Some 3HDM potentials with U(1) symmetry and DM

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based on arXiv:2408.02728
with/by: Anton Kuncinas, M. (Gui) N. Rebelo

Familiar faces!

Are we becoming a sect?

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sect

noun

a group of people with somewhat different religious beliefs (typically regarded as heretical ("BSM")) from those of a larger group (SM) to which they belong.

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95 GeV scalar

etc

stop here (Code of Conduct)

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Sect: Prophet accumulates a large wealth

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Sect: Prophet accumulates a large wealth

Thanks to the organiser(s) for keeping a lid on the conference fee!

Original scalar DM model: IDM

It is a 2HDM with a Z_2 symmetry

Notation used for the potential: $\boxed{V = \sum_{i,j} \mu_{ij}^2\, h_{ij} + \sum_{i,j} \lambda_{ijkl}\, h_{ij}\, h_{kl}}$

$$\boxed{h_{ij} \equiv h_i^\dagger h_j}$$

$$\begin{aligned} V_{\mathrm{IDM}} &= \mu_{11}^2 h_{11} + \mu_{22}^2 h_{22} \\ &\quad + \lambda_{1111} h_{11}^2 + \lambda_{2222} h_{22}^2 + \lambda_{1122} h_{11} h_{22} + \lambda_{1221} h_{12} h_{21} + \lambda_{1212}\left(h_{12}^2 + h_{21}^2\right) \end{aligned}$$

$$\mathbb{Z}_2 : \quad h_1 \to h_1, \quad h_2 \to -h_2$$

“Generalize”:

2HDM $\longrightarrow$ 3HDM

Z_2 $\longrightarrow$ U(1)

More terms in potential, with 3 doublets

More constraints, with higher symmetry

Come to talk by Anton Kuncinas!

U(1) symmetry:

2HDM context:

$$\{h_1, h_2\} \to \{e^{i\theta} h_1, h_2\} \quad \text{or} \quad \{h_1, e^{i\theta} h_2\} \quad \text{or} \quad \{e^{i\theta} h_1, e^{-i\theta} h_2\}$$

(differ by hypercharge rotation)

3HDM context:

$$\{h_1, h_2, h_3\} \to \{e^{i\theta} h_1, h_2, h_3\} \quad \text{or} \quad \{h_1, e^{i\theta} h_2, h_3\} \quad \text{or} \quad \{h_1, h_2, e^{i\theta} h_3\}$$

(all different!)

Ivanov, Keus, Vdovin:

$$U(1)_1: \quad \{h_1, h_2, h_3\} \to \{e^{i\theta} h_1, e^{-i\theta} h_2, h_3\}$$
$$U(1)_2: \quad \{h_1, h_2, h_3\} \to \{h_1, h_2, e^{i\theta} h_3\}$$

modulo hypercharge transformations

2HDM:

Enlarging the symmetry from Z_2 to U(1) leads to a reduction in the number of parameters: from n=7 to 6

Consequence: some physicsl parameter (mass or coupling) vanishes, or two are related

3HDM:

The Z_2 symmetric potential has
4 bilinear parameters and 17 quartic ones

$U(1)_1$: 3 bilinear and 10 quartic

$U(1)_2$: 4 bilinear and 14 quartic

U(1) X U(1): 3 bilinear and 9 quartic most symmetric among these

Symmetry: broken and unbroken

2HDM: $\mathbb{Z}_2:$ vacuum $(v, 0)$ unbroken IDM

3HDM (examples):

$$
\begin{array}{lll}
U(1)_1: & \text{vacuum } (0,0,v) & \text{unbroken} \\
U(1)_2: & \text{vacuum } (v_1,v_2,0) & \text{unbroken} \\
U(1)_2: & \text{vacuum } (v,0,0) & \text{unbroken} \\
U(1)\otimes U(1): & \text{vacuum } (0,0,v) & \text{unbroken}
\end{array}
$$

Comparing potentials:

$$V_{\mathrm{IDM}} = \mu_{11}^2 h_{11} + \mu_{22}^2 h_{22} + \lambda_{1111} h_{11}^2 + \lambda_{2222} h_{22}^2 + \lambda_{1122} h_{11} h_{22} + \lambda_{1221} h_{12} h_{21} \qquad (1 \leftrightarrow 2)$$
$$+ \lambda_{1212} \left(h_{12}^2 + h_{21}^2 \right)$$

$$V_{U(1)\otimes U(1)} = \frac{1}{2}\mu_{11}^2 h_{11} + \mu_{22}^2 h_{22} + \frac{1}{2}\lambda_{1111} h_{11}^2 + \lambda_{2222} h_{22}^2 + \lambda_{1122} h_{11} h_{22} + \lambda_{1221} h_{12} h_{21} \qquad (1 \leftrightarrow 2)$$
$$+ \frac{1}{2}\mu_{11}^2 h_{11} + \mu_{33}^2 h_{33} + \frac{1}{2}\lambda_{1111} h_{11}^2 + \lambda_{3333} h_{33}^2 + \lambda_{1133} h_{11} h_{33} + \lambda_{1331} h_{13} h_{31} \qquad (1 \leftrightarrow 3)$$
$$+ \lambda_{2233} h_{22} h_{33} + \lambda_{2332} h_{23} h_{32}. \qquad (2 \leftrightarrow 3)$$

Note the similarity 2HDM vs 3HDM!

