Workshop on Multi-Higgs Models

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शिक्षा मंत्रालय
MINISTRY OF EDUCATION
सत्यमेव जयते

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CUSTODIAL SYMMETRY AND RHO PARAMETER IN THE GEORGI-MACHACEK MODEL

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CUSTODIAL SYMMETRY

- **Spontaneous breaking of $SU(2)$ gauge symmetry** → **non-Abelian nature**

.... consider an interaction with the scalar field ϕ

$$D^{\mu}\phi = (\partial_{\mu} - igA^{\mu}_{a}T_{a})\,\phi$$

- **Doublet representation —**

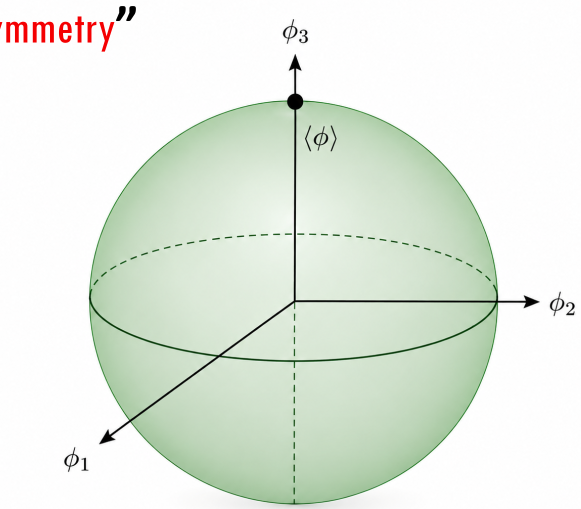
$$\langle\phi\rangle = \begin{pmatrix} 0 \\ v \end{pmatrix} \quad ; \quad m^2_{A_1} = m^2_{A_2} = m^2_{A_3} = \frac{1}{4}g^2v^2$$

.... invariant under global $SU(2)$ → **"Custodial $SU(2)$ Symmetry"**

- **Triplet representation —**

$$\langle\phi\rangle = \begin{pmatrix} 0 \\ 0 \\ v \end{pmatrix} \quad ; \quad m^2_{A_1} = m^2_{A_2} = g^2v^2 \;\; , \;\; m^2_{A_3} = 0$$

→ **there is a symmetry $A_1 \leftrightarrow A_2$, but not $SU(2)$**



CUSTODIAL SYMMETRY

- **Spontaneous breaking of $SU(2)_L \times U(1)_Y$ gauge symmetry**

.... consider an interaction with the scalar field ϕ

$$D^\mu \phi = (\partial_\mu - igA_a^\mu T_a - ig' B^\mu Y)\,\phi$$

- **Real triplet with $Y = 0$:**

$$\langle \phi \rangle = \begin{pmatrix} 0 \\ v \\ 0 \end{pmatrix} \quad ; \quad m_{A_1}^2 = m_{A_2}^2 = g^2 v^2 \ , \ m_{A_3}^2 = 0 \ , \ m_B^2 = 0$$

- **Complex triplet with $Y = 1$:**

$$\langle \phi \rangle = \begin{pmatrix} 0 \\ 0 \\ v \end{pmatrix} \quad ; \quad \boxed{m_{A_1}^2 = m_{A_2}^2 = g^2 v^2 \ , \ m_{A_3}^2 = 2g^2 v^2}$$

→ **SU(2) mass degeneracy is lifted**

$$m_B^2 = 2(g')^2 v^2 \ , \ m_{A_3 B}^2 = -\,2g' g v^2$$

CUSTODIAL SYMMETRY

- **Spontaneous breaking of $SU(2)_L \times U(1)_Y$ gauge symmetry**

…. consider an interaction with the triplet scalar field Δ

$$D^\mu \Delta = (\partial_\mu - igA_a^\mu T_a + ig' B^\mu T_3)\,\Delta$$

- **Real triplet ($Y=0$) + Complex triplet ($Y=1$) :**

$$\langle \Delta \rangle = \begin{pmatrix} v & 0 & 0 \\ 0 & v & 0 \\ 0 & 0 & v \end{pmatrix} \quad ; \quad m_{A_1}^2 = m_{A_2}^2 = 2g^2v^2 \,,\; m_{A_3}^2 = 2g^2v^2 \;\Rightarrow$$

