Precise Probe of the Higgs Potential via Gravitational Wave Observations

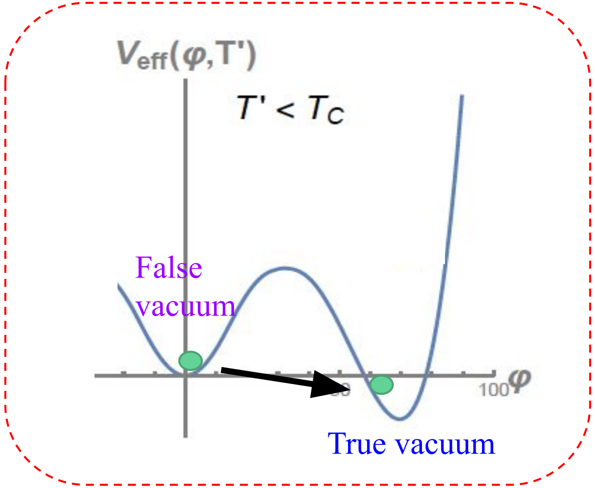
Katsuya Hashino (KOSEN/Fukushima Coll.)

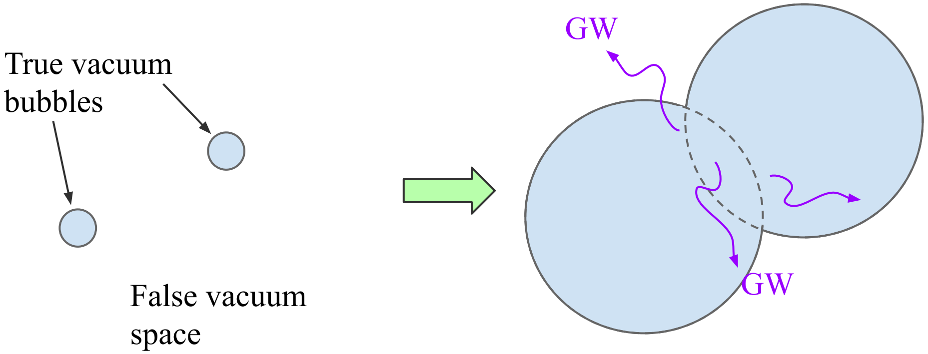
Collaborators: Daiki Ueda (Technion)

[K. H., Daiki Ueda, Phys.Rev.D 107 (2023) 9, 095022]
[K. H., Daiki Ueda, Ongoing work]

Introduction

★ The gravitational wave (GW) are produced by the first-order phase transition.





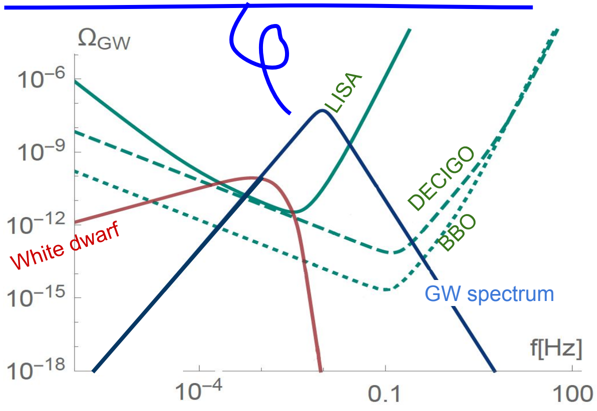
★ GW observations can be used to explore the new physics (NP) effects for the first-order phase transition.

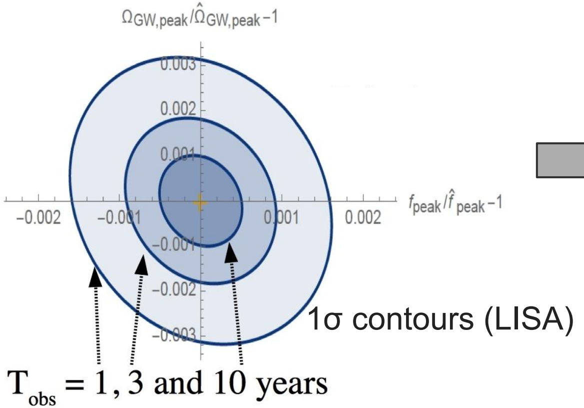
Introduction

★ We can quantitatively discuss how precisely obtain the NP parameter at the GW observation by the Fisher matrix analysis. [K. Hashino, R. Jinno, M. Kakizaki, S. Kanemura, T. Takahashi and M. Takimoto, PRD 99 (2019) no.7, 075011]

(The Fisher matrix corresponds to the inverse of the covariance matrix.)

$\hat{\Omega}_{\rm peak} = 10^{-7}, \hat{f}_{\rm peak} = 10^{-2} Hz$





$\hat{\Omega}_{\rm peak} = 10^{-7} \pm 3 \times 10^{-10},$

$\hat{f}_{\rm peak} = 10^{-2} \pm 1.5 \times 10^{-5} Hz$

LISA (1 year)

LISA, White dwarf : [A. Klein et al., Phys. Rev. D93 no. 2, (2016) 024003],
DECIGO, BBO : [K. Yagi and N. Seto, Phys. Rev. D83 (2011) 044011]

We can precisely measure the NP effects by GW observation.

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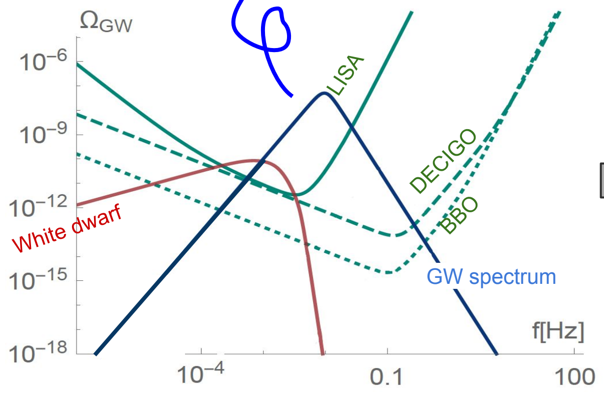

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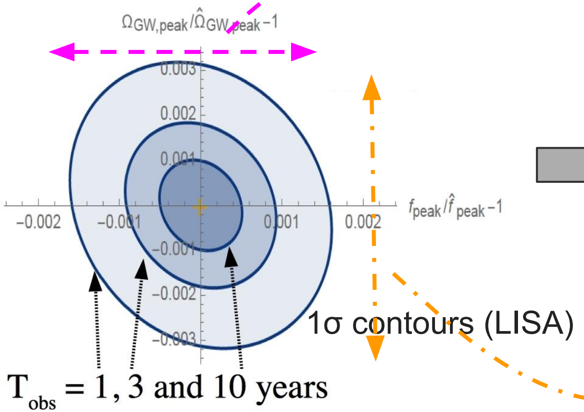

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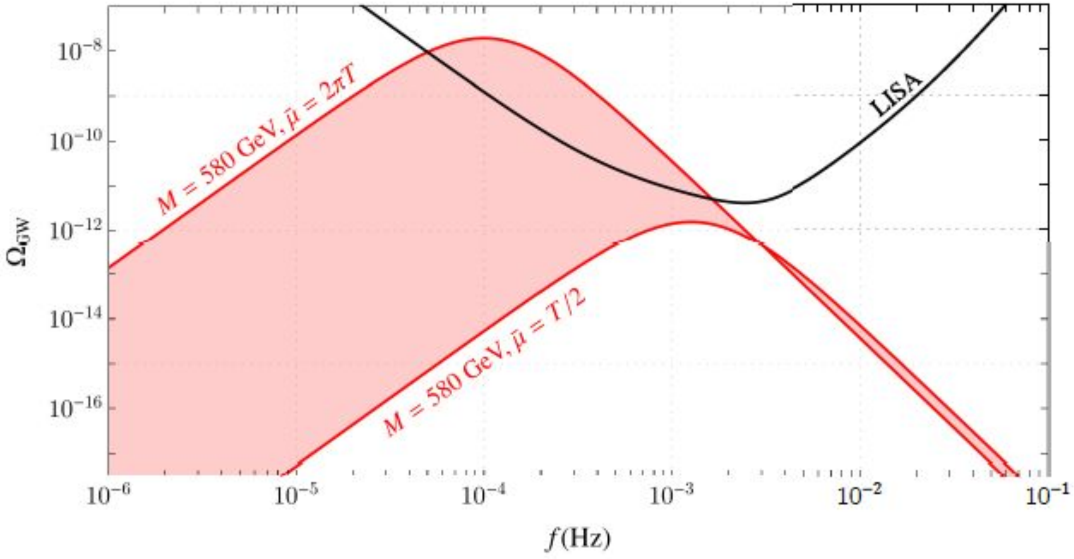
LISA (1 year)

The magnitude of 1σ confidence intervals

We can precisely measure the NP effects by GW observation.

