$U(1)$-charged Dark Matter in three-Higgs-doublet models

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In collaboration with: **P. Osland**, **M. N. Rebelo**
based on [2408.02728].

Workshop on Multi-Higgs Models

September 3–6

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Philosophy

Identify DM in 3HDMs with continuous symmetries.

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Neutral masses: $\mathcal{M}_N^2 = \mathrm{diag}\left(\mathcal{M}^2_{\mathrm{active}},\ \mathcal{M}^2_{\mathrm{dark}} \right) .$

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Allow for spontaneous symmetry breaking, complex couplings, soft terms.

Approach

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Different symmetries may lead to the same $V_{\mathcal{G}}$ (but likely not $\mathcal{L}_Y$), *e.g.*,

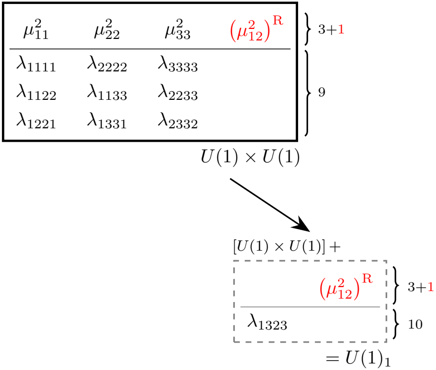
$$\left.\begin{array}{l} S_3 \otimes \mathbb{Z}_2 \cong D_6 \\ [U(1) \otimes U(1)] \rtimes S_3 \\ Q_8 \;(g_1 \in \mathbb{Z}_4) \\ \{O(2),\, D_4,\, \mathrm{CP}_2,\, \mathrm{CP}_4\} \rtimes \mathbb{Z}_3 \end{array}\right\} \quad O(2) \otimes U(1).$$

Example of Allowed Couplings

μ^2_{11}	μ^2_{22}	μ^2_{33}	$\left(\mu^2_{12}\right)^{\mathrm{R}}$	3+1
λ_{1111}	λ_{2222}	λ_{3333}		
λ_{1122}	λ_{1133}	λ_{2233}		9
λ_{1221}	λ_{1331}	λ_{2332}		

$U(1) \times U(1)$

Example of Allowed Couplings



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3+1

9

$U(1) \times U(1)$

$[U(1) \times U(1)] +$

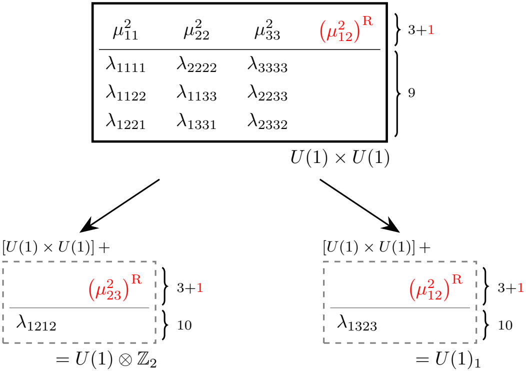
	$\left(\mu^2_{12}\right)^{\mathrm{R}}$
λ_{1323}	

3+1

10

$= U(1)_1$

Example of Allowed Couplings



μ_{11}^2	μ_{22}^2	μ_{33}^2	$\left(\mu_{12}^2\right)^{\mathrm{R}}$	$\}$ 3+1
λ_{1111}	λ_{2222}	λ_{3333}		
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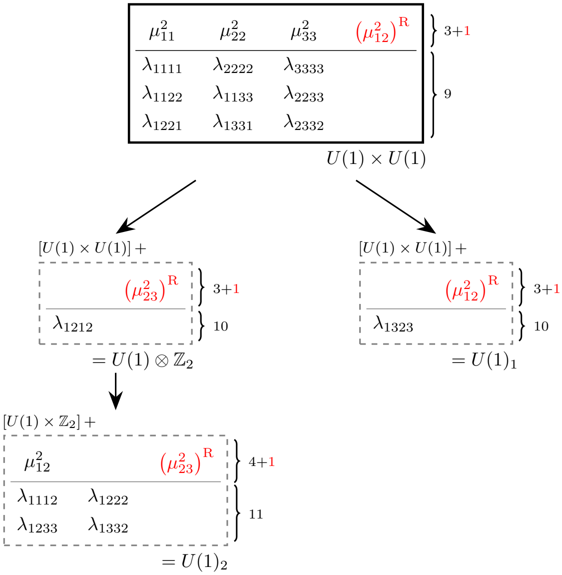
$= U(1)_1$

