_2 H_2^\dagger H_2$



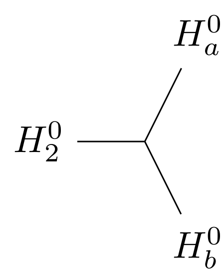
$$= \frac{iv}{\sqrt{2}} \begin{pmatrix} Z_1 & Z_6 \\ Z_6 & Z_3 \end{pmatrix}_{ab},$$



$$= \frac{iv}{\sqrt{2}} \begin{pmatrix} Z_1 & 2Z_6 \\ 2Z_6 & Z_5 \end{pmatrix}_{ab},$$



$$= \frac{iv}{\sqrt{2}} \begin{pmatrix} Z_6 & Z_5 \\ Z_4 & Z_7 \end{pmatrix}_{ab},$$



$$= \frac{iv}{\sqrt{2}} \begin{pmatrix} Z_6 & Z_3 + Z_4 \\ Z_3 + Z_4 & Z_7 \end{pmatrix}_{ab},$$

Amplitude dependence on quartic Couplings

$$M_1^s = \begin{pmatrix} Z_1^2 & Z_1Z_6 & Z_1Z_6 & 0 \\ Z_1Z_6 & Z_6^2 & Z_6^2 & 0 \\ Z_1Z_6 & Z_6^2 & Z_6^2 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}, \qquad M_1^u = \begin{pmatrix} Z_1^2 & Z_1Z_6 & Z_1Z_6 & Z_6^2 \\ Z_1Z_6 & Z_6^2 & 0 & 0 \\ Z_1Z_6 & 0 & Z_6^2 & 0 \\ Z_6^2 & 0 & 0 & 0 \end{pmatrix},$$

$$M_2^s = \begin{pmatrix} Z_6^2 & 0 & Z_3Z_6 & Z_6^2 \\ 0 & 0 & 0 & 0 \\ Z_3Z_6 & 0 & Z_3^2 & Z_3Z_6 \\ Z_6^2 & 0 & Z_3Z_6 & Z_6^2 \end{pmatrix}, \qquad M_2^u = \begin{pmatrix} Z_6^2 & 0 & Z_3Z_6 & 0 \\ 0 & 0 & Z_6^2 & 0 \\ Z_3Z_6 & Z_6^2 & Z_3^2 & Z_3Z_6 \\ 0 & 0 & Z_3Z_6 & Z_6^2 \end{pmatrix},$$

$M_{11,22} = M_{12,21} = 0$ then requires $\quad Z_6 = 0$

$$M_1^t = \begin{pmatrix} 8Z_1^2 s_{\tilde\alpha}^2 & -2Z_1Z_3c_{\tilde\alpha}s_{\tilde\alpha} & 0 & 0 \\ -2Z_1Z_3c_{\tilde\alpha}s_{\tilde\alpha} & 4Z_1Z_3s_{\tilde\alpha}^2 & 0 & 0 \\ 0 & 0 & 8Z_1Z_3s_{\tilde\alpha}^2 & -2Z_3^2c_{\tilde\alpha}s_{\tilde\alpha} \\ 0 & 0 & -2Z_3^2c_{\tilde\alpha}s_{\tilde\alpha} & 4Z_3^2s_{\tilde\alpha}^2 \end{pmatrix}$$

$$M_2^t = \begin{pmatrix} 8Z_1^2 c_{\tilde\alpha}^2 & 2Z_1Z_3c_{\tilde\alpha}s_{\tilde\alpha} & 0 & 0 \\ 2Z_1Z_3c_{\tilde\alpha}s_{\tilde\alpha} & 4Z_1Z_3c_{\tilde\alpha}^2 & 0 & 0 \\ 0 & 0 & 8Z_1Z_3c_{\tilde\alpha}^2 & 2Z_3^2c_{\tilde\alpha}s_{\tilde\alpha} \\ 0 & 0 & 2Z_3^2c_{\tilde\alpha}s_{\tilde\alpha} & 4Z_3^2c_{\tilde\alpha}^2 \end{pmatrix},$$