Theoretical uncertainties

★ However, the GW spectra have some theoretical uncertainties...



(RG scale dependence is the largest theoretical uncertainty.)

[D. Croon, O. Gould, P. Schicho, T. V. I. Tenkanen and G. White,JHEP 04 (2021), 055]

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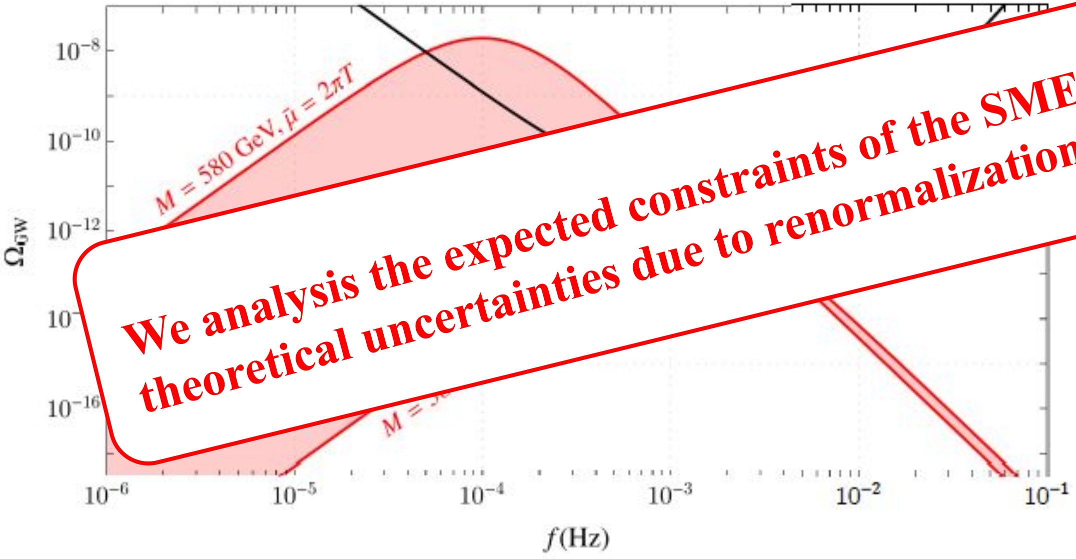

We analysis the expected constraints of the SMEFT with the theoretical uncertainties due to renormalization scale dependence.

is the

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[D. Croon, O. Gould, P. Schicho, T. V. I. Tenkanen and G. White,JHEP 04 (2021), 055]

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SMEFT

★ The Lagrangian of the SMEFT [B. Grzadkowski, M. Iskrzynski, M. Misiak, and J. Rosiek, JHEP 10 (2010) 085]

$$\mathcal{L}_{\mathrm{SMEFT}} = \mathcal{L}_{\mathrm{SM}} + \sum_i C_i \mathcal{O}_i$$

$$\mathcal{O}_H = (H^\dagger H)^3,$$

$$\mathcal{O}_{H\Box} = (H^\dagger H)\Box(H^\dagger H),$$

$$\mathcal{O}_{HD} = (H^\dagger D^\mu H)^*(H^\dagger D_\mu H),$$

$$(\mathcal{O}_{uH})_{ij} = (H^\dagger H)(\bar{q}_i u_j \tilde{H}),$$

In this time, we consider the SMEFT operators involving Higgs and top-quarks.

$$\sqrt{2}H^T = (0, \varphi)$$

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$$\sqrt{2}H^T = (0, \varphi)$$

$$\Delta\mathcal{L}_{\rm SMEFT} = \frac{1}{8} C_H \varphi^6$$

It is a dominant effects to realize the first-order phase transition.

[C. Grojean, G. Servant, and J. D. Wells, Phys. Rev. D 71 (2005) 036001, D. Bodeker, L. Fromme, S. J. Huber, and M. Seniuch, JHEP 02 (2005) 026 and so on.]

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$$\Delta\mathcal{L}_{\mathrm{SMEFT}} = \frac{1}{4}C_{HD}\varphi^2(\partial_\mu\varphi)^2 - C_{H\Box}\varphi^2(\partial_\mu\varphi)^2$$

These effects contribute to the Higgs potential by the wave function renormalization.

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In this time, we consider the SMEFT operators involving Higgs and top-quarks.

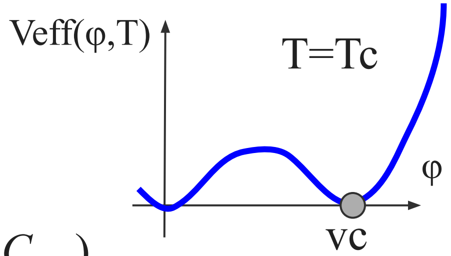
$$\sqrt{2}H^T = (0, \varphi)$$

$$\Delta\mathcal{L}_{\rm SMEFT} = C_{uH} \cdot \frac{3}{32\pi^2} Y_t \left(14 - 6\ln\frac{m_t^2}{v^2}\right) \cdot \frac{1}{2}\varphi^2(\partial_\mu\varphi)^2 - \Delta V_{c_{uH}}$$

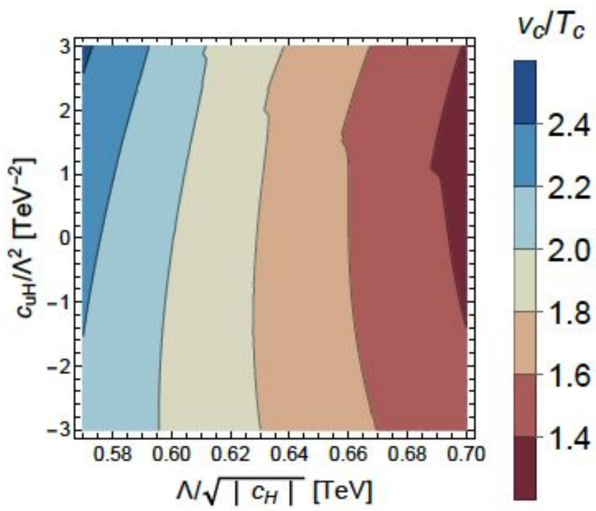
$$\Delta V_{c_{uH}} = -C_{uH} \cdot \frac{3}{32\pi^2} Y_t^3 \varphi^6 \left(-1 + \ln\frac{Y_t^2\varphi^2}{2v^2}\right)$$

It contributes to the Higgs potential by the top-quark one-loop effects.

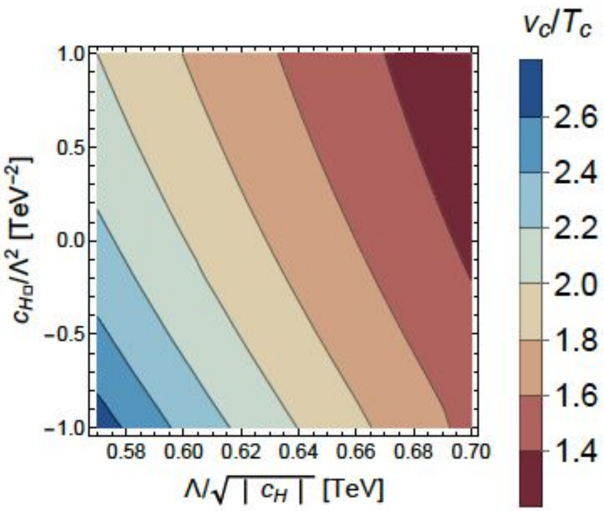
First-order EWPT



★ The results of vc/Tc for normalized (C_H, C_{uH}), (C_H, $C_{H\Box}$) and (C_H, C_{HD}).



