BSMPTv3: A Tool for Phase Transitions and Primordial Gravitational Waves in Extended Higgs Sectors

Margarete Mühlleitner, KIT
Multi-Higgs Models 2024
Lisboa, Portugal

Work in Collaboration with:
Ph. Basler, L. Biermann, J. Müller,
R. Santos, J. Viana

Outline

☐ Introduction

☐ Physics Problem

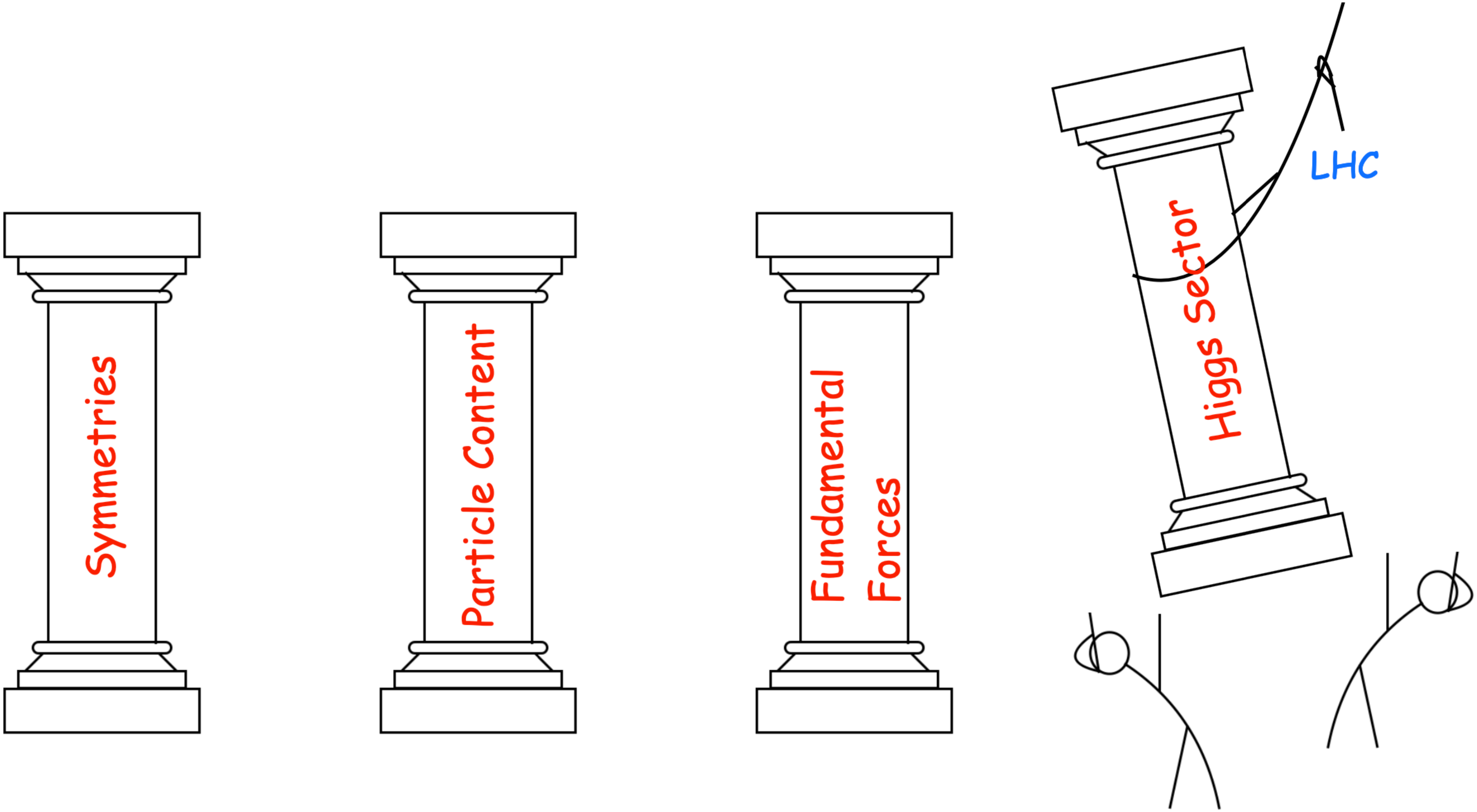
☐ BSMPTv1/v2

☐ BSMPTv3 : Code Structure, new classes, functionalities

☐ Results and Comparisons

☐ Conclusions

The Standard Model is Structurally Complete



Symmetries

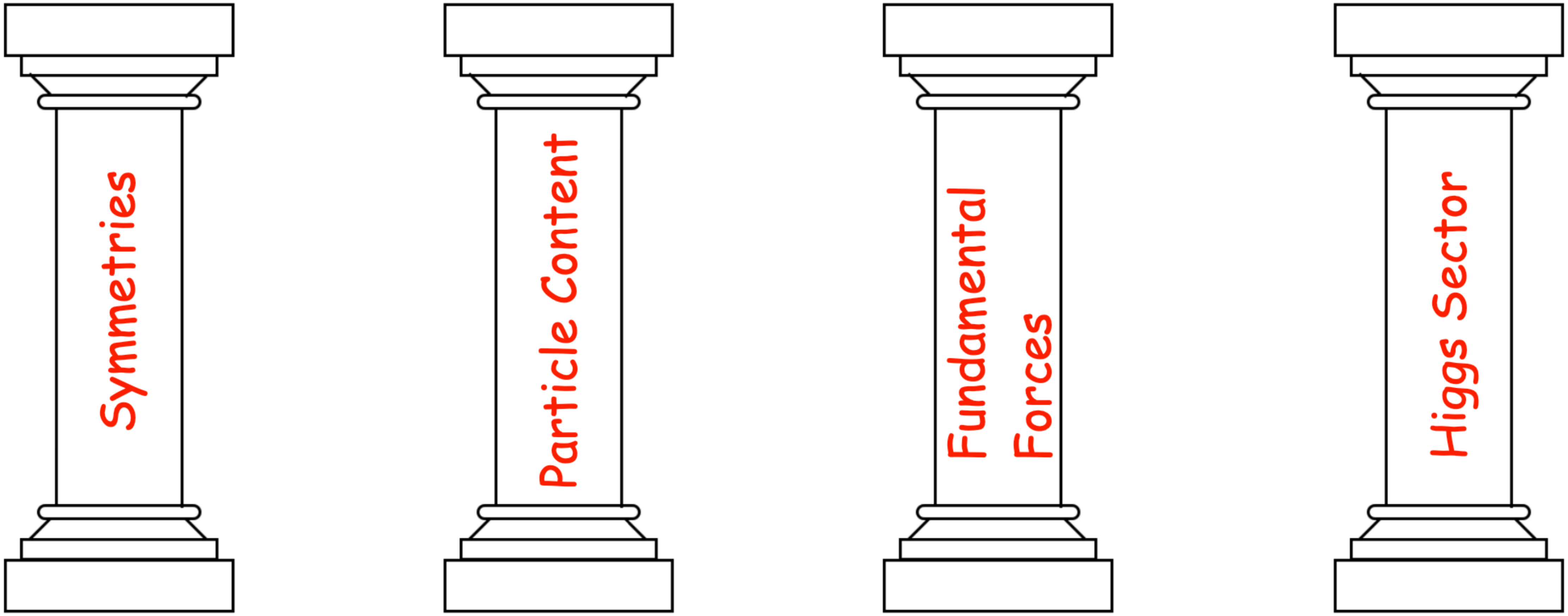
Particle Content

Fundamental Forces

Higgs Sector

LHC

The Standard Model is Structurally Complete



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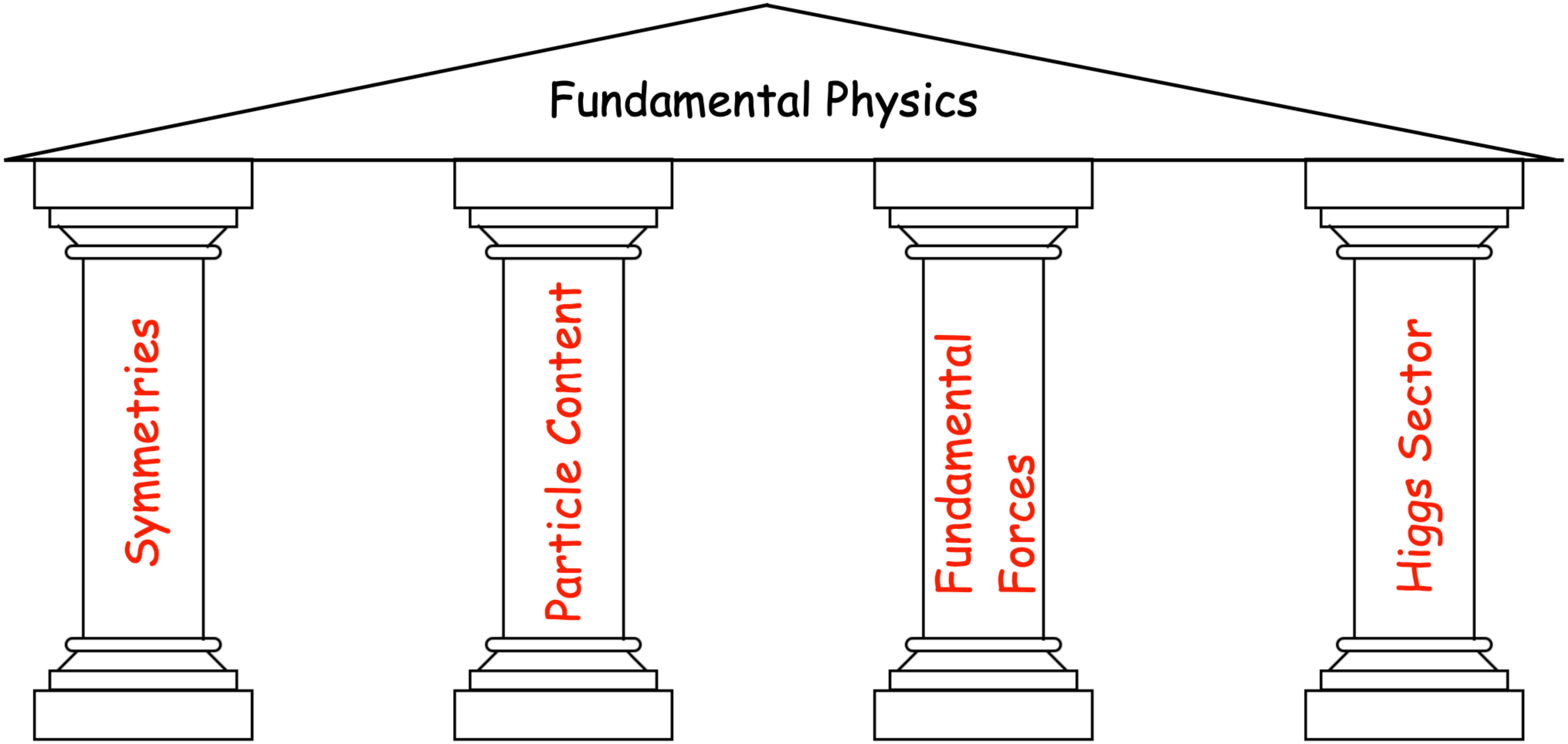


Fundamental Physics

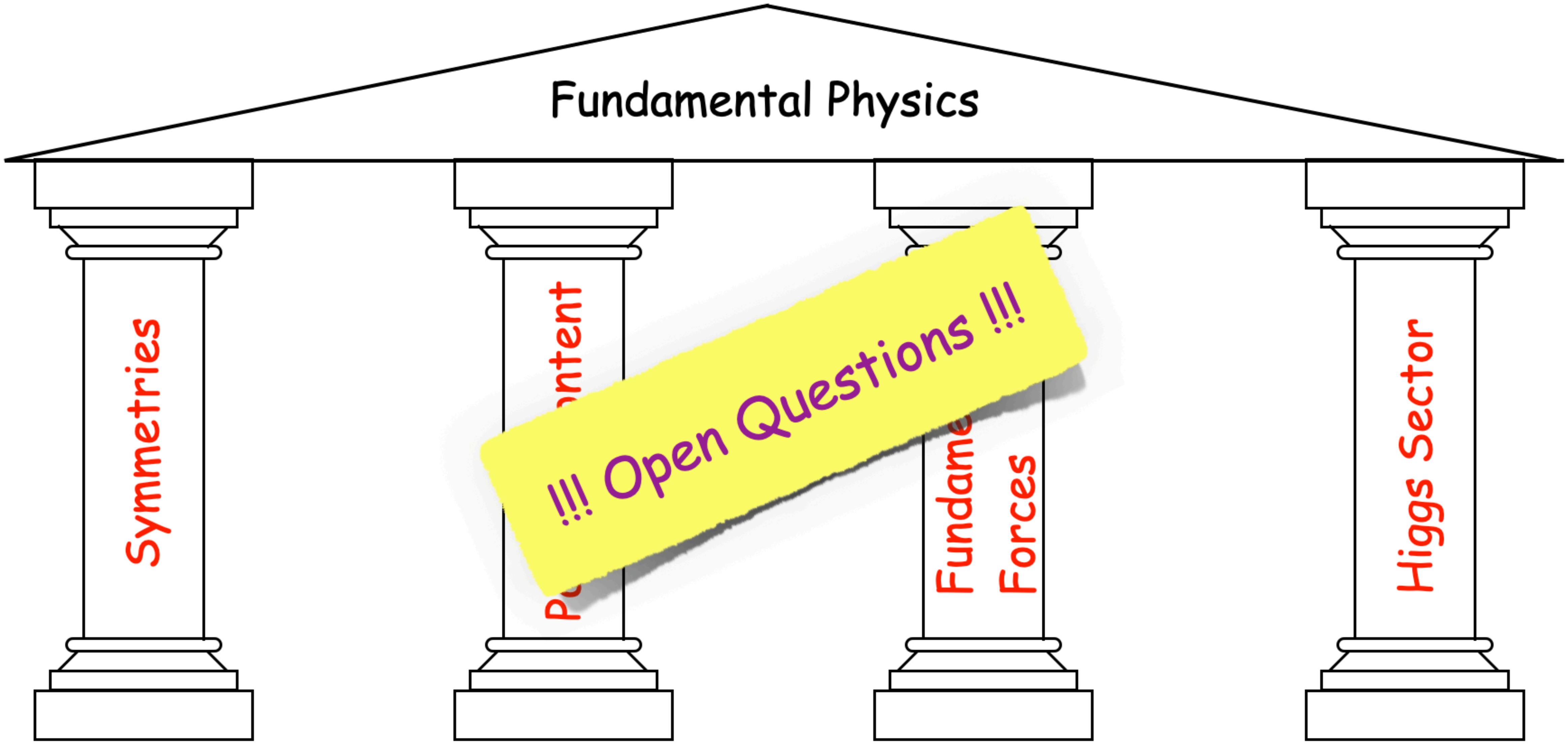
Symmetries

Particle Content

Fundamental Forces

Higgs Sector

The Standard Model is Structurally Complete - But



Fundamental Physics

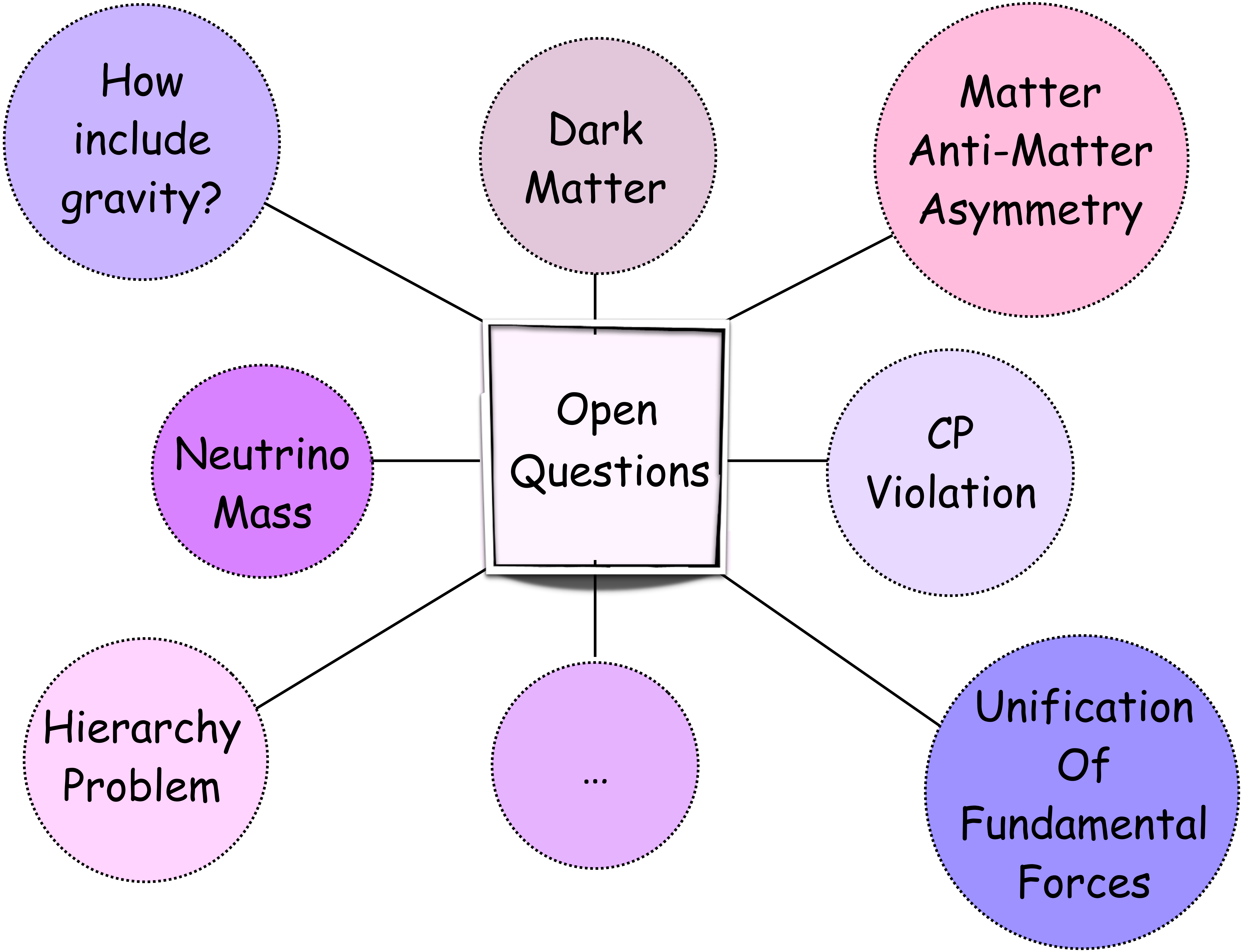
Symmetries

P... ontent

Fundame... Forces

Higgs Sector

!!! Open Questions !!!



Open Questions

How include gravity?

Dark Matter

Matter Anti-Matter Asymmetry

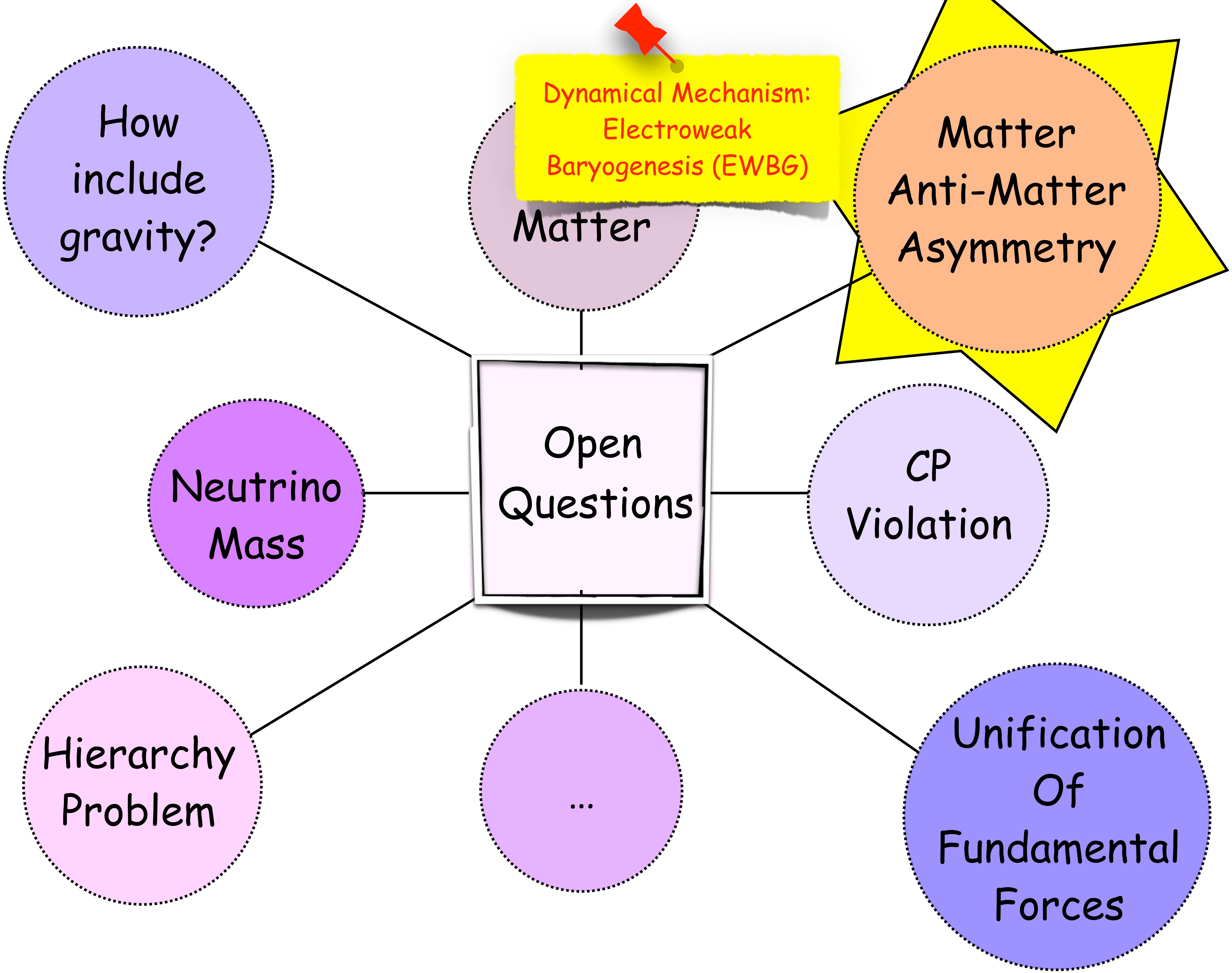
Neutrino Mass

CP Violation

Hierarchy Problem

...