$$M_3^t = M_4^t = 0 \, .$$

$M_{11,12} = M_{21,22}$ we get $Z_1 = Z_3$

Charged Higgs Masses

$$M_{11,11} - M_{12,12} - M_{21,21} + M_{22,22} = 0.$$

is automatically fulfilled in the t channel.
However, in the s and u channels,
the previously constrained Z-couplings imply

$$M_1^s = M_1^u = \begin{pmatrix} Z^2 & & & \\ & 0 & & \\ & & 0 & \\ & & & 0 \end{pmatrix}, \quad M_2^s = M_2^u = \begin{pmatrix} 0 & & & \\ & 0 & & \\ & & Z^2 & \\ & & & 0 \end{pmatrix},$$

Therefore, this condition can only be fulfilled if the two charged Higgs masses are the same

Entanglement Suppression Conditions

$$Z_1 = Z_2 = Z_3 \equiv Z \ , \qquad Z_i = 0 \ , \quad i \neq 1,2,3$$
$$Y_1 = Y_2 \equiv Y = -Zv^2/2 \ , \quad Y_3 = 0 \ ,$$

This leads to an extended symmetry, namely an SO(8) symmetry broken spontaneously to SO(7)

Carena, Low, C.W., Xiao, arXiv:2307.08112

This extended symmetry ensures the alignment of the Higgs sector.
It leads to

$$\lambda_1 = \lambda_2 = \lambda_3 = Z, \qquad \lambda_i = 0, i \neq 1,2,3, \text{ in any basis}$$

that is one of the ways of getting alignment.

Bhupal and Pilaftsis, 1408.3405

Entanglement Suppression and Alignment

$$Z_1 = Z_2 = Z_3 \equiv Z \;, \qquad Z_i = 0 \;, \quad i \neq 1,2,3$$
$$Y_1 = Y_2 \equiv Y = -Zv^2/2 \;, \quad Y_3 = 0 \;,$$

This leads to an extended symmetry, namely an SO(8) symmetry broken spontaneously to SO(7)

$$\mathcal{V} = Y(H_1^\dagger H_1 + H_2^\dagger H_2) + \frac{Z}{2}(H_1^\dagger H_1 + H_2^\dagger H_2)^2$$
$$= \frac{Z}{2}\left(|H_1^0|^2 + |H_2^0|^2 + G^+G^- + H^+H^- - \frac{v^2}{2}\right)^2$$

All non-standard Higgs bosons acquire masses degenerate with the Goldstone boson masses, namely zero !

This phenomenologically unacceptable, of course. A way of fixing this problem is to add a soft mass Y_2, that lift all the non-standard Higgs Boson masses, but keeps the alignment conditions.

$$M^2_{\rm NSM} = Y_2 + Z_3 v^2 = M^2_{H^+} \qquad \text{for } Z_4 = Z_5 = Z_6 = 0$$

Carena, Low, C.W., Xiao, arXiv:2307.08112

Entanglement Enhancement

$$\lambda_1 = \lambda_2 = \lambda_3 = \lambda_4 = \lambda_5, \quad \lambda_6 = \lambda_7 = 0$$

If, in addition, we asked for

$$m_{11}^2 = m_{22}^2$$

Six Massless (not three, not seven) bosons appear. It turns out that one can describe this systems in terms of SU(4). Symmetry with respect to eight generators is found, and two of these symmetries remain after symmetry breaking. 6 are broken. More, later ….