SU(2) mass degeneracy is restored

$$m_B^2 = 2(g')^2v^2 \,,\; m_{A_3B}^2 = -2g'gv^2$$

follows from the global $SU(2)_L \times SU(2)_R$ symmetry

$$\boxed{\text{Gauging } T_3 \;\Rightarrow\; SU(2)_R \text{ breaking} \;\Rightarrow\; \text{mass splitting}}$$

RHO PARAMETER

- **Consequence of custodial $SU(2)$ symmetry**

$$W^{\mu}_{\pm} = \frac{1}{\sqrt{2}}(A^{\mu}_1 \mp A^{\mu}_2)$$

$$\begin{pmatrix} Z \\ \gamma \end{pmatrix} = \begin{pmatrix} c_w & -s_w \\ s_w & c_w \end{pmatrix} \begin{pmatrix} A_3 \\ B \end{pmatrix}$$

rho parameter, $\rho \equiv \frac{m_W^2}{m_Z^2 c_w^2} = 1$

  - **If $\rho \neq 1$ at tree level, ρ or c_w^2 becomes an additional free parameter.**

[Ross and Veltman '75]

...the case of a real / complex scalar triplet under $SU(2)_L$

- **Georgi-Machacek (GM) model**

[Georgi and Machacek '85]
[Chanowitz and Golden '85]

$$\langle \Phi \rangle = \begin{pmatrix} v_\phi & 0 \\ 0 & v_\phi \end{pmatrix} \quad \langle \Delta \rangle = \begin{pmatrix} v_\chi & 0 & 0 \\ 0 & v_\chi & 0 \\ 0 & 0 & v_\chi \end{pmatrix}$$

$$v^2 = v_\phi^2 + 8 v_\chi^2 \approx (246\ \text{GeV})^2$$

...under the global $SU(2)_L \times SU(2)_R$ $\xrightarrow{\text{SSB}}$ **custodial** $SU(2)_V$ $\rightarrow$ $\rho = 1$ **at tree level** **(prediction)**

What happens at one loop?

RHO

PARAMETER

- **At One Loop :**

$$\boxed{\rho = 1 + \Delta\rho}$$

- **What is the one-loop correction $\Delta\rho$?**
- **Is $\Delta\rho$ UV finite?**
- **If divergent, what is its structure (e.g., $\propto \frac{1}{\epsilon}$) ?**
- **How large is it numerically?**

Is $\Delta\rho$ UV finite?

If UV divergent

What additional renormalization condition is required?

$$\boxed{\Delta\rho = \frac{1}{m_W^2}\mathrm{Re}\left[\Sigma_{WW}(0) - c_w^2\Sigma_{ZZ}(0) - 2s_w c_w \Sigma_{\gamma Z}(0) - s_w^2\Sigma_{\gamma\gamma}(0)\right]}$$

[Peskin and Takeuchi ’90]

[Degrassi and Sirlin ’92]

[Chanowitz and Golden ’85]

[Gunion, Vega and Wudka ’91]

In the SM, $\Delta\rho$ is UV finite in the Feynman-’t Hooft gauge ($\xi = 1$)

But, in the GM model, $\Delta\rho$ is UV divergent in the Feynman-’t Hooft gauge ($\xi = 1$)

…despite the fact that both the GM model and the SM satisfy $\rho = 1$ at tree level