Unification Of Fundamental Forces



Dynamical Mechanism:
Electroweak
Baryogenesis (EWBG)

Open
Questions

How
include
gravity?

Matter

Matter
Anti-Matter
Asymmetry

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Unification
Of
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Forces

Electroweak Baryogenesis

- **Electroweak Baryogenesis (EWBG):** generation of the observed baryon-antibaryon asymmetry in the electroweak phase transition (EWPT) [Riemer-Sorensen, Jenssen '17]

$$5.8 \cdot 10^{-10} < \frac{n_B - n_{\bar{B}}}{n_\gamma} < 6.6 \cdot 10^{-10}$$

- **Sakharov Conditions:** [Sakharov '67]
  - (i) B number violaton (sphaleron processes)
  - (ii) C and CP violation
  - (iii) Departure from thermal equilibrium

- **Additional constraint:** EW phase transition must be strong first order PT [Quiros '94; Moore '99]

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For M_H=125 GeV:
smooth cross-over in SM

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Electroweak Baryogenesis

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Requires beyond the SM (BSM) physics: extended Higgs sectors

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Strong-First-Order Phase Transitions (SFOPT) and Gravitational Waves

Laser Interferometer Gravitational-Wave Observato
Supported by the National Science Founda
Operated by Caltech and MIT

Image credit: The SXS (Simulating eXtreme Spacetimes) Project

Gravitational Waves Detected 100 Years After Einstein's Prediction

News Release • February 11, 2016

SFOEWPT
(for EWBG)

can source

Gravitational Waves
(GW)

sensitive to

Pulsar timing arrays, LISA
(space bound interferometer)

Strong-First-Order Phase Transitions (SFOPT) and Gravitational Waves

Directly probe echo of Cosmological FOPT

Discovery of Physics Beyond the SM

SFOEWPT
(for EWBG)

↓ can source

Gravitational Waves
(GW)

↑ sensitive to

Pulsar timing arrays, LISA
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Image credit: The SXS (Simulating eXtreme Spacetimes) Project

Gravitational Waves Detected 100 Years After Einstein's Prediction

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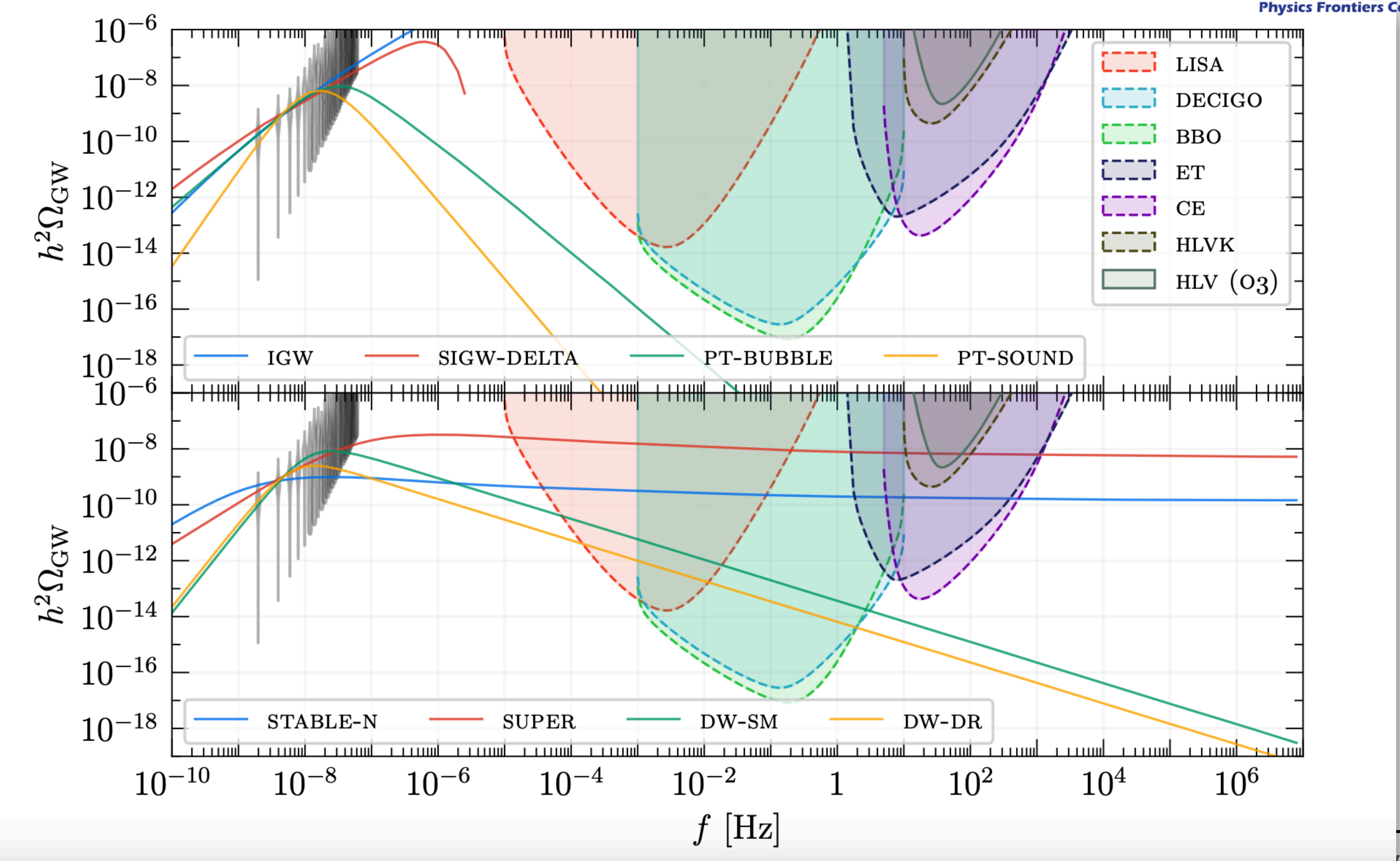
Experiments Sensitive to GWs from SFOPT

[Afzal et al.,'23]

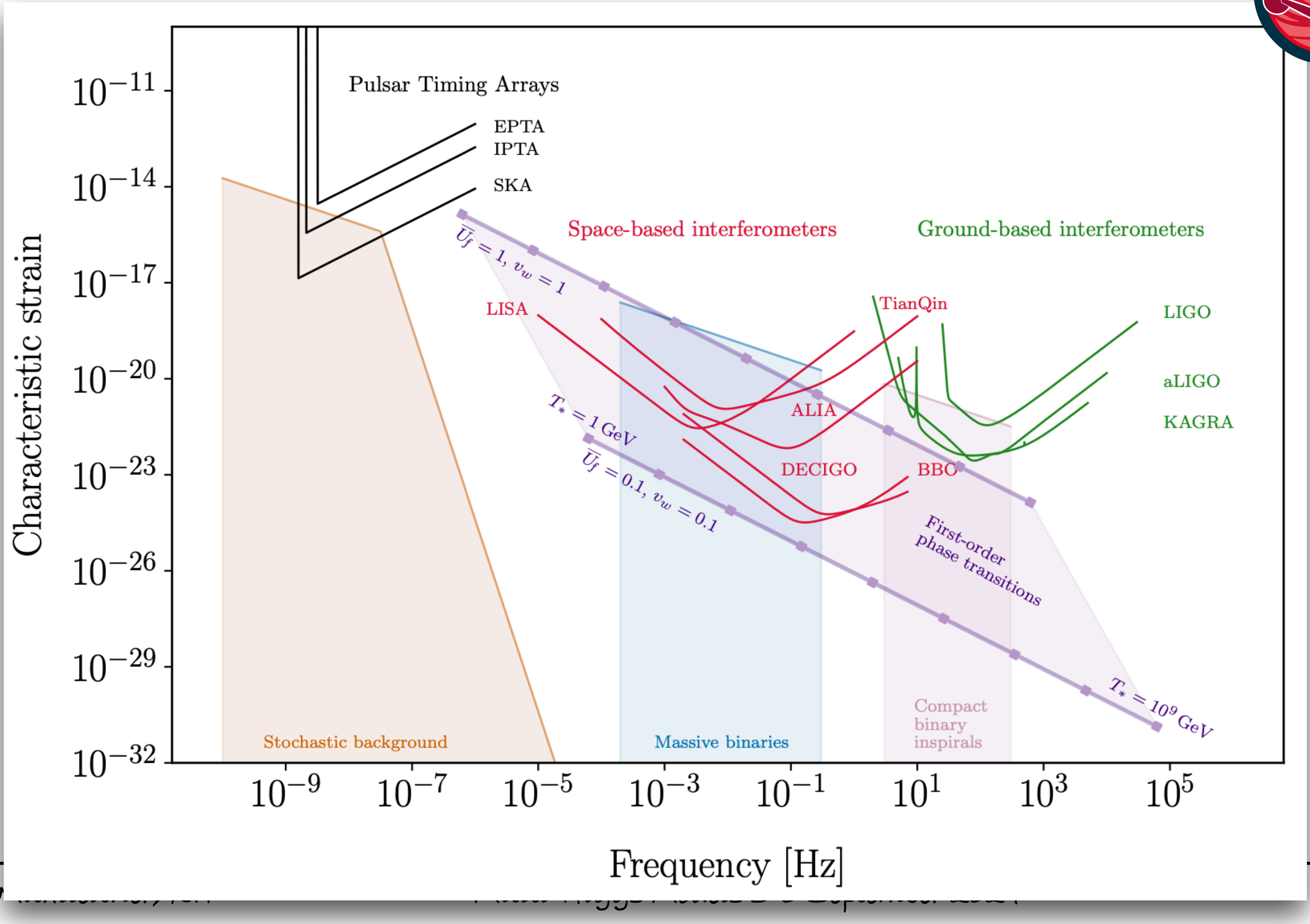
NANOGrav: 3 evidence for stochastic GW background which could come from SFOPT



Experiments Sensitive to GWs from SFOPT

LISA: expected sensitivity in frequency range associated w/ SFOPT

[Amaro-Seoane et al.,'17]



Physics Problem

The Challenge

- Combination of collider phenomenology and cosmological observations: insights into true physics underlying nature

- Challenge: spans over large range of energy scales involving different physics

- Requires: consistent combination of information from collider observables and gravitational waves observation

- Complexity: numerical solution of problem

- Needed: Code performing whole chain from particle physics model to gravitational waves*

*For an excellent review on the topic, see P, Athron, C. Balazs, A. Fowlie, L. Morris, L. Wu, „Cosmological phase transitions: From perturbative particle physics to gravitational waves“ Prog. Part. Nucl. Phys. 135 (2024) 104094

From Particle Physics Model to Gravitational Waves

- BSM model with extended Higgs sector at T=0:
  derive allowed parameter regions compatible w/ theoretical and experimental constraints

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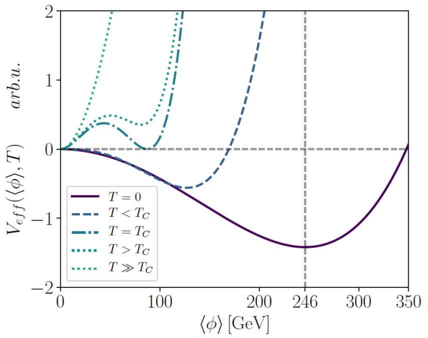
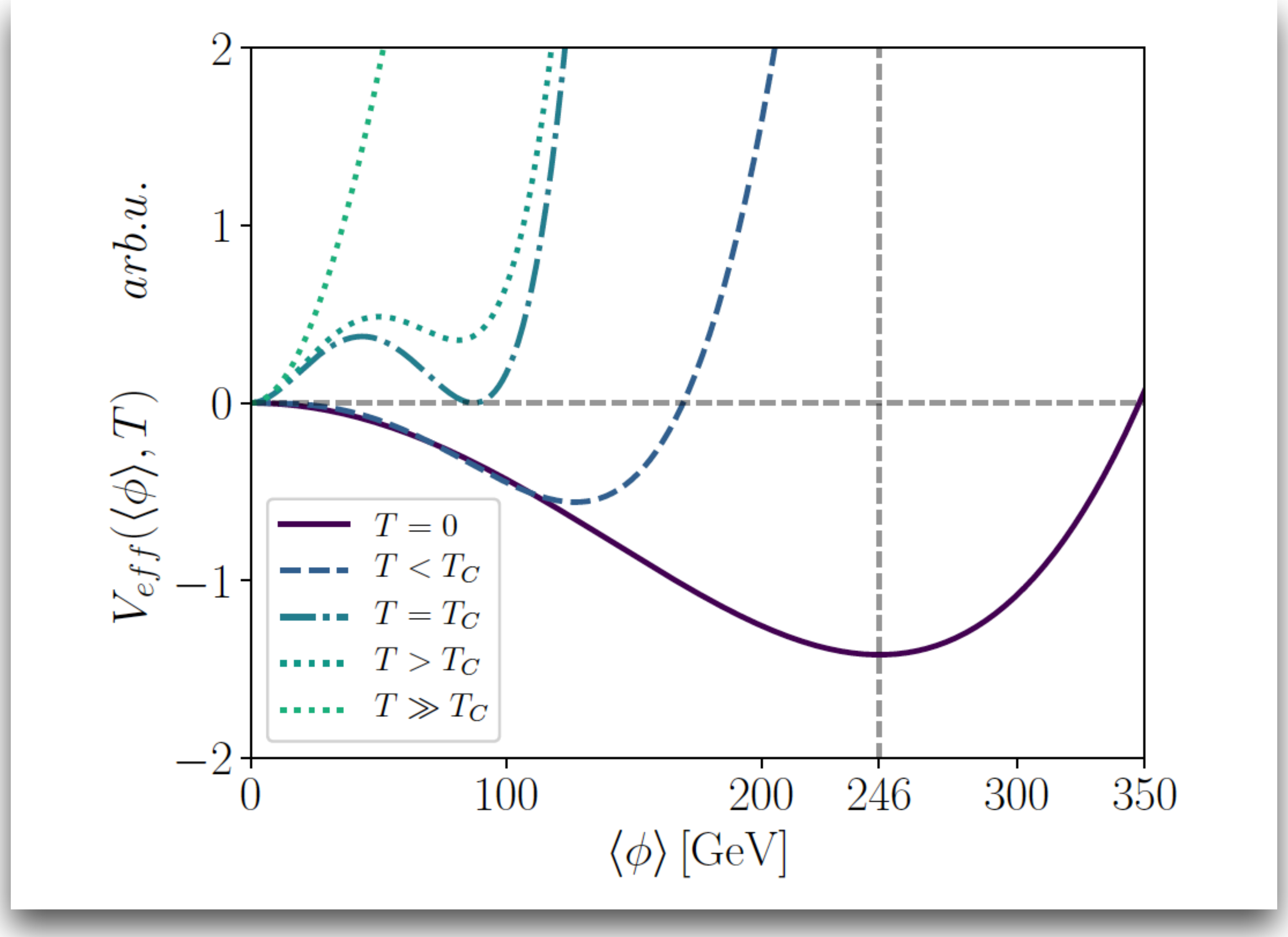
  - determine effective potential at non-zero temperature
  - trace vacuum phases as function of T
  - collect all coexisting phase pairs w/ their critical temperatures T_c



Strong First-Order Electroweak Phase Transition (SFOEWPT)

[From Ph. Basler, PhD Thesis]



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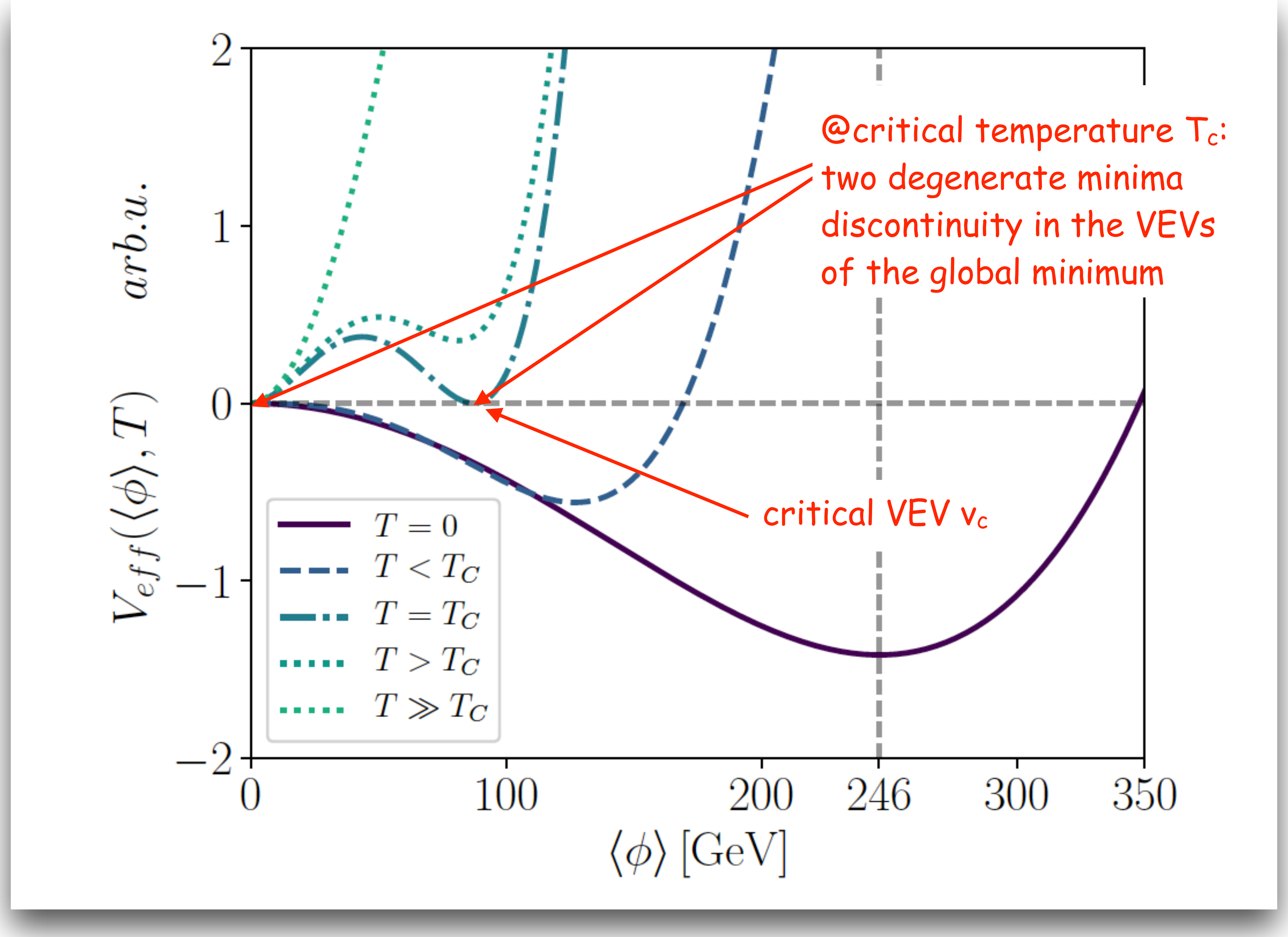


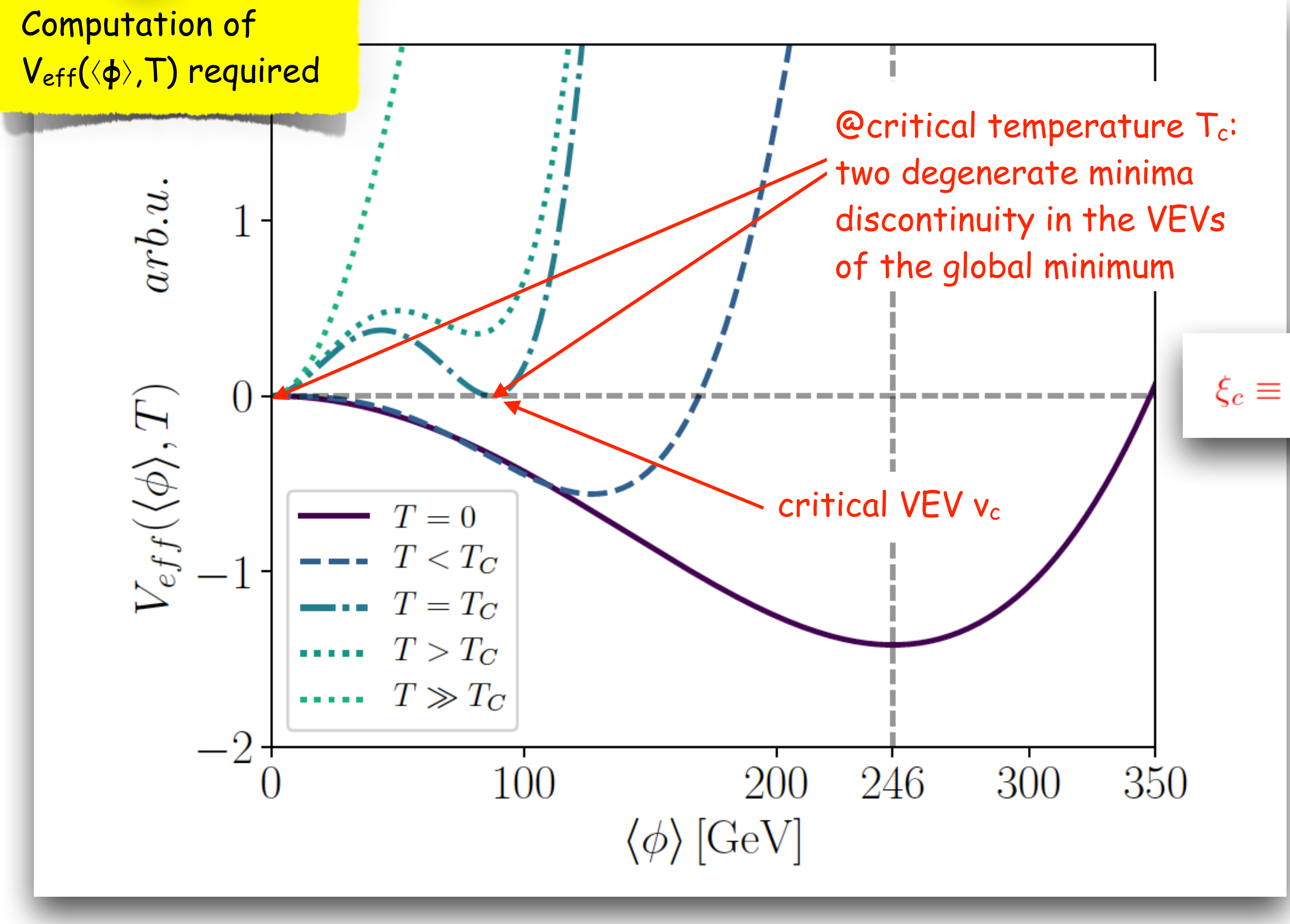
@critical temperature T_c:
two degenerate minima
discontinuity in the VEVs
of the global minimum

critical VEV v_c

Strong First-Order Electroweak Phase Transition (SFOEWPT)

