Automatising dark matter relic density calculations for models with discrete symmetries

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Introduction

- **Dark Matter (DM) relic density** has been measured with **very high accuracy**, allowing to test and strongly constrain new models with a DM candidate.

$\Omega_{DM}h^2 = 0.120 \pm 0.001$ Planck collaboration – 1807.06209

- There is a plethora of models proposing DM particles. **Weakly Interacting Massive Particles (WIMPs)** are particularly popular. Sophisticated programs have been developed to perform precise relic density computations within models with discrete symmetries (mainly for WIMPs).

- A certain level of automation is desirable, since this computation can involve the contribution of a large number of processes. **Major public codes capable of computing dark matter observables: SuperIso Relic, DarkSUSY, micrOMEGAs** and **MadDM**.

Arbey et al. - 1806.11489, Bringmann et al. - 1802.03399, Bélanger et al. - 1801.03509, Arina et al. - 2107.04598

- **DarkTree (DT):** new code to **calculate DM's relic density**, assuming the standard **freeze-out** mechanism. Can be used for a generic model where a **discrete symmetry (Z_2)** is present. **Main focal point of the code will be parameter searches.**

Freeze-out in a nutshell

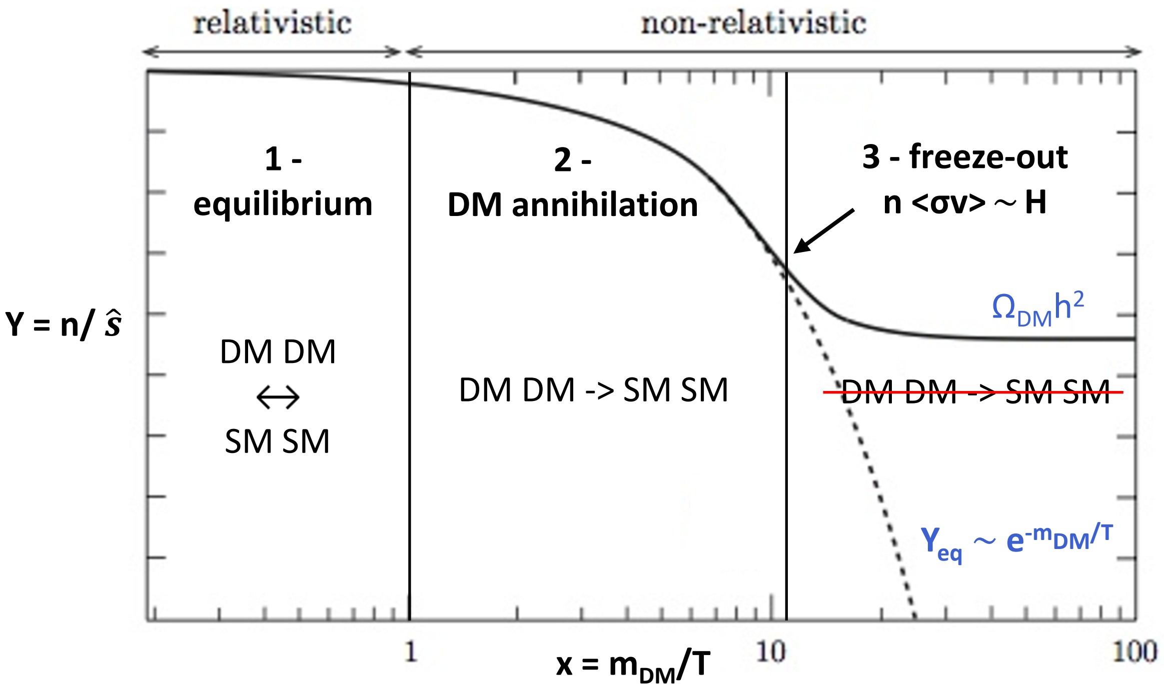


Image source: Daniel D. Baumann, Lecture notes on Cosmology

1 - T > m_{DM}, **equilibrium** between DM and SM particles.