$c_{H\Box} = c_{HD} = 0.$



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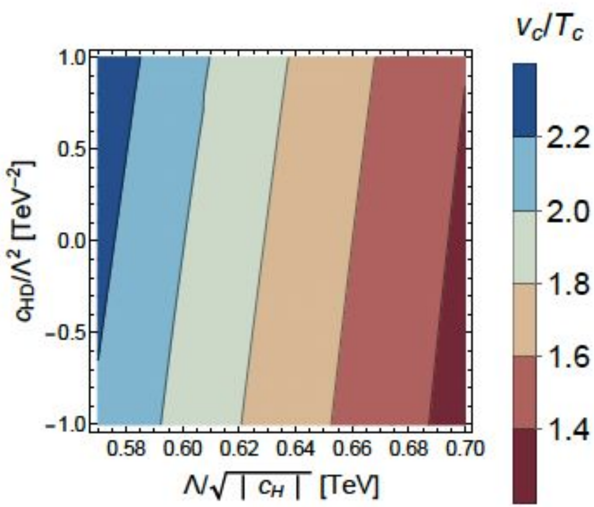


$c_{uH} = c_{H\Box} = 0$

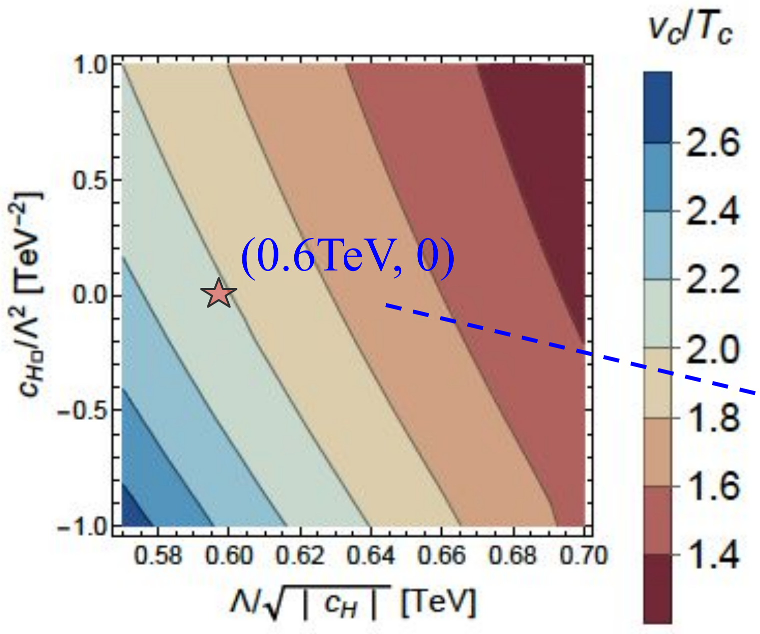
C_{uH} is insensitive as a source for the first-order EWPT.

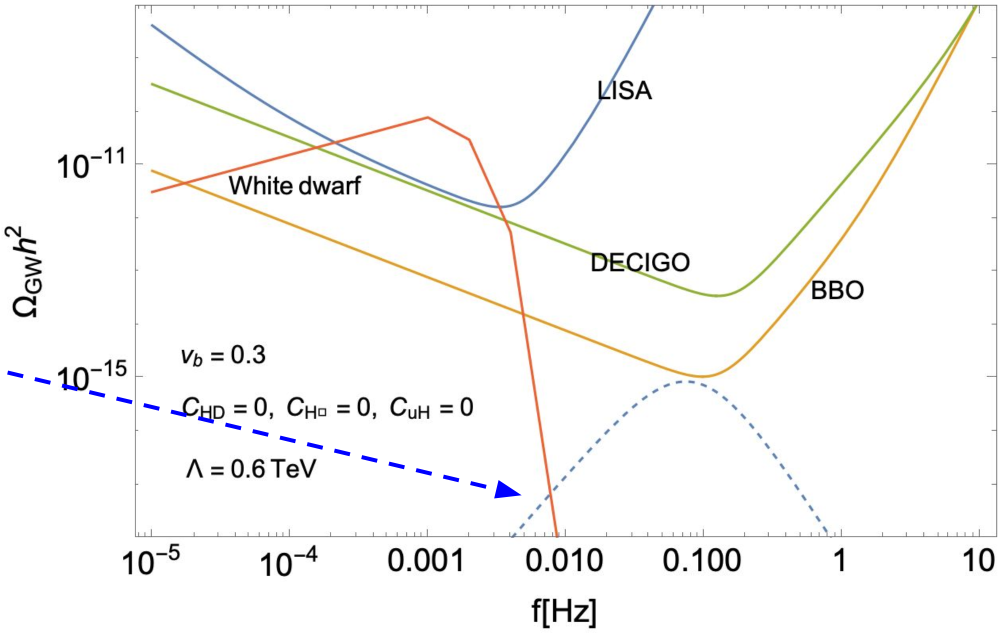
$C_{H\Box}$ and C_{HD} could be a source for the first-order EWPT.

[K. H., Daiki Ueda, Phys.Rev.D 107 (2023) 9, 095022]

First-order EWPT

★ Example of the expected uncertainty for $(C_H, C_{H\Box})$ at the GW observation





$C_H = 0.6$ TeV, $C_{H\Box} = 0$

First-order EWPT

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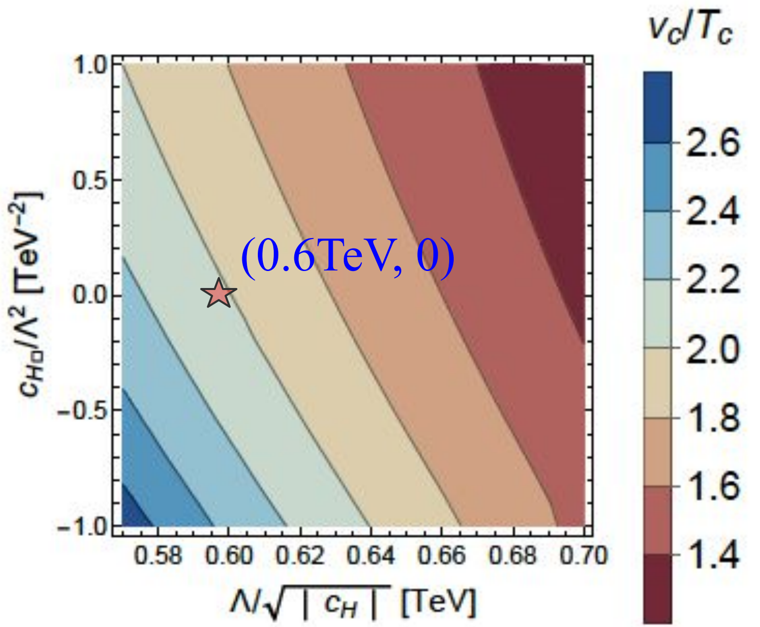
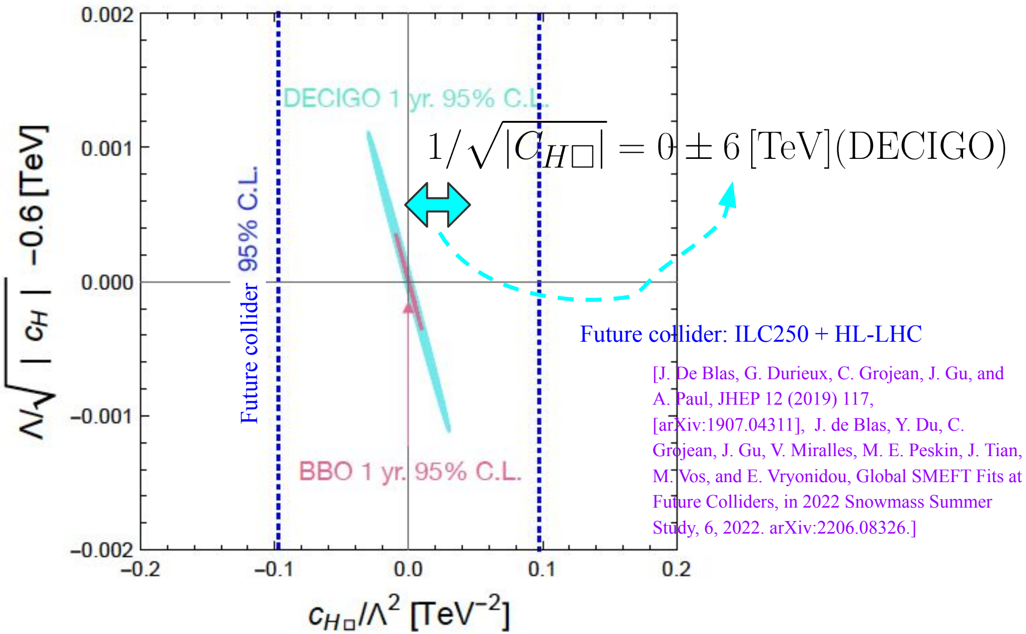