Computation of $V_{eff}(\langle\phi\rangle,T)$ required

[From Ph. Basler, PhD Thesis]



@critical temperature T_c:
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critical VEV v_c

$$\xi_c \equiv \frac{\langle\Phi_c\rangle}{T_c} \geq 1$$

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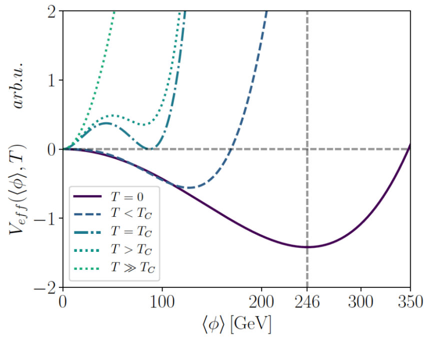

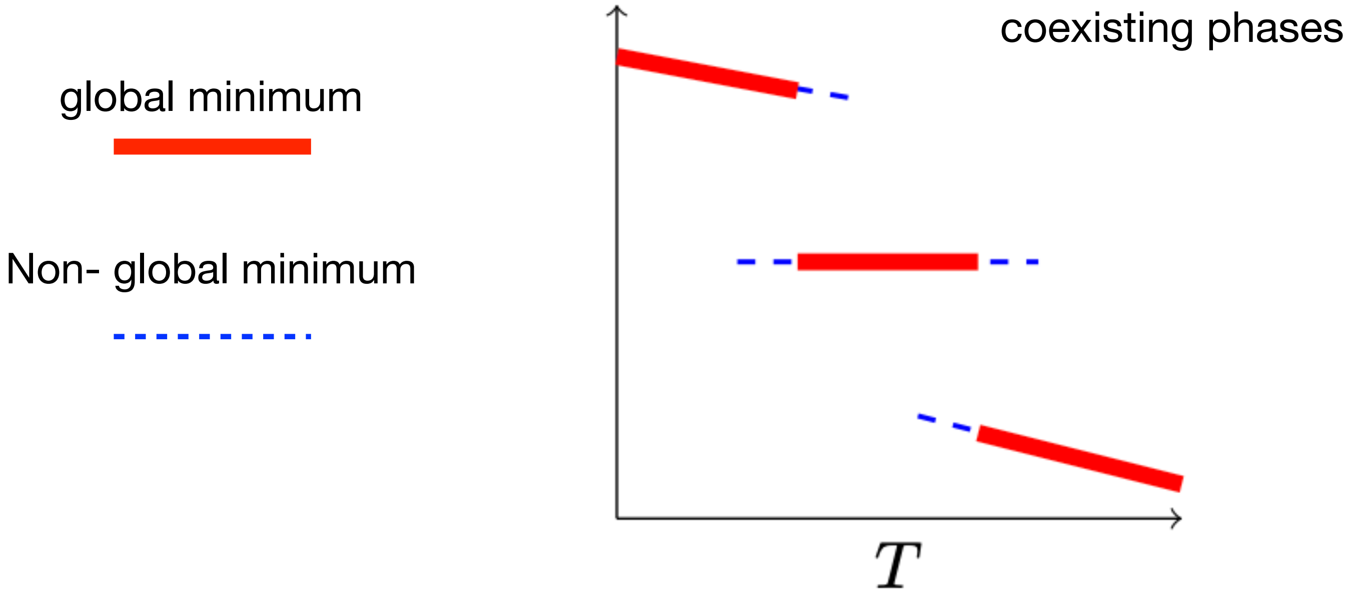

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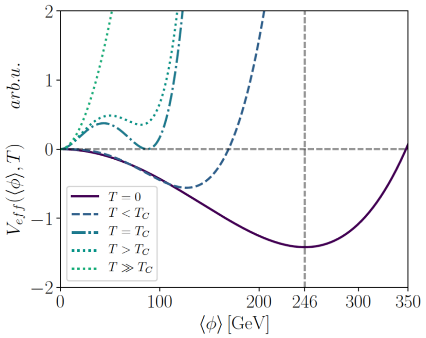
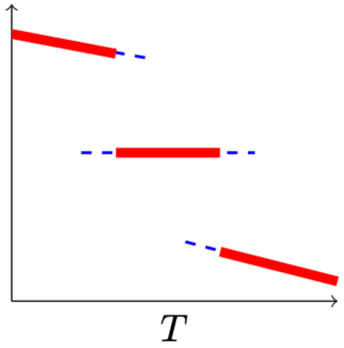


- Realization of phase transition between false and true minimum?:
  - computation of bounce action and tunneling rate for coexisting pairs w/ T_c



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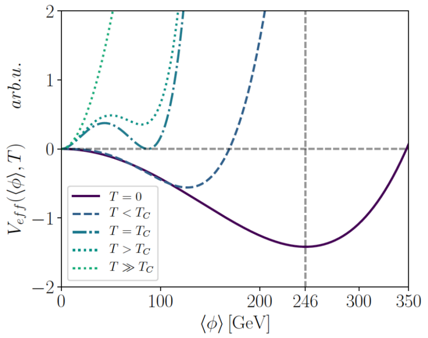

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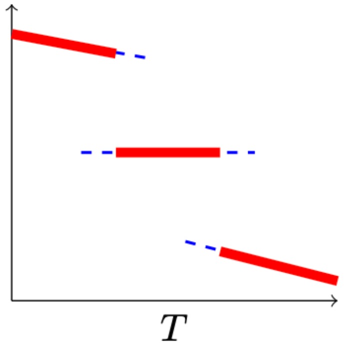

- Determination of temperatures characteristic for phase transition:
  - nucleation T_n, percolation T_p, completion T_f temperature

- Gravitational waves spectrum:
  - peak amplitude and frequency for each phase pair with a first-order phase transition
  - signal-to-noise ratio at LISA

Available Codes

- CosmoTransitions [Wainwright,'11]: minima tracing, bounce solution via path deformation
- BubbleProfiler [Athron eal,'19]: find bounce solution via semi-analytic algorithm [Akula eal,'16]
- PhaseTracer [Athron eal,'20]: similar algorithm as CosmoTransitions; handles discrete symmetries; can be linked to potentials in FlexibleSUSY [Athron eal] and BSMPT
- Vevacious [Camargo-Molino eal,'14/15]: gradient-based minimization of one-loop potential; wrapper of CosmoTransitions interfaced w/ models in SARAH [Staub] framework
- AnyBubble [Masoumi,'17]: via a multiple shooting algorithm
- EVADE [Hollik eat,'19]: minima of scalar potential via polynomial homotopy continuation, estimate of decay rate from bounce solution in one-field case
- SimpleBounce [Sato,'19]: via gradient flow method
- FindBounce [Guada eal,'18,'20]: via polygonal multifield method
- OptiBounce [Bardsley,'21]: vial solving the 'reduced' minimization problem [Coleman,'77]
- TransitionListener [Kahlhöfer,Tasillo,'21]: dark sector PT, GW spectra, signal-to-noise ratios

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New Code BSMPTv3 [Basler,Biermann,MM,Müller,Santos,Viana,]

- **Self-contained C++ code for the whole chain**
- **Tracing the phases of extended Higgs sectors**
- **Calculation of the bounce action**
- **Computation of the transition rate**
- **Computation of the characteristic temperatures (critical, nucleation, percolation, completion)**
- **Determination of latent heat release, inverse time scale**
- **Computation of gravitational waves spectrum**
- **Computation of Signal-to-Noise ratio at LISA**

Able to treat multistep phase transitions, discrete symmetries, flat directions, check for EWSR, reports transition history

The Code BSMPTV1/V2

The Code BSMPTv1

[v1:Basler,MM,'18]

Beyond-the-Standard-Model Phase Transitions - BSMPT:
A tool for electroweak phase transitions in extended Higgs sectors

- Computation of the loop-corrected effective potential V_{eff} at finite temperature, including thermal masses
- For extended Higgs sectors
- Determination of VEV $\langle\phi\rangle(T) \rightsquigarrow \xi_c = v_c/T_c$
- In on-shell (OS) renormalization scheme: masses and mixing angles from V_{eff}^{loop} are equal to leading-order values ⇝ efficient scan of parameter space of the models
- Computation of loop-corrected trilinear Higgs self-couplings in OS scheme
- Easy implementation of new models
- Programming language: C++

The Effective Potential

$$V^{(1)}(\omega,T) = \underbrace{V^{(0)}(\omega,T)}_{\text{tree-level}} + \underbrace{V^{CW}(\omega)}_{\substack{\text{T-indep.}\\ \text{Coleman-Weinberg}\\ \text{potential}\\ \text{MSbar renormalized}}} + \underbrace{V^{T}(\omega,T)}_{\substack{\text{T-dep. UV fin.}\\ \text{IR fin. after resumm.}\\ m^2 \text{ -> } m^2 + \Pi^{(1)}(0)}} + \underbrace{V^{CT}(\omega)}_{\substack{\text{finite shift of}\\ \text{scalar masses}\\ \text{\& mixing angles}}}$$

[Coleman,Weinberg,'73]

[Carrington,'92]
[Parwani,'92]
[Arnold,Espinosa,'93]

[Basler eal,'17]

BSMPT: Global minimization of loop-corrected effective potential at T ϵ {0,300} GeV in all possible field directions ω at T $\neq$ 0 GeV.

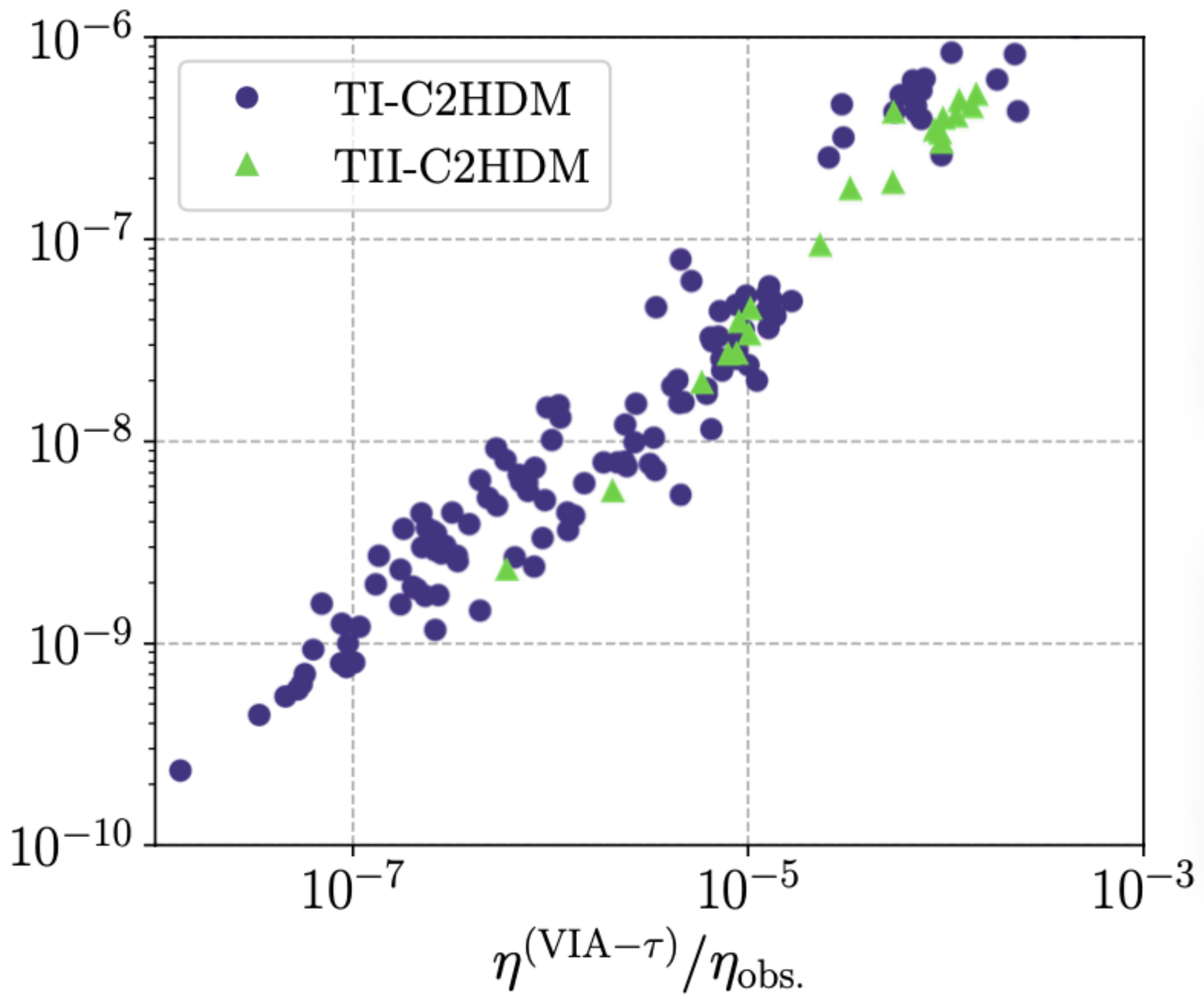
SFOEWPT: $\xi = v_c/T_c > 1$

The Code BSMPTV2

- Calculation of the BAU for the CP-violating 2HDM (C2HDM), using
  - * the semiclassical force approximation (FH)
  - * the VEV-insertion approximation (VIA)

  (wall profile is approximated by the Kink profile)
- Implementation of a new model: CxSM - further code improvements

[Basler,MM,Müller,'20, +Biermann,'21]



Take with caution VIA: [Postma eal,'22]: LO source term in VIA vanishes

Update of computation of baryogenesis, cf. talk by Joao Viana on Tuesday afternoon

Code available at: https://github.com/phbasler/BSMPT

Program: BSMPT version 2.6.0

Released by: Philipp Basler and Lisa Biermann and Margarete Mühlleitner and Jonas Müller

Manual: version 2.0

BSMPT - Beyond the Standard Model Phase Transitions: The C++ program package BSMPT calculates the strength of the electroweak phase transition in extended Higgs sectors. For this the loop-corrected effective potential at finite temperature is calculated including the daisy resummation of the bosonic masses. The program computes the vacuum expectation value (VEV) \f$ v \f$ of the potential as a function of the temperature, and in particular the critical VEV \f$v_c\f$ at the temperature \f$T_c\f$ where the phase transition takes place. In addition, the loop-corrected trilinear Higgs self-couplings are provided. We apply an 'on-shell' renormalization scheme in the sense that the loop-corrected masses and mixing angles are required to be equal to their tree-level input values. This allows for efficient scans in the parameter space of the models.

The models implemented so far are

- CP-conserving 2-Higgs-Doublet Models (R2HDM)
- CP-violating 2-Higgs-Doublet Models (C2HDM)
- Next-to-Minimal 2HDM (N2HDM)
- CP in the Dark (arXiv 1807.10322, arXiv 2204.13425)
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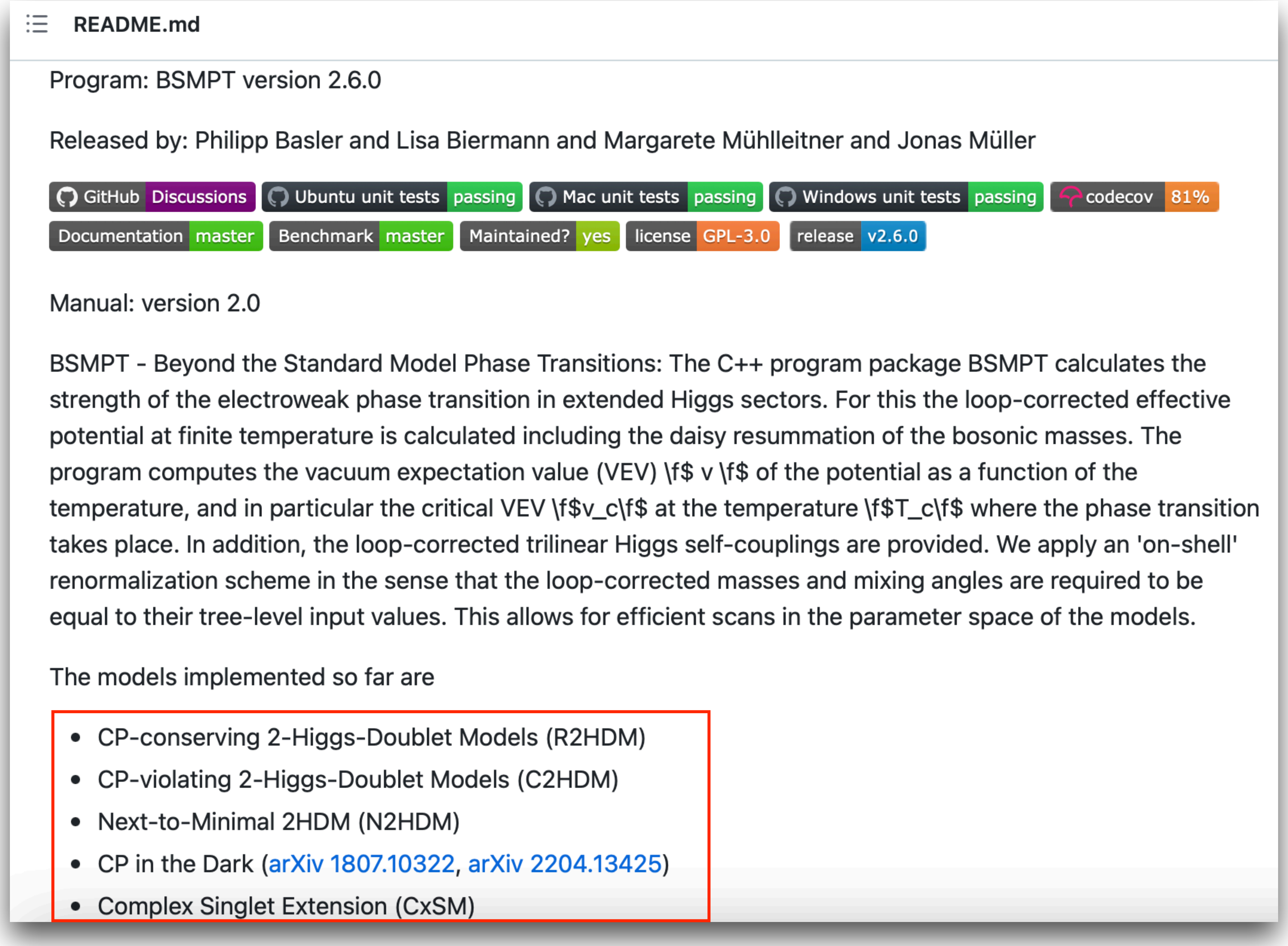

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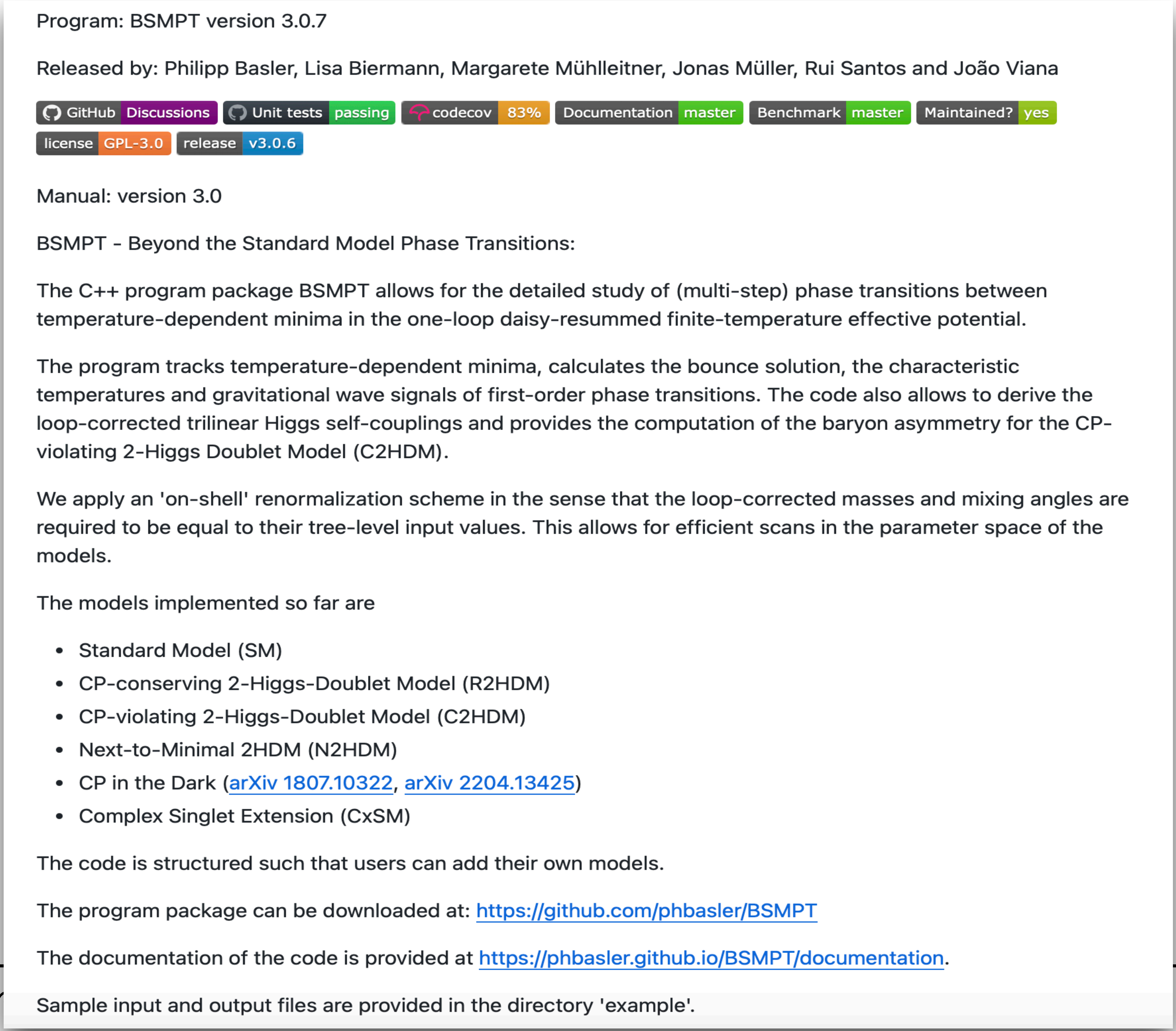
The Code BSMPTv3

Version 3: https://github.com/phbasler/BSMPT



Program: BSMPT version 3.0.7

Released by: Philipp Basler, Lisa Biermann, Margarete Mühlleitner, Jonas Müller, Rui Santos and João Viana

Manual: version 3.0

BSMPT - Beyond the Standard Model Phase Transitions:

The C++ program package BSMPT allows for the detailed study of (multi-step) phase transitions between temperature-dependent minima in the one-loop daisy-resummed finite-temperature effective potential.