2 - Universe cools off, DM production is disfavoured (T ~ m_{DM}). **DM annihilation dominates**.

3 - **freeze-out:** DM annihilation rate ~ Universe expansion rate. Annihilation heavily suppressed, DM number density "freezes".

- Number density given by the **Boltzmann equation:**

$$\boxed{\dot{n}(t) + \overbrace{3H(t)n(t)}^{\text{dilution from expansion}} = \overbrace{-\langle\sigma v\rangle(n(t)^2}^{\text{DM DM -> SM SM}} - \overbrace{n_{eq}^2)}^{\text{SM SM -> DM DM}}}$$

- **⟨σv⟩: thermal average DM annihilation cross section times relative velocity (TAC)**.

$$\langle\sigma v\rangle = \frac{\int d^3p_1 d^3p_2 f_1(E_1) f_2(E_2)\,\sigma v_{12}}{\int d^3p_1 d^3p_2 f_1(E_1) f_2(E_2)} \Longrightarrow \langle\sigma v\rangle = \frac{\int_{4m_{DM}^2}^{\infty} ds\sqrt{s}\left(s-4m_{DM}^2\right)K_1(\sqrt{s}/T)\sigma}{8m_{DM}^4 T K_2^2\left(m_{DM}/T\right)}$$

Gondolo, Gelmini – DOI:10.1016/0550-3213(91)90438-4, Edsjo, Gondolo - hep-ph/9704361

Freeze-out in a nutshell

- Further simplifications can be made to the Boltzmann equation, by defining Y = n/$\hat{s}$ and x = m_{DM}/T:

$$\boxed{\frac{dY}{dx} = -\sqrt{\frac{\pi}{45G}}\frac{g_*^{1/2} m_{DM}}{x^2}\langle\sigma v\rangle\left(Y^2 - Y_{\rm eq}^2\right)}$$

$$\boxed{\Omega_{DM}h^2 \approx 2.742\cdot 10^8 \frac{m_{DM}}{\rm GeV} Y_0}$$

- **Two special cases:** **co-annihilations** and **resonant annihilations**.

➢ Additional dark sector particles also contribute to DM annihilation. This changes Y_{eq} and ⟨σv⟩:

$$Y_{\rm eq} = \sum_{i=1}^{N} Y_{{\rm eq},i} = \frac{45x^2}{4\pi^4 h_{\rm eff}(x)}\sum_{i=1}^{N} g_i\left(\frac{m_i}{m_1}\right)^2 K_2\left(\frac{m_i}{m_1}x\right)$$

$$\langle\sigma v\rangle_{\rm eff} = \frac{\sum_{i,j=1}^{N} g_i g_j \int_{(m_i+m_j)^2}^{\infty} ds\sqrt{s}p_{ij}^2\sigma_{ij}(s)K_1\left(\frac{\sqrt{s}x}{m_1}\right)}{2T\left(\sum_{i=1}^{N} g_i m_i^2 K_2\left(\frac{m_i}{m_1}x\right)\right)^2}$$

➢ **Co-annihilations can significantly increase TAC calculation time**. **Boltzmann suppression factor** can be used to speed up the calculation.

$$\frac{K_1\left(\frac{(m_i+m_j)x}{m_1}\right)}{K_1\left(\frac{2m_1 x}{m_1}\right)} \propto e^{-\frac{x}{m_1}(m_i+m_j-2m_1)}$$

➢ DM annihilation near a resonance is extremely efficient. **Estimating the contribution of the resonance peak is challenging**, particularly for narrow peaks since we might miss them.

DarkTree description

Hahn et al. - hep-ph/0012260, Shtabovenko et al. - 2001.04407

- Divided into **two sub-programs**: a **Mathematica code**, integrated with **FeynArts+FeynCalc**, and a **C++ code**. The first allows the user to implement new models, the second is used to calculate the relic density for the new or already implemented models. **All tree-level annihilation and co-annihilation channels are included.**

- **Mathematica code:**

➢ **New model implementation:** user must provide Feynman rules for the model to be implemented, in **FeynRules** format. These are automatically converted into **FeynArts+CalcHep** files to extract the relevant information about the model.