Example of Allowed Couplings



$$\left.\begin{array}{cccc} \mu_{11}^2 & \mu_{22}^2 & \mu_{33}^2 & \left(\mu_{12}^2\right)^{\mathrm{R}} \end{array}\right\} 3+1$$

$$\left.\begin{array}{ccc} \lambda_{1111} & \lambda_{2222} & \lambda_{3333} \\ \lambda_{1122} & \lambda_{1133} & \lambda_{2233} \\ \lambda_{1221} & \lambda_{1331} & \lambda_{2332} \end{array}\right\} 9$$

$$U(1) \times U(1)$$

$$[U(1) \times U(1)] +$$

$$\left.\left(\mu_{23}^2\right)^{\mathrm{R}}\right\} 3+1$$

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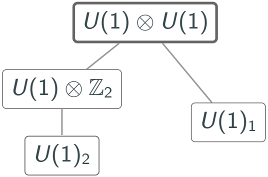
$$[U(1) \times \mathbb{Z}_2] +$$

$$\left.\begin{array}{cc} \mu_{12}^2 & \left(\mu_{23}^2\right)^{\mathrm{R}} \end{array}\right\} 4+1$$

$$\left.\begin{array}{cc} \lambda_{1112} & \lambda_{1222} \\ \lambda_{1233} & \lambda_{1332} \end{array}\right\} 11$$

$$= U(1)_2$$

Considered Symmetries



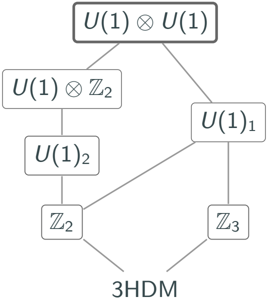
$U(1) \otimes U(1)$

$U(1) \otimes \mathbb{Z}_2$

$U(1)_1$

$U(1)_2$

Considered Symmetries



$U(1) \otimes U(1)$

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$U(1)_1$

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$\mathbb{Z}_2$

$\mathbb{Z}_3$

3HDM

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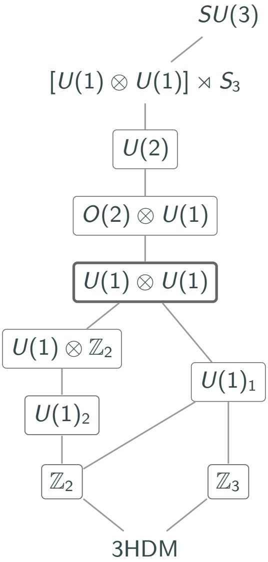

$SU(3)$

$[U(1) \otimes U(1)] \rtimes S_3$

$U(2)$

$O(2) \otimes U(1)$

$U(1) \otimes U(1)$

$U(1) \otimes \mathbb{Z}_2$

$U(1)_1$

$U(1)_2$

$\mathbb{Z}_2$

$\mathbb{Z}_3$

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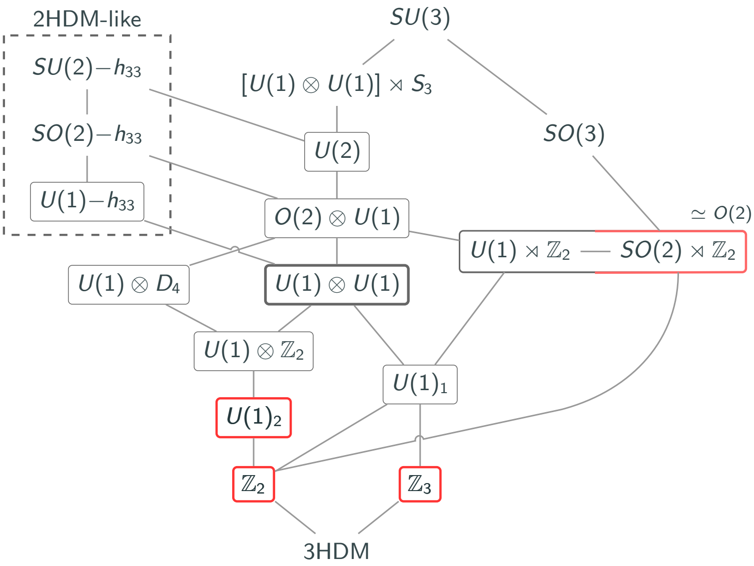

2HDM-like

$SU(3)$

$SU(2)-h_{33}$

$[U(1) \otimes U(1)] \rtimes S_3$

$SO(2)-h_{33}$

$U(2)$

$SO(3)$

$U(1)-h_{33}$

$O(2) \otimes U(1)$

$\simeq O(2)$

$U(1) \rtimes \mathbb{Z}_2$ — $SO(2) \rtimes \mathbb{Z}_2$

$U(1) \otimes D_4$

$U(1) \otimes U(1)$

$U(1) \otimes \mathbb{Z}_2$

$U(1)_1$

$U(1)_2$

$\mathbb{Z}_2$

$\mathbb{Z}_3$

3HDM

Implementations within $U(1) \otimes U(1)$-3HDM

$U(1) \otimes U(1)$-3HDM:

$$V_0 = \sum_i \mu_{ii}^2 h_{ii} + \sum_i \lambda_{iiii} h_{ii}^2 + \sum_{i<j} \lambda_{iijj} h_{ii} h_{jj} + \sum_{i<j} \lambda_{ijji} h_{ij} h_{ji}, \text{ where } h_{ij} \equiv h_i^\dagger h_j.$$