The program tracks temperature-dependent minima, calculates the bounce solution, the characteristic temperatures and gravitational wave signals of first-order phase transitions. The code also allows to derive the loop-corrected trilinear Higgs self-couplings and provides the computation of the baryon asymmetry for the CP-violating 2-Higgs Doublet Model (C2HDM).

We apply an 'on-shell' renormalization scheme in the sense that the loop-corrected masses and mixing angles are required to be equal to their tree-level input values. This allows for efficient scans in the parameter space of the models.

The models implemented so far are

- Standard Model (SM)
- CP-conserving 2-Higgs-Doublet Model (R2HDM)
- CP-violating 2-Higgs-Doublet Model (C2HDM)
- Next-to-Minimal 2HDM (N2HDM)
- CP in the Dark (arXiv 1807.10322, arXiv 2204.13425)
- Complex Singlet Extension (CxSM)

The code is structured such that users can add their own models.

The program package can be downloaded at: https://github.com/phbasler/BSMPT

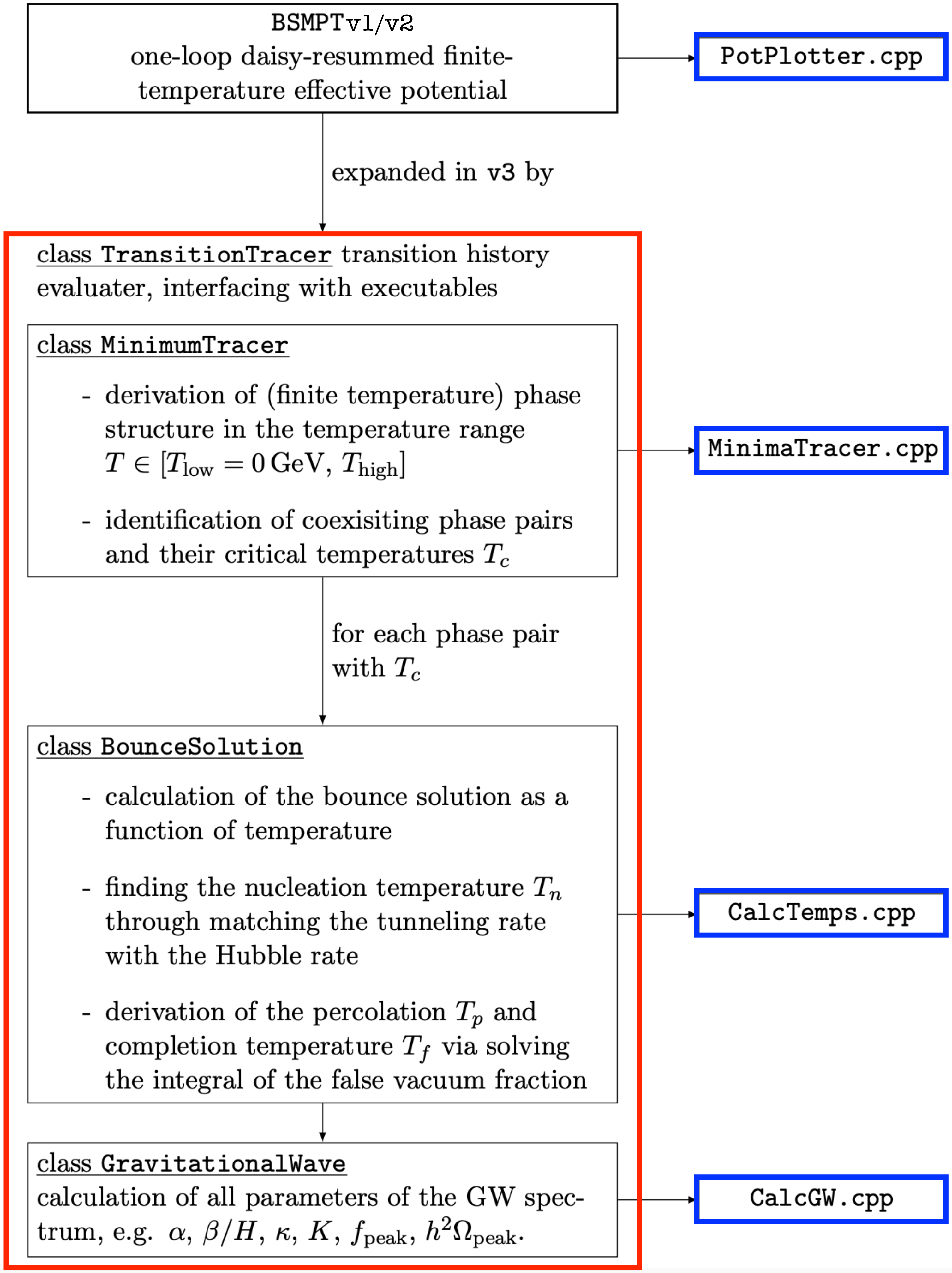
The documentation of the code is provided at https://phbasler.github.io/BSMPT/documentation.

Sample input and output files are provided in the directory 'example'.

Upgrade to BSMPTv3

Code structure
Classes
Functionalities

BSMPTv3
4 classes
4 executables



`BSMPTv1/v2`
one-loop daisy-resummed finite-temperature effective potential → `PotPlotter.cpp`

expanded in v3 by

class `TransitionTracer` transition history evaluater, interfacing with executables

class `MinimumTracer` → `MinimaTracer.cpp`

- derivation of (finite temperature) phase structure in the temperature range $T \in [T_{\mathrm{low}} = 0\,\mathrm{GeV},\, T_{\mathrm{high}}]$
- identification of coexisiting phase pairs and their critical temperatures T_c

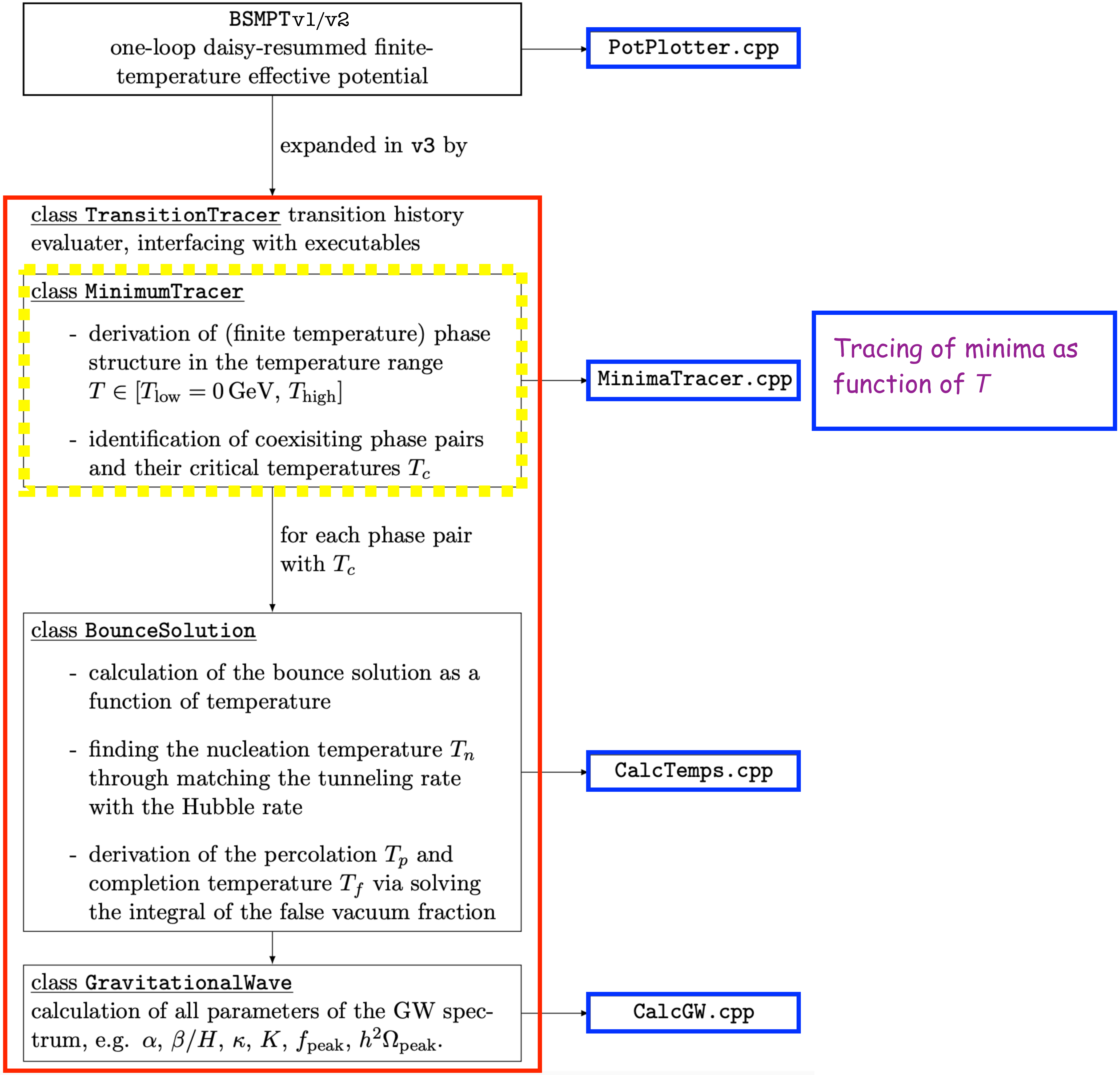
for each phase pair with T_c

class `BounceSolution` → `CalcTemps.cpp`

- calculation of the bounce solution as a function of temperature
- finding the nucleation temperature T_n through matching the tunneling rate with the Hubble rate
- derivation of the percolation T_p and completion temperature T_f via solving the integral of the false vacuum fraction

class `GravitationalWave` → `CalcGW.cpp`
calculation of all parameters of the GW spectrum, e.g. α, β/H, κ, K, f_{peak}, $h^2\Omega_{\mathrm{peak}}$.

BSMPTv3
4 classes
4 executables



`BSMPTv1/v2`
one-loop daisy-resummed finite-temperature effective potential

→ `PotPlotter.cpp`

expanded in `v3` by

class **`TransitionTracer`** transition history evaluater, interfacing with executables

class **`MinimumTracer`**

- derivation of (finite temperature) phase structure in the temperature range $T \in [T_{\text{low}} = 0\,\text{GeV},\, T_{\text{high}}]$
- identification of coexisiting phase pairs and their critical temperatures T_c

→ `MinimaTracer.cpp`

Tracing of minima as function of T

for each phase pair with T_c

class **`BounceSolution`**

- calculation of the bounce solution as a function of temperature
- finding the nucleation temperature T_n through matching the tunneling rate with the Hubble rate
- derivation of the percolation T_p and completion temperature T_f via solving the integral of the false vacuum fraction

→ `CalcTemps.cpp`

class **`GravitationalWave`**
calculation of all parameters of the GW spectrum, e.g. α, β/H, κ, K, f_{peak}, $h^2\Omega_{\text{peak}}$.

→ `CalcGW.cpp`

The Class MinimumTracer

- Phase Tracker: find coexisting phase pairs and their critical temperature
  seed points for global minimum search w/ gradient-free methods GSL, libcmaes, NLopt
  to find minimum: search for points w/ vanishing gradient w/ Newton-Raphson method

- Discrete Symmetries: models w/ discrete $\mathbb{Z}_2$ symmetries increase # of possible minima°
  BSMPTv3 identifies symmetries: * save computational time * symmetry-related minima may have different transition rates to other minima, take transition w/ lowest action

---

°Model w/ spontan. broken discrete symmetries ⇝ domain walls; domain wall effects not considered presently.

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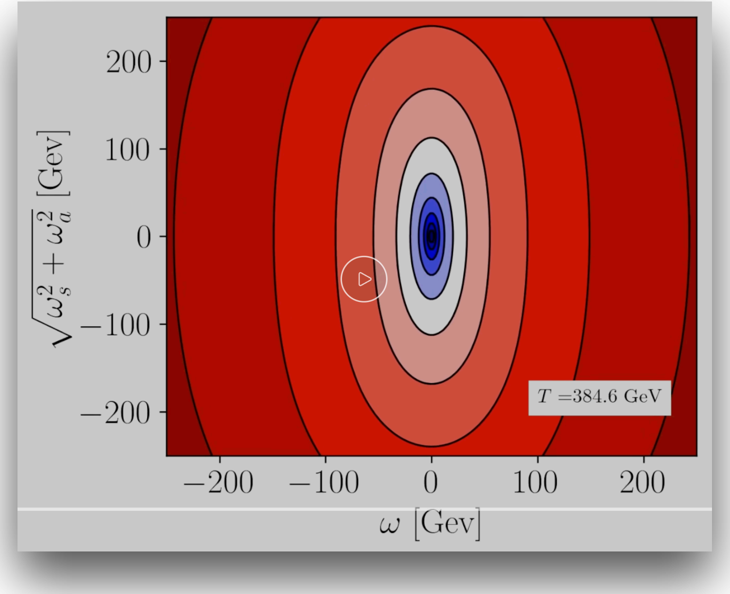
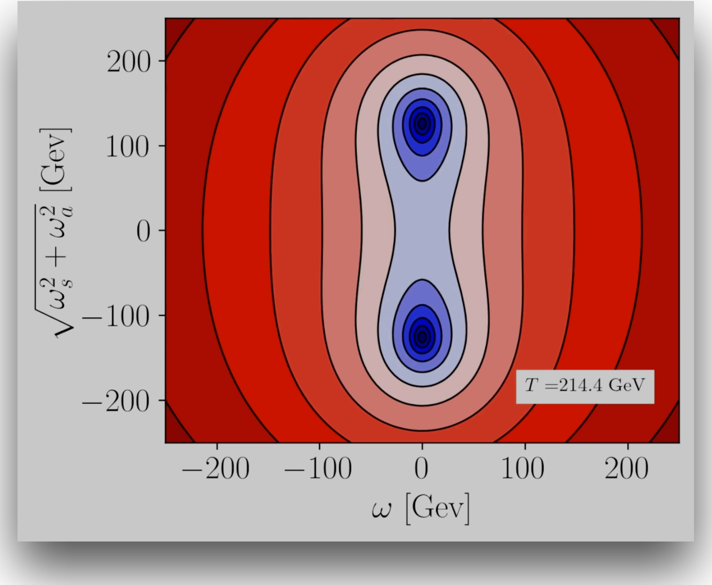
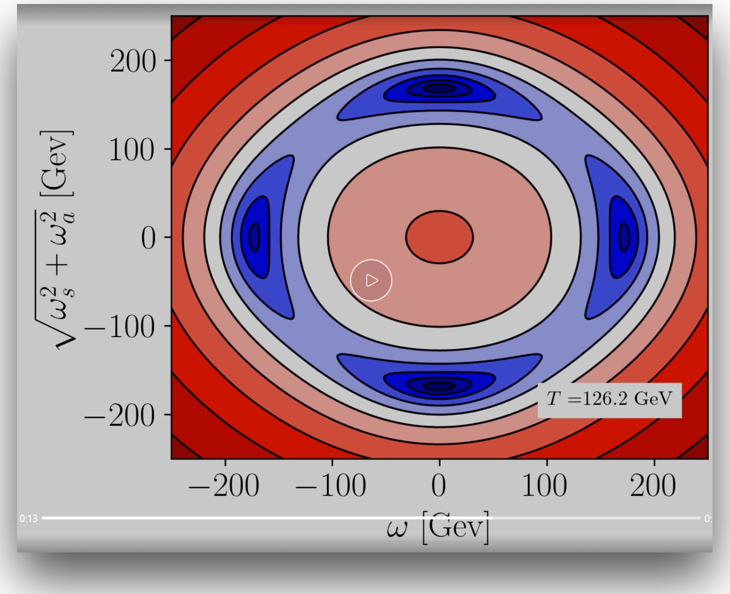
  BSMPTv3 identifies symmetries: * save computational time * symmetry-related minima
  may have different transition rates to other minima, take transition w/ lowest action







°Model w/ spontan. broken discrete symmetries → domain walls; domain wall effects not considered presently.

The Class MinimumTracer

- Phase Tracker: find coexisting phase pairs and their critical temperature
  seed points for global minimum search w/ gradient-free methods GSL, libcmaes, NLopt
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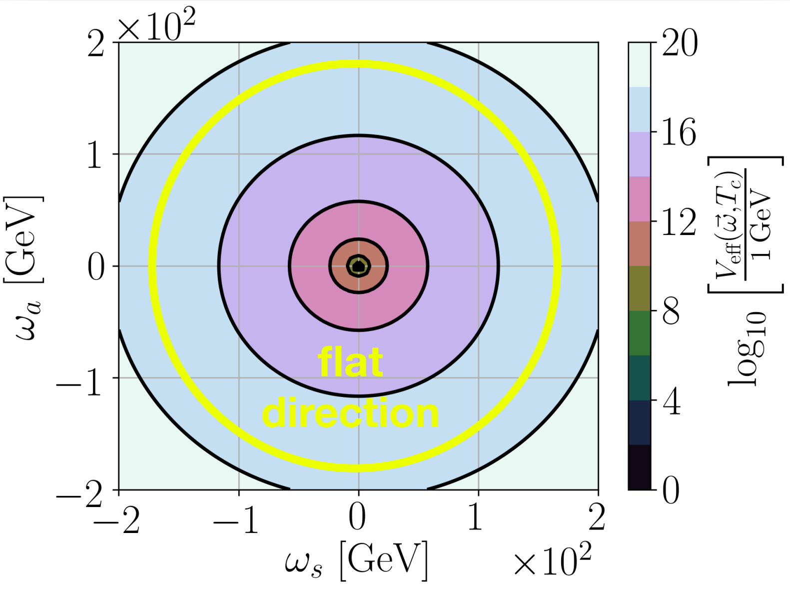

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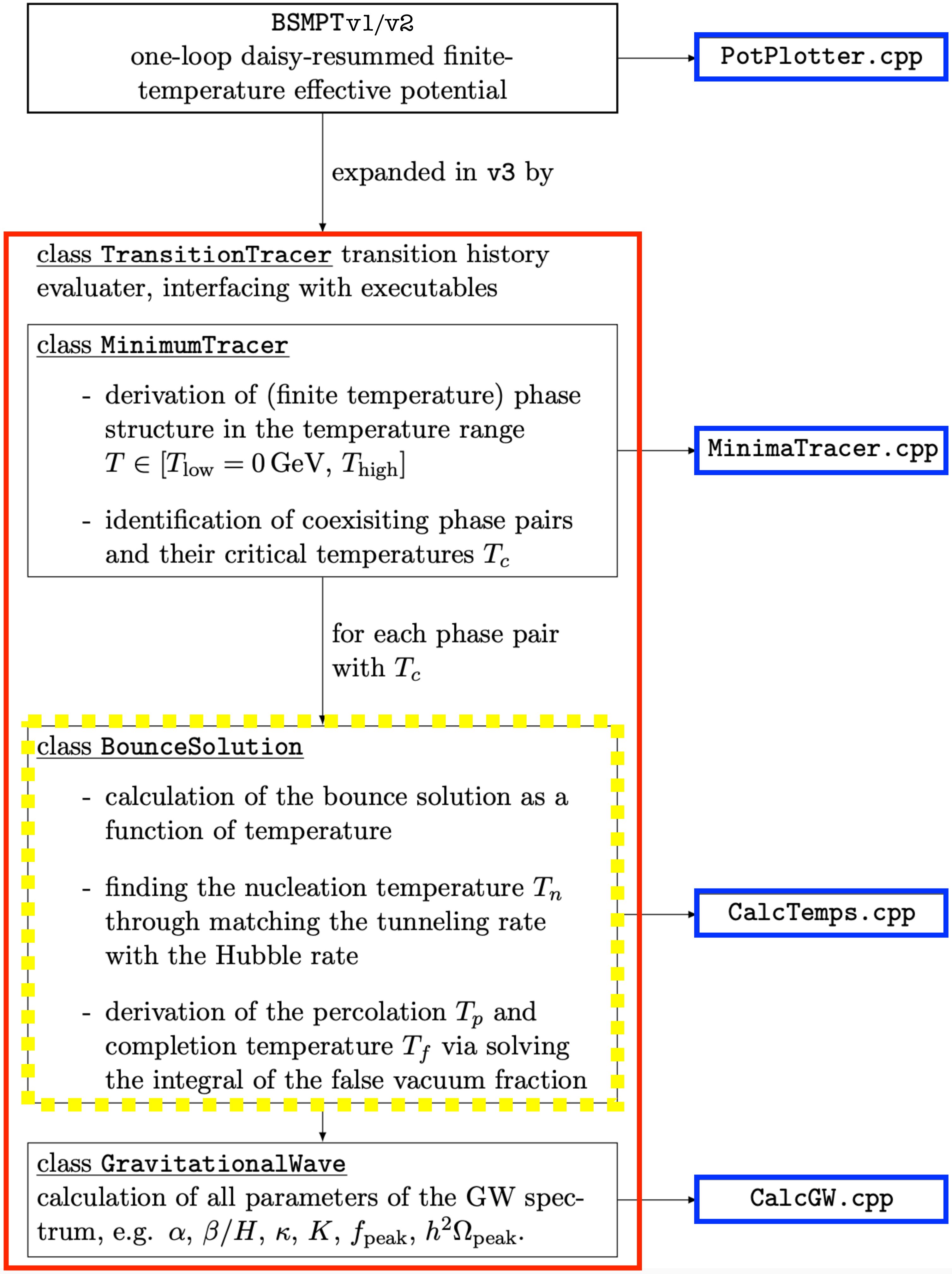
- Electroweak symmetry restoration: symmetry might not be restored at high T
  [Meade,Ramani,'18;Baldes,Servant,'18;Matsedonskyi,Servant,'20;Carena eal,'21;Biekötter eal,'21,'22]

  high-T behavior depending on implementation of thermal masses:
  ~ T^2 (Arnold-Espinosa) resp. ~ $T^2/\log T^2$ (Parwani)

  rescaled potential V_{eff}/T^2 resp. $V_{eff}/(T^2 \log T^2)$ only quadratic function in field directions
  ~> minimization allows for check of electroweak symmetry restoration

