Electroweak Scale Hierarchy from Custodial Symmetry

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based on:

arXiv:2407.15920 w/ **Thede de Boer** and Manfred Lindner

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Outline

- Hierarchy problem
- General idea of "Custodial Naturalness"
- Minimal model
- Numerical analysis, experimental constraints and predictions
- Extensions and embeddings
- Conclusions

Disclaimer: For this talk in 4D, scale invariance $\sim$ conformal invariance.

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Miguel Levy: “Still better than “$N = 1 + 2 + 3 + 4 + \cdots = -\frac{1}{12}$ NHDM.”

Electroweak scale hierarchy problem

Not a problem *in* the Standard Model (SM). [Bardeen '95]

However, in presence of heavy scales Λ_{high}, it remains puzzling that

(see, however, [Mooij, Shaposhnikov '21], [K.-S. Choi '24])

$$m_h^2 \;\propto\; \Lambda_{\text{high}}^2 \;,$$

which, in case e.g. $\Lambda_{\text{high}} \sim M_{\text{Pl}}$, is not supported by observation.

Symmetry based solutions:

- Supersymmetry.
- Composite Higgs ($h =$ pNGB of some new strongly coupled sector).

However, neither is the SM close-to supersymmetric,
nor do the Higgs measurements hint at compositeness.
No top-partners observed.

But: SM *is* close to **scale invariant**, *explicitly* broken only by $\mu_H\,(\sim m_h \sim v_{\text{EW}})_{\text{SM}}$.

Conformal “solution”

- The SM exhibits classical scale symmetry, only explicitly broken by $\mu_H^2 |H|^2$.
- Quantum corrections *could* spontaneously generate $\mu_H^2 \sim \Lambda_{\mathrm{CW}}^2 \sim \mathrm{e}^{-\frac{\lambda}{g^4}} \Lambda_{\mathrm{high}}^2$,
  [Coleman, Weinberg '73]
- . . . But in SM this parametrically only works for $m_h \sim m_t \sim \mathcal{O}(10\,\mathrm{GeV})$. [Weinberg '76]

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New here: Higgs as pNGB of spontaneosuly broken **custodial symmetry** avoids this problem.

✓ Technically natural suppression of EW scale.

✓ Only elementary fields, no compositeness.

✓ No top partners, marginal top Yukawa like in SM.

"Custodial Naturalness" – General Idea

Assumptions:

1. Classical scale invariance.
2. New complex scalar Φ + new $\mathrm{U}(1)_\mathrm{X}$ gauge symmetry. $\mathrm{SU}(3)_\mathrm{c}\times\mathrm{SU}(2)_\mathrm{L}\times\mathrm{U}(1)_\mathrm{Y}\times\mathrm{U}(1)_\mathrm{X}$
3. High-scale $\mathrm{SO}(6)$ **custodial** symmetry of scalar potential:

$$\Rightarrow \qquad V(H,\Phi) = \lambda\left(|H|^2+|\Phi|^2\right)^2 \quad \text{at } \mu=\Lambda_\mathrm{high}\equiv M_\mathrm{Pl}\,.$$

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- ***If*** $\mathrm{SO}(6)$ *were* classically exact $\rightarrow$ [Coleman, Weinberg '73] $\rightarrow$ VEVs $\langle\Phi\rangle\&\langle H\rangle$.

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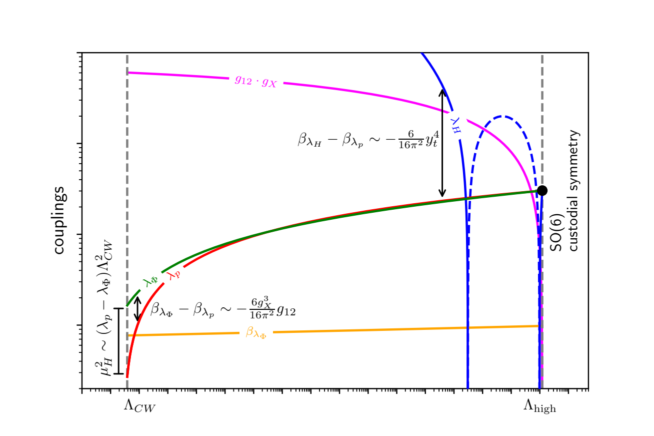
- Realistically: $\mathrm{SO}(6)$ explicitly broken by: y_t, g_Y & g_X, g_{12}, ..., e.g. y_new

$\Rightarrow$ $\mathrm{SO}(6) \xrightarrow{\langle\mathbf{6}\rangle} \mathrm{SO}(5)$: massive dilaton + 4 *would-be* NGBs + massive pNGB "h".