Vacuum	SYM	V	Mixing of the neutral states	Comments
$(\hat{v}_1, \hat{v}_2, 0)$	$-$	V_0	$\{\eta_1, \eta_2\}-\{\eta_3\}$ $-\{\chi_1\}-\{\chi_2\}-\{\chi_3\}$	$m_{\eta_3} = m_{\chi_3}$, $m_{\chi_1} = m_{\chi_2} = 0$
$(\hat{v}_1, \hat{v}_2, 0)$	$-$	$V_0 + \{(\mu_{12}^2)^{\mathrm{R}}\}$	$\{\eta_1, \eta_2\}-\{\chi_1, \chi_2\}$ $-\{\eta_3\}-\{\chi_3\}$	$m_{\eta_3} = m_{\chi_3}$
$(v, 0, 0)$	✓	V_0	diagonal	$m_{\eta_2} = m_{\chi_2}$, $m_{\eta_3} = m_{\chi_3}$

Pitfalls: Example of $O(2)$-3HDM

Consider two bases of $O(2)$:

$$V_{SO(2)\rtimes\mathbb{Z}_2}: \begin{pmatrix}\cos\theta & \sin\theta & 0\\ \sin\theta & -\cos\theta & 0\\ 0 & 0 & 1\end{pmatrix} \quad \text{and} \quad V_{U(1)_1\rtimes\mathbb{Z}_2}: \begin{pmatrix}0 & e^{-i\theta} & 0\\ e^{i\theta} & 0 & 0\\ 0 & 0 & 1\end{pmatrix}.$$

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$SO(2) \rtimes \mathbb{Z}_2 : (\hat{v}_1 e^{i\sigma},\, \hat{v}_2,\, 0)$ and $U(1)_1 \rtimes \mathbb{Z}_2 : (v_1,\, v_2,\, 0)$ require $2\lambda_{1111} - \lambda_{1122} - \lambda_{1221} = 0$ and there are $3 \times\, m_N^2 = 0$. Symmetry increase!?

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CPV in $SO(2)\rtimes\mathbb{Z}_2:\ (0,\ \hat{v}_2,\ \hat{v}_3)+\mu_{23}^2$ (soft term) but no CPV in $U(1)_1\rtimes\mathbb{Z}_2$.

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DM might be *hidden* within $\langle h_i\rangle\neq 0$.

The Dark Matter Candidates

Cases *without* spontaneous symmetry breaking:

$U(1) \otimes U(1)$	$(v,\, 0,\, 0)$		
$U(1)_1$	$(v,\, 0,\, 0)$	$(0,\, 0,\, v)$	
$U(1) \otimes \mathbb{Z}_2$	$(v,\, 0,\, 0)$	$(0,\, 0,\, v)$	
$U(1)_2$	$(v,\, 0,\, 0)$	$(0,\, 0,\, v)$	$(v_1,\, v_2,\, 0)$
$O(2) \otimes U(1)$		$(0,\, 0,\, v)$	
$O(2)$		$(0,\, 0,\, v)$	
$U(1) \otimes D_4$		$(0,\, 0,\, v)$	
$U(2)$		$(0,\, 0,\, v)$	
$U(1){-}h_{33}$	$(v,\, 0,\, 0)$	$(0,\, 0,\, v)$	
$SO(2){-}h_{33}$		$(0,\, 0,\, v)$	
$SU(2){-}h_{33}$		$(0,\, 0,\, v)$	

$[U(1) \otimes U(1)] \rtimes S_3$, $SO(3)$, $SU(3)$ result in spontaneous symmetry breaking.