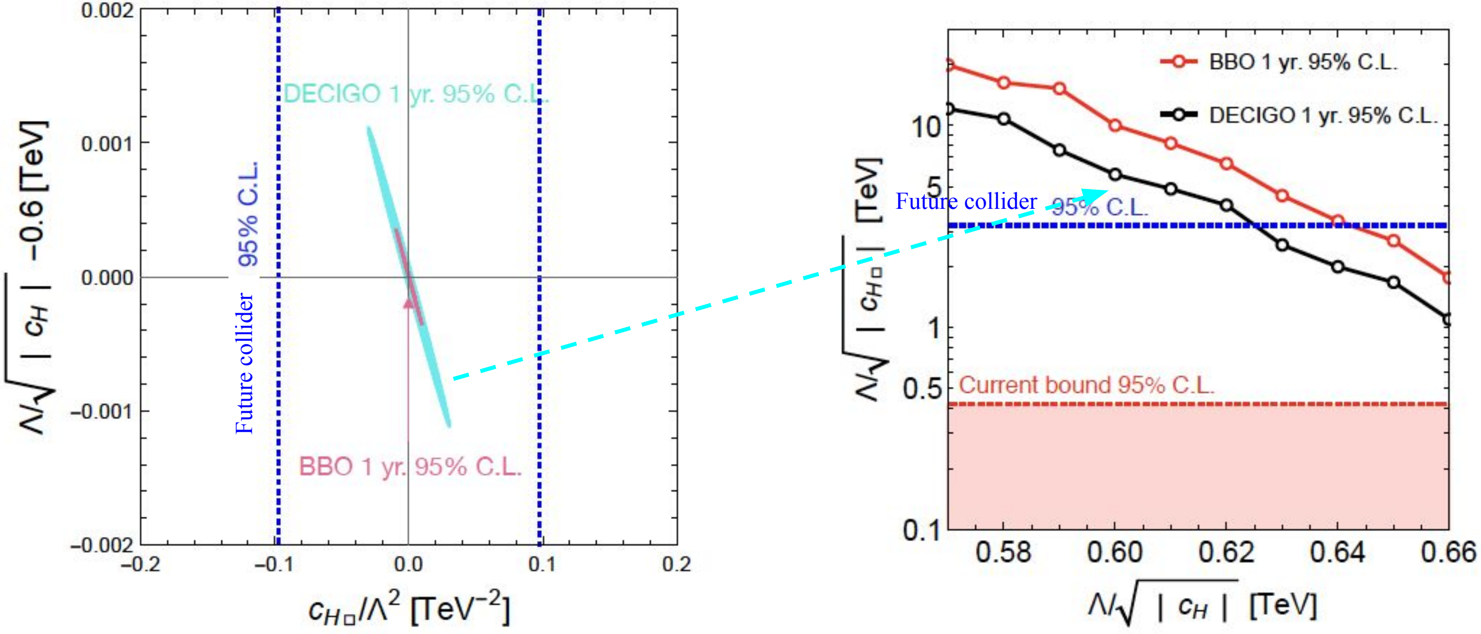


95% C.L. confidence regions for DECIGO and BBO with 1-year statistics, assuming the central values of the benchmark point.

[K. H., Daiki Ueda, Phys.Rev.D 107 (2023) 9, 095022]

Results of the last work

★ Example of the expected constraints for $(C_H, C_{H\Box})$ at the GW observation

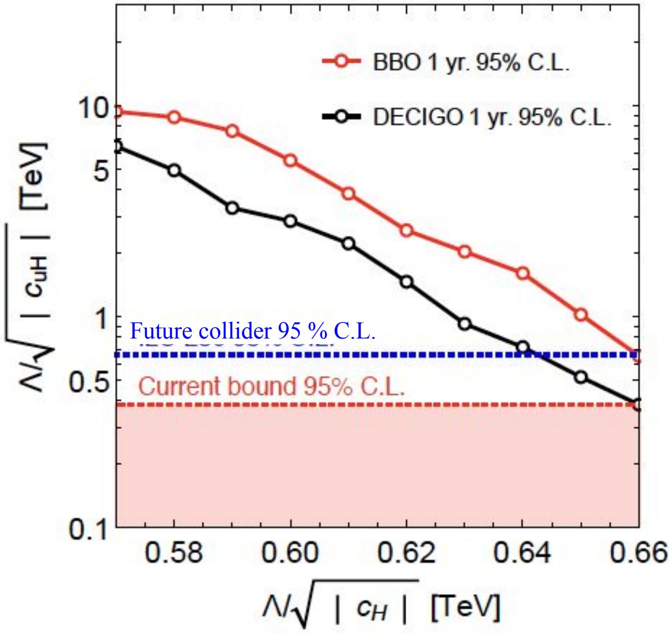


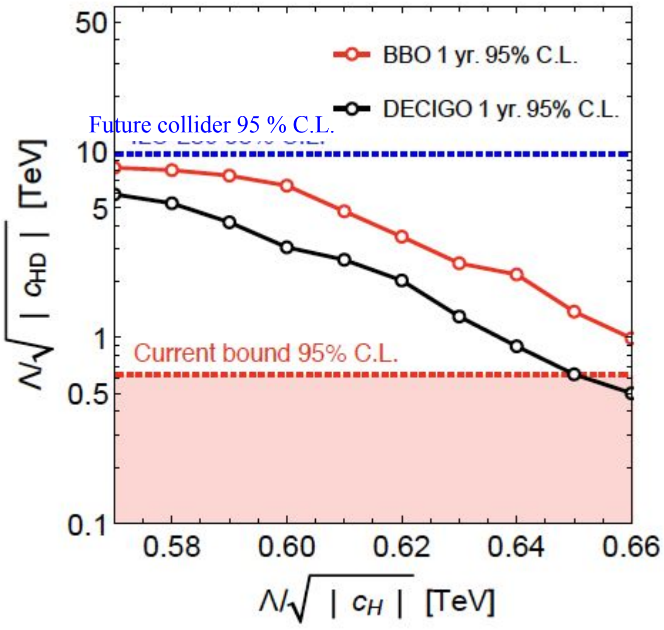
The magnitude of 95% C.L. confidence intervals of vertical axis ($C_{H\Box}$).

[K. H., Daiki Ueda, Phys.Rev.D 107 (2023) 9, 095022]

Results of the last work

★ Expected constraints for other effects at the GW observation





DECIGO and the BBO experiments can be sensitive to the SMEFT effects, such as C_{uH} and $C_{H\Box}$, once the SFO-EWPT arises.

[K. H., Daiki Ueda, Phys.Rev.D 107 (2023) 9, 095022]

Results with theoretical uncertainty

★ Results of C_{uH}

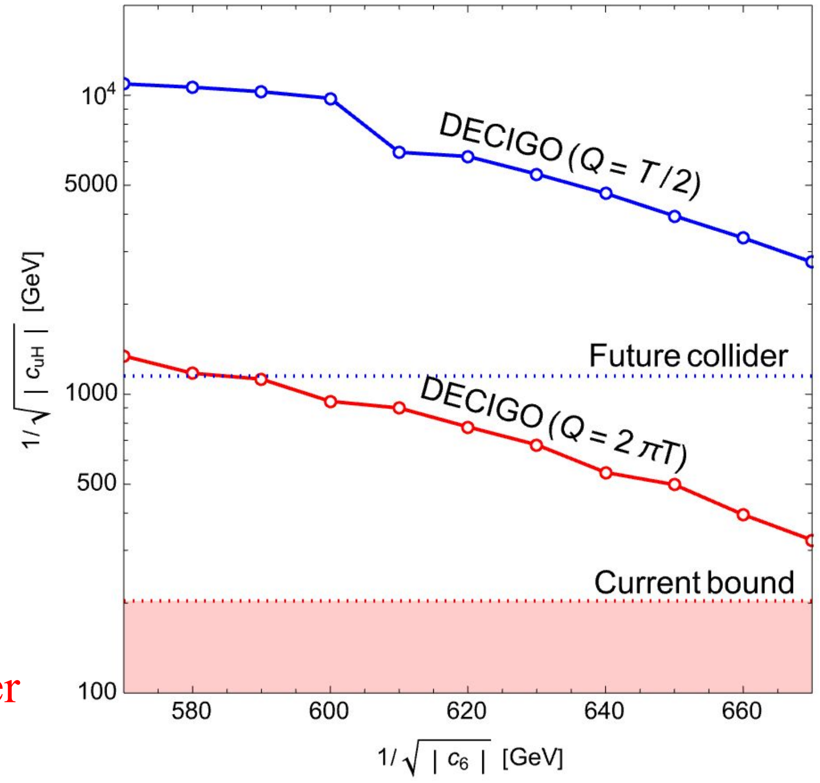
Blue line: Q = T/2

Red line: Q = 2πT

[K. H., Daiki Ueda, Ongoing work]

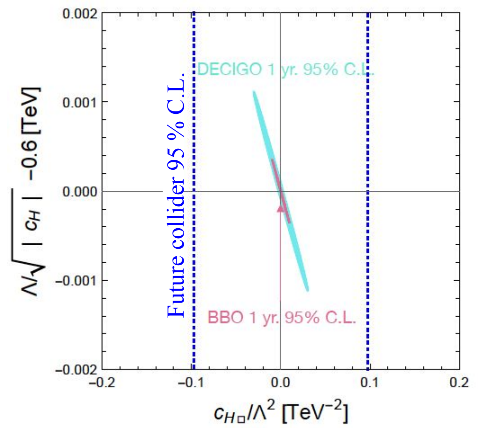
★ Although the uncertainties show up in the GW spectra, we may be able to obtain the precise value of NP parameter.