Backup Slides II

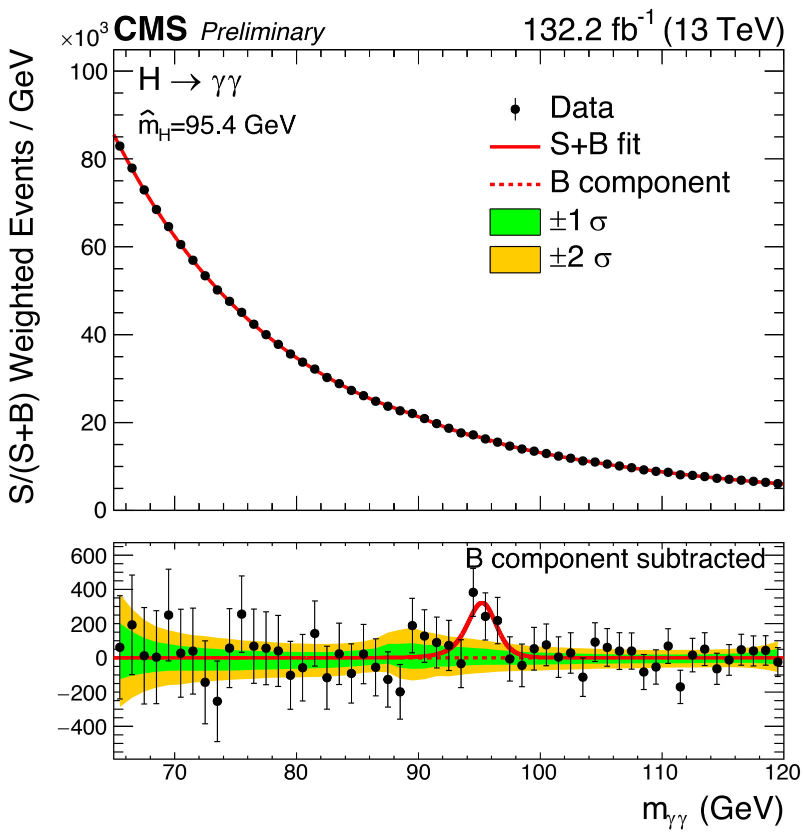
Hints of New Scalars ?

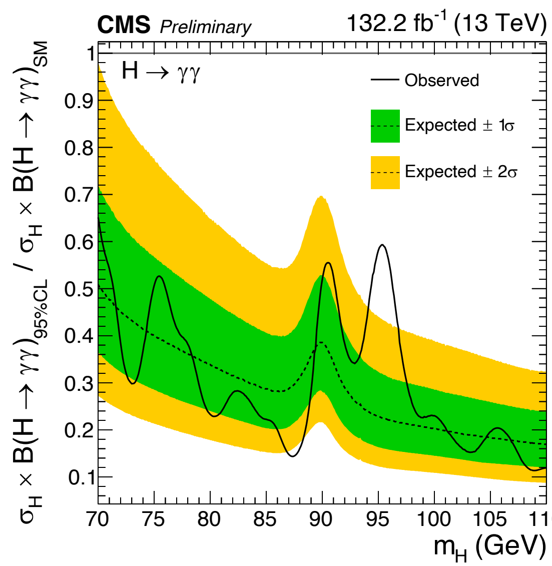
S. Tkaczyk

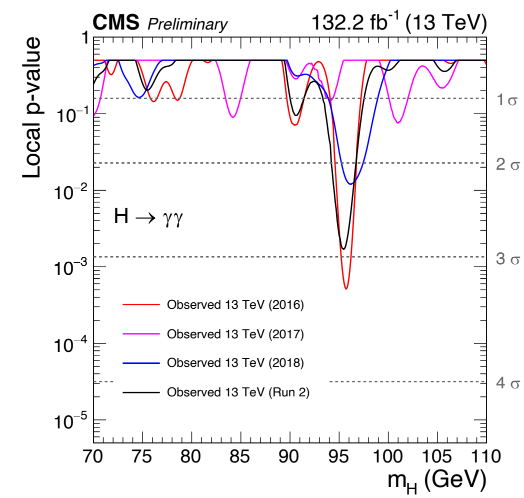
Search for Light H → γγ

CMS-PAS-HIG-020-002

Search for additional light H → γγ decays below H(125)