The Dark Matter Candidates: Mass Degeneracies

<table>
<tr><th>Model</th><th>Vacuum</th><th>Symmetry</th><th>h_{SM}</th></tr>
<tr><td colspan="4">$m_{\eta_3} = m_{\chi_3}$</td></tr>
<tr><td>I-a</td><td>$(v,\, 0,\, 0)$</td><td>$U(1) \otimes \mathbb{Z}_2$</td><td>$\eta_1$</td></tr>
<tr><td>I-b</td><td>$(v,\, 0,\, 0)$</td><td>$U(1)_2$</td><td>$\{\eta_1,\, \eta_2,\, \chi_2\}$</td></tr>
<tr><td>I-c</td><td>$(v,\, 0,\, 0)$</td><td>$U(1){-}h_{33}$</td><td>η_1</td></tr>
<tr><td>I-d</td><td>$(\hat{v}_1,\, \hat{v}_2,\, 0)$</td><td>$U(1)_2$</td><td>$\{\eta_1,\, \eta_2,\, \chi_1,\, \chi_2\}$</td></tr>
<tr><td>I-e</td><td>$(\hat{v}_1 e^{i\sigma},\, \hat{v}_2,\, 0)$</td><td>$U(1)_2$</td><td>$\{\eta_1,\, \eta_2,\, \chi_1,\, \chi_2\}$</td></tr>
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<tr><td>II-c</td><td>$(0,\, 0,\, v)$</td><td>$U(1) \otimes \mathbb{Z}_2$</td><td>$\eta_3$</td></tr>
<tr><td colspan="4">$m_{\{\eta_1,\eta_2\}} = m_{\{\chi_1,\chi_2\}}$, η_i and χ_i mix separately</td></tr>
<tr><td>II-d</td><td>$(0,\, 0,\, v)$</td><td>$U(1)_1$</td><td>η_3</td></tr>
<tr><td>II-e</td><td>$(0,\, 0,\, v)$</td><td>$O(2)$</td><td>η_3</td></tr>
<tr><td colspan="4">$m_{\{\eta_1,\eta_2,\chi_1,\chi_2\}}$, all states mix together</td></tr>
<tr><td>II-f</td><td>$(0,\, 0,\, v)$</td><td>$U(1)_2$</td><td>η_3</td></tr>
<tr><td colspan="4">$m_{\eta_1} = m_{\eta_2} = m_{\chi_1} = m_{\chi_2}$</td></tr>
<tr><td>II-g</td><td>$(0,\, 0,\, v)$</td><td>$O(2) \otimes U(1)$</td><td>η_3</td></tr>
<tr><td>II-h</td><td>$(0,\, 0,\, v)$</td><td>$U(1) \otimes D_4$</td><td>η_3</td></tr>
<tr><td>II-i</td><td>$(0,\, 0,\, v)$</td><td>$U(2)$</td><td>η_3</td></tr>
</table>

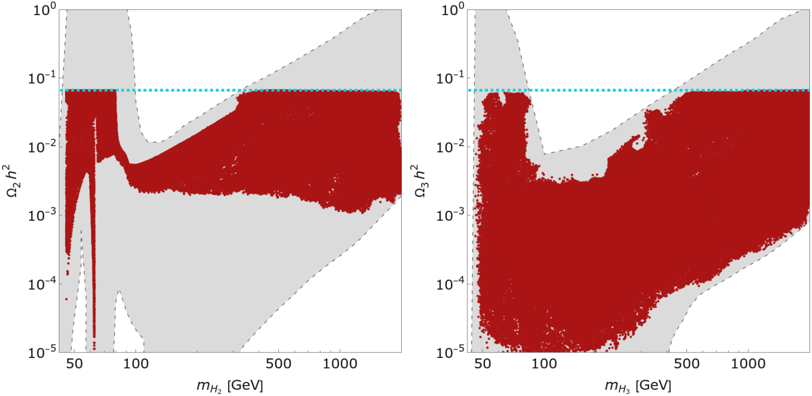
[*See P. Osland's talk.*]

The Dark Matter Candidates: Scalar Couplings

Symmetry	$(v, 0, 0)$	$(0, 0, v)$	$(v_1, v_2, 0)$
$U(1) \otimes U(1)$	$7 + 32\ (5 + 10)$		
$U(1)_1$	$15 + 40\ (8 + 13)$	$9 + 53\ (6 + 24)$	
$U(1) \otimes \mathbb{Z}_2$	$7 + 32\ (6 + 11)$	$7 + 38\ (5 + 16)$	
$U(1)_2$	$24 + 55\ (20 + 40)$	$12 + 73\ (8 + 36)$	$\mathbb{R} : 16 + 41\ (13 + 24)$ $\mathbb{C} : 24 + 55\ (20 + 40)$
$O(2) \otimes U(1)$		$7 + 32\ (3 + 7)$	
$O(2)$		$7 + 39\ (4 + 12)$	
$U(1) \otimes D_4$		$7 + 34\ (3 + 11)$	
$U(2)$		$7 + 32\ (3 + 6)$	
$U(1){-}h_{33}$	$4 + 16\ (3 + 5)$	$1 + 26\ (1 + 6)$	
$SO(2){-}h_{33}$		$1 + 26\ (1 + 5)$	
$SU(2){-}h_{33}$		$1 + 26\ (1 + 4)$	

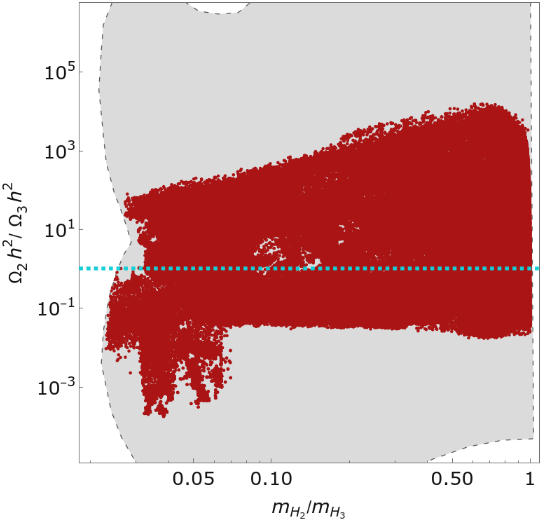
Dark Matter in $U(1) \otimes U(1)$-3HDM

There are 4 (2) DM candidates, $\Omega_{\rm DM} h^2 = \Omega_2 h^2 + \Omega_3 h^2$.