ULTRAVIOLET DIVERGENCES

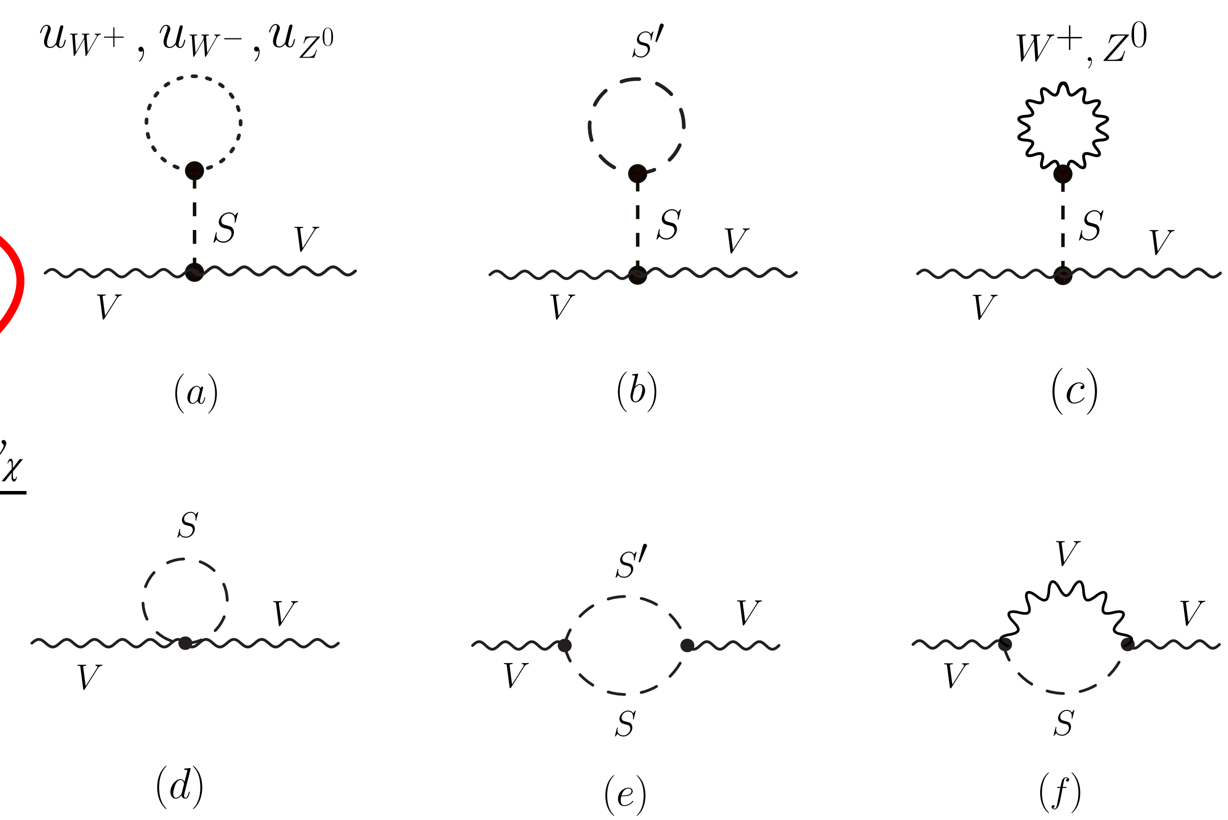
[Chowdhury, Kundu, Mondal, **SS**, Srivastava, arXiv: 2601.06252]

$$\Delta\rho = \frac{1}{m_W^2}\mathrm{Re}\Big[\Sigma_{WW}(0) - c_w^2\Sigma_{ZZ}(0) - 2s_w c_w \Sigma_{\gamma Z}(0) - s_w^2\Sigma_{\gamma\gamma}(0)\Big]$$

- **Unlike the SM, the GM model has a non-vanishing tadpole contribution to $\Delta\rho$**

$$\Delta\rho\Big|_{\text{a+b+c}} = -\frac{1}{\epsilon}\frac{(g')^2 s_\beta^2}{32\pi^2 s_w^2 m_{H_5}^2}\Big[6m_W^2 s_w^2(2+\tan^2\theta_W) + m_{H_5}^2(\xi_Z - c_w^2\xi_W)\Big]$$

...where $s_\beta = \frac{2\sqrt{2}v_\chi}{v}$



$V = W, Z, \gamma$

Tadpole contribution is gauge dependent and must be combined with the self-energy contribution to obtain a gauge-invariant result:

$$\Delta\rho\Big|_{\text{d+e+f}} = \frac{1}{\epsilon}\frac{(g')^2 s_\beta^2}{32\pi^2 s_w^2}\Big[3s_w^2 + \xi_Z - c_w^2\xi_W\Big]$$

One loop correction to $\Delta\rho$ is gauge-independent, but remains UV divergent

ONE LOOP
RENORMALIZATION

- **Input parameters:**
  - EM fine structure constant α_{em}
  - Fermi constant G_μ
  - Z-boson mass m_Z

Important: UV divergence

We find

$$\boxed{\Delta\rho \propto \frac{1}{\epsilon}}$$

i.e., $\Delta\rho$ is UV divergent ($\frac{1}{\epsilon}$ arises in dimensional regularization).