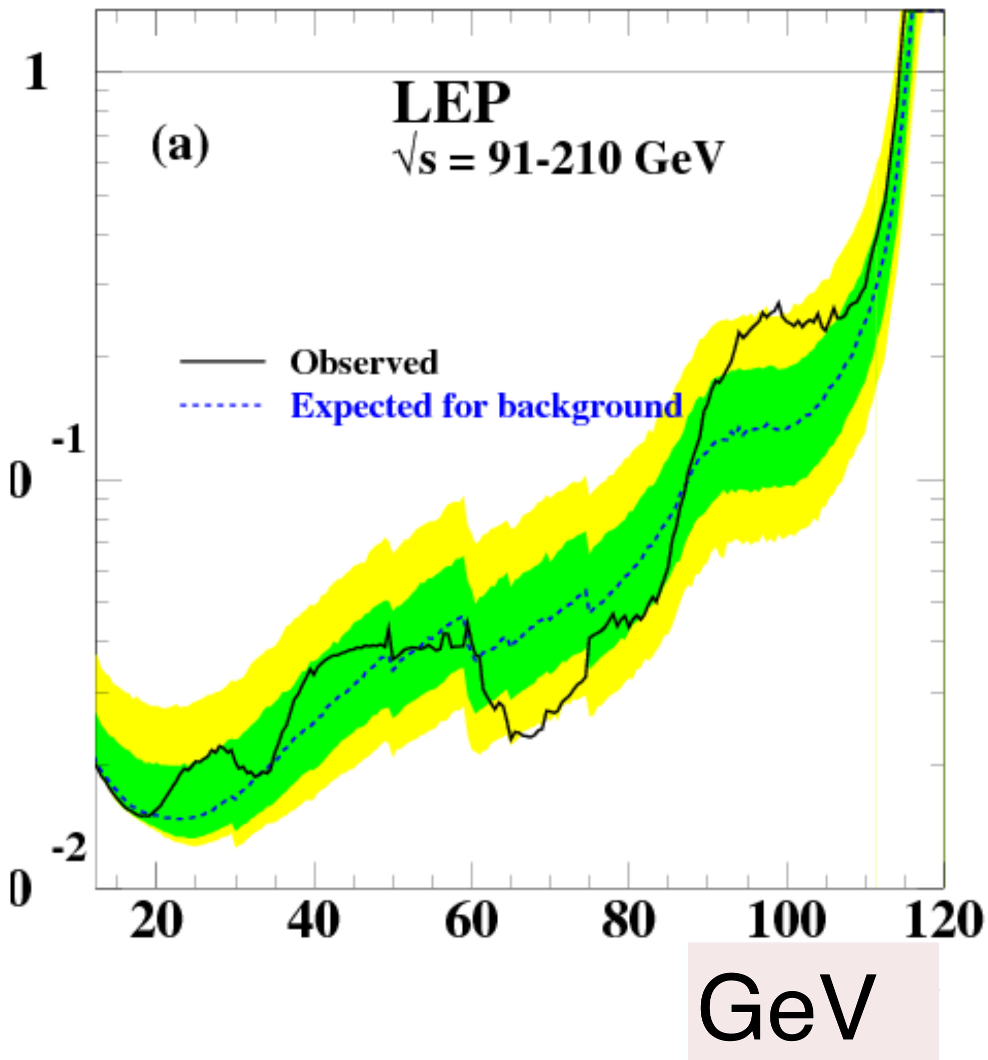


Data compatible with background-only-hypothesis
Observed Upper Limit on σ x BF : from 73 to 15 fb-1
Largest deviation M=**94.5** GeV w/ Local (Global) **2.9**(**1.3**)σ

ATLAS results not inconsistent with the CMS excess, arXiv:2306.03889

KOTLARSKI, BANK

Searches at LEP



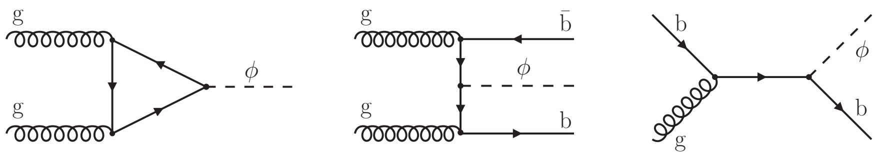
Excess at 96 GeV ?