BSMPTv3
4 classes
4 executables



BSMPTv1/v2
one-loop daisy-resummed finite-temperature effective potential

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expanded in v3 by

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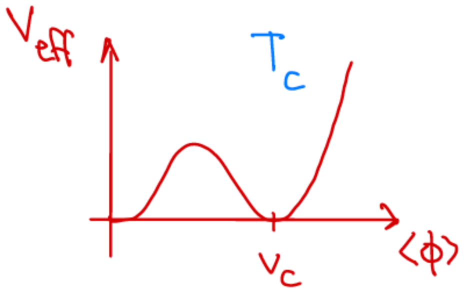
`CalcGW.cpp`

Calculation of the bounce solution and characteristic temperatures for first-order phase transitions between pairs of coexisting phases

The Class BounceSolution Solution of Bounce Equation & Temp. Scales

- Critical Temperature T_c: true (v≠0) and false (v=0) vacuum are degenerate, PT starts via quantum tunneling



- Nucleation Temperature T_n: one bubble nucleated per cosmological horizon
  if not matched → vacuum trapped in false vacuum
  [see e.g. Baum eal,'21; Biekötter eal,'23]

calculated from (tunneling decay rate per Hubble volume matches Hubble rate)

$$\frac{\Gamma(T_n)}{H^4(T_n)} = 1$$

or: approximate temperature from (used in CosmoTransitions)

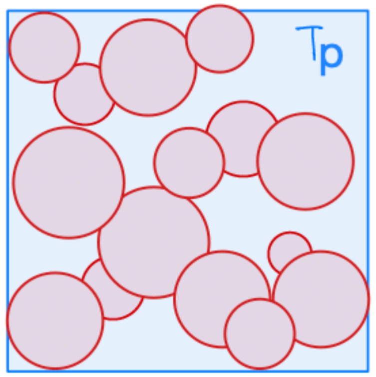
$$\frac{S_3(T_n)}{T_n} \sim 140$$

The Class BounceSolution Solution of Bounce Equation & Temp. Scales

- **Percolation temperature T_p:** probability of finding point in false vacuum is 70%

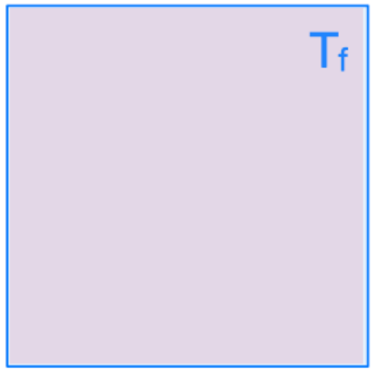
[Ellis eal,'19]

$$P_f(T_*) = \mathrm{e}^{-I(T_*)} \equiv 0.7 \text{ with } I(T) = \frac{4\pi v_b^3}{3} \int_T^{T_c} \frac{\Gamma(T')dT'}{T'^4 H(T')} \left(\int_T^{T'} \frac{d\tilde{T}}{H(\tilde{T})} \right)^3$$



- **Completion temperature T_f:** probability of finding point in false vacuum is 1% transition completes

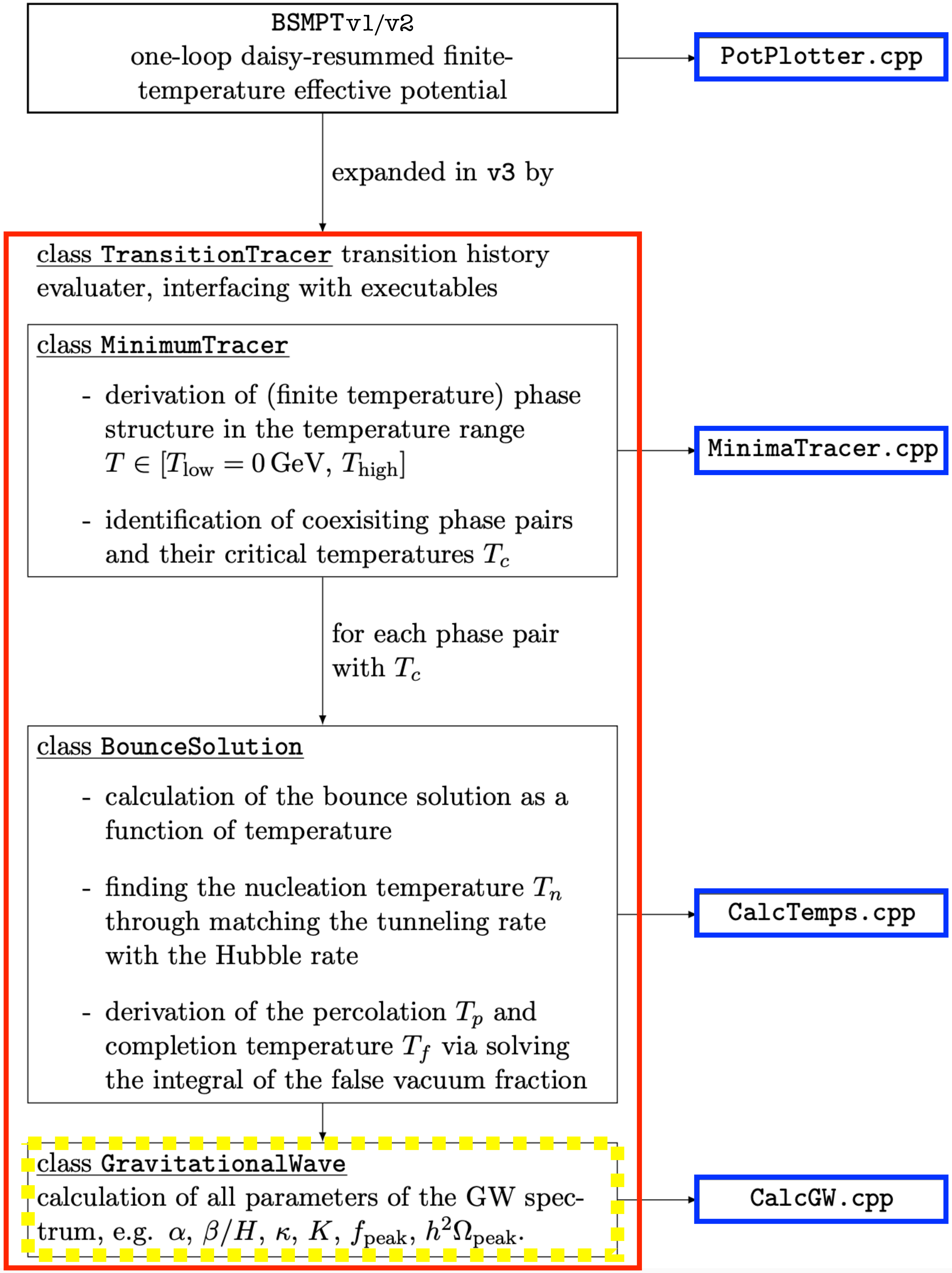
$$P_f(T_f) = \epsilon_f \qquad \epsilon_f = 0.01$$



BSMPTv3

4 classes

4 executables



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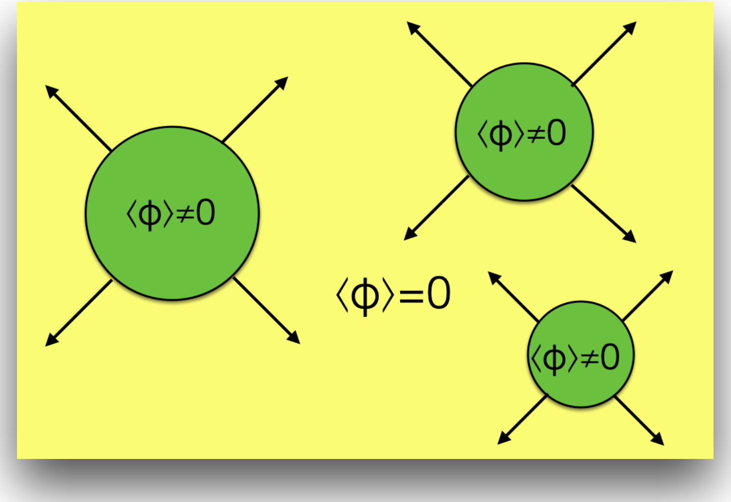
`CalcGW.cpp`

Calculation of the gravitational waves spectrum sourced by first-order phase transitions

Gravitational Waves Spectrum



- FOEWPT ⤳ bubble nucleation

- Bubble wall expands into hot plasma
  ⇒ spherical symmetry breaking ⤳ gravitational waves

- Source of gravitational waves:

  - bubble collisions and mergers [Kosowsky et al.,'92,93,94]

  - magnetohydronamic turbulence (shocks in fluid) [Caprini,Durer,'06;Kahniashvili eal,'08/10]

  - sound waves (bubble-wall accelerated plasma) [Giblin,Mertens,'13/14;Hindmarsh eal,'14/15]

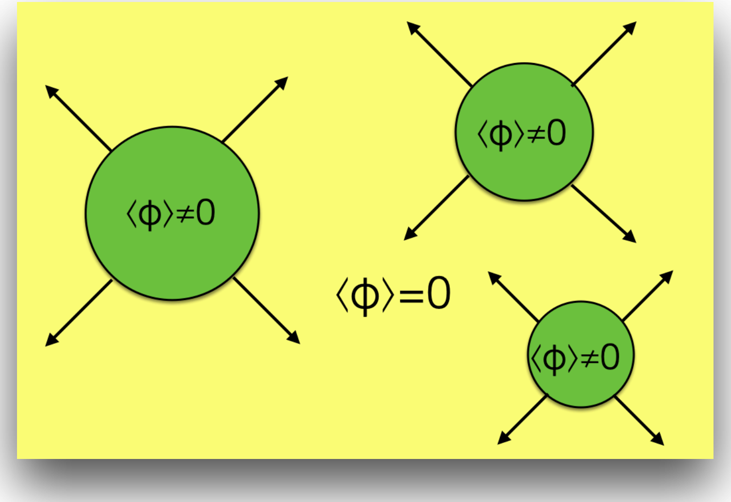
- Sound waves dominant contribution:

  if bubble walls interact w/ surrounding plasma, hence for

  - enough friction with the plasma ⇒ non-runaway bubbles, reaching a terminal velocity v_b

  - no early onset of turbulent regime, released latent heat $\alpha < 1$

Gravitational Waves Spectrum



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implemented
in BSMPTv3

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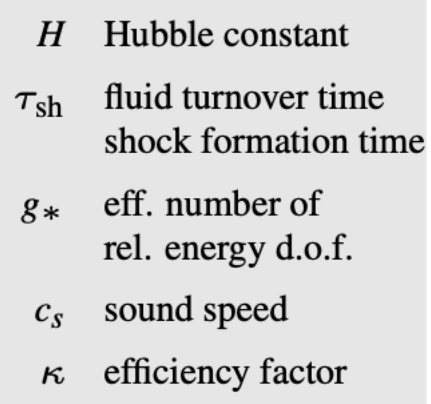
Computation of the Gravitational Waves Spectrum

- Relevant quantities for GW spectrum:
  - PT strength, resp. released latent heat during PT

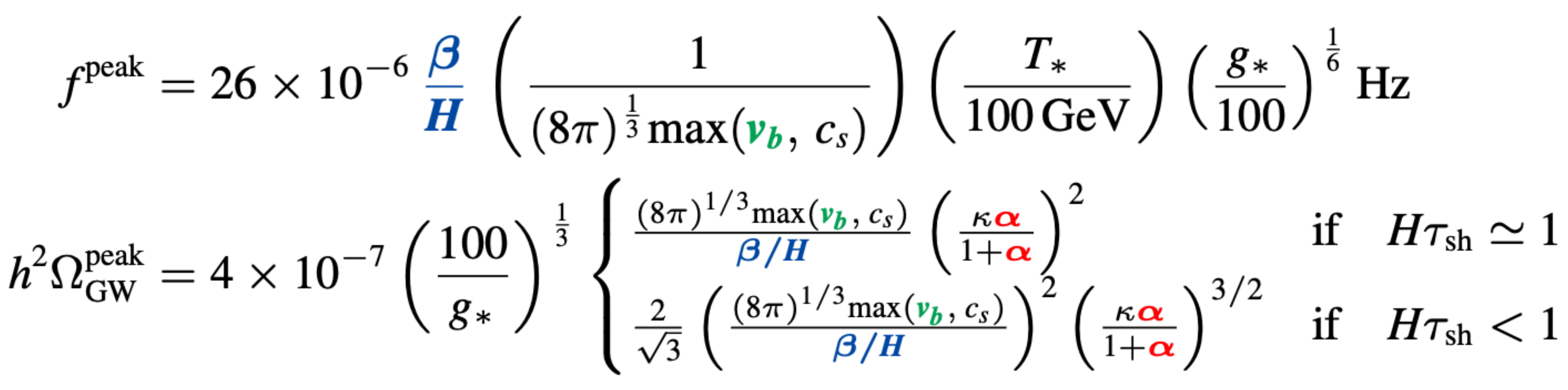
$$\alpha = \frac{1}{\rho_\gamma}\left[V(\vec{\phi}_f) - V(\vec{\phi}_t) - \frac{T}{4}\left(\frac{\partial V(\vec{\phi}_f)}{\partial T} - \frac{\partial V(\vec{\phi}_t)}{\partial T}\right)\right]_{T=T_*}$$

  - inverse time scale of the PT $\frac{\beta}{H} = T_* \frac{d}{dT}\left(\frac{\hat{S}_3(T)}{T}\right)\Big|_{T_*}$
  - bubble wall velocity v_b



H	Hubble constant
$\tau_{\rm sh}$	fluid turnover time shock formation time
g_*	eff. number of rel. energy d.o.f.
c_s	sound speed
κ	efficiency factor

- Peak frequency and amplitude of acoustic GWs [Hindmarsh eal,'17;Caprini eal,'20]



$$f^{\rm peak} = 26 \times 10^{-6}\, \frac{\beta}{H}\left(\frac{1}{(8\pi)^{\frac{1}{3}}\max(v_b,\, c_s)}\right)\left(\frac{T_*}{100\,{\rm GeV}}\right)\left(\frac{g_*}{100}\right)^{\frac{1}{6}} {\rm Hz}$$

$$h^2\Omega^{\rm peak}_{\rm GW} = 4 \times 10^{-7}\left(\frac{100}{g_*}\right)^{\frac{1}{3}} \begin{cases} \frac{(8\pi)^{1/3}\max(v_b, c_s)}{\beta/H}\left(\frac{\kappa\alpha}{1+\alpha}\right)^2 & \text{if} \quad H\tau_{\rm sh} \simeq 1 \\ \frac{2}{\sqrt{3}}\left(\frac{(8\pi)^{1/3}\max(v_b, c_s)}{\beta/H}\right)^2\left(\frac{\kappa\alpha}{1+\alpha}\right)^{3/2} & \text{if} \quad H\tau_{\rm sh} < 1 \end{cases}$$

The Signal-to-Noise Ratio

- Signal-to-noise ratio at LISA:

$$\mathrm{SNR} = \sqrt{\mathcal{T} \int_{f_{\min}}^{f_{\max}} \mathrm{d}f \left[\frac{h^2\Omega_{\mathrm{GW}}(f)}{h^2\Omega_{\mathrm{Sens}}(f)} \right]^2}$$

$h^2\Omega_{\mathrm{Sens}}$ nominal sensitivity of a given LISA configuration to stochastic sources

$\mathcal{T}$ experimental acquisition time in seconds (4 years, min. duty cycle of 75%)

$f_{\min}$, $f_{\max}$ minimum and maximum frequency to which LISA is sensitive

- In BSMPTv3:

$$\text{SNR in } \mathbf{BSMPTv3}\text{: } \mathrm{SNR}(3\,\text{years})$$

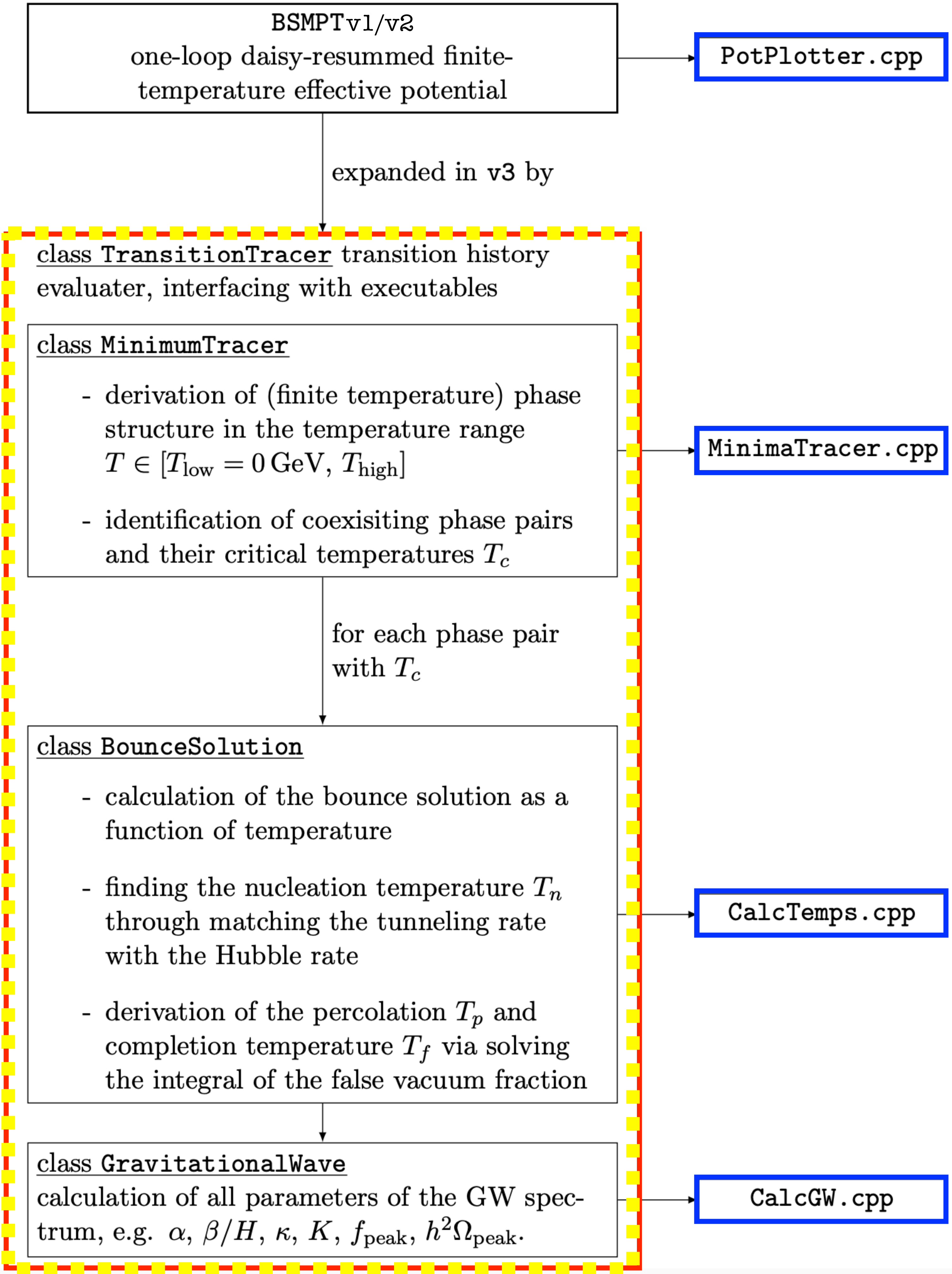
for Y years:

$$\mathrm{SNR}(\mathcal{Y}) = \sqrt{\frac{\mathcal{Y}}{3}}\,\mathrm{SNR}(3\,\text{years})$$

BSMPTv3

4 classes

4 executables



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The Class TransitionTracer

- Management of the calculation:
  interfaces the classes MinimumTracer, BounceSolution, GravitationalWave with the executables

- Reports transition history for each point

Results and Comparisons

Results: Benchmark Points

- Benchmark points `BP1-BP3`: for the models 2HDM and CxSM
  both models exhibit several minimum directions

- Benchmark point `BP1` in the 2-Higgs-Doublet-Model: four minimum directions

$$V_{\text{tree}} = m_{11}^2 \Phi_1^\dagger \Phi_1 + m_{22}^2 \Phi_2^\dagger \Phi_2 - \left[m_{12}^2 \Phi_1^\dagger \Phi_2 + \text{h.c.} \right] + \frac{1}{2}\lambda_1 (\Phi_1^\dagger \Phi_1)^2 + \frac{1}{2}\lambda_2 (\Phi_2^\dagger \Phi_2)^2 \\ + \lambda_3 (\Phi_1^\dagger \Phi_1)(\Phi_2^\dagger \Phi_2) + \lambda_4 (\Phi_1^\dagger \Phi_2)(\Phi_2^\dagger \Phi_1) + \left[\frac{1}{2}\lambda_5 (\Phi_1^\dagger \Phi_2)^2 + \text{h.c.} \right] .$$

$$\Phi_1 = \frac{1}{\sqrt{2}} \begin{pmatrix} \rho_1 + i\,\eta_1 \\ \zeta_1 + \omega_1 + i\,\psi_1 \end{pmatrix} , \quad \Phi_2 = \frac{1}{\sqrt{2}} \begin{pmatrix} \rho_2 + \omega_{\text{CB}} + i\,\eta_2 \\ \zeta_2 + \omega_2 + i\,(\psi_2 + \omega_{\text{CP}}) \end{pmatrix}$$

$$\{\omega_{\text{CB}}, \omega_1, \omega_2, \omega_{\text{CP}}\}\big|_{T=0} = \{0, v_1, v_2, 0\} , \text{ with}$$

$$\omega_{\text{EW}}\big|_{T=0} \equiv \sqrt{\omega_1^2 + \omega_2^2 + \omega_{\text{CB}}^2 + \omega_{\text{CP}}^2}\Bigg|_{T=0} = \sqrt{v_1^2 + v_2^2} \equiv v = 246 \text{ GeV}$$