Alloul et al. - 1310.1921, Belyaev et al. - 1207.6082

➢ **Automatic computation of 2 -> 2 tree-level squared amplitudes** for all freeze-out processes and **total decay widths for s-channel mediators** contributing to the relic density. **Non-scalar mediators:** only 1-to-2 decays at leading order. **Scalar mediators:** QCD corrections, decays into gluons and off-shell W/Z bosons implemented by using analytical formulae from **HDECAY**.

Djouadi et al. - 1801.09506

➢ Squared amplitudes, widths and other necessary information saved in C++ files that will be compiled and accessed by the C++ code to calculate the relic density for the implemented model. Only needs to be compiled once.

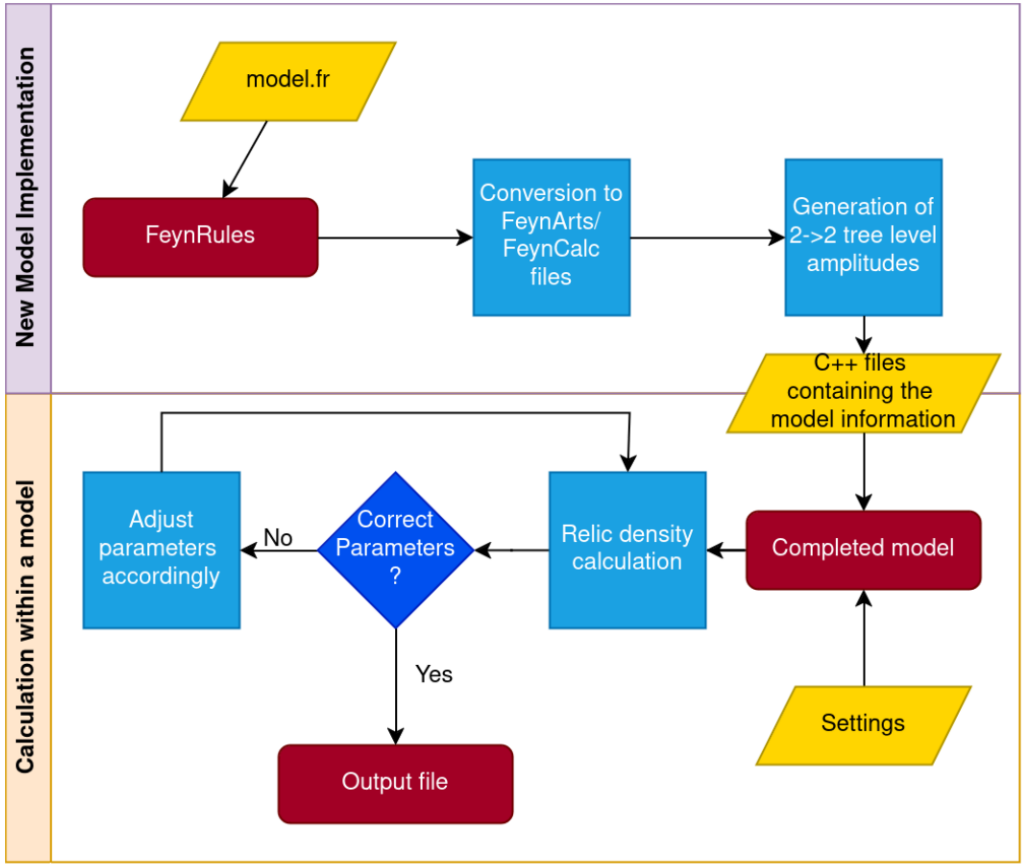
DarkTree description

- **C++ code:**

➢ **Computation of tree-level cross sections and TACs** for all relevant freeze-out processes. **Solving Boltzmann equation** to obtain the relic density.