S. Tkaczyk

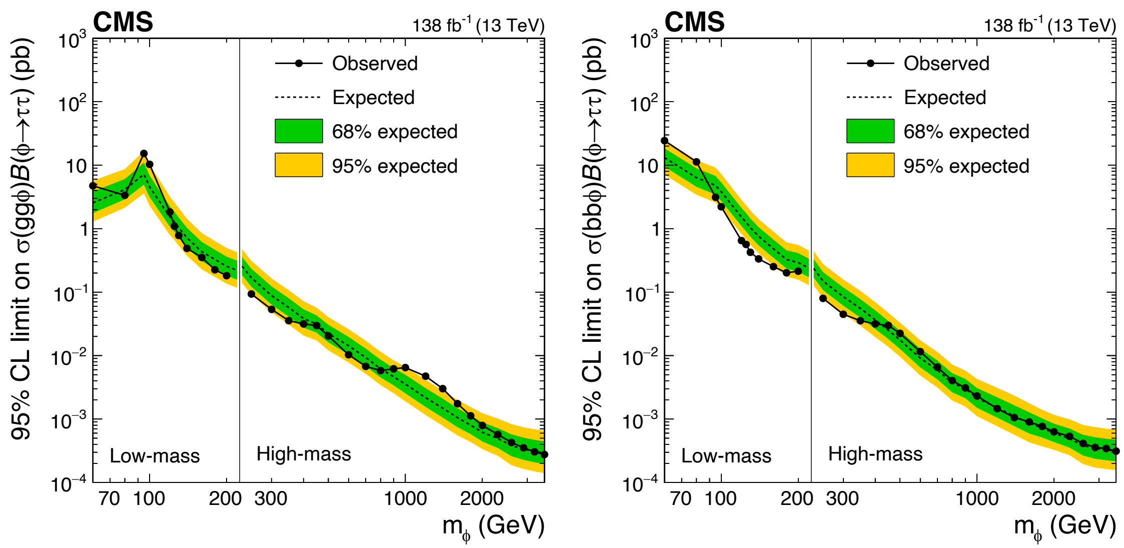
Search for neutral higgs ϕ

JHEP07(2023)073

Neutral higgs ϕ in ggF or in association with b-quark(s)