We are studying the expected constraints of other operators with theoretical uncertainty.

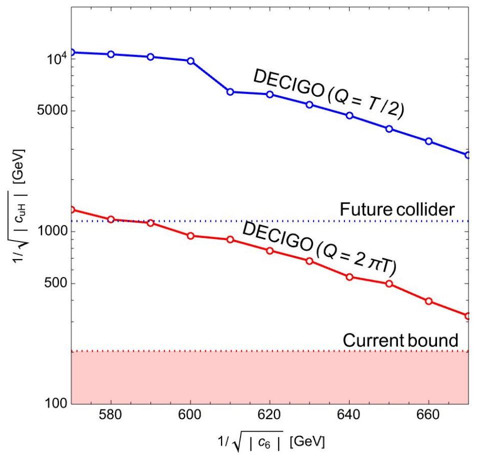


Summary

★ In this time, we consider the SMEFT and discuss the dimension-six operator effects on the spectrum of GWs.



★ DECIGO and the BBO experiments may be sensitive to the SMEFT effects, once the SFO-EWPT arises.

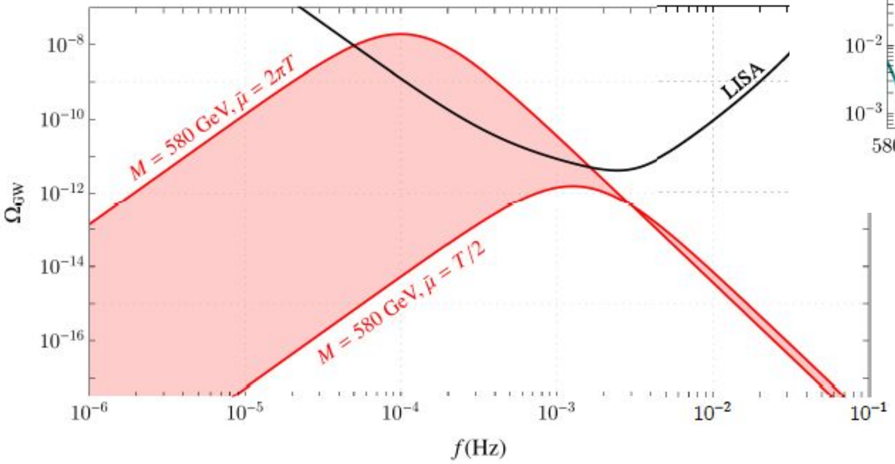


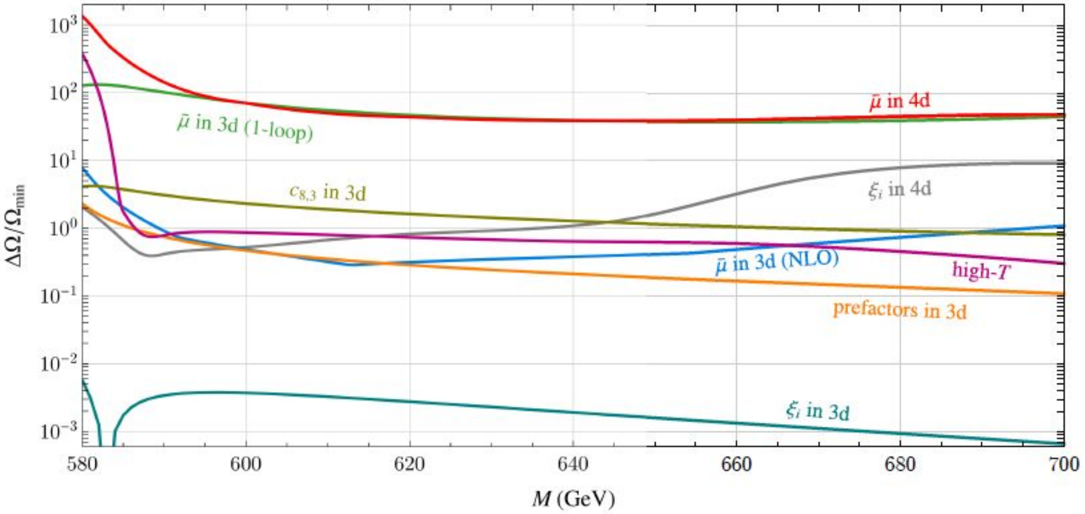
★ Although the theoretical uncertainties show up in the potential, we may be able to obtain the precise value of parameter at the GW observation.

Backup

Theoretical uncertainties

$\Delta\Omega_{GW}/\Omega_{GW}$	4d approach	3d approach
RG scale dependence	$\mathcal{O}(10^2 - 10^3)$	$\mathcal{O}(10^0 - 10^1)$
Gauge dependence	$\mathcal{O}(10^1)$	$\mathcal{O}(10^{-3})$
High-T approximation	$\mathcal{O}(10^{-1} - 10^0)$	$\mathcal{O}(10^0 - 10^2)$
Higher loop orders	unknown	$\mathcal{O}(10^0 - 10^1)$
Nucleation corrections	unknown	$\mathcal{O}(10^{-1} - 10^0)$
Nonperturbative corrections	unknown	unknown





[D. Croon, O. Gould, P. Schicho, T. V. I. Tenkanen and G. White,JHEP 04 (2021), 055]

Fisher matrix analysis

❖ Likelihood function

$$\delta\chi^2(\{p\},\{\hat{p}\}) = 2T_{\rm obs}\sum_{(I,I')}\int_0^\infty df\,\frac{\Gamma^2_{II'}(f)\left[S_h(f,\{p\})-S_h(f,\{\hat{p}\})\right]^2}{\sigma^2_{II'}(f)}.$$

$$S_h(f) = \frac{3H_0^2}{2\pi^2}\frac{1}{f^3}\Omega_{\rm GW}(f)$$

Noise for detection of GW : $\sigma^2_{II'}(f) = [S_I(f)+\Gamma_{II}(f)S_h(f,\{\hat{p}\})]\,[S_{I'}(f)+\Gamma_{I'I'}(f)S_h(f,\{\hat{p}\})]+\Gamma^2_{II'}(f)S_h^2(f,\{\hat{p}\})$

S_h (f, {p}) : GW spectrum for parameter set {p},
S_{eff} : Effective sensitivity of interferometers,
$\{\hat{p}\}$: Fiducial parameter set, T_{obs} : Observation period

$\Gamma_{II'}$ is the overlap reduction function.

Taylor expansion $\delta\chi^2(\{p\},\{\hat{p}\}) \simeq \mathcal{F}_{ab}(p_a-\hat{p}_a)(p_b-\hat{p}_b)$

$$\mathcal{F}_{ab} = 2T_{\rm obs}\sum_{(I,I')}\int_0^\infty df\,\frac{\Gamma^2_{II'}(f)\partial_{p_a}S_h(f,\{\hat{p}\})\partial_{p_b}S_h(f,\{\hat{p}\})}{\sigma^2_{II'}(f)}$$

[N. Seto, Phys. Rev. D 73, 063001 (2006)]

$$S_{\rm eff}(f) = \left[\sum_{(I,I')}\frac{\Gamma^2_{II'}(f)}{\sigma^{({\rm null})2}_{II'}(f)}\right]^{-1/2}$$

[E. Thrane and J. D. Romano, Phys. Rev. D 88, no. 12, 124032 (2013)]

Fisher matrix analysis

❖ Likelihood function

$$\delta\chi^2(\{p\},\{\hat{p}\}) = 2T_{\rm obs}\int_0^\infty df\,\frac{[S_h(f,\{p\}) - S_h(f,\{\hat{p}\})]^2}{[S_{\rm eff}(f) + S_h(f,\{\hat{p}\})]^2}$$

Taylor expansion

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$S_{\rm eff}$: Effective sensitivity of interferometers,
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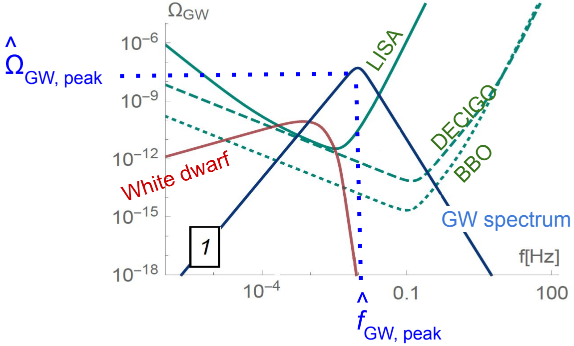
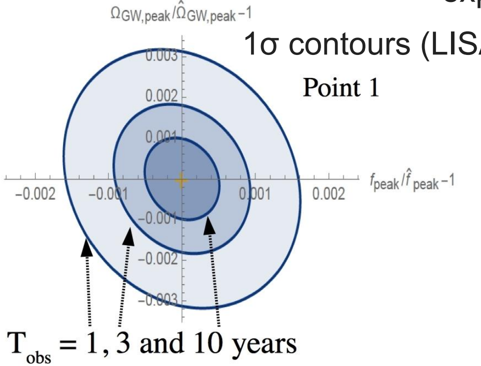
$$\mathcal{F}_{ab} = 2T_{\rm obs}\int_0^\infty df\,\frac{\partial_{p_a}S_h(f,\{\hat{p}\})\partial_{p_b}S_h(f,\{\hat{p}\})}{[S_{\rm eff}(f) + S_h(f,\{\hat{p}\})]^2}$$

The inverse matrix of F_{ab} is the covariance matrix.