General Idea – RGE evolution is key

$$\text{below } M_{\rm Pl}: \quad V_{\text{tree}}(H,\Phi) \;=\; \lambda_H|H|^4+2\,\lambda_p|\Phi|^2|H|^2+\lambda_\Phi|\Phi|^4\;.$$



Actual running for a benchmark point. Dashed=negative.
β_i: Beta function coefficients.

Custodial sym. breaking:

- dominant breaking: y_t

  $\Rightarrow \quad \langle H\rangle \ll \langle\Phi\rangle$

- splitting $\lambda_\Phi-\lambda_p$ **requires** a new breaking of C.S.

Minimal C.S. breaking:

$\mathrm{U}(1)_{\mathrm{X}}-\mathrm{U}(1)_{\mathrm{Y}}$
gauge kinetic mixing g_{12}.

This generates "$\lambda_\Phi-\lambda_p$."

General Idea – Masses and EW scale

Masses of physical real scalars $h_\Phi \subset \Phi$ and $h \subset H$: $\langle\Phi\rangle = \frac{v_\Phi}{\sqrt{2}}, \langle H\rangle = \frac{v_h}{\sqrt{2}}$

Dilaton: $$m_{h_\Phi}^2 \approx \frac{3\,g_X^4}{8\pi^2}\,v_\Phi^2$$

pNGB Higgs: $$m_h^2 \approx 2\left[\lambda_\Phi\left(1+\frac{g_{12}}{2\,g_X}\right)^2 - \lambda_p\right] v_\Phi^2\ .$$

- This corresponds to $m_{h_\Phi}^2 \approx \beta_{\lambda_\Phi} v_\Phi^2$ and $m_h^2 \approx 2\left(\lambda_\Phi\beta_{\lambda_p}/\beta_{\lambda_\Phi} - \lambda_p\right) v_\Phi^2$.
- λ_H can stay at its SM value.
- EW scale VEV gets to keep the SM relation

$$v_H^2 \approx \frac{m_h^2}{2\lambda_H}\ .$$

⇒ The **EW scale is custodially suppressed** compared to the intermediate scale v_Φ of spontaneous scale and custodial symmetry violation.

Minimal Model

Field	#Gens.	$\mathrm{SU}(3)_\mathrm{c}\times\mathrm{SU}(2)_\mathrm{L}\times\mathrm{U}(1)_\mathrm{Y}$	$\mathrm{U}(1)_\mathrm{X}$	$\mathrm{U}(1)_\mathrm{B-L}$
Q	3	$(\mathbf{3},\mathbf{2},+\frac{1}{6})$	$-\frac{2}{3}$	$+\frac{1}{3}$
u_R	3	$(\mathbf{3},\mathbf{1},+\frac{2}{3})$	$+\frac{1}{3}$	$+\frac{1}{3}$
d_R	3	$(\mathbf{3},\mathbf{1},-\frac{1}{3})$	$-\frac{5}{3}$	$+\frac{1}{3}$
L	3	$(\mathbf{1},\mathbf{2},-\frac{1}{2})$	$+2$	-1
e_R	3	$(\mathbf{1},\mathbf{1},-1)$	$+1$	-1
ν_R	3	$(\mathbf{1},\mathbf{1},\ 0)$	$+3$	-1
H	1	$(\mathbf{1},\mathbf{2},+\frac{1}{2})$	$+1$	0
Φ	1	$(\mathbf{1},\mathbf{1},\ 0)$	$+1$	$q_\Phi^\mathrm{B-L}=-\frac{1}{3}$

$$\boxed{Q^{(\mathrm{X})} \equiv 2\,Q^{(\mathrm{Y})} + \frac{1}{q_\Phi^\mathrm{B-L}}\,Q^{(\mathrm{B-L})}}$$

- The only free parameter of the charge assignment is $q_\Phi^\mathrm{B-L}$.
- Constrained to $\frac{1}{3} \lesssim |q_\Phi^\mathrm{B-L}| \lesssim \frac{5}{11}$; special value: $q_\Phi^\mathrm{B-L} = -\frac{16}{41}$. Let us fix $q_\Phi^\mathrm{B-L} = -\frac{1}{3}$.