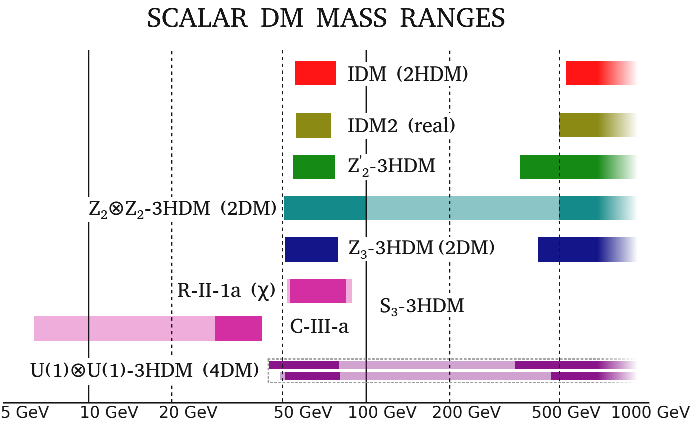


- No points when $m_{H_2} \in [100;\ 300]$ GeV and $m_{H_3} \in [100;\ 450]$ GeV;
- In $[50;\ 80] \cup [300;\ 2000]$ GeV comparable $m^2_{H_i}$ and $\Omega_i h^2$;
- Regions where Ωh^2 is dominated by a single mass scale, $m_{H_2} = [45;\ 80] \cup [330;\ 2000]$ GeV and $m_{H_3} = [52;\ 80] \cup [470;\ 2000]$ GeV;

Dark Matter in $U(1) \otimes U(1)$-3HDM



Updated Stock of Dark Matter in 3HDMs



IDM: **(A. Belyaev, G. Cacciapaglia, I. P. Ivanov, F. Rojas-Abatte, M. Thomas, 2016)**,
(J. Kalinowski, W. Kotlarski, T. Robens, D. Sokolowska, A. F. Zarnecki, 2018);

IDM2: **(M. Merchand, M. Sher, 2019)**;

$\mathbb{Z}_n$-3HDM: **(A. Cordero-Cid, J. Hernández-Sánchez, V. Keus, S. F. King, S. Moretti, D. Rojas, D. Sokołowska, 2014-2022)**;

$\mathbb{Z}_2 \otimes \mathbb{Z}_2$-3HDM: **(R. Boto, P. N. Figueiredo, J. C. Romão, J. Silva, 2024)**;

S_3-3HDM: **(A. Kunčinas, O. M. Øgreid, P. Osland, M. N. Rebelo, 2021-2023)**;

Conclusions

- We classified and identified models, embedding $U(1)$-stabilised DM in 3HDMs. These models contain mass-degenerate pairs of DM candidates, due to the unbroken $U(1)$. Such classification and identification of models is useful for model builders interested in three-Higgs-doublet models stabilised by continuous symmetries;
- We performed a numerical scan of the $U(1) \otimes U(1)$-3HDM. Within the model there is a multi-component DM sector, with two different mass scales. We found that there are possible solutions throughout a broad DM mass range, 45–2000 GeV;

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Appendix: on $\mathrm{CP}_2 \to D_4$

CP_2: $\{h_1 \to h_2^*,\ h_2 \to -h_1^*,\ h_3 \to h_3^*\}$.

$$\begin{aligned} V_{\mathrm{CP}_2} = {} & \mu_{11}^2(h_{11}+h_{22}) + \mu_{33}^2 h_{33} + \lambda_{1111}(h_{11}^2+h_{22}^2) + \lambda_{3333} h_{33}^2 \\ & + \lambda_{1122} h_{11} h_{22} + \lambda_{1133}(h_{11} h_{33} + h_{22} h_{33}) + \lambda_{1221} h_{12} h_{21} \\ & + \lambda_{1331}(h_{13} h_{31} + h_{23} h_{32}) + \lambda_{1212}(h_{12}^2 + h_{21}^2) + \left\{\lambda_{1313}(h_{13}^2 + h_{23}^2) + \mathrm{h.c.}\right\} \\ & + \lambda_{1112}(h_{11} h_{12} + h_{11} h_{21} - h_{12} h_{22} - h_{21} h_{22}). \end{aligned}$$

Perform a basis transformation $\begin{pmatrix} h_1 \\ h_2 \\ h_3 \end{pmatrix} = \frac{1}{\sqrt{2}} \begin{pmatrix} i & 1 & 0 \\ 1 & i & 0 \\ 0 & 0 & \sqrt{2} \end{pmatrix} \begin{pmatrix} h_1' \\ h_2' \\ h_3' \end{pmatrix}$.