Benchmark Point BP1

- BP1 input parameters:

From [Aoki,Biermann,Borschensky,Ivanov,MM,Sakurai,'23]

$$\texttt{BP1:} \quad \text{type} = 1\,,\ \lambda_1 = 6.931\,,\ \lambda_2 = 0.2631\,,\ \lambda_3 = 1.287\,,\ \lambda_4 = 4.772\,,\ \lambda_5 = 4.728\,,$$

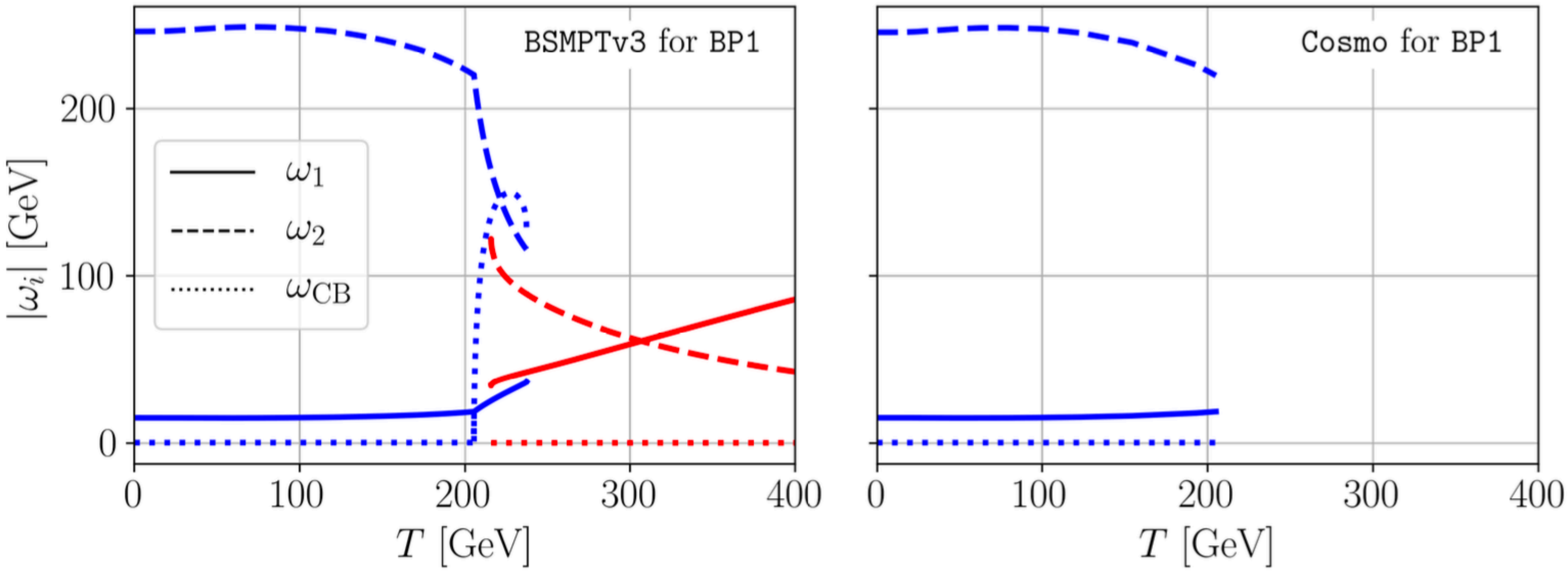
$$m_{12}^2 = 1.893 \times 10^4\,\mathrm{GeV}^2\,,\ \tan\beta = 16.578\,.$$

- In the following comparisons w/ CosmoTransitions

Transition History - Comparison w/ CosmoTransitions

high-T phase, low-T phase

[Basler,Biermann,MM,Müller,Santos,Viana,´24]



History:

BSMPTv3

- first-order PT from neutral (red) to charge-breaking CB phase (blue)
- second-order PT into a neutral minimum

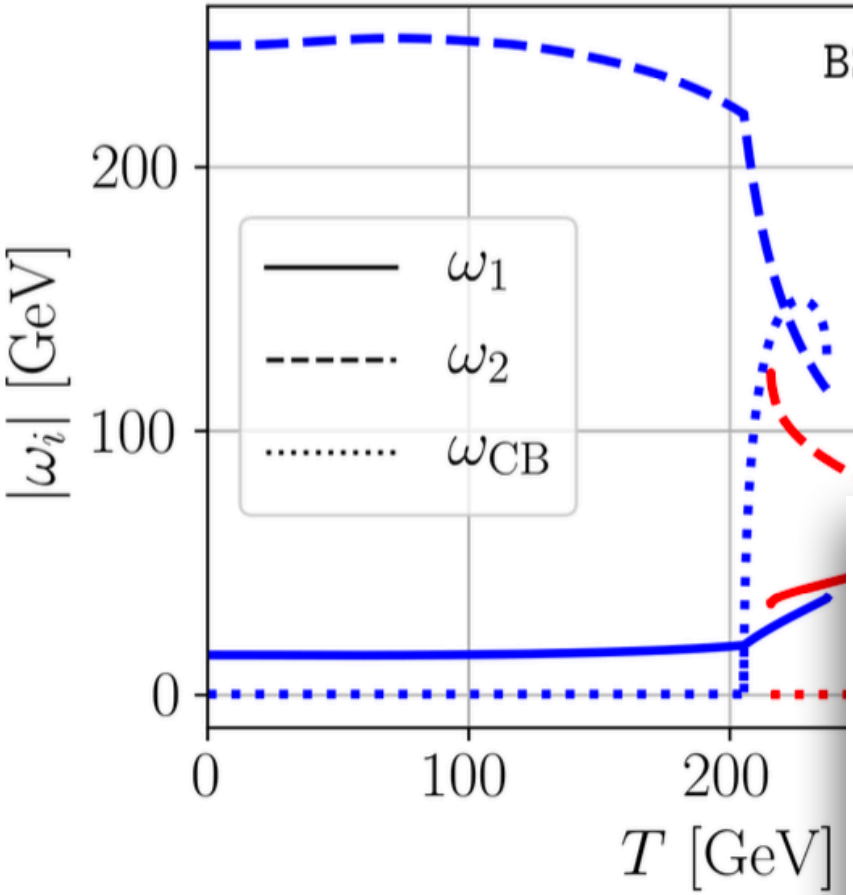
CosmoTransitioins

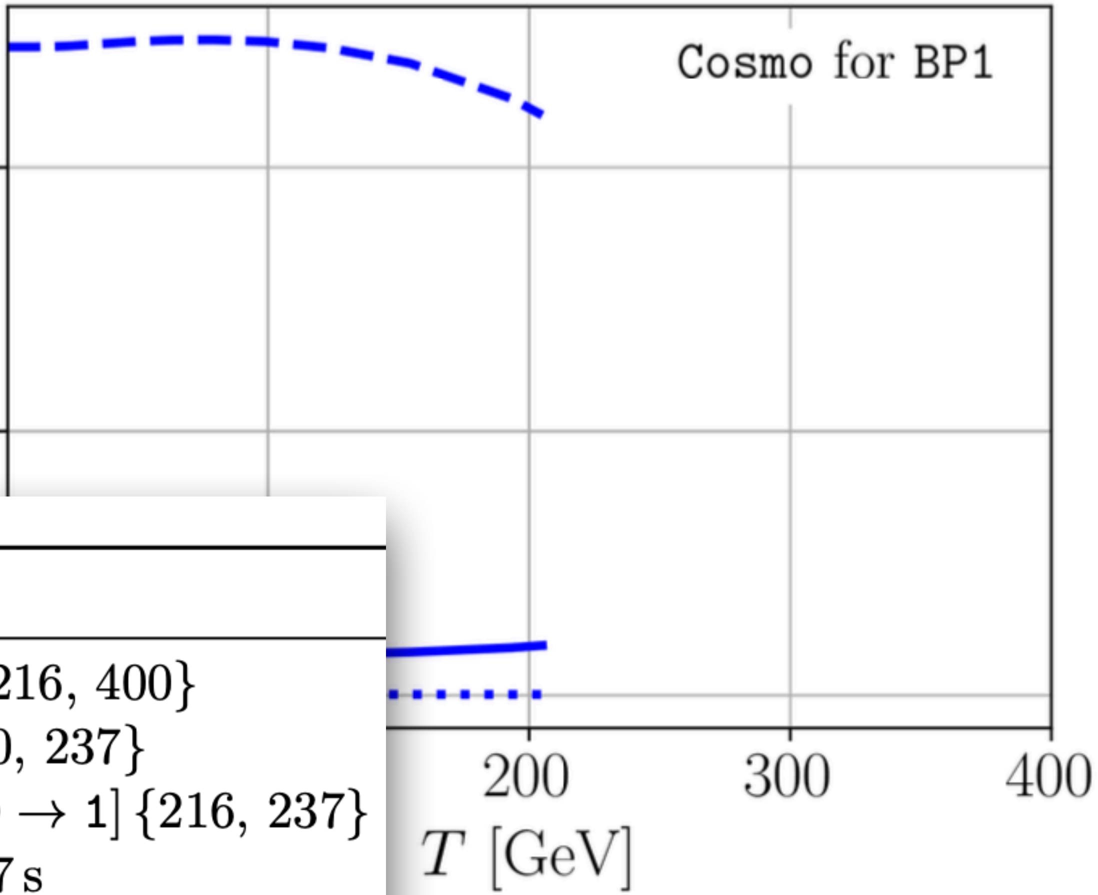
- agrees w/ low-T phase until T~200 GeV
- fails to trace any minima for higher temperatures

Transition History - Comparison w/ CosmoTransitions

high-T phase, low-T phase

[Basler,Biermann,MM,Müller,Santos,Viana,´24]





	BP1
phases$_{\texttt{BSMPT}}$	0: {216, 400} 1: {0, 237}
pairs$_{\texttt{BSMPT}}$	0: [0 → 1] {216, 237}
$t_{\texttt{MinimaTracer}}$	41.47 s
T_c	{226.3}
T_n	{222.9, 222.9}
T_p	{222.6}
T_f	{222.6}
$t_{\texttt{CalcTemps}}$	6.87 min
history	0 — (0) → 1
phases$_{\texttt{Cosmo}}$	{0, 206}
$T_{\text{crit},\texttt{Cosmo}}$	{−}
$T^{\text{approx}}_{\text{nucl},\texttt{Cosmo}}$	{−}
$t_{\texttt{Cosmo}}$	3.95 s

only finds one phase though

BSMPTv3

- first-order PT from neu
  to charge-breaking CB p
- second-order PT into a
  minimum

smoTransitioins

es w/ low-T phase

T~200 GeV

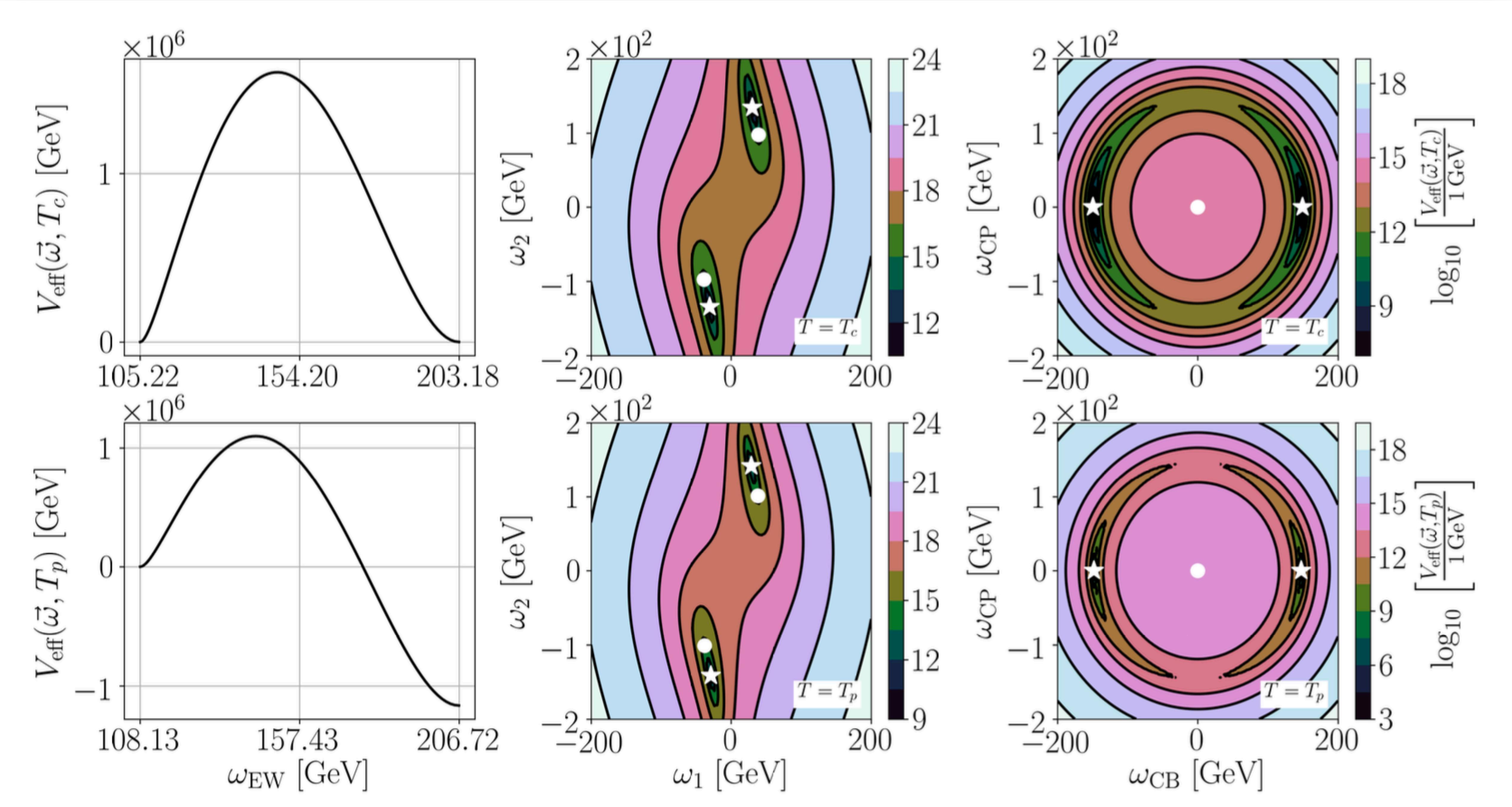
to trace any minima

igher temperatures

Potential Contours generated w/ PotPlotter

- BP1 (2HDM):

[Basler,Biermann,MM,Müller,Santos,Viana,´24]



$$\omega_{\rm EW} \equiv \sqrt{\sum_{i=1,2,{\rm CB},{\rm CP}} \omega_i^2}$$

White dots (asterisks) = wrong (true) minimum

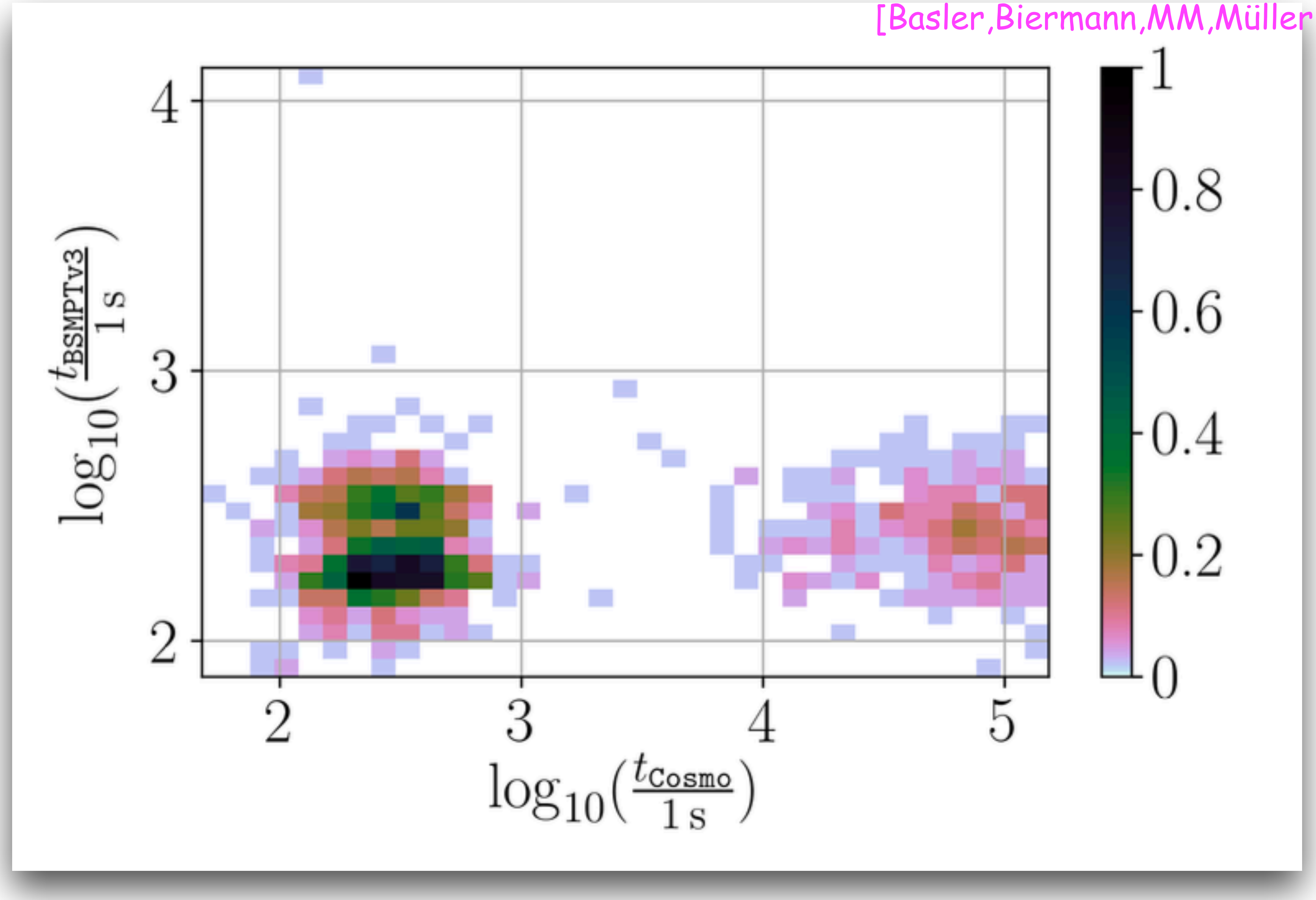
Broader Comparison based on Parameter Scan

- Scan of 2HDM parameter space: four field directions for minimization
  with ScannerS [Coimbra eal,'13;MM eal,'22]
  check for relevant theoretical and experimental constraints [cf. Azevedo eal,'23]

- Comparison BSMPTv3 and CosmoTransitions: for subset of points for which both codes find the same transitions

Runtime Comparison

[Basler,Biermann,MM,Müller,Santos,Viana,´24]



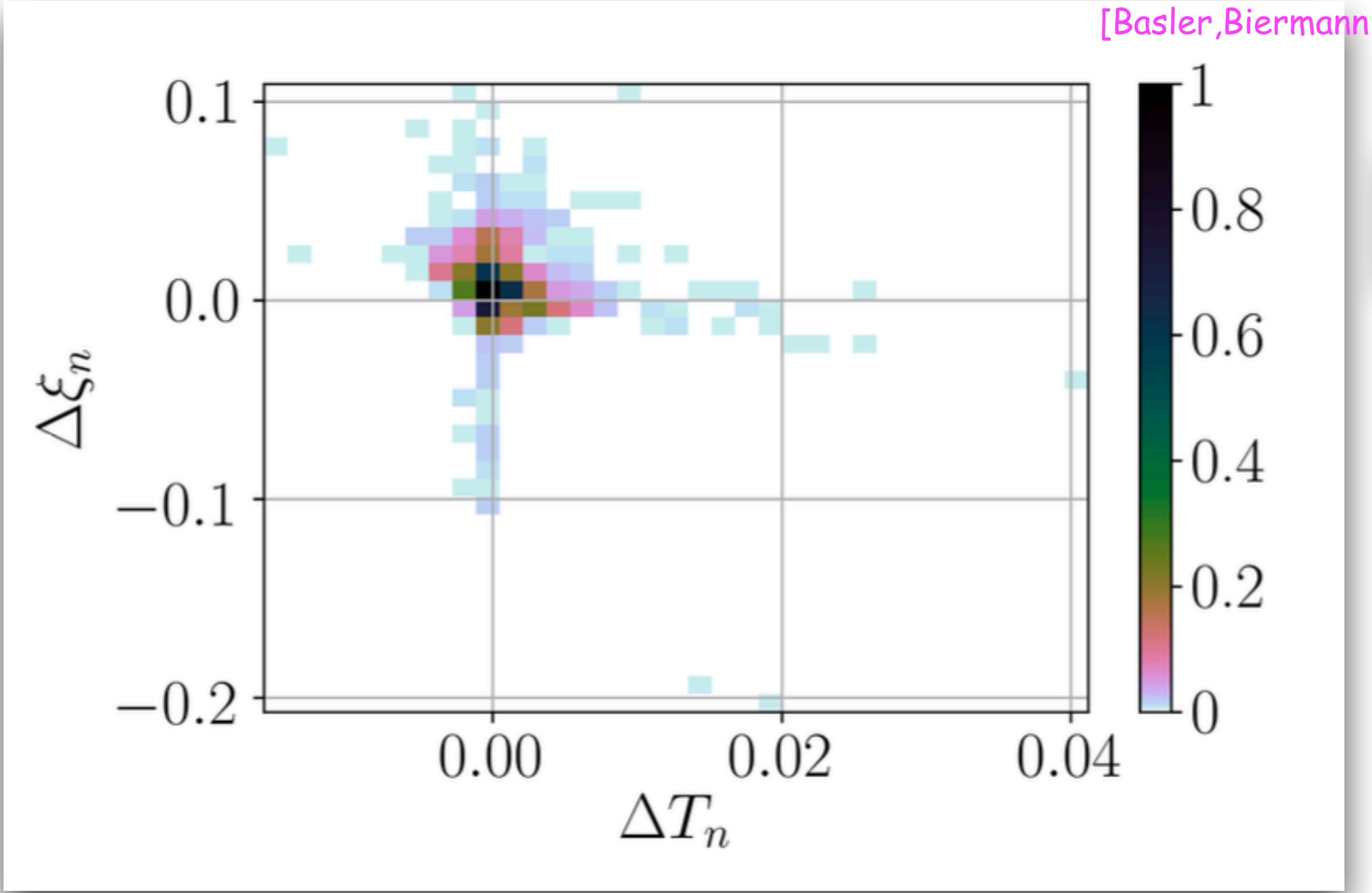
BSMPTv3 up to 10^3 faster than CosmoTransitions