- **Counterterms:**

$$\left\{\begin{aligned}
\frac{\delta\alpha_{em}}{\alpha_{em}} &= \frac{d}{dp^2}\Sigma_{\gamma\gamma}(p^2)\Big|_{p^2=0} + \frac{2s_w}{c_w}\frac{\Sigma_{\gamma Z}(0)}{m_Z^2}\,,\\
\frac{\delta G_\mu}{G_\mu} &= \frac{\delta\alpha_{em}}{\alpha_{em}} - \frac{\delta m_W^2}{m_W^2} - \frac{\delta s_w^2}{s_w^2}\,,\\
\delta m_Z^2 &= \mathrm{Re}\,\Sigma_{ZZ}(m_Z^2)\,,\quad \delta m_W^2 = \mathrm{Re}\,\Sigma_{WW}(m_W^2)\,,\\
\frac{\delta s_w^2}{s_w^2} &= \frac{c_w^2}{s_w^2}\left(\frac{\delta m_Z^2}{m_Z^2} - \frac{\delta m_W^2}{m_W^2}\right).
\end{aligned}\right.$$

ONE LOOP RENORMALIZATION

- **Input parameters:**
  - EM fine structure constant α_{em}
  - Fermi constant G_μ
  - Z-boson mass m_Z

- **Counterterms:**

$$\left\{\begin{aligned}
\frac{\delta\alpha_{em}}{\alpha_{em}} &= \frac{d}{dp^2}\Sigma_{\gamma\gamma}(p^2)\Big|_{p^2=0} + \frac{2s_w}{c_w}\frac{\Sigma_{\gamma Z}(0)}{m_Z^2}\,,\\
\frac{\delta G_\mu}{G_\mu} &= \frac{\delta\alpha_{em}}{\alpha_{em}} - \frac{\delta m_W^2}{m_W^2} - \frac{\delta s_w^2}{s_w^2}\,,\\
\delta m_Z^2 &= \mathrm{Re}\ \Sigma_{ZZ}(m_Z^2)\,,\quad \delta m_W^2 = \mathrm{Re}\ \Sigma_{WW}(m_W^2)\,,\\
\frac{\delta s_w^2}{s_w^2} &= \cancel{\frac{c_w^2}{s_w^2}}\Big(\cancel{\frac{\delta m_Z^2}{m_Z^2}} - \frac{\delta m_W^2}{m_W^2}\Big)\,.
\end{aligned}\right.$$

Important: UV divergence

We find

$$\boxed{\Delta\rho \propto \frac{1}{\epsilon}}$$

i.e., $\Delta\rho$ is UV divergent ($\frac{1}{\epsilon}$ arises in dimensional regularization).

$$\boxed{\Delta\rho\Big|_{\mathrm{CT}} = \frac{\delta m_W^2}{m_W^2} - \frac{\delta m_Z^2}{m_Z^2} + \frac{\delta s_w^2}{c_w^2}\,.}$$

➡ **EW renormalization scheme must be extended in the GM model**

Additional input

Tadpole

$$\Delta\rho = \Delta\rho\Big|_{\mathrm{SE}} + \Delta\rho\Big|_{\mathrm{tad}} + \Delta\rho\Big|_{\mathrm{CT}}$$

Self-energy

Counterterm

[Chowdhury, Kundu, Mondal, **SS**, Srivastava, arXiv: 2601.06252]