➢ **Co-annihilations with Boltzmann suppression factor < 10^{-6} excluded** by default. For **resonant annihilations**, we identify the positions and widths of the resonance peaks, and if their contribution to the TAC is significant, an adaptive Simpson's method is used to integrate the peaks, and between peaks, an adaptive Gauss-Kronrod method.

➢ Designed such that it can be run with different functions and options via a **settings file** where the user can specify how to scan and **perform searches in the parameter space** of a model with the provided functions.



New Model Implementation

model.fr

FeynRules

Conversion to FeynArts/ FeynCalc files

Generation of 2->2 tree level amplitudes

C++ files containing the model information

Calculation within a model

Settings

Completed model

Relic density calculation

Correct Parameters ?

No

Adjust parameters accordingly

Yes

Output file

DarkTree validation

- **Four models currently implemented in DarkTree:**

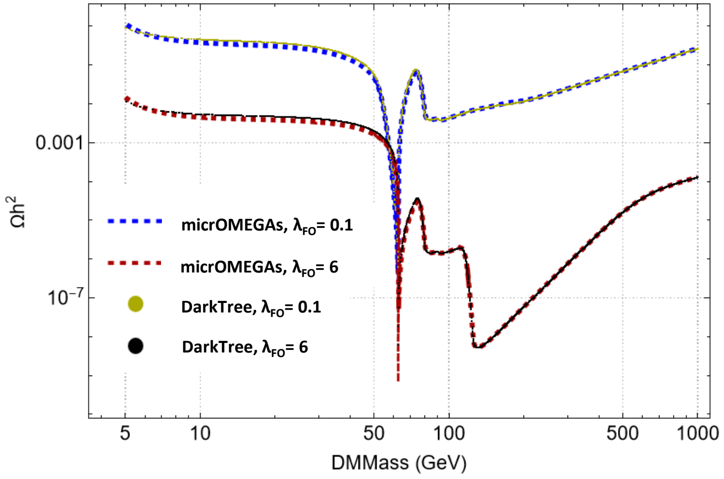
➢ **Complex singlet scalar extension of the Standard Model (CxSM)**