(We assume that we can apply the expression to one detector (LISA).)

❖ Sample point





LISA, White dwarf : [A. Klein et al., Phys. Rev. D93 no. 2, (2016) 024003]

DECIGO, BBO : [K. Yagi and N. Seto, Phys. Rev. D83 (2011) 044011]

❖ The expected constraints on the GW spectrum propagate to the parameters in the model.

Fisher matrix analysis

❖ Effective sensitivity

- LISA

$$S_{\rm eff}(f) = \frac{20}{3}\frac{4S_{\rm acc}(f) + S_{\rm sn}(f) + S_{\rm omn}(f)}{L^2}\left[1 + \left(\frac{f}{0.41c/2L}\right)^2\right],$$

with $L = 5 \times 10^9$ m and

$$S_{\rm acc}(f) = 9 \times 10^{-30}\frac{1}{(2\pi f/1{\rm Hz})^4}\left(1 + \frac{10^{-4}}{f/1{\rm Hz}}\right)\ {\rm m^2Hz^{-1}},$$
$$S_{\rm sn}(f) = 2.96 \times 10^{-23}\ {\rm m^2Hz^{-1}},$$
$$S_{\rm omn}(f) = 2.65 \times 10^{-23}\ {\rm m^2Hz^{-1}}.$$

[A. Klein et al., Phys. Rev. D93 no. 2, (2016) 024003]

- DECIGO

$$S_{\rm eff}(f) = \left[7.05 \times 10^{-48}\left[1 + (f/f_p)^2\right] + 4.8 \times 10^{-51}\frac{(f/1{\rm Hz})^{-4}}{1 + (f/f_p)^2} + 5.33 \times 10^{-52}(f/1{\rm Hz})^{-4}\right]\ {\rm Hz^{-1}},$$

with $f_p = 7.36$ Hz.

- BBO

$$S_{\rm eff}(f) = \left[2.00 \times 10^{-49}(f/1{\rm Hz})^2 + 4.58 \times 10^{-49} + 1.26 \times 10^{-52}(f/1{\rm Hz})^{-4}\right]\ {\rm Hz^{-1}}.$$

[K. Yagi and N. Seto, Phys. Rev. D83 (2011) 044011]

Fisher matrix analysis

- Noise for the white dwarf [A. Klein et al., Phys. Rev. D93 no. 2, (2016) 024003]

$$S'_{\rm WD}(f) = \begin{cases} (20/3)(f/1\ {\rm Hz})^{-2.3} \times 10^{-44.62}\ {\rm Hz}^{-1} & \equiv S^{(1)}_{\rm WD}(f) \quad (10^{-5}\ {\rm Hz} < f < 10^{-3}\ {\rm Hz}), \\ (20/3)(f/1\ {\rm Hz})^{-4.4} \times 10^{-50.92}\ {\rm Hz}^{-1} & \equiv S^{(2)}_{\rm WD}(f) \quad (10^{-3}\ {\rm Hz} < f < 10^{-2.7}\ {\rm Hz}), \\ (20/3)(f/1\ {\rm Hz})^{-8.8} \times 10^{-62.8}\ {\rm Hz}^{-1} & \equiv S^{(3)}_{\rm WD}(f) \quad (10^{-2.7}\ {\rm Hz} < f < 10^{-2.4}\ {\rm Hz}), \\ (20/3)(f/1\ {\rm Hz})^{-20.0} \times 10^{-89.68}\ {\rm Hz}^{-1} & \equiv S^{(4)}_{\rm WD}(f) \quad (10^{-2.4}\ {\rm Hz} < f < 10^{-2}\ {\rm Hz}). \end{cases}$$

$$S_{\rm WD}(f) = \frac{1}{1/S^{(1)}_{\rm WD}(f) + 1/S^{(2)}_{\rm WD}(f) + 1/S^{(3)}_{\rm WD}(f) + 1/S^{(4)}_{\rm WD}(f)}$$

$$S_{\rm WD} \simeq \max(S^{(1)}_{\rm WD}, S^{(2)}_{\rm WD}, S^{(3)}_{\rm WD}, S^{(4)}_{\rm WD})$$

Fisher matrix analysis

❖ Noise for binary neutron stars and binary black holes (Stochastic GW)

[Virgo, LIGO Scientific Collaboration, B. P. Abbott et al., arXiv:1710.05837 [gr-qc]]

$$S_{\rm NSBH}(f) = \frac{3H_0^2}{2\pi^2}\frac{1}{f^3} \times 1.8 \times 10^{-8} \left(\frac{f}{25\ {\rm Hz}}\right)^{\frac{2}{3}} \qquad 10\ {\rm Hz} < f < 10^3\ {\rm Hz}$$

$\longrightarrow$ This noise does not affect our analysis, because frequency of the noise is small.

(We tried to extrapolate the noise to 1 Hz, but our result doesn't change.)

SMEFT

★ The Higgs potential up to the first order of the Wilson coefficients

$$V = \frac{1}{2}\mu^2\varphi^2 + \frac{1}{4}\left(\lambda - \frac{4}{3}c_{\text{kin}}^{(0)}\mu^2\right)\varphi^4 - \frac{1}{4}c_{\text{kin}}^{(1)}\mu^2\varphi^5 + \frac{1}{6}\left(-\frac{3}{4}C_H - 2c_{\text{kin}}^{(0)}\lambda - \frac{6}{5}c_{\text{kin}}^{(2)}\mu^2\right)\varphi^6 + \Delta V_{c_{uH}}$$

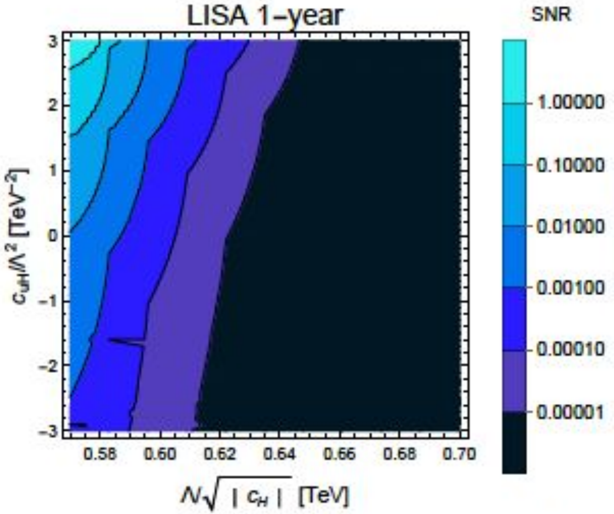
$$c_{\text{kin}}^{(0)} = \frac{1}{4}C_{HD} - C_{H\square} + \frac{1}{2}C_{uH}\cdot\frac{3}{32\pi^2}Y_t\left(14 - 6\ln\frac{m_t^2}{v^2}\right),$$