BSMPTv3: mean (median) runtime 4.15 min (3.47 min)

CosmoTransitions: mean (median) runtime 41.46 min (5.61 min)

Temperature Comparison

[Basler,Biermann,MM,Müller,Santos,Viana,´24]



$$\Delta T_i = \frac{\left(T_i^{\texttt{BSMPTv3}} - T_i^{\texttt{Cosmo}}\right)}{T_i^{\texttt{BSMPTv3}}}$$

$$\xi_i = \frac{\sqrt{\sum_k \omega_k^2(T_i)}}{T_i}$$

$$\omega_k \in \{\omega_{\rm CB},\, \omega_1,\, \omega_2,\, \omega_{\rm CP}\}$$

$$\Delta\xi_i = \frac{\left(\xi_i^{\texttt{BSMPTv3}} - \xi_i^{\texttt{Cosmo}}\right)}{\xi_i^{\texttt{BSMPTv3}}}$$

Outliers: up to 4.1% rel. difference in ΔT_n
up to -20.7% rel. difference in $\Delta\xi_n$
outliers correlated w/ rapidly changing potential in small T interval

The Model „CP in the Dark“

- Next-to-Minimal 2-Higgs Doublet Model: [Azevedo,Ferreira,MM,Patel,Santos,Wittbrodt,'18]

$$
\begin{aligned}
V^{(0)} = \; & m_{11}^2|\Phi_1|^2 + m_{22}^2|\Phi_2|^2 + \frac{m_S^2}{2}\Phi_S^2 + \left(A\Phi_1^\dagger\Phi_2\Phi_S + \text{ h.c.}\right) \\
& + \frac{\lambda_1}{2}|\Phi_1|^4 + \frac{\lambda_2}{2}|\Phi_2|^4 + \lambda_3|\Phi_1|^2|\Phi_2|^2 + \lambda_4|\Phi_1^\dagger\Phi_2|^2 + \frac{\lambda_5}{2}[(\Phi_1^\dagger\Phi_2)^2 + (\Phi_2^\dagger\Phi_1)^2] \\
& + \frac{\lambda_6}{4}\Phi_S^4 + \frac{\lambda_7}{2}|\Phi_1|^2\Phi_S^2 + \frac{\lambda_8}{2}|\Phi_2|^2\Phi_S^2 \,.
\end{aligned}
$$

- with one discrete $\mathbb{Z}_2$ symmetry: $\Phi_1 \to \Phi_1\,, \quad \Phi_2 \to -\Phi_2\,, \quad \Phi_S \to -\Phi_S$

  one SM-like Higgs plus dark sector: $h_1, h_2, h_3, H^\pm$

- trilinear coupling A is complex: dark sector with explicit CP violation <- not constrained by electric dipole moment

Vacuum Structure of „CP in the Dark“

- General vacuum structure at T≠0:

$$\Phi_1 = \frac{1}{\sqrt{2}}\begin{pmatrix} \rho_1 + i\eta_1 \\ \zeta_1 + \omega_1 + i\Psi_1 \end{pmatrix}, \Phi_2 = \frac{1}{\sqrt{2}}\begin{pmatrix} \rho_2 + \omega_{CB} + i\eta_2 \\ \zeta_2 + \omega_2 + i(\Psi_2 + \omega_{CP}) \end{pmatrix}, \Phi_S = \zeta_S + \omega_S$$

electroweak VEVs: ω_1, ω_2 , CP-violating VEV: ω_{CP}

charge-breaking VEV: ω_{CB} (unphysical; found to be zero for all of our scan points)

Z_2-symmetry breaking VEV: ω_S

- General vacuum structure at T=0:

$$\Phi_1 = \frac{1}{\sqrt{2}}\begin{pmatrix} \rho_1 + i\eta_1 \\ \zeta_1 + v_1 + i\Psi_1 \end{pmatrix}, \Phi_2 = \frac{1}{\sqrt{2}}\begin{pmatrix} \rho_2 + i\eta_2 \\ \zeta_2 + i\Psi_2 \end{pmatrix}, \Phi_S = \zeta_S$$

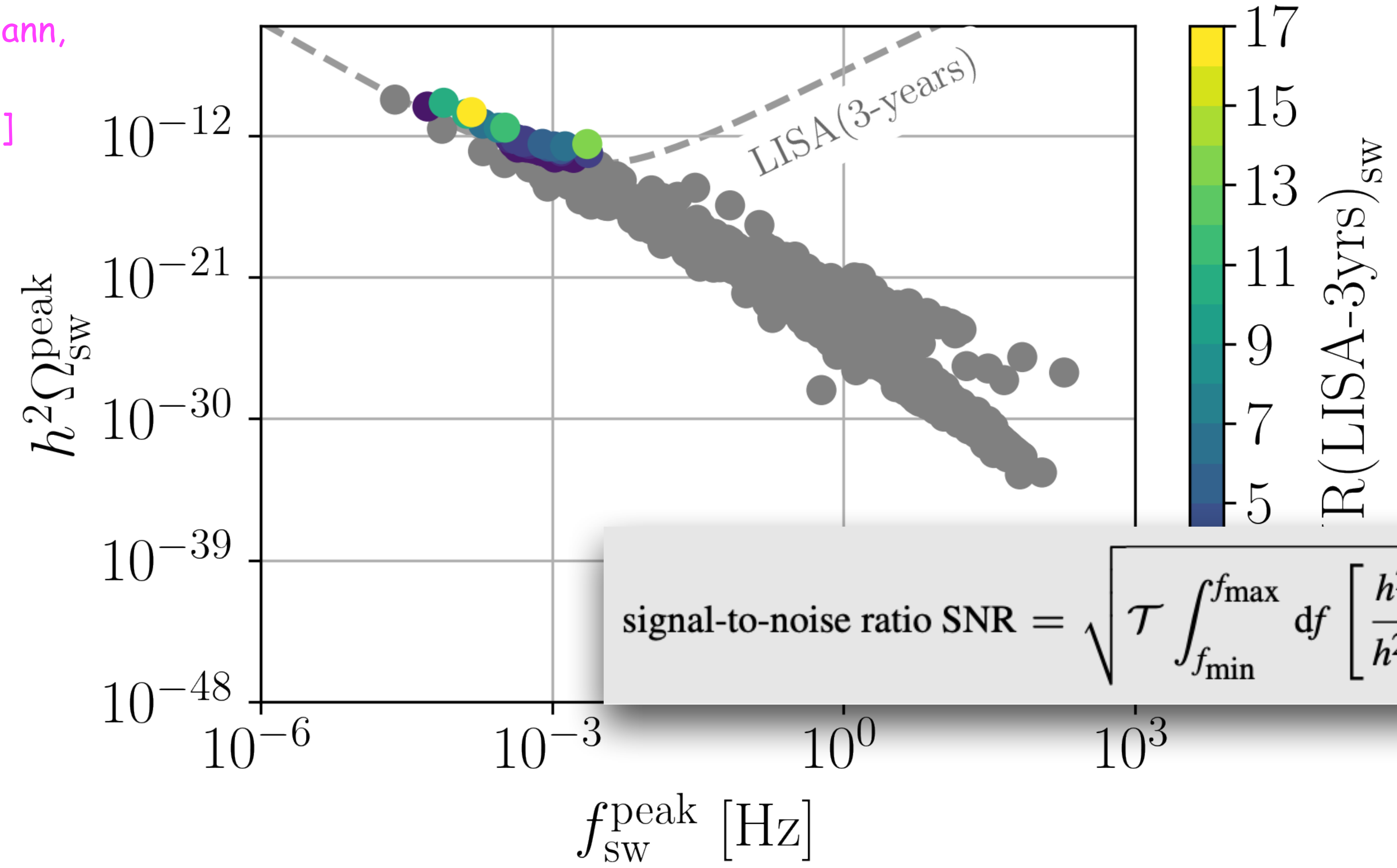
$$\langle\Phi_1\rangle|_{T=0} = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 \\ v_1 \end{pmatrix}, \langle\Phi_2\rangle|_{T=0} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}, \langle\Phi_S\rangle|_{T=0} = 0$$

$$\omega_1\big|_{T=0\ \mathrm{GeV}} = v1 \equiv v = 246.22\ \mathrm{GeV}$$

GW from (S)FOEWPT in „CP in the Dark“*

[Basler,Biermann, MM,Müller, Santos,Viana]

*[Azevedo,Ferreira, MM,Patel,Santos, Wittbrodt]

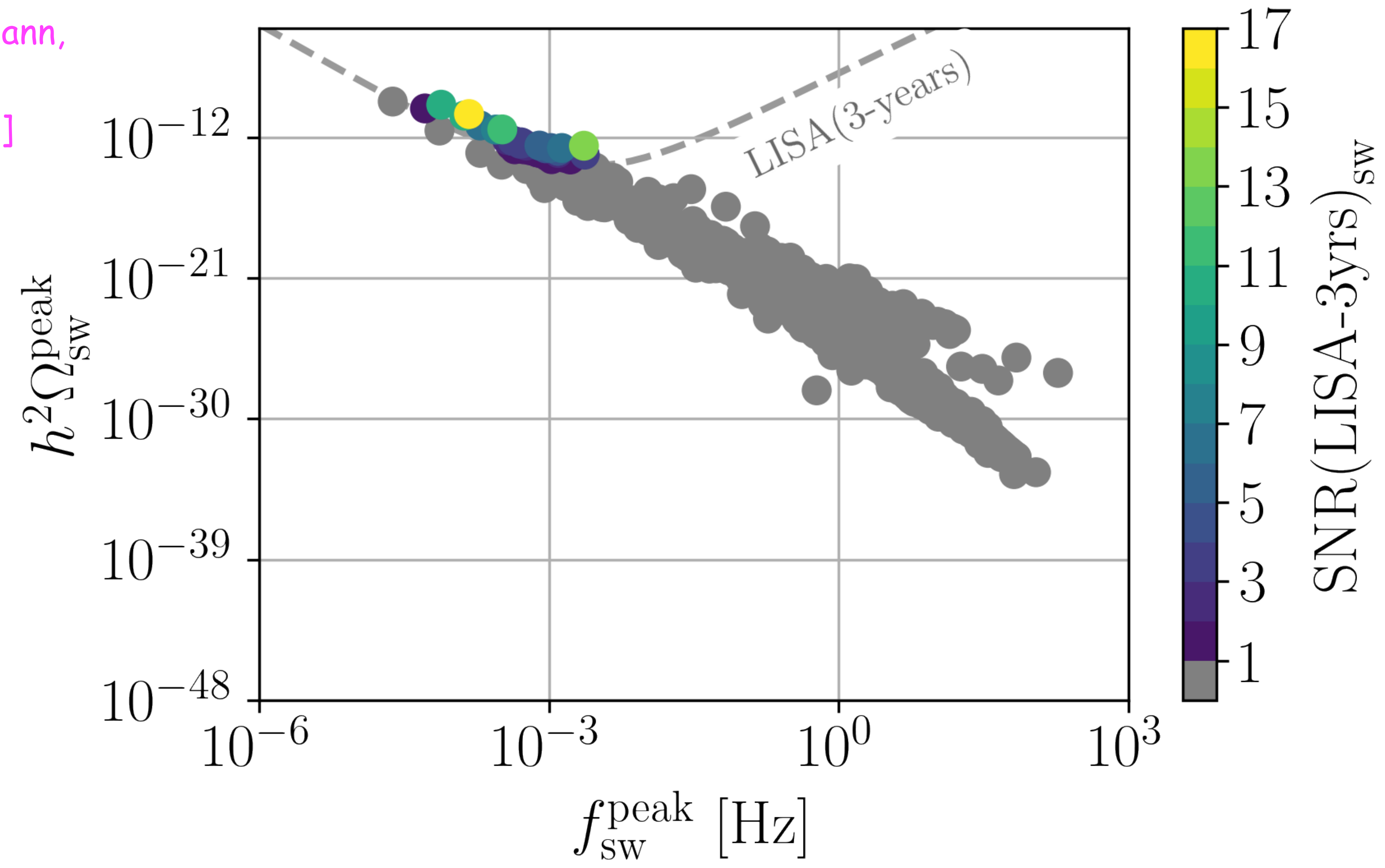


$$\text{signal-to-noise ratio SNR} = \sqrt{\mathcal{T}\int_{f_{\rm min}}^{f_{\rm max}} {\rm d}f \left[\frac{h^2\Omega_{\rm GW}(f)}{h^2\Omega_{\rm Sens}(f)}\right]^2}$$

- ∃ points w/ SNR(LISA-3yrs)>10, compatible w/ all relevant theor. and exp. constraints
- all points lead to EW minimum at T=0 (no vacuum trapping)
- all of the LISA-sensitive points (colored points) have SFOEWPT: ξ_c>1

GW from (S)FOEWPT in „CP in the Dark"*

[Basler,Biermann, MM,Müller, Santos,Viana]



*[Azevedo,Ferreira, MM,Patel,Santos, Wittbrodt]

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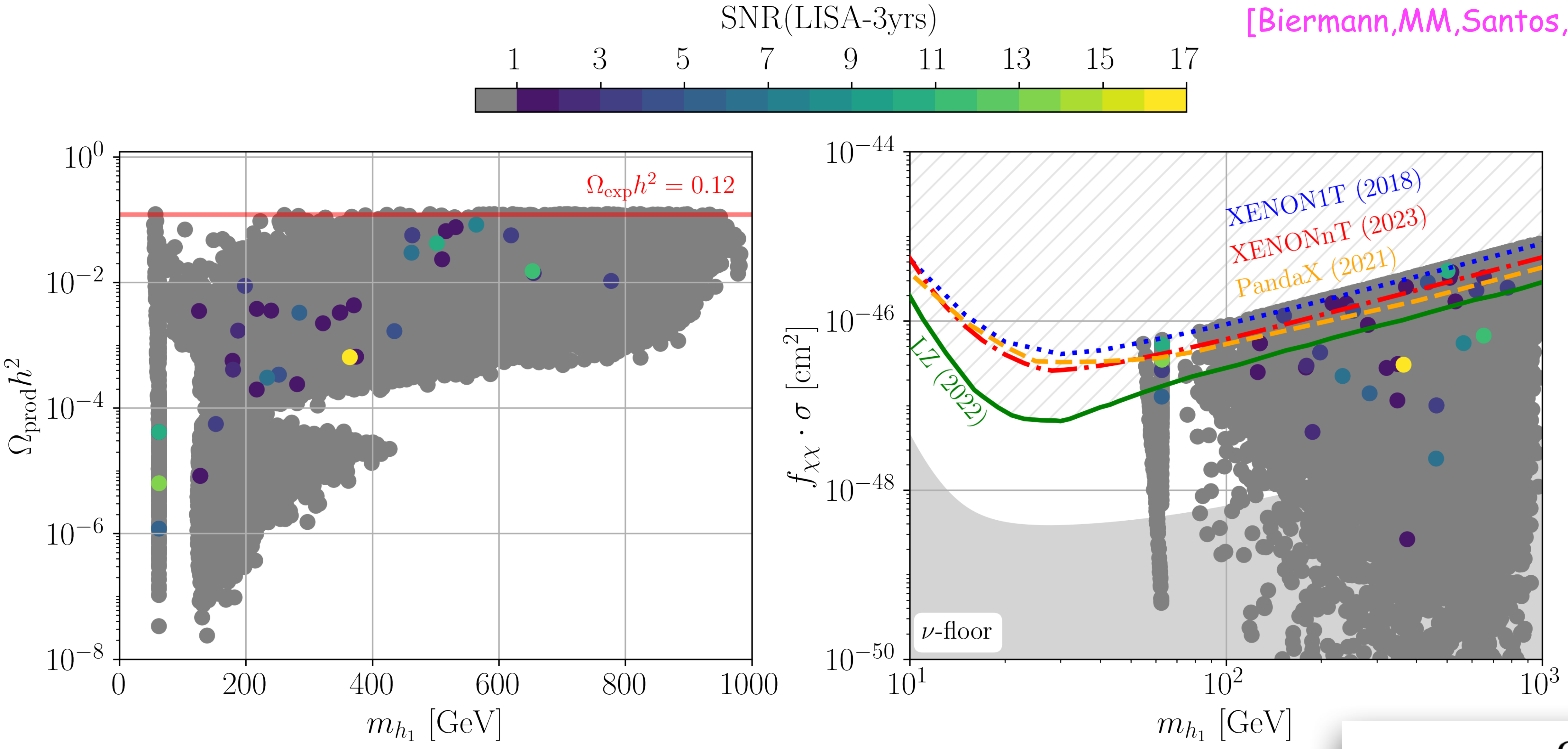
DM Observables and GW

[Biermann,MM,Santos,Viana]



$$\sigma \cdot f_{\chi\chi} \equiv \sigma \cdot \frac{\Omega_{\rm prod}h^2}{\Omega_{\rm obs}h^2}$$

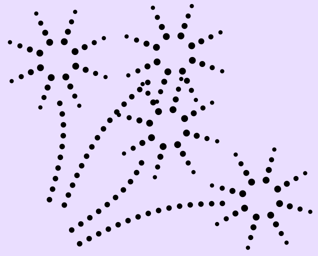
- Viable GW points (SNR(LISA-3yrs)>1 - colored points):
  compatible w/ relic density (<Ωh^2)
  above neutrino floor
  testable at future direct detection experiments

Conclusions

Summary: Why BSMPTv3 ? Therefore:

[Basler,MM,v1,'18] [Basler,MM,Müller,v2,'20] [Basler,Biermann,MM,MüllerSantos,Viana,v3,'24]

- Optimized minimum tracing & tracking of temperature-dependent coexisting minimum phases over any temperature interval
- Numerical derivation of the bounce solution for any number of field dimensions
- Besides critical and nucleation temperatures, calculation of the percolation and completion temperatures (not implemented in CosmoTransitions)
- Able to treat multistep phase transitions, discrete symmetries, flat directions, check for EWSR, reports transition history
- Calculation of the α, β/H, of f^{peak} and $h^2\Omega^{peak}$ of the (acoustic & turbulence) GW spectrum
- Computation of signal-to-noise-ratio at LISA
- For all implemented models (CxSM, R2HDM, C2HDM, N2HDM, CP in the Dark) and beyond*
- Embedded in the framework of the existing BSMPT code
  -> consistent computation of all EWPT-related observables
  -> *easy user interface for implementing a new model
  -> designed to use input from ScannerS



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Performs whole chain from particle physics model to gravitational waves in a consistent self-contained code!

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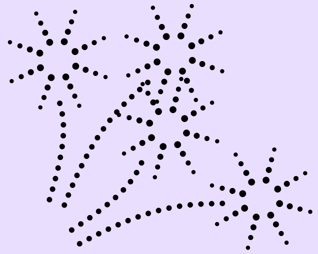
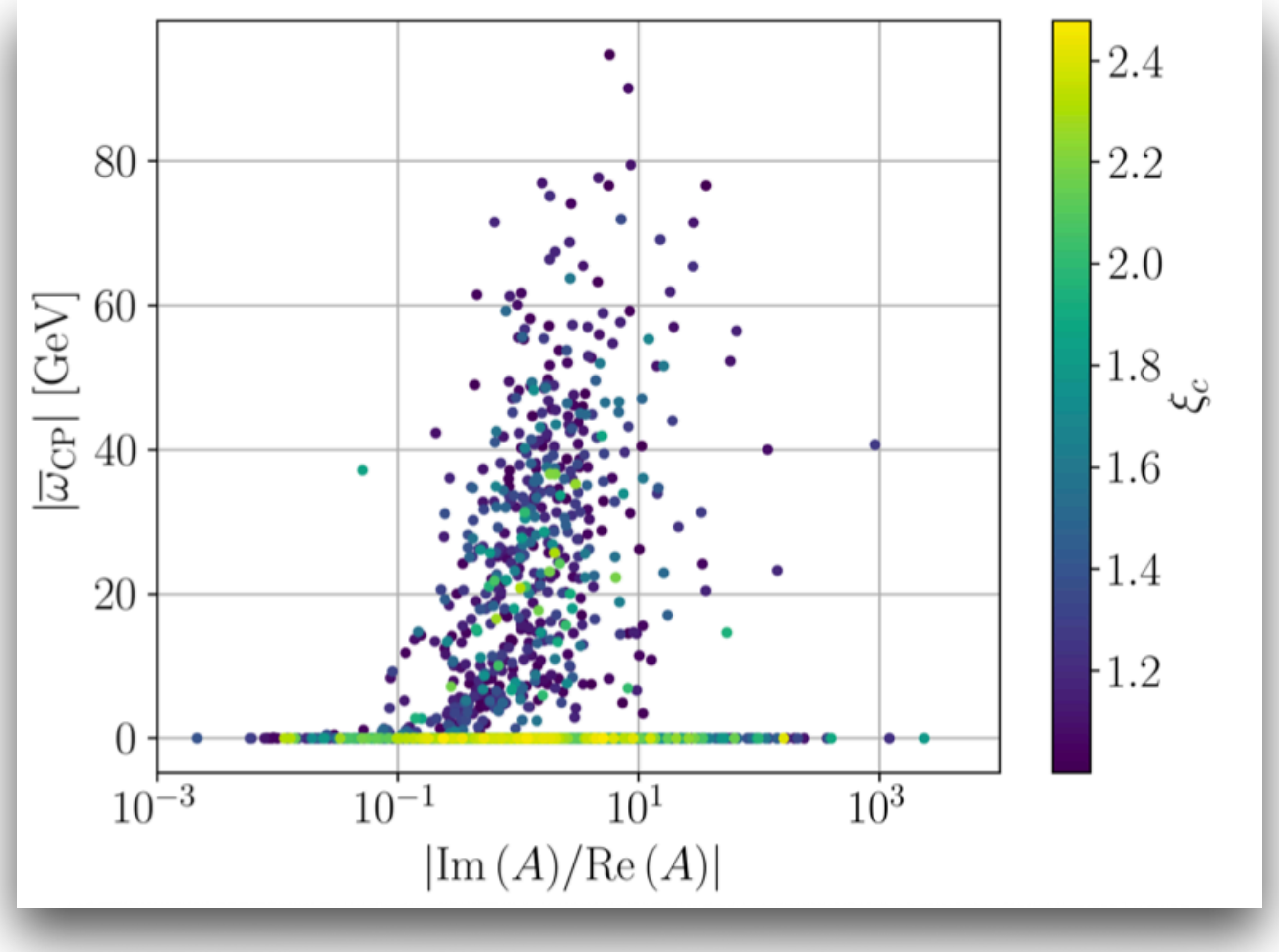


Thank you for your attention!