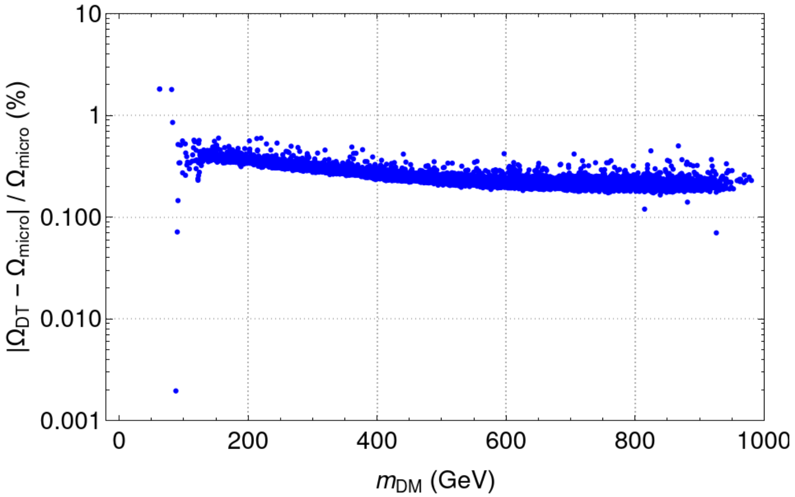
➢ **CP in the dark (CPD)**

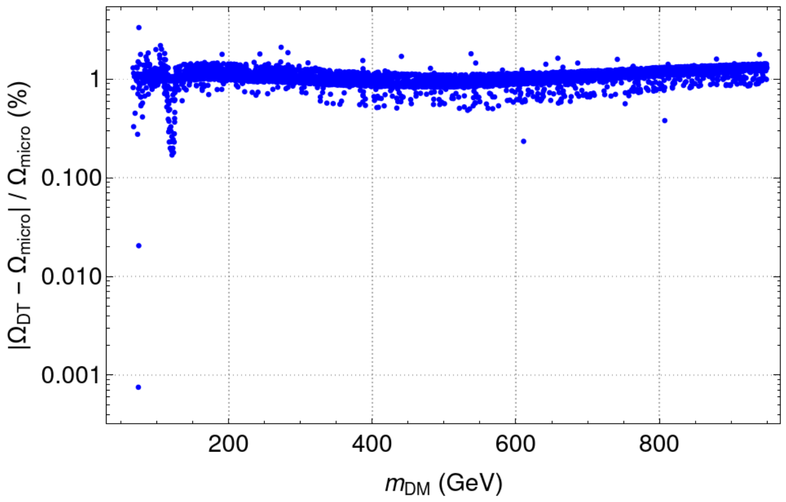
➢ **Two-real-scalar-singlet model (TRSM)**

➢ **Dark doublet phase (DDP) of the next-to-two-Higgs-doublet model (N2HDM)**

Coimbra et al. - 1301.2599
Azevedo et al. - 1807.10322
Ghorbani et al. - 1501.00206
Engeln et al. - 2004.05382


Real singlet scalar


CPD


DDP

- DarkTree was tested, using these models, by comparing the relic density obtained with DarkTree and micrOMEGAs for several random points, showing **excellent agreement between DarkTree and micrOMEGAs**. **Only significant differences (up to 10%) occur at resonances, due to a different treatment of resonant annihilations** between the two codes.

Parameter searches

- Depending on the model complexity, finding parameter regions sharing the same relic density can be quite difficult and time consuming. **DarkTree provides a few different methods to search for such regions.** **To the best of my knowledge, parameter searches are not implemented in other major public DM codes**.

- **Find parameter:** only a single parameter is changed until the desired relic density is obtained. The followed direction attempts to minimize the absolute value of the difference between the calculated and desired relic densities, $\Delta\Omega = \Omega_c - \Omega_d$, until the root or minimum of $|\Delta\Omega|$ is found.

- **Random walk:** multiple parameters scanned simultaneously using a random walk algorithm to change the parameters until the desired relic density is found.

- **Tracker:** the code lays a grid over the scanned region consisting of N cells, keeps track of N_{best} "best cells" (with respect to *w*), and saves them for future scans. Once these cells are found, we can generate new parameter points from them with a certain probability p_b, and random points with a probability $p_r = 1 - p_b$. **Can be combined with the random walk.**

$$w = \begin{cases} \left(\frac{\Omega_d}{\Omega_c}\right)^2, & \Omega_d < \Omega_c \\ \left(\frac{\Omega_c}{\Omega_d}\right)^2, & \Omega_d > \Omega_c \end{cases}$$

Under development!

Parameter searches – Find parameter

- **CxSM –** SM extended by a complex singlet scalar field S with zero hypercharge.

$$\mathbb{S} = \frac{1}{\sqrt{2}}(v_S + S + i(v_A + A))$$

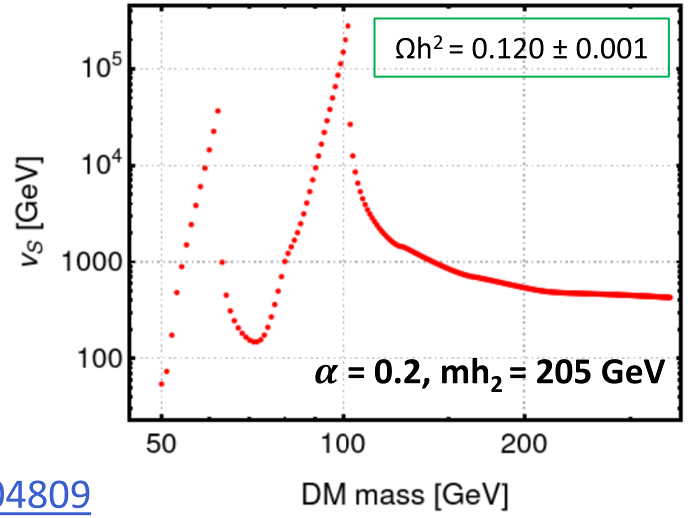
- Setting **v_A = 0** and choosing **two separate Z_2 symmetries S → −S and A → −A**, under which we impose invariance of the potential, **A is stable and becomes the DM candidate.** The Higgs field (H) and S mix with each other.