Note: Our model is very similar to "classical conformal extension of minimal $B-L$ model", but $q_\Phi^\mathrm{B-L} \neq -2$.
[Iso, Okada, Orikasa '09]

Numerical analysis

- SM parameters G_F, m_h, m_t $\longleftrightarrow$ parameters λ, g_X and y_t (@$\Lambda_\mathrm{high} \sim M_\mathrm{Pl}$).
- Remaining free parameter: g_{12}. Can fix $g_{12}|_{M_\mathrm{Pl}} = 0 \quad \Leftrightarrow \quad$ C.S. fixes all d.o.f.'s.

Same number of parameters as the SM!

$\rightarrow$ Properties of Z' and h_Φ are predictions of the model.

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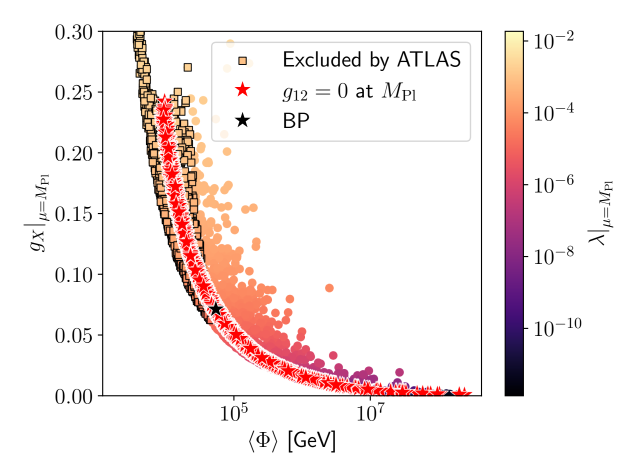
Parameter scan

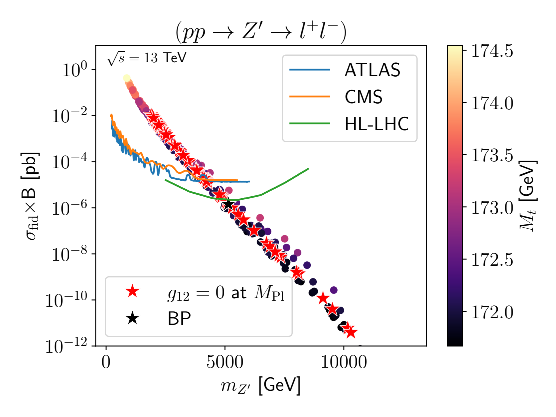
- Impose $\mathrm{SO}(6)$ symmetric BC's $@M_\mathrm{Pl}$: $\lambda_{H,\Phi,p}|_{M_\mathrm{Pl}} = \lambda|_{M_\mathrm{Pl}}$ and $g_{12}|_{M_\mathrm{Pl}} = 0$.
- 2-loop running with `PyR@TE`. [Sartore, Schienbein '21]
- Iteratively determine intermediate scale Φ_0, match to SM at $\mu_0 \sim \mathcal{O}(g_X \Phi_0)$.
- Numerically minimize 1-loop V_eff (at μ_0), compute v_Φ and v_H, m_{h_Φ}, m_h, $\lambda_{H,\Phi,p}$, match to 1-loop V_eff^SM (+dilaton hidden scalar, corrections negligible).
- From μ_0 down to m_t 2-loop running.
- Require $v_H^\text{exp} = 246.2 \pm 0.1\,\mathrm{GeV}$, as well as g_L, g_Y, g_3 and y_t within SM errors.
- Low scale new couplings g_X, g_{12} and masses $m_{Z'}$, m_{h_Φ} are predictions.

Parameter space



Parameters at $\mu = M_{\rm Pl}$. All points shown reproduce the correct EW scale. New scale $\langle\Phi\rangle = v_\Phi/\sqrt{2}$ is prediction. (m_h, M_t not imposed as constraint).