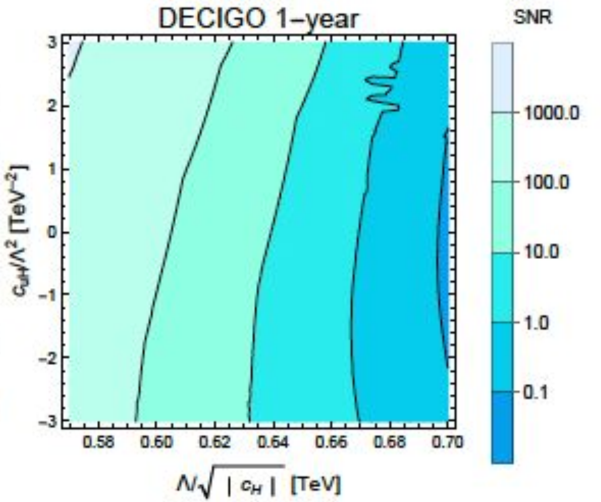
$$c_{\text{kin}}^{(1)} = \frac{1}{2}C_{uH}\cdot\frac{3}{32\pi^2}Y_t\left(-\frac{28}{v}\right),$$

$$c_{\text{kin}}^{(2)} = \frac{1}{2}C_{uH}\cdot\frac{3}{32\pi^2}Y_t\left(\frac{8}{v^2}\right).$$

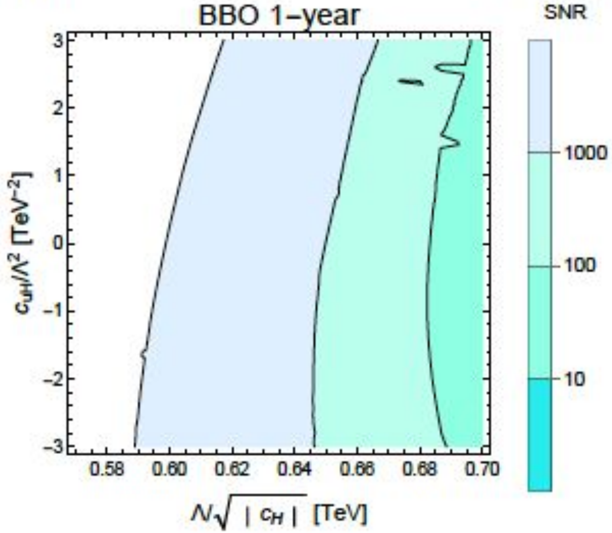
$$\Delta V_{c_{uH}} = -C_{uH}\cdot\frac{3}{32\pi^2}Y_t^3\varphi^6\left(-1 + \ln\frac{Y_t^2\varphi^2}{2v^2}\right)$$

★ SNR results with respect to CH and CuH

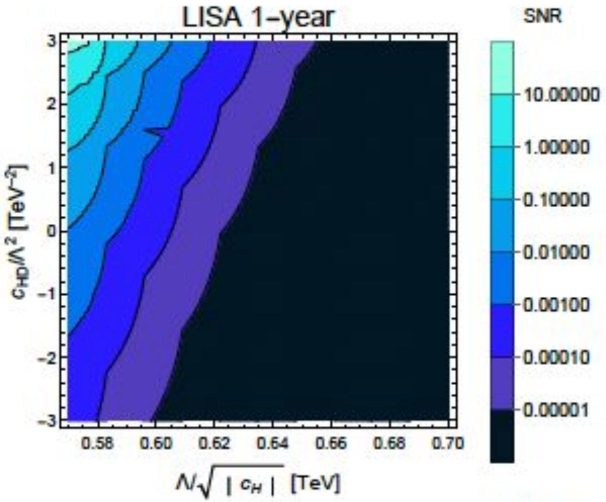


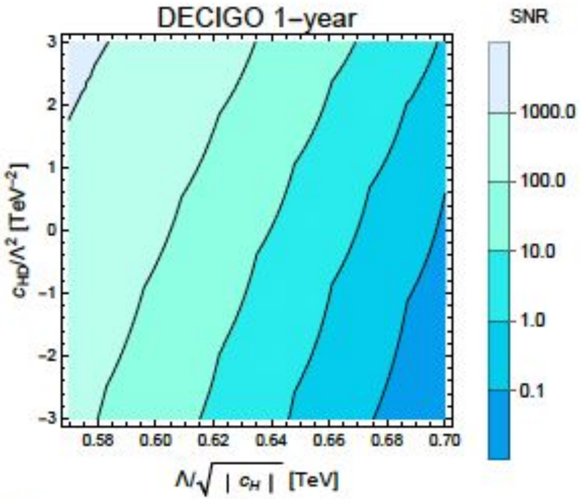


$$\mathrm{SNR} = \sqrt{\delta \times T_{\rm obs} \int_0^\infty df \left[\frac{\Omega_{\rm GW}(f)}{\Omega_{\rm sen}(f)}\right]^2}$$

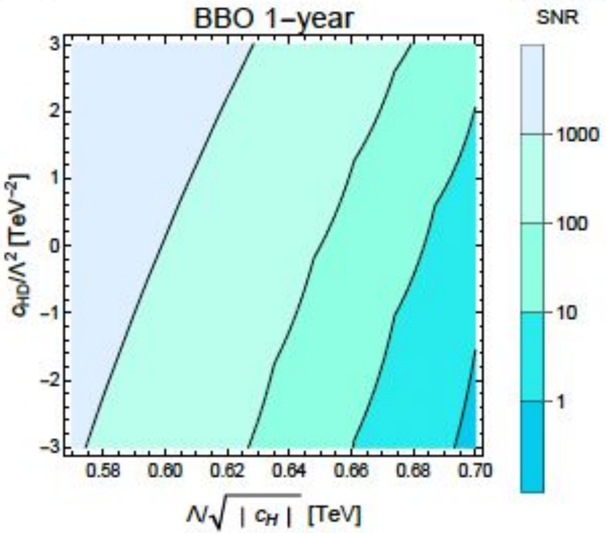


★ SNR results with respect to CH and CHD

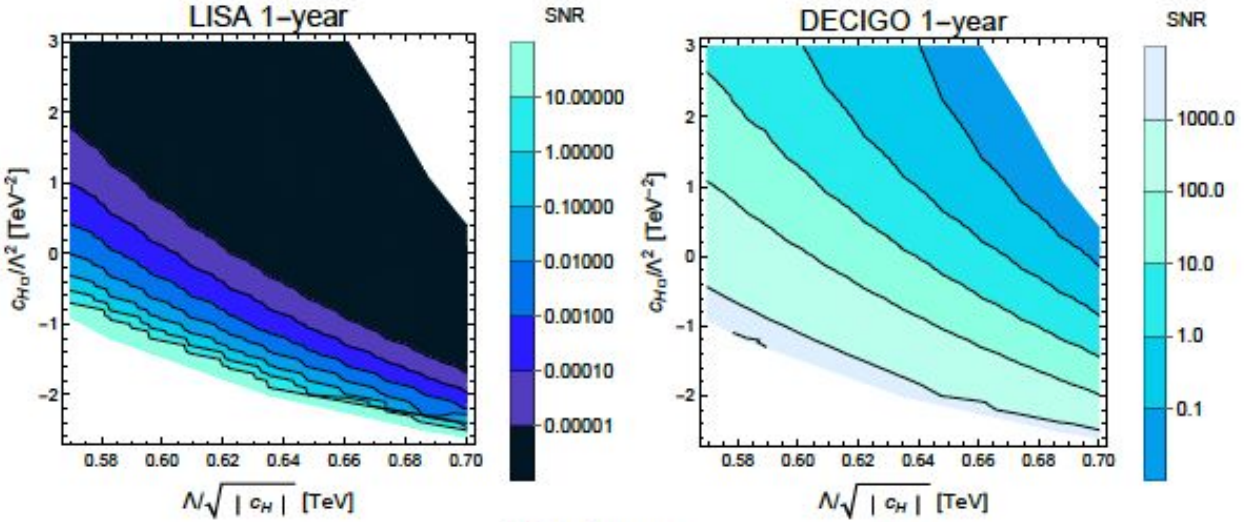




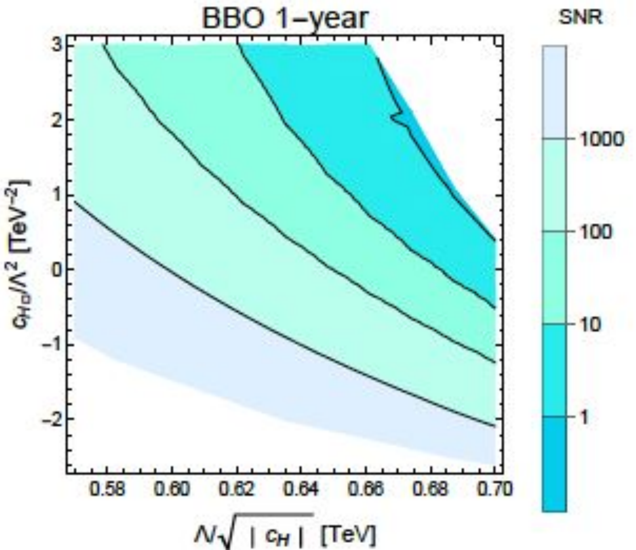
$$\mathrm{SNR} = \sqrt{\delta \times T_{\mathrm{obs}} \int_0^\infty df \left[\frac{\Omega_{\mathrm{GW}}(f)}{\Omega_{\mathrm{sen}}(f)}\right]^2}$$