$$V = \frac{m^2}{2}\Phi^\dagger\Phi + \frac{\lambda}{4}(\Phi^\dagger\Phi)^2 + \frac{\delta_2}{2}\Phi^\dagger\Phi|\mathbb{S}|^2 + \frac{b_2}{2}|\mathbb{S}|^2 + \frac{d_2}{4}|\mathbb{S}|^4 + \left(\frac{b_1}{4}\mathbb{S}^2 + c.c.\right) \qquad \begin{pmatrix} h_1 \\ h_2 \end{pmatrix} = R_\alpha \begin{pmatrix} H \\ S \end{pmatrix} \qquad v\,,\; v_S\,,\; \alpha\,,\; m_{h_1}\,,\; m_{h_2}\,,\; m_A$$

- We scan over the DM mass and DarkTree automatically finds the singlet VEV value such that we obtain the requested relic density. **Much faster than a random scan!**

- Two peaks at the resonances ($m_{DM} = m_{hi}/2$). The portal coupling is proportional to v_S^{-1} , thus v_S needs to be large to compensate for the Higgs resonances.

- Recently we used DarkTree to study **freeze-in as a complementary process to freeze-out**, using a version of the TRSM with two DM candidates.



Capucha et al. - 2407.04809

Parameter searches – Tracker + Random walk

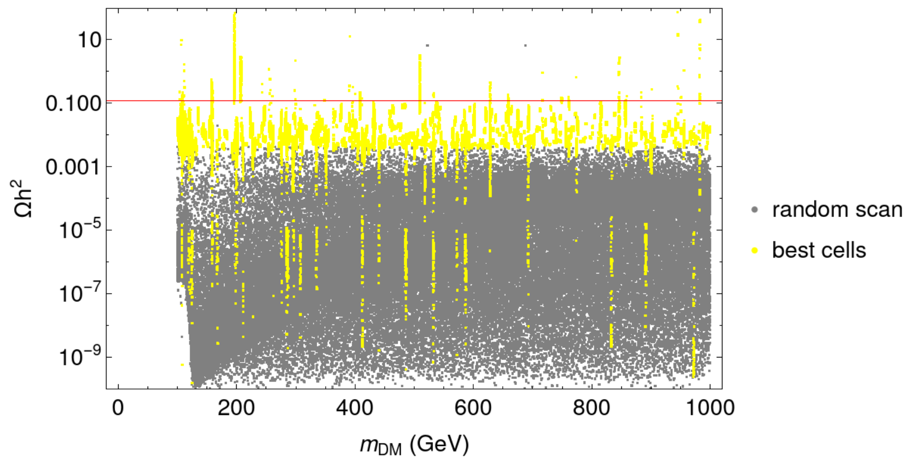
- **CPD –** two doublets + one real singlet. One Z_2 symmetry is introduced, from which one DM candidate emerges. This model allows for **CP-violation in the dark scalar sector.**