Phenomenological constraints



- $Z' \to l^+l^-$ resonance searches require $m_{Z'} \gtrsim 4\,\mathrm{TeV}$. (di-jets are weaker)
- EW precision: Additional custodial breaking shifts m_Z:

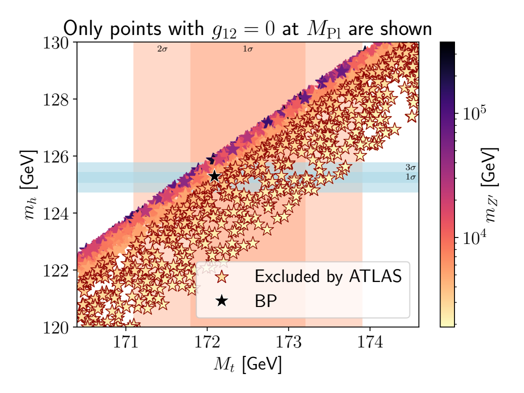
  $$\Delta m_Z \propto -m_Z \langle H \rangle^2 / (2\langle \Phi \rangle^2)\ .$$

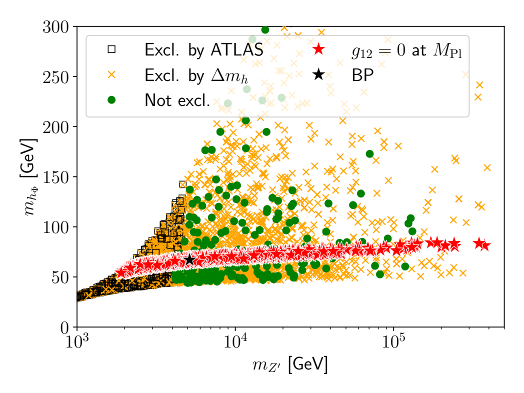
  Constraint: $\langle \Phi \rangle \gtrsim 18\,\mathrm{TeV}$, weaker than direct Z' searches.
- Dilaton-higgs mixing:

  $$\mathcal{O}_{h_\Phi} \approx \sin\theta \times \mathcal{O}^{\rm SM}_{h \to h_\Phi}\ .$$

  For $m_{h_\Phi} \sim 75\,\mathrm{GeV}$, $\sin\theta \lesssim 10^{-1}$ is a-OK.
  (typical values for us are BP: $\sin\theta \sim 10^{-2.5}$)
- Neglect dilaton-gauge^2 coupling from trace anomaly, suppressed by v_h/v_Φ.

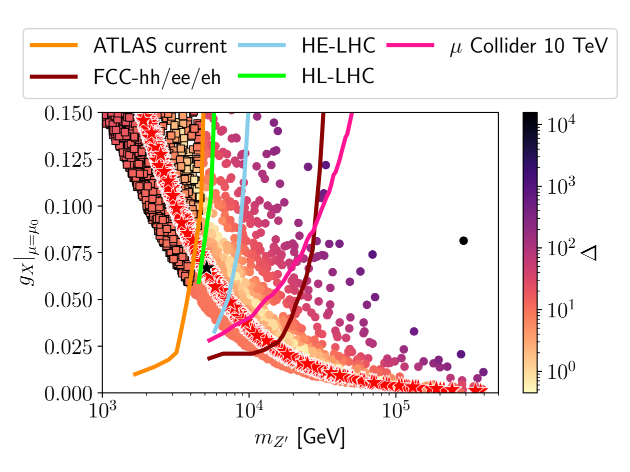
Reproductions and predictions





All points shown reproduce the correct EW scale. M_t: top pole mass.

Fine tuning and Future collider projections



Projections are for hypercharge universal Z' from [R.K. Ellis et al. '20]

Fine tuning:

$$\Delta \;:=\; \max_{g_i} \left| \frac{\partial \ln \frac{\langle H\rangle}{\langle \Phi\rangle}}{\partial \ln g_i} \right| .$$

Barbiere-Giudice measure.
[Barbieri, Giudice '88]

The choice of $\langle H\rangle/\langle\Phi\rangle$ automatically subtracts the shared sensitivity of VEVs to variation of g_i. [Anderson, Castano '95]

Red stars: $g_{12}|_{M_{\mathrm{Pl}}} = 0$.

Black star: benchmark point.