$\phi \rightarrow \tau\tau$ in lepton or hadron decays

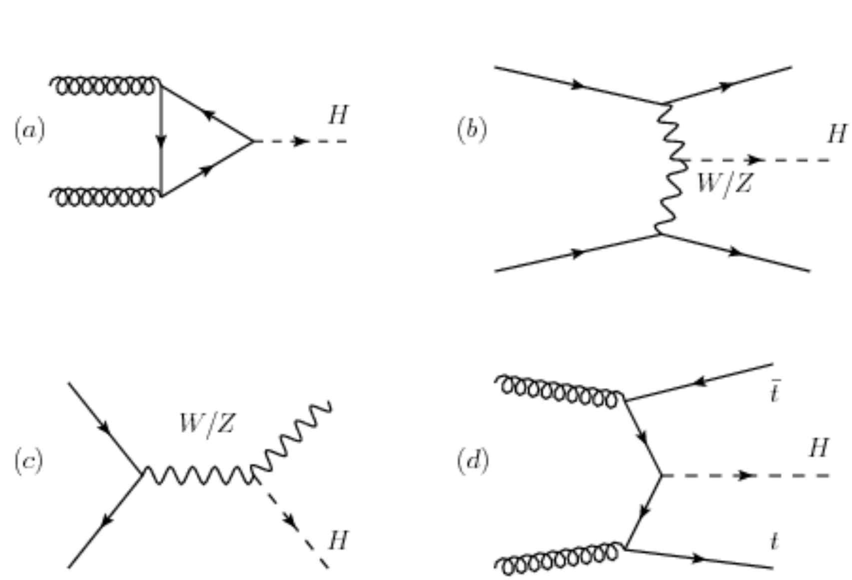


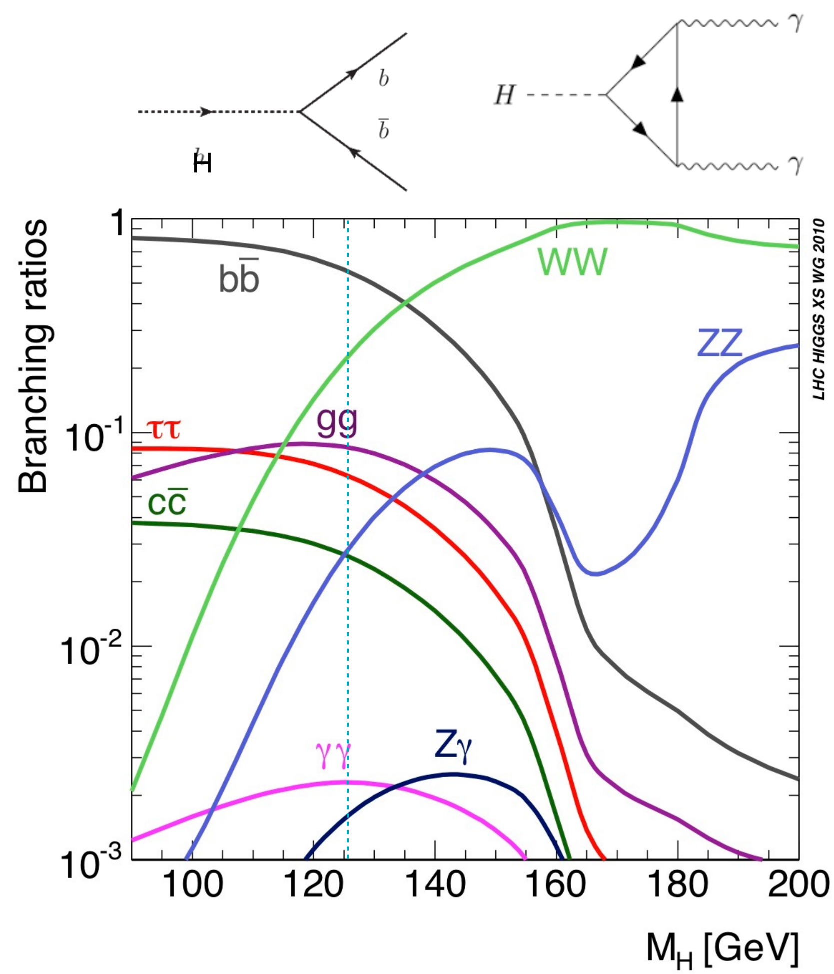
Limits set [60 - 3500 GeV] ranging from 10pb to 0.3fb
e.g. two excesses in ggϕ at 0.1 and 1.2 TeV with ~3σ

In MSSM scenarios M_h^{125} & $M_{h,\ EFT}^{125}$ additional Higgs bosons with masses below 350 GeV excluded

We collide two protons (quarks and gluons) at high energies :

LHC Higgs Production Channels and Decay Branching Ratios





A Higgs with a mass of about 125 GeV allows to study many decay channels

Why we should be surprised

- The Higgs potential suffers from a problem of stability under ultraviolet corrections, namely, given any sector that couples to the Higgs sector with gauge invariant masses, the Higgs mass parameter will be affected

$$\Delta m_H^2 \propto (-1)^{2S} \frac{k^2 N_g}{16\pi^2}\ m_{\rm new}^2$$

- These are physical corrections, regularization independent and shows that unless the new physics is lighter than the few TeV scale of very weakly coupled to the Higgs sector, the presence of a weak scale mass parameter is hard to understand.

- This is particularly true in models that try to connect the Higgs with the ultraviolet physics, like Grand Unified Theories.