$$\begin{aligned} V_{D_4} = {} & \mu_{11}^2(h_{11}+h_{22}) + \mu_{33}^2 h_{33} + \lambda_{1111}(h_{11}^2+h_{22}^2) + \lambda_{3333} h_{33}^2 + \lambda_{1122} h_{11} h_{22} \\ & + \lambda_{1133}(h_{11} h_{33} + h_{22} h_{33}) + \lambda_{1221} h_{12} h_{21} + \lambda_{1331}(h_{13} h_{31} + h_{23} h_{32}) \\ & + \lambda_{1212}(h_{12}^2 + h_{21}^2) + \left\{\lambda_{1313}(h_{13}^2 + h_{23}^2) + \mathrm{h.c.}\right\}. \end{aligned}$$

CP_2-symmetric 3HDM has a redundant quartic coupling λ_{1112}.

Appendix: $O(2) \otimes U(1)$

Invariant under:

$$\begin{pmatrix} h_1 \\ h_2 \\ h_3 \end{pmatrix} = \begin{pmatrix} 0 & e^{i\theta_1} & 0 \\ e^{i\theta_2} & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} h_1' \\ h_2' \\ h_3' \end{pmatrix}.$$

$$V = \mu_{11}^2(h_{11} + h_{22}) + \mu_{33}^2 h_{33} + \lambda_{1111}(h_{11}^2 + h_{22}^2) + \lambda_{3333} h_{33}^3 + \lambda_{1122} h_{11} h_{22}$$
$$+ \lambda_{1133}(h_{11} h_{33} + h_{22} h_{33}) + \lambda_{1221} h_{12} h_{21} + \lambda_{1331}(h_{13} h_{31} + h_{23} h_{32}).$$

$S_3 \otimes \mathbb{Z}_2 \simeq D_6$. However, D_n, with $n \geq 5$ result in identical V.

Perform a basis change:

$$\begin{pmatrix} h_1 \\ h_2 \\ h_3 \end{pmatrix} = \frac{1}{\sqrt{2}} \begin{pmatrix} -e^{-i\frac{\pi}{4}} & -e^{i\frac{\pi}{4}} & 0 \\ e^{-i\frac{\pi}{4}} & -e^{i\frac{\pi}{4}} & 0 \\ 0 & 0 & \sqrt{2} \end{pmatrix} \begin{pmatrix} h_1' \\ h_2' \\ h_3' \end{pmatrix}.$$

$$V = \mu_{11}^2(h_{11} + h_{22}) + \mu_{33}^2 h_{33} + \lambda_a(h_{11}^2 + h_{22}^2) + \lambda_{3333} h_{33}^3 + \lambda_b h_{12} h_{21}$$
$$+ \lambda_{1331}(h_{13} h_{31} + h_{23} h_{32}) + \lambda_c h_{11} h_{22} + \lambda_{1133}(h_{11} h_{33} + h_{22} h_{33}) - \frac{1}{2}\Lambda(h_{12}^2 + h_{21}^2).$$

Invariance under $g = \begin{pmatrix} 0 & e^{i\alpha} & 0 \\ -e^{-i\alpha} & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix}$.

Appendix: $[U(1) \otimes U(1)] \rtimes S_3$

Invariant under:

$$\begin{pmatrix} h_1 \\ h_2 \\ h_3 \end{pmatrix} = \begin{pmatrix} 0 & e^{i\theta_1} & 0 \\ 0 & 0 & e^{i\theta_2} \\ 1 & 0 & 0 \end{pmatrix} \begin{pmatrix} h_1' \\ h_2' \\ h_3' \end{pmatrix} .$$

$$\begin{aligned} V &= \mu_{11}^2 (h_{11} + h_{22} + h_{33}) + \lambda_{1111} (h_{11}^2 + h_{22}^2 + h_{33}^2) \\ &\quad + \lambda_{1122} (h_{11} h_{22} + h_{22} h_{33} + h_{33} h_{11}) + \lambda_{1221} (h_{12} h_{21} + h_{23} h_{32} + h_{31} h_{13}) \\ &= \mu_{11}^2 \sum_i h_{ii} + \lambda_{1111} \sum_i h_{ii}^2 + \lambda_{1122} \sum_{i<j} h_{ii} h_{jj} + \lambda_{1221} \sum_{i<j} h_{ij} h_{ji} . \end{aligned}$$

$$g = \begin{pmatrix} 0 & e^{i\theta_1} & 0 \\ 0 & 0 & e^{i\theta_2} \\ e^{-i(\theta_1+\theta_2)} & 0 & 0 \end{pmatrix} .$$