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Differences:

(1) The IDM has the λ_{1212} (λ_5) term;

(2) $U(1) \otimes U(1)$ potential has couplings between the two inert sectors of h_2 and h_3

Spectrum - 2HDM

Unbroken $U(1)$ in the 2HDM [Ferreira et al]:

$M_H = M_A,$ Mass degeneracy

both decouple from gauge bosons and charged scalars

"Peccei-Quinn DM"

Some 3HDM Potentials:

$U(1) \otimes U(1)$ symmetric potential (real):

$$\boxed{V_0 = \sum_i \mu_{ii}^2 h_{ii} + \sum_i \lambda_{iiii} h_{ii}^2 + \sum_{i<j} \lambda_{iijj} h_{ii} h_{jj} + \sum_{i<j} \lambda_{ijji} h_{ij} h_{ji}}$$

$$V_{U(1)_1} = V_0 + \{\lambda_{1323} h_{13} h_{23} + \text{h.c.}\}$$

$$V_{U(1)_2} = V_0 + \Big\{\mu_{12}^2 h_{12} + \lambda_{1212} h_{12}^2 + \lambda_{1112} h_{11} h_{12} + \lambda_{1222} h_{12} h_{22} + \lambda_{1233} h_{12} h_{33} + \lambda_{1332} h_{13} h_{32} + \text{h.c.}\Big\}$$

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$$\boxed{V_{U(1)_1} = V_0 + \{\lambda_{1323} h_{13} h_{23} + \text{h.c.}\}}$$

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Some 3HDM Potentials:

$U(1) \otimes U(1)$ symmetric potential (real):

$$V_0 = \sum_i \mu_{ii}^2 h_{ii} + \sum_i \lambda_{iiii} h_{ii}^2 + \sum_{i<j} \lambda_{iijj} h_{ii} h_{jj} + \sum_{i<j} \lambda_{ijji} h_{ij} h_{ji}$$

$$V_{U(1)_1} = V_0 + \{\lambda_{1323} h_{13} h_{23} + \text{h.c.}\}$$

$$\boxed{\begin{aligned} V_{U(1)_2} = V_0 + \Big\{ & \mu_{12}^2 h_{12} + \lambda_{1212} h_{12}^2 + \lambda_{1112} h_{11} h_{12} + \lambda_{1222} h_{12} h_{22} \\ & + \lambda_{1233} h_{12} h_{33} + \lambda_{1332} h_{13} h_{32} + \text{h.c.} \Big\} \end{aligned}}$$

Spectrum - 3HDM $U(1)_2$

Notation: $$h_i = e^{i\sigma_i}\begin{pmatrix} h_i^+ \\ \frac{1}{\sqrt{2}}\left(\hat{v}_i + \eta_i + i\chi_i\right)\end{pmatrix}$$

$U(1)_2$: vacuum $(v_1, v_2, 0)$ unbroken

DM?

Potential contains terms like
$\lambda_{1233}h_{12}h_{33}$ and $\lambda_{1332}h_{13}h_{32}$
both lead to mass terms for neutral fields in h_3:
$\lambda_{1233}v_1v_2(\eta_3^2+\chi_3^2)$ and $\lambda_{1332}v_1v_2(\eta_3+i\chi_3)(\eta_3-i\chi_3)$

$$M^2(\eta_3^2+\chi_3^2)$$

η_3 and χ_3 are mass-degenerate!

Spectrum - 3HDM $U(1)_2$

$$U(1)_2: \quad \text{vacuum } (v_1, v_2, 0) \quad \text{unbroken}$$

DM?

Higgs basis:

$$U(1)_2: \quad \text{vacuum } (v, 0, 0) \quad \text{unbroken}$$

DM?

Same story:

$$M^2(\eta_3^2 + \chi_3^2)$$

η_3 and χ_3 are mass-degenerate!

Spectrum - 3HDM $U(1)_2$

$$U(1)_2: \quad \text{vacuum } (v_1, v_2, 0) \quad \text{unbroken}$$

DM?