Prime target: Z' at FC, Dilaton production(+displaced dec.) at Higgs factories.

Extensions and embeddings

"Custodial Naturalness" is reasonably stable under variation of boundary conditions, charge assignments, addition of extra particles.

[de Boer, Lindner, AT 'XX]

Minimal model: $|\Phi|^2|H|^2$ and $X^{\mu\nu}Y_{\mu\nu}$, in extensions also $\overline{L}\tilde{H}\Psi_{\text{new}}$.

Additional fermions can:

- Provide ingredients for neutrino mass generation,
  [Iso, Okada, Orikasa '09], [Foot, Kobakhidze, McDonald, Volkas '07]
- Be part of the dark matter, [S. Okada '18]
- "Cure" SM vacuum instability. [(Das), Oda, Okada, Takahashi '15('16)]

- Custodial symmetry could originate from UV fixed point $\leftrightarrow$ quantum criticality.
  e.g. [Litim, Sannino '14]
- GUT embeddings $G_{\text{cust.}} \subset G_{\text{GUT}}$ allow to constrain $q^{\Phi}_{\text{B}-\text{L}}$ and compute the size of gauge-kinetic mixing g_{12}.
- Note: We have ignored finite-T effects here, this is yet to be done!
- CW transition is known to be first order $\rightarrow$ Gravitational wave signals.
  [Litim, Wetterich, Tetradis '97],[Dasgupta, Dev, Ghoshal, Mazumdar '22],[Huang, Xie '22]

Conclusions

- Classical scale invariance + extended custodial symmetry (here $\mathrm{SO}(6)$)

$\Rightarrow$ New mechanism to explain large scale separation and little hierarchy problem.

- Minimal model: Φ + $\mathrm{U}(1)_\mathrm{X}$ gauge: same number of parameters as the SM.
- Predicts light scalar dilaton $m_\Phi \sim 75\,\mathrm{GeV}$ + Z' at $4 - 100\,\mathrm{TeV}$.
- Top mass at lower end of currently allowed 1σ region.
- Perfect model to motivate new colliders + Higgs factory.
- Many extensions to explore, e.g. scale invariant 2HDM + $\mathrm{SO}(8)$?

Thank You!

Backup slides

Details of the potential and matching

Effective potential for background fields H_b and Φ_b @1-loop $\overline{\text{MS}}$:

$(-1)^{2s_i} \equiv {}^{+}_{(-)}1$ for bosons(fermions), $n_i \equiv \#\text{d.o.f}$

$C_i = \frac{5}{6}\left(\frac{3}{2}\right)$ for vector bosons(scalars/fermions).

$$V_{\text{eff}} = V_{\text{tree}} + \sum_i \frac{n_i(-1)^{2s_i}}{64\pi^2} m_{i,\text{eff}}^4 \left[\ln\left(\frac{m_{i,\text{eff}}^2}{\mu^2}\right) - C_i\right] .$$

Two different *analytical* expansions: First

$$V_{\text{EFT}}(H_b) \ := \ V_{\text{eff}}\left(H_b, \tilde{\Phi}(H_b)\right) , \qquad \text{with} \qquad \left.\frac{\partial V_{\text{eff}}}{\partial \Phi_b}\right|_{\Phi_b = \tilde{\Phi}(H_b)} = 0 .$$

Using $\Phi_0 := \Phi(H_b/\Phi_b = 0)$, we expand V_{EFT} in $H_b \ll \Phi_0$, $\curvearrowright$ RG-scale independent expression

$$V_{\text{EFT}} \approx 2\left[\lambda_p - \left(1 + \frac{g_{12}}{2\,g_X}\right)^2 \lambda_\Phi\right] \Phi_0^2 H_b^2 + \frac{\lambda_p \lambda_H}{16\pi^2}[...] .$$

This expression illustrates the origin of the Higgs mass and EW scale suppression.