Appendix: $U(1) \otimes D_4$

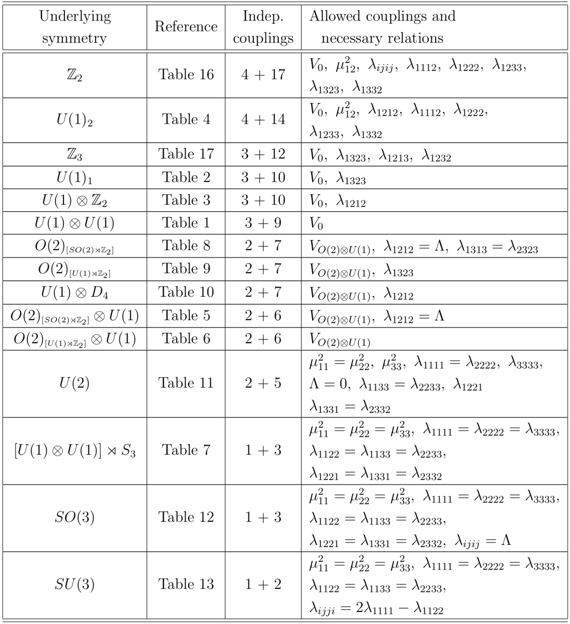
$$V^{\mathrm{ph}}_{U(1)\otimes Z_2} = \lambda_{1212} h_{12}^2 + \mathrm{h.c.},$$

$$\begin{aligned} V = {} & \mu_{11}^2 (h_{11} + h_{22}) + \mu_{33}^2 h_{33} + \lambda_{1111}(h_{11}^2 + h_{22}^2) + \lambda_{3333} h_{33}^2 + \lambda_{1122} h_{11} h_{22} \\ & + \lambda_{1133}(h_{11}h_{33} + h_{22}h_{33}) + \lambda_{1221} h_{12} h_{21} + \lambda_{1331}(h_{13}h_{31} + h_{23}h_{32}) \\ & + \lambda_{1212}(h_{12}^2 + h_{21}^2). \end{aligned}$$

Invariance under $\begin{pmatrix} h_1 \\ h_2 \\ h_3 \end{pmatrix} = \begin{pmatrix} 0 & -e^{i\alpha} & 0 \\ e^{i\alpha} & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} h_1' \\ h_2' \\ h_3' \end{pmatrix}$ suggests that $\{\lambda_{1112}\,(h_{11}h_{12} - h_{21}h_{22}) + \mathrm{h.c.}\}$ is allowed. Change of basis $\begin{pmatrix} h_1 \\ h_2 \\ h_3 \end{pmatrix} = \frac{1}{\sqrt{2}} \begin{pmatrix} e^{\frac{i\pi}{4}} & e^{\frac{i\pi}{4}} & 0 \\ -e^{-\frac{i\pi}{4}} & e^{-\frac{i\pi}{4}} & 0 \\ 0 & 0 & \sqrt{2} \end{pmatrix} \begin{pmatrix} h_1' \\ h_2' \\ h_3' \end{pmatrix}$ yields $\lambda_{1212} \in \mathbb{C}$.

As pointed out by I. Ivanov, this should better be denoted as a quotient group.

Appendix: Different Models



Underlying symmetry	Reference	Indep. couplings	Allowed couplings and necessary relations
$\mathbb{Z}_2$	Table 16	$4 + 17$	$V_0,\ \mu_{12}^2,\ \lambda_{ijij},\ \lambda_{1112},\ \lambda_{1222},\ \lambda_{1233},$ $\lambda_{1323},\ \lambda_{1332}$
$U(1)_2$	Table 4	$4 + 14$	$V_0,\ \mu_{12}^2,\ \lambda_{1212},\ \lambda_{1112},\ \lambda_{1222},$ $\lambda_{1233},\ \lambda_{1332}$
$\mathbb{Z}_3$	Table 17	$3 + 12$	$V_0,\ \lambda_{1323},\ \lambda_{1213},\ \lambda_{1232}$
$U(1)_1$	Table 2	$3 + 10$	$V_0,\ \lambda_{1323}$
$U(1) \otimes \mathbb{Z}_2$	Table 3	$3 + 10$	$V_0,\ \lambda_{1212}$
$U(1) \otimes U(1)$	Table 1	$3 + 9$	V_0
$O(2)_{[SO(2)\rtimes\mathbb{Z}_2]}$	Table 8	$2 + 7$	$V_{O(2)\otimes U(1)},\ \lambda_{1212} = \Lambda,\ \lambda_{1313} = \lambda_{2323}$
$O(2)_{[U(1)\rtimes\mathbb{Z}_2]}$	Table 9	$2 + 7$	$V_{O(2)\otimes U(1)},\ \lambda_{1323}$
$U(1) \otimes D_4$	Table 10	$2 + 7$	$V_{O(2)\otimes U(1)},\ \lambda_{1212}$
$O(2)_{[SO(2)\rtimes\mathbb{Z}_2]} \otimes U(1)$	Table 5	$2 + 6$	$V_{O(2)\otimes U(1)},\ \lambda_{1212} = \Lambda$
$O(2)_{[U(1)\rtimes\mathbb{Z}_2]} \otimes U(1)$	Table 6	$2 + 6$	$V_{O(2)\otimes U(1)}$
$U(2)$	Table 11	$2 + 5$	$\mu_{11}^2 = \mu_{22}^2,\ \mu_{33}^2,\ \lambda_{1111} = \lambda_{2222},\ \lambda_{3333},$ $\Lambda = 0,\ \lambda_{1133} = \lambda_{2233},\ \lambda_{1221}$ $\lambda_{1331} = \lambda_{2332}$
$[U(1) \otimes U(1)] \rtimes S_3$	Table 7	$1 + 3$	$\mu_{11}^2 = \mu_{22}^2 = \mu_{33}^2,\ \lambda_{1111} = \lambda_{2222} = \lambda_{3333},$ $\lambda_{1122} = \lambda_{1133} = \lambda_{2233},$ $\lambda_{1221} = \lambda_{1331} = \lambda_{2332}$
$SO(3)$	Table 12	$1 + 3$	$\mu_{11}^2 = \mu_{22}^2 = \mu_{33}^2,\ \lambda_{1111} = \lambda_{2222} = \lambda_{3333},$ $\lambda_{1122} = \lambda_{1133} = \lambda_{2233},$ $\lambda_{1221} = \lambda_{1331} = \lambda_{2332},\ \lambda_{ijij} = \Lambda$
$SU(3)$	Table 13	$1 + 2$	$\mu_{11}^2 = \mu_{22}^2 = \mu_{33}^2,\ \lambda_{1111} = \lambda_{2222} = \lambda_{3333},$ $\lambda_{1122} = \lambda_{1133} = \lambda_{2233},$ $\lambda_{ijji} = 2\lambda_{1111} - \lambda_{1122}$