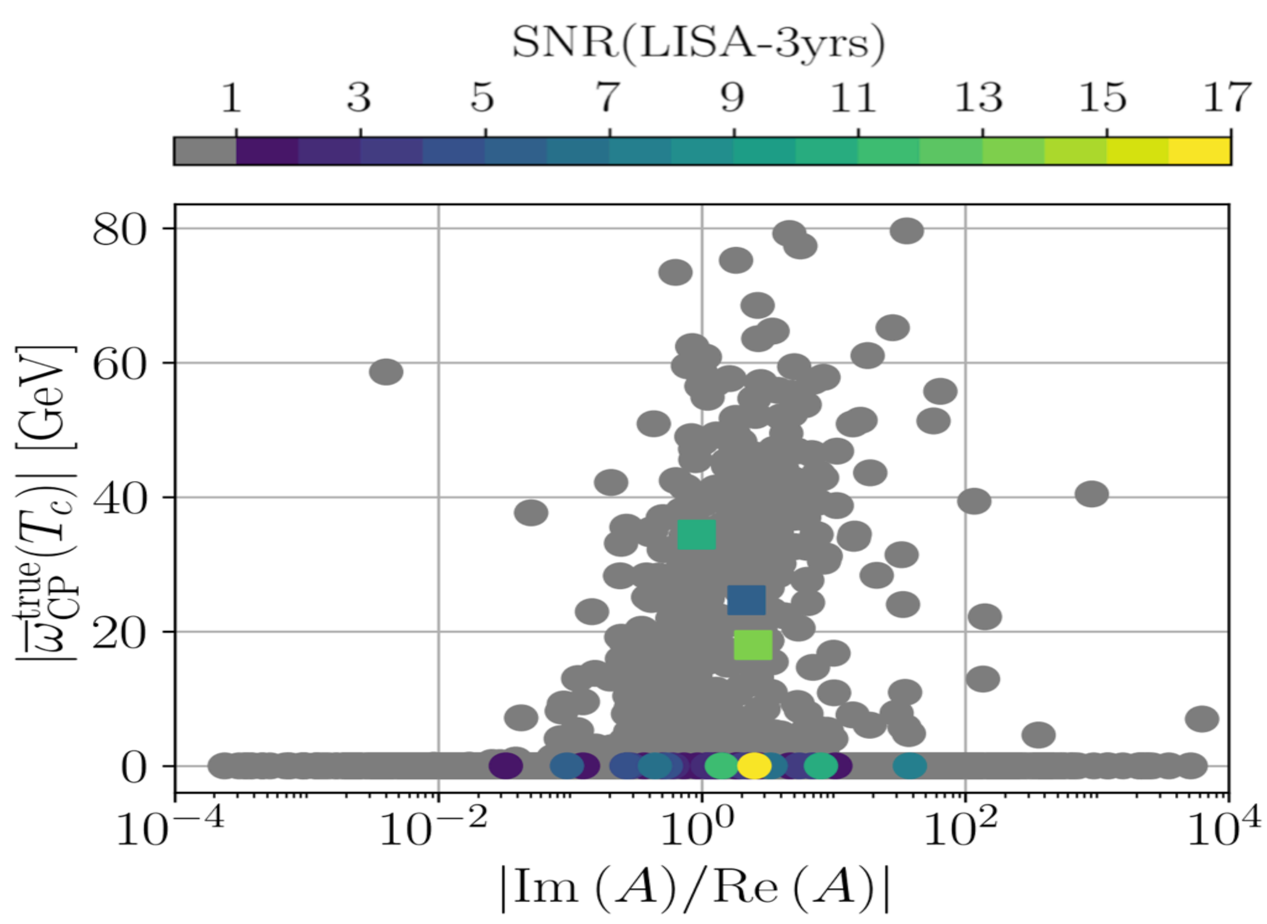
Spontaneous CP Violation

[Biermann,MM,Müller,'22]



- possibility of SFOEWPT & spontaneous CP violation (CPV)
- spontaneous Z_2 violation also possible => non-standard CPV transferred to visible sector
- interesting for EWBG!

[Biermann,MM,Santos,Viana]

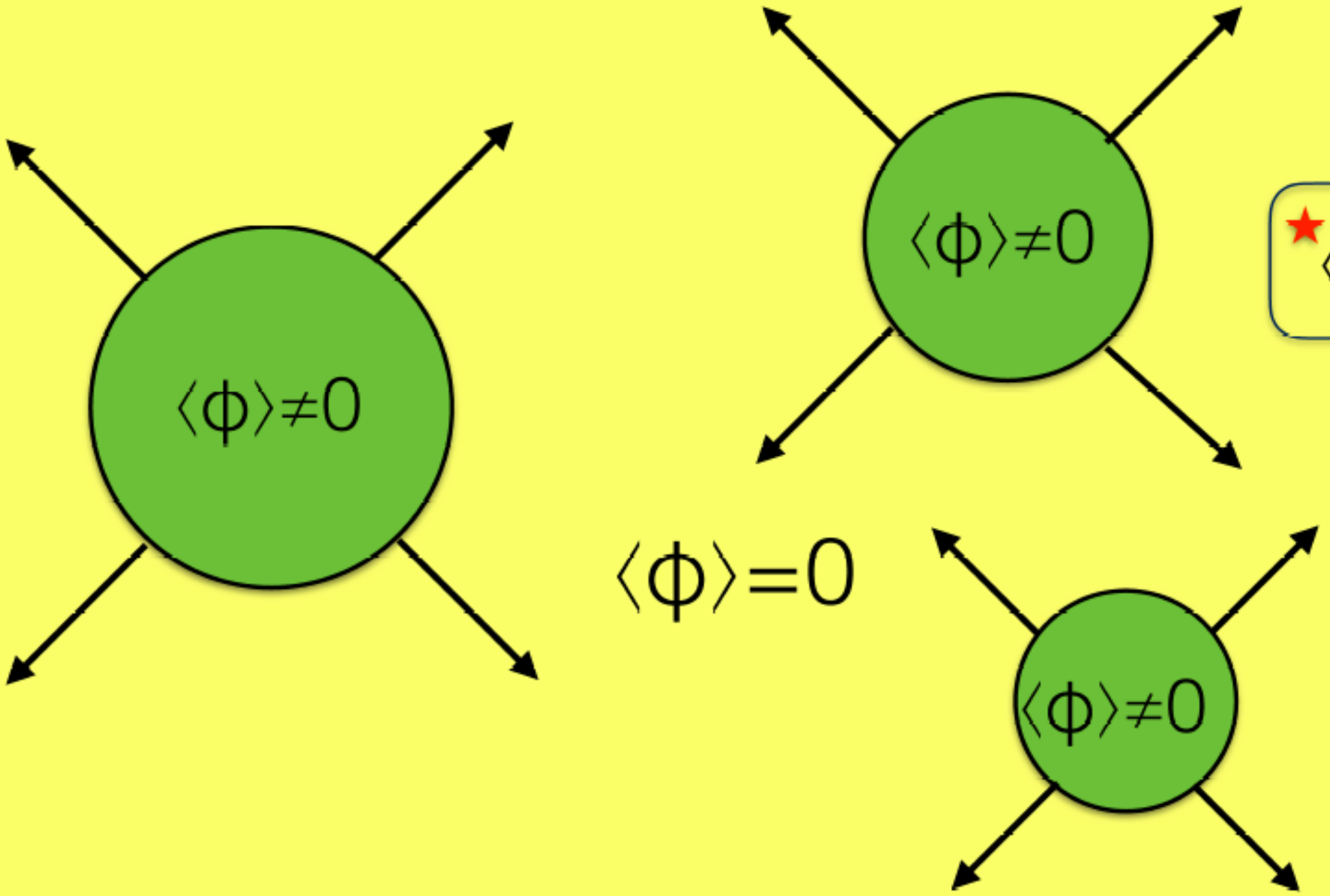


- SNR(LISA-3yrs)>1 (colored) for max. $|\omega_{CP}|=O(10^{-1})$
- spontaneous Z_2 violation leads to plasma friction w/ (former) DM direction =>
- spontaneous CPV may escape run-away

EWBG in a Nutshell

Bubbles of the non-zero Higgs field VEV nucleate from the symmetric vacuum

They expand & particles in plasma interact with the phase interface in a CP-violating way



★ $\langle\phi_c(T_c)\rangle/T_c \geq 1$

ϕ_c, T_c
VEV and T at EW phase transition

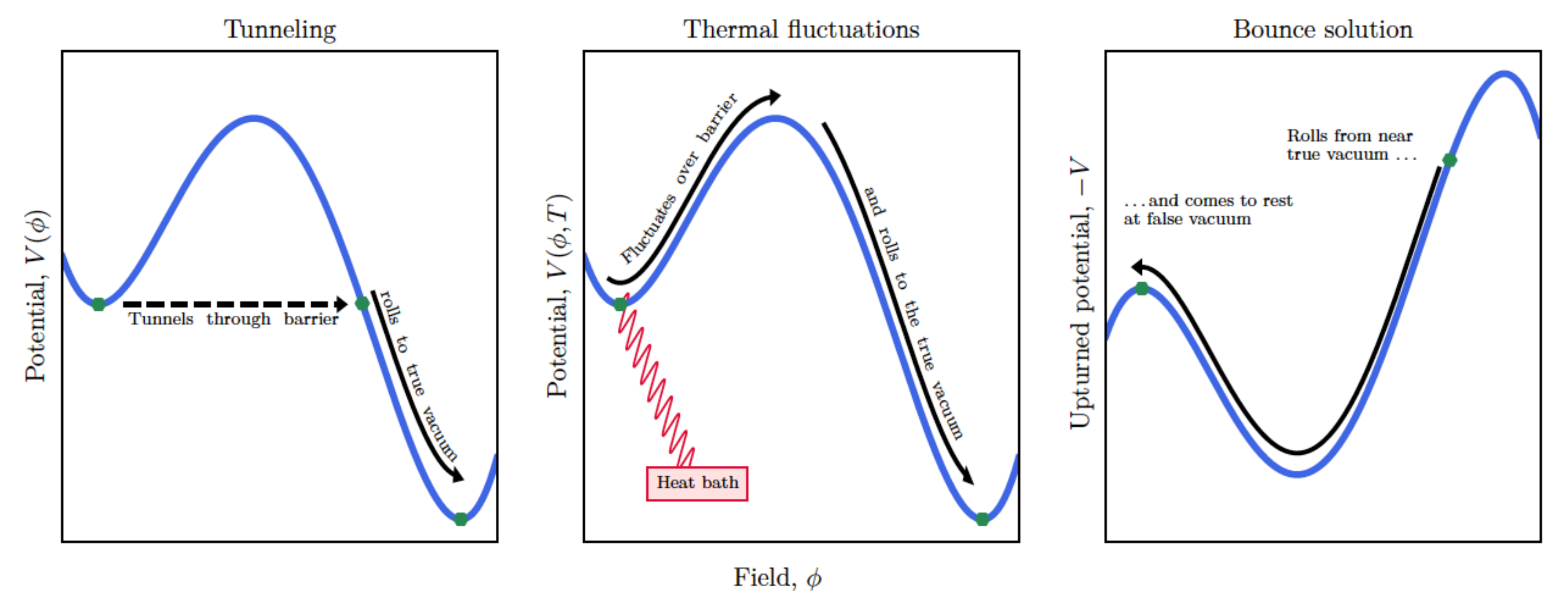
CP-asymmetry is converted into a baryon asymmetry by sphalerons in the symmetric phase in front of bubble wall

Produced baryons muss not be washed out by sphaleron processes in symmetric phase in front of bubble wall ★

Vacuum Decay

- Vacuum decay: transition from false to true vacuum through quantum tunneling or thermal fluctuations

[Plot from Athron eal,'23]



- Tunneling rate per unit volume:

$$\Gamma(T) = A(T)\, \mathrm{e}^{\frac{\hat{S}_3}{T}} \simeq T^4 \left(\frac{\hat{S}_3}{2\pi T}\right)^{\frac{3}{2}} \mathrm{e}^{\frac{\hat{S}_3}{T}}$$

$\hat{S}_3$ minimized O3-symmetric Euclidean action (action of the bounce solution)

Expanding bubbles w/ true vacuum, racing against expanding universe, interactions with plasma in front of bubble wall

Parameter Scan

- ✦ Scan in parameter space of the model w/ ScannerS [Coimbra eal,'13;MM eal,'20]

- ✦ Keep only points compatible w/

theoretical constraints: bounded-from-below, perturbative unitarity, EW vacuum

experimental constraints: (EDMs automatically fulfilled)
- EW precision tests
  SM-like Higgs h compatibility w/:
- m_h=125 GeV
- Higgs Data [HiggsSignals]
(- Higgs exclusion limits [HiggsBounds])
- BR(h->inv) < 0.11 [ATLAS,'19]
- $\mu(h\rightarrow\gamma\gamma)=1.12\pm0.09$ [CMS,'21]

DM observables (through MicrOMEGAs):
- relic density $\Omega_{obs}h^2=0.1200\pm0.0012$ [Aghanim eal,'18] (require it to below)
- XENON1T exclusion limit [Aprile eal,'18]
- new LUX-ZEPLIN exclusion limit [Aalbers eal,'22]

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SM-like Higgs h has SM couplings by construction w/ exception of:
- hH^+H^- coupling modifies loop-ind. $h\gamma\gamma$ coupling
- $h\to h_ih_j$ decay ($h_{i,j}$ dark sector particles), modifies total width & hence BRs

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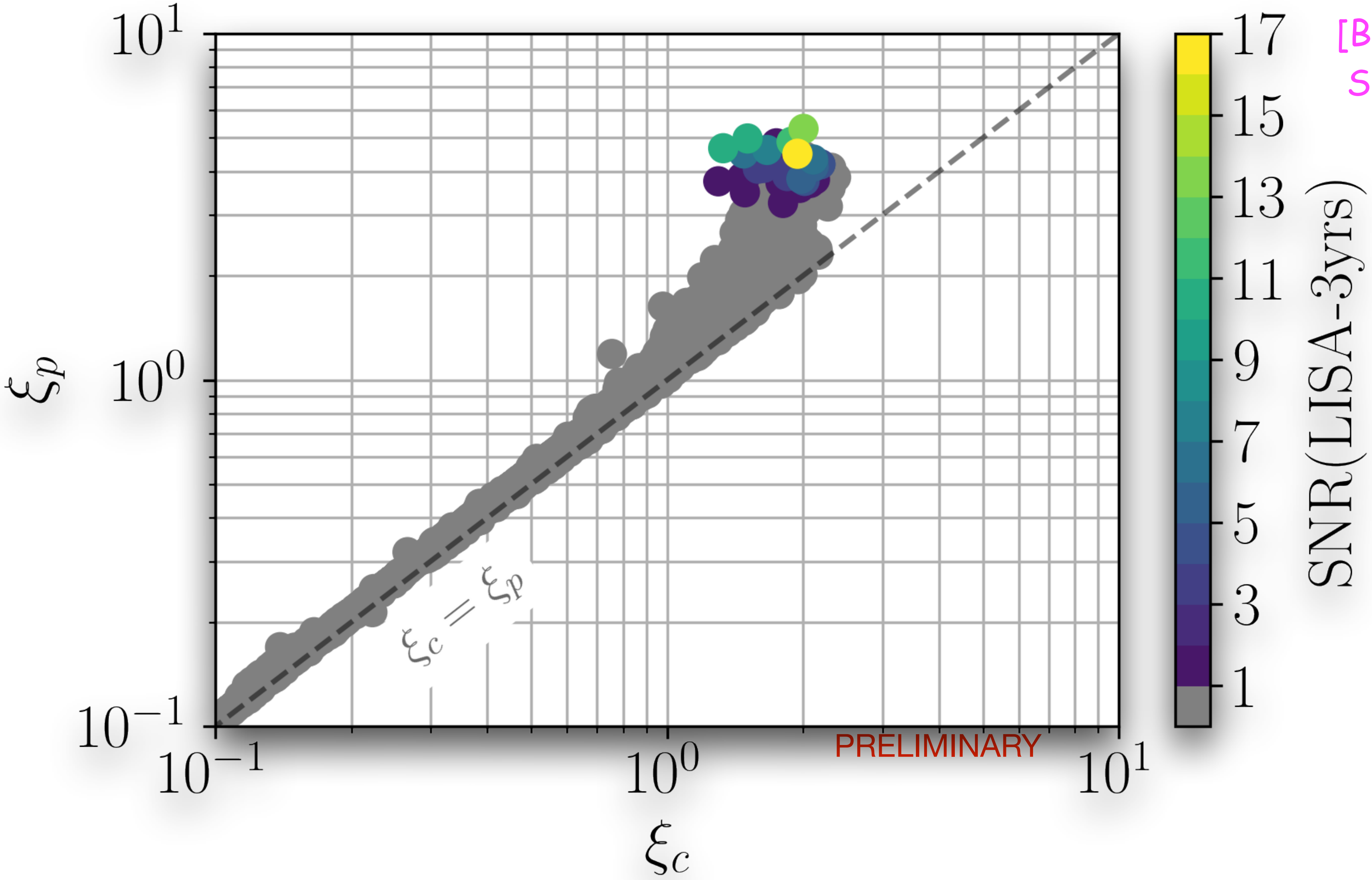
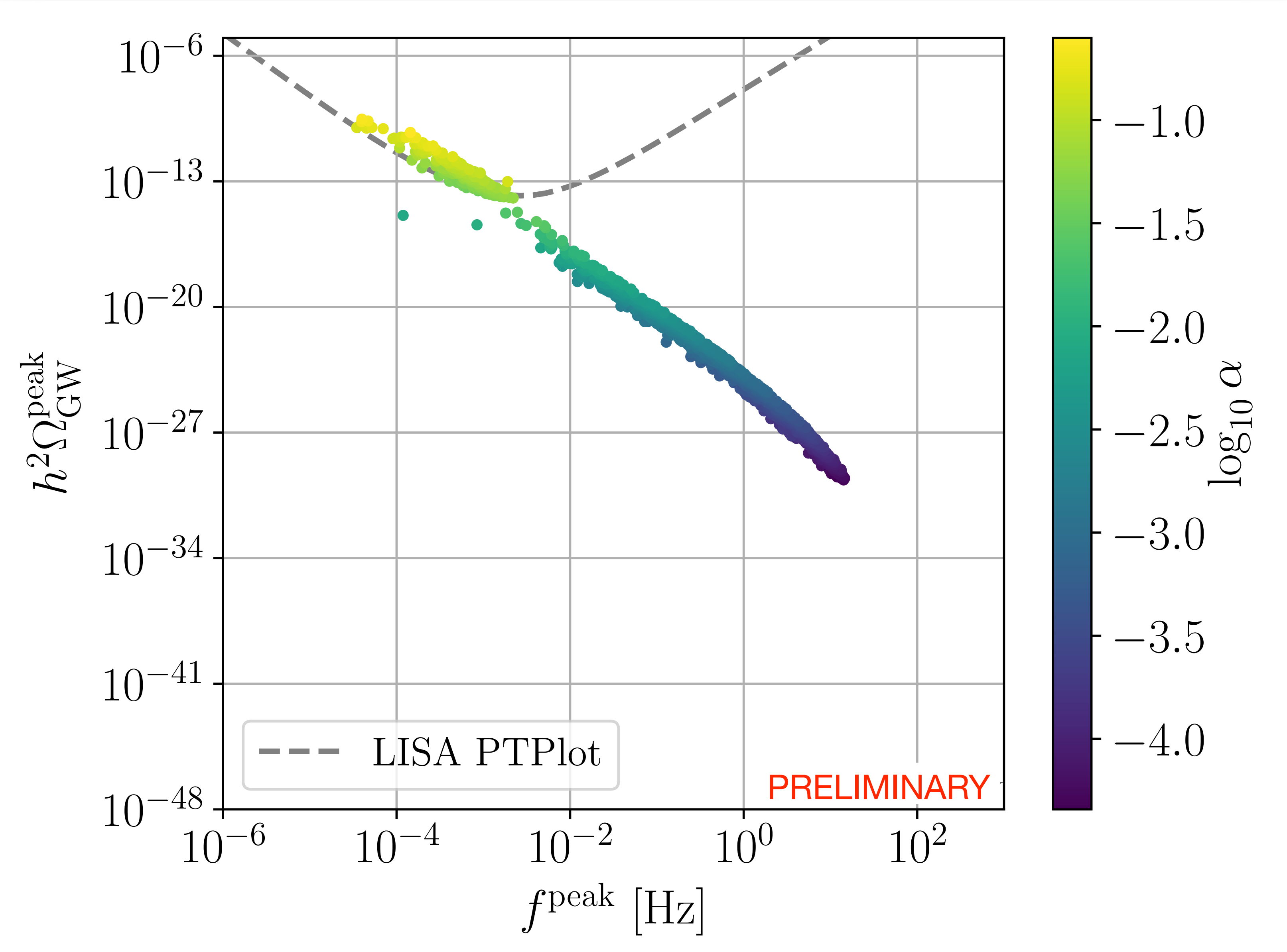


[Biermann,MM, Santos,Viana]

- comparison of ξ_{per}=v_*/T_* and ξ_c=v_c/T_c
- colored points SNR(LISA-3yrs)>1
- (almost) all of the LISA-sensitive points have SFOEWPT: ξ_c>1

GW from (S)FOEWPT in „CP in the Dark“

[Biermann,MM,
Santos,Viana]



Comparison with released latent heat during PT