- In such a case, we need a delicate cancellation of corrections, that only an extension like Supersymmetry can provide.

Relation between couplings in Higgs and general bases

$$\langle\Phi_1\rangle = \frac{1}{\sqrt{2}}\begin{pmatrix}0\\ v_1\end{pmatrix}\,,\quad \langle\Phi_2\rangle = \frac{1}{\sqrt{2}}\begin{pmatrix}0\\ v_2 e^{i\eta}\end{pmatrix}$$

δ=η

The opposite relation between quartic couplings in the Higgs basis and those in the weak basis can be obtained by changing β by -β

$$\lambda_1 = Z_1 c_\beta^4 + Z_2 s_\beta^4 + \frac{1}{2} Z_{345} s_{2\beta}^2 - 2 s_{2\beta}\left(\mathrm{Re}[Z_6 e^{i\delta}] c_\beta^2 + \mathrm{Re}[Z_7 e^{i\delta}] s_\beta^2\right)\,,$$

$$\lambda_2 = Z_1 s_\beta^4 + Z_2 c_\beta^4 + \frac{1}{2} Z_{345} s_{2\beta}^2 + 2 s_{2\beta}\left(\mathrm{Re}[Z_6 e^{i\delta}] s_\beta^2 + \mathrm{Re}[Z_7 e^{i\delta}] c_\beta^2\right)\,,$$

$$\lambda_3 = \frac{1}{4}\left(Z_1 + Z_2 - 2Z_{345}\right) s_{2\beta}^2 + Z_3 + \mathrm{Re}[(Z_6 - Z_7) e^{i\delta}] s_{2\beta} c_{2\beta}\,,$$

$$\lambda_4 = \frac{1}{4}\left(Z_1 + Z_2 - 2Z_{345}\right) s_{2\beta}^2 + Z_4 + \mathrm{Re}[(Z_6 - Z_7) e^{i\delta}] s_{2\beta} c_{2\beta}\,,$$

$$\begin{aligned}\lambda_5 e^{2i\delta} &= \frac{1}{4}(Z_1 + Z_2 - 2Z_{345}) s_{2\beta}^2 + \mathrm{Re}[Z_5 e^{2i\delta}] + i\,\mathrm{Im}[Z_5 e^{2i\delta}] c_{2\beta} \\ &\quad + \mathrm{Re}[(Z_6 - Z_7) e^{i\delta}] s_{2\beta} c_{2\beta} + i\,\mathrm{Im}[(Z_6 - Z_7) e^{i\delta}] s_{2\beta}\,,\end{aligned}$$

$$\begin{aligned}\lambda_6 e^{i\delta} &= \frac{1}{2}(Z_1 c_\beta^2 - Z_2 s_\beta^2 - Z_{345} c_{2\beta} - i\,\mathrm{Im}[Z_5 e^{2i\delta}]) s_{2\beta} \\ &\quad + \mathrm{Re}[Z_6 e^{i\delta}] c_\beta c_{3\beta} + i\,\mathrm{Im}[Z_6 e^{i\delta}] c_\beta^2 + \mathrm{Re}[Z_7 e^{i\delta}] s_\beta s_{3\beta} + i\,\mathrm{Im}[Z_7 e^{i\delta}] s_\beta^2\,,\end{aligned}$$

$$\begin{aligned}\lambda_7 e^{i\delta} &= \frac{1}{2}(Z_1 s_\beta^2 - Z_2 c_\beta^2 + Z_{345} c_{2\beta} + i\,\mathrm{Im}[Z_5 e^{2i\delta}]) s_{2\beta} \\ &\quad + \mathrm{Re}[Z_6 e^{i\delta}] s_\beta s_{3\beta} + i\,\mathrm{Im}[Z_6 e^{i\delta}] s_\beta^2 + \mathrm{Re}[Z_7 e^{i\delta}] c_\beta c_{3\beta} + i\,\mathrm{Im}[Z_7 e^{i\delta}] c_\beta^2\,,\end{aligned}$$