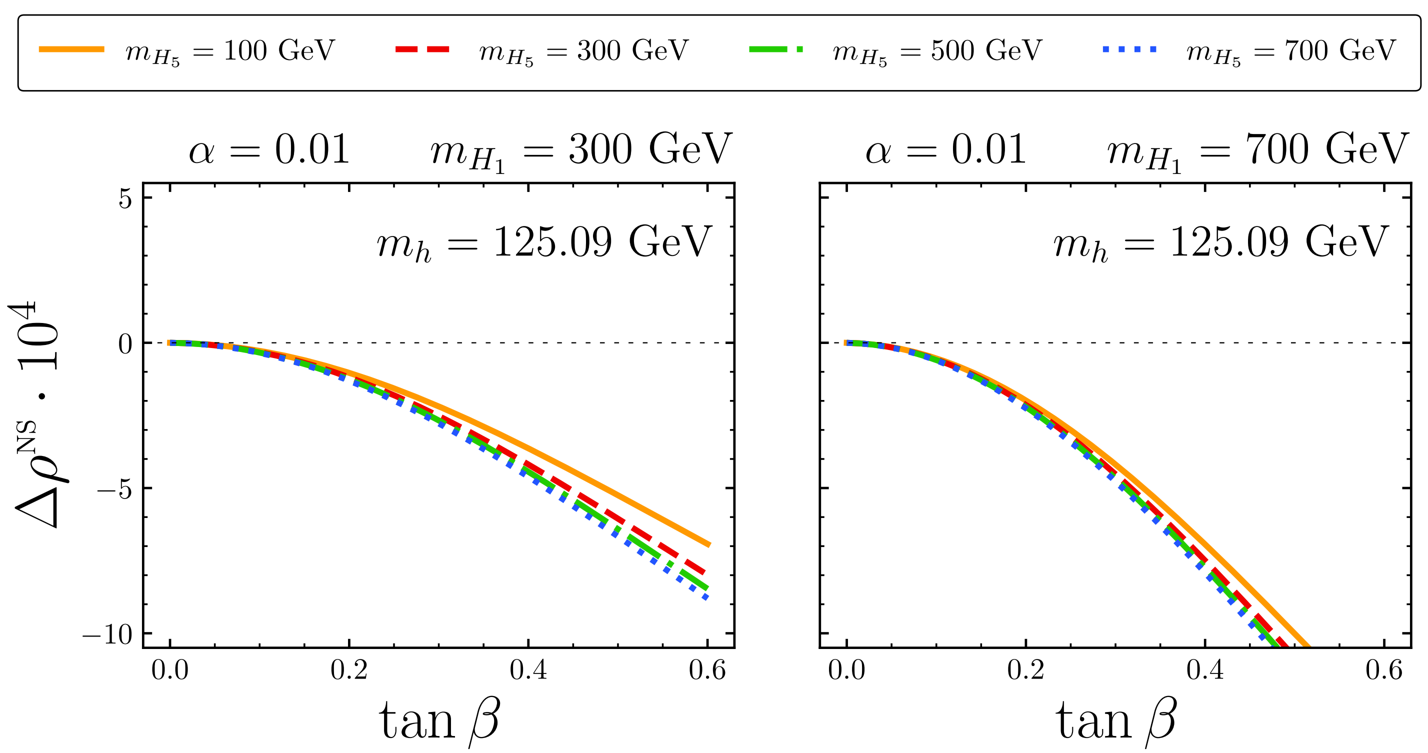
ONE LOOP CORRECTIONS

- **New Physics (NP) parameters —**

$$\boxed{\tan\beta\,,\quad \alpha\,,\quad m_{H_1}\,,\quad m_{H_3}\,,\quad m_{H_5}}$$

…where $\tan\beta = \dfrac{2\sqrt{2}v_\chi}{v_\phi}$

- Triplet VEV v_χ
- CP-even mixing angle α
- Custodial singlet mass m_{H_1}
- Custodial triplet mass m_{H_1}
- Custodial fiveplet mass m_{H_1}



Dependence of $\tan\beta$ on $\Delta\rho^{\mathbf{NS}}$

→ **One loop non-standard (NS) correction to $\Delta\rho$ is remains negative.**

→ **There is a no dependence on m_{H_3}**

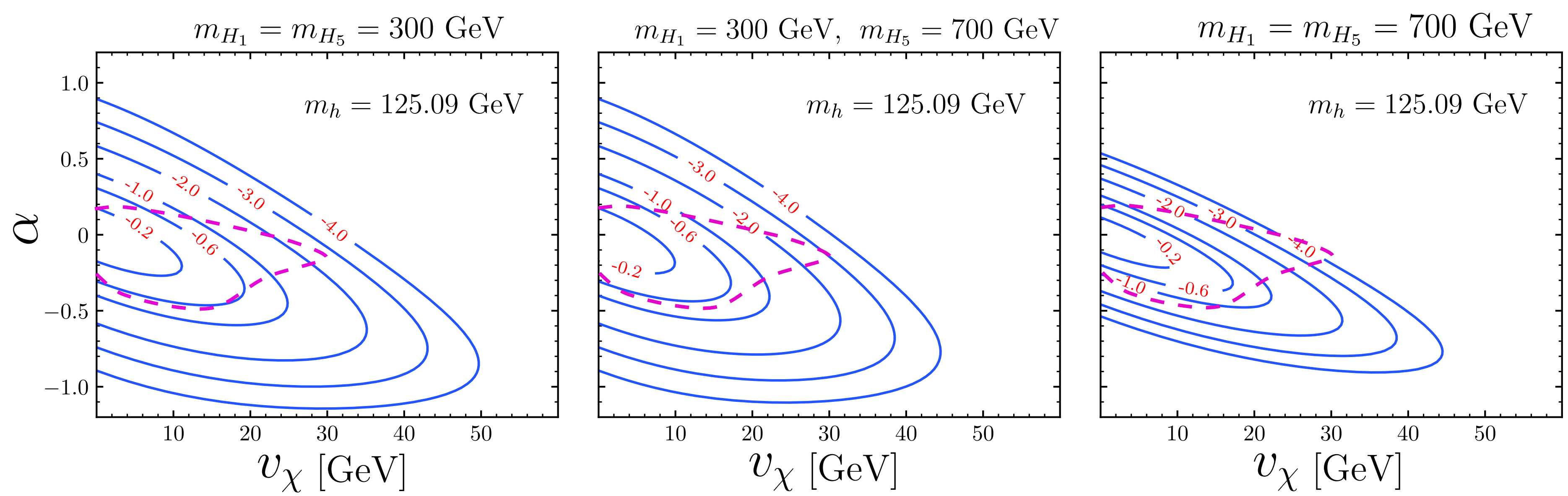
→ **$\Delta\rho^{\mathbf{NS}}$ approaches zero in the limit of $\tan\beta \to 0$ or in the limit of $g' \to 0$.**