Alternatively, take $\mu = \mu_0 := \sqrt{2} g_X \Phi_0 \mathrm{e}^{-1/6} \sim \langle\Phi\rangle$ and "t Hooft-like" expansion $\frac{\lambda_p}{\lambda_H} \sim \frac{H_b^2}{\Phi_0^2} \sim \epsilon^2 \to 0$,

$$V_{\text{EFT}} = -\frac{6\,g_X^4}{64\pi^2}\Phi_0^4 + 2\,\lambda_p \Phi_0^2 H_b^2 + \lambda_H H_b^4 + \sum_{i=\text{SM}} \frac{n_i(-1)^{2s_i}}{64\pi^2} m_{i,\text{eff}}^4 \left[\ln\left(\frac{m_{i,\text{eff}}^2}{\mu_0^2}\right) - C_i\right] .$$

This expression facilitates matching to the SM at scale μ_0.

Details of the potential and matching II

For all practical purpose the usual CW relation holds:

$$\Phi_0^2 \approx \exp\left\{-\frac{16\pi^2\lambda_\Phi}{3g_X^4} - \ln(2g_X^2) + \frac{1}{3} + \ldots\right\}\mu^2 \,. \tag{1}$$

Analytically we can use $H_b \ll \tilde{\Phi}(0) := \Phi_0$ and the leading order expression for Φ_0 reads

$$\frac{1}{16\pi^2}\ln\left(\frac{\Phi_0^2}{\mu^2}\right) = -\frac{\lambda_\Phi + \frac{1}{16\pi^2}\left\{q_\Phi^4 g_X^4\left[3\ln\left(2q_\Phi^2 g_X^2\right) - 1\right] + 4\,\lambda_p^2\left(\ln 2\lambda_p - 1\right)\right\}}{3\,q_\Phi^4 g_X^4 + 4\,\lambda_p^2} \,. \tag{2}$$

Alternatively, we can use the ϵ expansion, and Φ_0 at $\mathcal{O}(\epsilon^0)$ reads

$$\frac{1}{16\pi^2}\ln\left(\frac{\Phi_0^2}{\mu^2}\right) = -\frac{\lambda_\Phi + \frac{1}{16\pi^2}\left\{q_\Phi^4 g_X^4\left[3\ln\left(2q_\Phi^2 g_X^2\right) - 1\right]\right\}}{3\,q_\Phi^4 g_X^4} \,. \tag{3}$$

This is an example for the difference between the two expansion schemes. Note that our quantitative analysis is not based on any of these expansions but uses a fully numerical minimization of the effective potential to compute $\langle\Phi\rangle$ and $\langle H\rangle$.

Integrating out scalar in non-conformal model

Consider a simple two complex scalar system with a potential given by

$$V = -m_H^2|H|^2 - m_\Phi^2|\Phi|^2 + \frac{\lambda_H}{2}|H|^4 + \lambda_p|H|^2|\Phi|^2 + \frac{\lambda_\Phi}{2}|\Phi|^4.$$

For $m_\Phi^2 > 0$ and $-m_H^2 + m_\Phi^2 \frac{\lambda_p}{\lambda_\Phi} > 0$, this potential has a minimum at $\langle\Phi\rangle := \frac{v_\Phi}{\sqrt{2}} = \sqrt{\frac{m_\Phi^2}{\lambda_\Phi}}, \langle H\rangle = 0$.

Integrating out the heavy field Φ at tree level, we find the low energy potential

$$\begin{aligned} V_{\text{EFT}} &= \left(-m_H^2 + \lambda_p \frac{v_\Phi^2}{2}\right)|H|^2 + \frac{1}{2}\left(\lambda_H + \frac{\lambda_p^2}{\lambda_\Phi}\right)|H|^4 \\ &= \left(-m_H^2 + \lambda_p \frac{m_\Phi^2}{\lambda_\Phi}\right)|H|^2 + \frac{1}{2}\left(\lambda_H + \frac{\lambda_p^2}{\lambda_\Phi}\right)|H|^4. \end{aligned}$$

The light field is massless at tree level if $\lambda_\Phi\, m_H^2 = \lambda_p\, m_\Phi^2$.
A special point fulfilling this condition is $m_H^2 = m_\Phi^2 := m^2$ and $\lambda_p = \lambda_\Phi := \lambda$. At this point the original potential is given by

$$V = -m^2\left(|H|^2 + |\Phi|^2\right) + \frac{\lambda}{2}\left(|H|^2 + |\Phi|^2\right)^2 + \frac{\lambda_H - \lambda}{2}|H|^4$$

This potential is symmetric up to the quartic term of H which can violate the symmetry badly without affecting the light mass term at tree level.