Appendix: Dark Matter in $U(1) \otimes U(1)$-3HDM

$$
\begin{aligned}
V_{\mathrm{IDM}} = {} & \mu_{11}^2 h_{11} + \mu_{22}^2 h_{22} + \lambda_{1111} h_{11}^2 + \lambda_{2222} h_{22}^2 + \lambda_{1122} h_{11} h_{22} + \lambda_{1221} h_{12} h_{21} \\
& + \lambda_{1212} \left(h_{12}^2 + h_{21}^2 \right). \\
V_{U(1)\otimes U(1)} = {} & \frac{1}{2}\mu_{11}^2 h_{11} + \mu_{22}^2 h_{22} + \frac{1}{2}\lambda_{1111} h_{11}^2 + \lambda_{2222} h_{22}^2 + \lambda_{1122} h_{11} h_{22} + \lambda_{1221} h_{12} h_{21} \\
& + \frac{1}{2}\mu_{11}^2 h_{11} + \mu_{33}^2 h_{33} + \frac{1}{2}\lambda_{1111} h_{11}^2 + \lambda_{3333} h_{33}^2 + \lambda_{1133} h_{11} h_{33} + \lambda_{1331} h_{13} h_{31} \\
& + \lambda_{2233} h_{22} h_{33} + \lambda_{2332} h_{23} h_{32}.
\end{aligned}
$$

Masses:

$$
\begin{aligned}
m_{h_i^+}^2 &= \mu_{ii}^2 + \frac{1}{2}\lambda_{11ii} v^2, \text{ for } i = \{2, 3\}, \\
m_{\eta_i}^2 &= m_{\chi_i}^2 \equiv m_{H_i}^2 = \mu_{ii}^2 + \frac{1}{2}\left(\lambda_{11ii} + \lambda_{1ii1}\right) v^2, \text{ for } i = \{2,\ 3\}.
\end{aligned}
$$

Z interactions: $\mathcal{L}_{VHH} = -\frac{g}{2\cos\theta_W} Z^\mu \eta_i \overleftrightarrow{\partial}_\mu \chi_i.$

Higgs portal: $g(\eta_i^2 h) = g(\chi_i^2 h) = v\left(\lambda_{11ii} + \lambda_{1ii1}\right).$

Appendix: $U(1) \otimes U(1)$ Scan

- All of the additional scalars can be as heavy as 2 TeV;
- The DM candidates are associated with the neutral states of the h_2 and h_3 doublets, $m_{H_2} \leq m_{H_3}$;
- LEP constraints. We allow for $m_{H_i^\pm} \geq 70$ GeV and assume $m_{H_i} > \frac{1}{2} m_Z$;
- Perturbativity, Unitarity (8π), Stability conditions;
- Electroweak precision observables S and T;
- $\text{Br}\left(h \rightarrow \text{invisible}\right) \leq 0.1$;
- LHC searches implemented in HiggsTools;
- DM relic density is evaluated as $\Omega_{\text{DM}} h^2 = 2\Omega_2 h^2 + 2\Omega_3 h^2$;
- Direct DM searches based on XENONnT and LUX-ZEPLIN;
- Indirect DM searches.

Appendix: Mass Scatter Plots of $U(1) \otimes U(1)$-3HDM