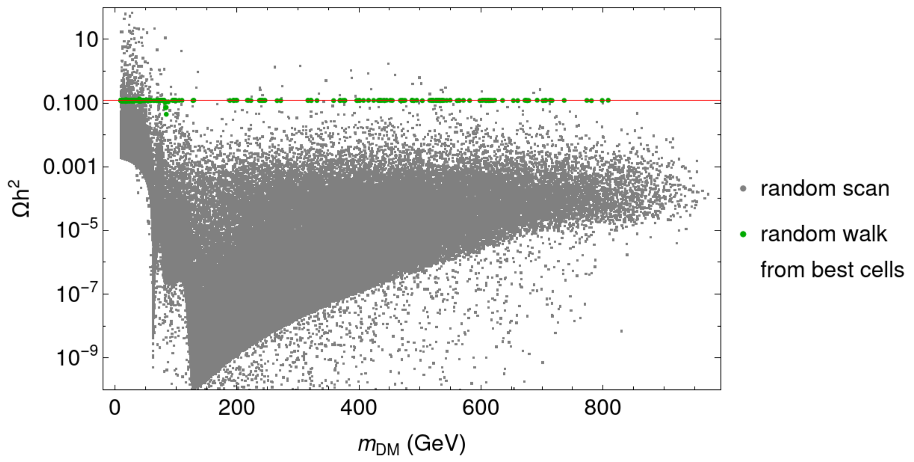
$$\mathbb{Z}_2^{(3)}: \quad \Phi_1 \to \Phi_1, \quad \Phi_2 \to -\Phi_2, \quad \Phi_S \to -\Phi_S$$

$$\Phi_1 = \begin{pmatrix} \phi_1^+ \\ \frac{1}{\sqrt{2}}(v_1 + \rho_1 + i\,\eta_1) \end{pmatrix}, \quad \Phi_2 = \begin{pmatrix} \phi_2^+ \\ \frac{1}{\sqrt{2}}(\rho_2 + i\,\eta_2) \end{pmatrix}, \quad \Phi_S = \rho_s$$

$$\begin{aligned} V_{\text{Scalar}} = {} & m_{11}^2 \Phi_1^\dagger \Phi_1 + m_{22}^2 \Phi_2^\dagger \Phi_2 + \frac{\lambda_1}{2}\left(\Phi_1^\dagger \Phi_1\right)^2 + \frac{\lambda_2}{2}\left(\Phi_2^\dagger \Phi_2\right)^2 \\ & + \lambda_3 \Phi_1^\dagger \Phi_1 \Phi_2^\dagger \Phi_2 + \lambda_4 \Phi_1^\dagger \Phi_2 \Phi_2^\dagger \Phi_1 + \frac{\lambda_5}{2}\left[\left(\Phi_1^\dagger \Phi_2\right)^2 + \text{h.c.}\right] \\ & + \frac{1}{2} m_s^2 \Phi_S^2 + \frac{\lambda_6}{8}\Phi_S^4 + \frac{\lambda_7}{2}\Phi_1^\dagger \Phi_1 \Phi_S^2 + \frac{\lambda_8}{2}\Phi_2^\dagger \Phi_2 \Phi_S^2 + \left(A \Phi_1^\dagger \Phi_2 \Phi_S + \text{h.c.}\right) \end{aligned}$$

- We start with a random scan over the parameter space of the CPD, keeping track of the best cells. **By using the random walk method on the best cells, we find parameter regions with the desired relic density much faster than by doing a simple blind/random scan**.





Conclusions and future work

- **New code to calculate the relic density of DM for freeze-out**, in a generic model with a Z_2 symmetry. **All tree-level annihilation and co-annihilation processes are included.**

- **DarkTree** is divided in two parts: a **Mathematica code to implement new models**, which automatically calculates all the squared matrix elements and widths necessary for freeze-out. The second part is a **C++ code to calculate the relic density for the new or already implemented models**. **Different methods to find parameter regions sharing the same relic density are also provided**.

- DarkTree was tested by comparing relic densities with **micrOMEGAs** for the already implemented models, showing **excellent agreement between the two codes**. **Most significant differences occur for resonant annihilations**. Co-annihilations and mass threshold effects are correctly reproduced.

- **Future work:**

➢ further development of parameter searches;

➢ implementation of generic models with other symmetries (Z_3, multiple Z_2, …), additional DM generation mechanisms (freeze-in?, others);

➢ 2 to 3/4 processes; cross sections at NLO for some models?

Ideas, suggestions, comments are much appreciated!

THE END.
THANK YOU!