Higgs basis:

$$U(1)_2: \quad \text{vacuum } (v, 0, 0) \quad \text{unbroken}$$

DM?

Same story:

Not protected by symmetry

$$M^2(\eta_3^2 + \chi_3^2)$$

η_3 and χ_3 are mass-degenerate!

Spectrum - 3HDM $U(1)_2$

Other

$$U(1)_2: \quad \text{vacuum } (0,0,v) \quad \text{unbroken}$$

DM?

Terms in the potential, $\langle h_1 \rangle = \langle h_2 \rangle = 0$:

$$h_{ii}h_{33} \sim v^2(\eta_i^2 + \chi_i^2)$$
$$h_{3i}h_{i3} \sim v^2(\eta_i^2 + \chi_i^2)$$
$$h_{3i}h_{j3} + \text{h.c.} \sim v^2(\eta_i\eta_j + \chi_i\chi_j)$$

Mass terms:

$$A(\eta_1^2 + \chi_1^2) + B(\eta_2^2 + \chi_2^2) + C(\eta_1\eta_2 + \chi_1\chi_2)$$

$$M^2 = \begin{array}{c c} & \begin{array}{cccc} \eta_1 & \chi_1 & \eta_2 & \chi_2 \end{array} \\ \begin{array}{c} \eta_1 \\ \chi_1 \\ \eta_2 \\ \chi_2 \end{array} & \begin{pmatrix} A & & C & \\ & A & & C \\ C & & B & \\ & C & & B \end{pmatrix} \end{array}$$

Spectrum - 3HDM $U(1)_2$

$$\det(M^2) = (A \cdot B - C^2)^2 \equiv (M_a^2 M_b^2)^2$$

Two pairs of mass-degenerate states!

Spectrum - 3HDM $U(1)_1$

may rotate phase

This was $U(1)_2$: $\{h_1, h_2, {\color{red}h_3}\}$

Consider $U(1)_1$: $\{{\color{blue}h_1}, {\color{blue}h_2}, h_3\}$

opposite rotations

$(v, 0, 0)$ double degeneracy

DM

$(0, 0, v)$ double degeneracy

DM

Spectrum - 3HDM $U(1)_1$

Consider $U(1)_1 :$ $\{h_1, h_2, h_3\}$

opposite rotations

$$(v, 0, 0) \quad \text{double degeneracy}$$
$$(0, 0, v) \quad \text{double degeneracy}$$

These cases are not related by a re-labelling. Note the potential:

$$V_{U(1)_1} = V_0 + \{\lambda_{1323} h_{13} h_{23} + \text{h.c.}\}$$

treats 3 differently from {1,2}

Spectrum - 3HDM $U(1) \otimes U(1)$

$$U(1) \otimes U(1): \quad \{h_1, h_2, h_3\}$$

using also hypercharge rotations

$$(v, 0, 0) \sim (0, v, 0) \sim (0, 0, v) \quad \text{double degeneracy}$$

DM

Spectrum - 3HDM $O(2) \otimes U(1)$

$$O(2) \otimes U(1): \quad \{\mathrm{h}_1, \mathrm{h}_2, h_3\}$$

constrained by O(2)

$(0, 0, v)$ quadruple degeneracy

Degeneracy: two states with same mass: CP even and odd

Double degeneracy: two pairs with same mass: each pair: one CP even and odd

Quadruple degeneracy: four states with same mass

but only one quantum number, CP, restricted to only two values

no way to make all four states mutually orthogonal

UNPHYSICAL

Comparing 3HDMs

Different symmetries lead to DM models with common spectra (in terms of structure)

However, couplings (for example number of independent ones) are different. See Anton’s talk.

Some implementations violate CP

If we just impose a vanishing vev, and allow the symmetry to be partly broken, then massless (Goldstone) states appear. Can be cured by soft symmetry-breaking terms. Such models could also accommodate DM stabilized by a remnant symmetry.

Symmetry breaking

Setting one or two vevs to zero, for example:

$$(v_1, v_2, 0) \quad \text{or} \quad (v, 0, 0)$$

typically breaks the underlying symmetry, and leads to massless (Goldstone) states

Exceptions: In some cases (of broken symmetries) no massless states

Instead, we observe a mass degeneracy

This is possible if “only” a discrete symmetry is broken

Summary

- DM models with unbroken U(1) symmetry exhibit mass degeneracies.
  One interpretation: under CP one even and the other odd
  Other interpretation: the two states carry opposite U(1) charge

- Different symmetries may lead to the same potential
  Different potentials may lead to the same spectrum
  (but different coupling structure)

- We studied only cases of unbroken symmetry
- Cases with broken symmetry might also be of interest
  (some remnant symmetry could stabilize DM)
- For a full discussion of 11 different symmetries, come to Anton's talk