★ SNR results with respect to CH and CH□

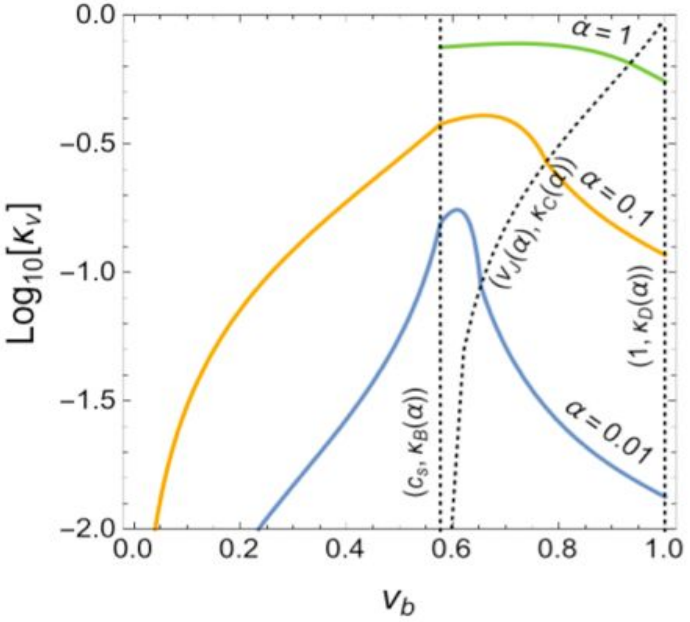


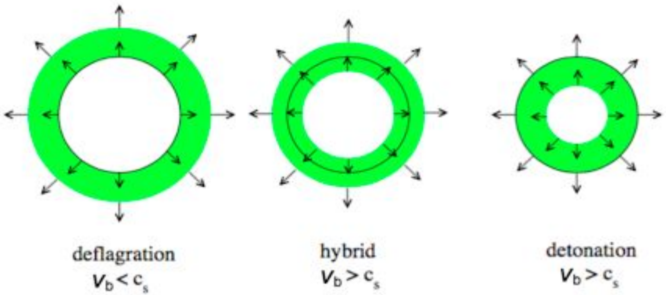
$$\text{SNR} = \sqrt{\delta \times T_{\text{obs}} \int_0^\infty df \left[\frac{\Omega_{\text{GW}}(f)}{\Omega_{\text{sen}}(f)}\right]^2}$$



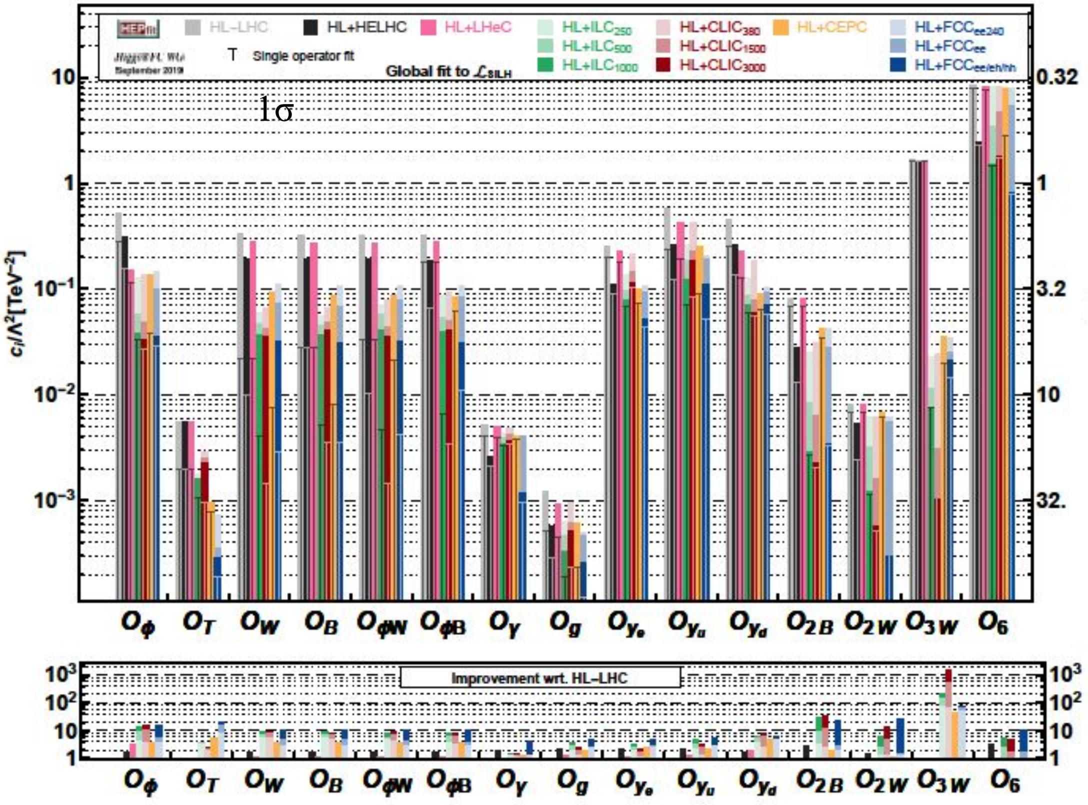
Efficiency factors

[J. R. Espinosa, T. Konstandin, J. M. No and G. Servant, JCAP 1006, 028 (2010)]





The black circle is bubble wall.
In green we show the region of non-zero fluid velocity.



J. de Blas, M. Cepeda, J. D'Hondt, R. K. Ellis, C. Grojean, B. Heinemann, F. Maltoni, A. Nisati, E. Petit and R. Rattazzi, et al., JHEP 01 (2020), 139 [arXiv:1905.03764 [hep-ph]].

$$
\begin{aligned}
\mathscr{L}_{\rm SILH} = & \frac{c_\phi}{\Lambda^2}\frac{1}{2}\partial_\mu(\phi^\dagger\phi)\partial^\mu(\phi^\dagger\phi) + \frac{c_T}{\Lambda^2}\frac{1}{2}(\phi^\dagger \overleftrightarrow{D}_\mu \phi)(\phi^\dagger \overleftrightarrow{D}^\mu \phi) - \frac{c_6}{\Lambda^2}\lambda(\phi^\dagger\phi)^3 + \left(\frac{c_{y_f}}{\Lambda^2} y^f_{ij}\phi^\dagger\phi\bar{\psi}_{Li}\phi\psi_{Rj} + \text{h.c.}\right) \\
& + \frac{c_W}{\Lambda^2}\frac{ig}{2}\left(\phi^\dagger \overleftrightarrow{D}^a_\mu \phi\right) D_\nu W^{a\,\mu\nu} + \frac{c_B}{\Lambda^2}\frac{ig'}{2}\left(\phi^\dagger \overleftrightarrow{D}_\mu \phi\right)\partial_\nu B^{\mu\nu} + \frac{c_{\phi W}}{\Lambda^2} ig D_\mu\phi^\dagger\sigma_a D_\nu\phi W^{a\,\mu\nu} + \frac{c_{\phi B}}{\Lambda^2} ig' D_\mu\phi^\dagger\sigma_a D_\nu\phi B^{\mu\nu} \\
& + \frac{c_\gamma}{\Lambda^2} g'^2 \phi^\dagger\phi B^{\mu\nu}B_{\mu\nu} + \frac{c_g}{\Lambda^2} g_s^2 \phi^\dagger\phi G^{A\,\mu\nu}G^A_{\mu\nu} \\
& - \frac{c_{2W}}{\Lambda^2}\frac{g^2}{2}(D^\mu W^a_{\mu\nu})(D_\rho W^{a\,\rho\nu}) - \frac{c_{2B}}{\Lambda^2}\frac{g'^2}{2}(\partial^\mu B_{\mu\nu})(\partial_\rho B^{\rho\nu}) - \frac{c_{2G}}{\Lambda^2}\frac{g_S^2}{2}(D^\mu G^A_{\mu\nu})(D_\rho G^{A\,\rho\nu}) \\
& + \frac{c_{3W}}{\Lambda^2} g^3 \epsilon_{abc} W^{a\,\nu}_\mu W^{b\,\rho}_\nu W^{c\,\mu}_\rho + \frac{c_{3G}}{\Lambda^2} g_S^3 f_{ABC} G^{A\,\nu}_\mu G^{B\,\rho}_\nu G^{C\,\mu}_\rho ,
\end{aligned}
$$