Benchmark point 1 (BP)

μ [GeV]	g_X	g_{12}	λ_H	λ_p	λ_Φ	y_t	m_{h_Φ} [GeV]	$m_{Z'}$ [GeV]	m_h [GeV]	v_H [GeV]
$1.2 \cdot 10^{19}$	0.0713	0.	$\lambda_H = \lambda_p = \lambda_\Phi = 3.3030 \cdot 10^{-5}$			0.377	-	-	-	-
4353	0.0668	0.0093	**0.084**	$-1.6 \cdot 10^{-6}$	$-2.5 \cdot 10^{-11}$	**0.795**	67.0	5143	**132.0**	**263.0**
172	-	-	0.13	-	-	0.930	-	-	125.3	246.1

Table: Input parameters of an example benchmark point (BP) at the high scale (top) and corresponding predictions at the matching scale μ_0 (middle) and M_t (bottom). At μ_0 the bold parameters also correspond to the parameters of the one-loop SM effective potential. The numerical result for the VEV of Φ is $\langle\Phi\rangle = v_\Phi/\sqrt{2} = 54407\,\mathrm{GeV}$.

One-loop RGE's

Neglect all Yukawas besides y_t and take general $\mathrm{U}(1)_{\mathrm{X}}$ charges $q_{H,\Phi}$.

$$
\begin{aligned}
\beta_{\lambda_H} &= \frac{1}{16\pi^2}\left[+\frac{3}{2}\left(\left(\frac{g_Y^2}{2}+\frac{g_L^2}{2}\right)+2\left(q_H g_X+\frac{g_{12}}{2}\right)^2\right)^2+\frac{6}{8}g_L^4-6y_t^4\right. \\
&\qquad\left.+24\lambda_H^2+4\lambda_p^2+\lambda_H\left(12y_t^2-3g_Y^2-12\left(q_H g_X+\frac{g_{12}}{2}\right)^2-9g_L^2\right)\right], \\
\beta_{\lambda_\Phi} &= \frac{1}{16\pi^2}\left(+6q_\Phi^4 g_X^4+20\lambda_\Phi^2+8\lambda_p^2-12\lambda_\Phi q_\Phi^2 g_X^2\right), \\
\beta_{\lambda_p} &= \frac{1}{16\pi^2}\left[+6q_\Phi^2 g_X^2\left(q_H g_X+\frac{g_{12}}{2}\right)^2+8\lambda_p^2\right. \\
&\qquad\left.+\lambda_p\left(8\lambda_\Phi+12\lambda_H-\frac{3}{2}g_Y^2-6q_\Phi^2 g_X^2-6\left(q_H g_X+\frac{g_{12}}{2}\right)^2-\frac{9}{2}g_L^2+6y_t^2\right)\right], \\
\beta_{g_{12}} &= \frac{1}{16\pi^2}\left[-\frac{14}{3}g_X g_Y^2-\frac{14}{3}g_X g_{12}^2+\frac{41}{3}g_Y^2 g_{12}+\frac{179}{3}g_X^2 g_{12}+\frac{41}{6}g_{12}^3\right].
\end{aligned}
$$

The dominant splitting of $\lambda_\Phi - \lambda_p$ via running (for benchmark charges) is given by

$$
\beta_{\lambda_\Phi}-\beta_{\lambda_p} = -\frac{6\,g_{12}\,g_X^2}{16\pi^2}\left(g_X+\frac{g_{12}}{4}\right)-\frac{\lambda_p}{16\pi^2}\left[6y_t^2-\frac{9}{2}g_L^2-\frac{3}{2}g_Y^2+12(\lambda_H-\lambda_p)\right]+\ldots,
$$

We do the numerical running with the full two-loop beta functions computed with `PyR@TE`.

Higgs-dilaton mixing

A crude analytic expression for the Higgs-dilaton mixing angle is

$$\tan\theta \approx \frac{2\left[\lambda_p - \left(1+\frac{g_{12}}{2g_X}\right)^2\left(\lambda_\Phi - \frac{3g_X^4}{16\pi^2}\right)\right]v_H v_\Phi}{m_h^2 - m_{h_\Phi}^2}\,.$$

Note: We use a fully numerical evaluation of all masses and mixings for our analysis which also confirms the analytic approximations.