[Chowdhury, Kundu, Mondal, **SS**, Srivastava, arXiv: 2601.06252]

SUMMARY AND OUTLOOK

In the GM model, the value of $\Delta\rho^{\text{NS}}$

- remains of order 10^{-4} across the parameter space, but **negative.**
- dependence on BSM masses is mild (**logarithmic**), while the dependence on v_χ is **quadratic.**



The contours of $\Delta\rho^{\text{NS}}$ (shown in blue color) in the v_χ vs. α plane. We denote the value of $\Delta\rho^{\text{NS}} \times 10^4$ for each contour in the red color. Magenta contours allowed from Higgs data.

→ **Current global EW fit (with 3 EW inputs) yields**

$$\rho = 1 + \Delta\rho^{\text{NS}} = 1.00031 \pm 0.00019 \rightarrow \text{Positive central value of } \Delta\rho^{\text{NS}}$$

→ **In the GM model,** ρ **parameter considered as an additional input.**

A full four-parameter EW fit is needed to reassess the constraints in the v_χ vs. α plane

Thank You!

BACKUP SLIDES

- **Higgs Potential:**

$$V_{\text{tree}}(\Phi,\Delta) = \frac{1}{2}m_1^2\text{Tr}[\Phi^\dagger\Phi] + \frac{1}{2}m_2^2\text{Tr}[\Delta^\dagger\Delta] + \lambda_1(\text{Tr}[\Phi^\dagger\Phi])^2 + \lambda_2(\text{Tr}[\Delta^\dagger\Delta])^2$$
$$+\lambda_3\text{Tr}[(\Delta^\dagger\Delta)^2] + \lambda_4\text{Tr}[\Phi^\dagger\Phi]\text{Tr}[\Delta^\dagger\Delta] + \lambda_5\text{Tr}[\Phi^\dagger\tau^a\Phi\tau^b]\text{Tr}[\Delta^\dagger t^a\Delta t^b]$$
$$+\mu_1\text{Tr}[\Phi^\dagger\tau^a\Phi\tau^b](P^\dagger\Delta P)_{ab} + \mu_2\text{Tr}[\Delta^\dagger t^a\Delta t^b](P^\dagger\Delta P)_{ab}$$

where $\Phi = \begin{pmatrix} \phi^{0*} & \phi^+ \\ -\phi^- & \phi^0 \end{pmatrix}$, and $\Delta = \begin{pmatrix} \chi^{0*} & \xi^+ & \chi^{++} \\ -\chi^- & \xi^0 & \chi^+ \\ \chi^{--} & -\xi^- & \chi^0 \end{pmatrix}$

$$\boxed{\mathscr{L}_H = \frac{1}{2}\text{Tr}[(D_\mu\Phi)^\dagger(D^\mu\Phi)] + \frac{1}{2}\text{Tr}[(D_\mu\Delta)^\dagger(D^\mu\Delta)] - V_{\text{tree}}(\Phi,\Delta)}$$

- **Electroweak Symmetry Breaking:**

$$\phi^0 \to \frac{v_\phi}{\sqrt{2}} + \frac{1}{\sqrt{2}}\left(\phi^0 + i\phi'\right) \; , \quad \chi^0 \to v_\chi + \frac{1}{\sqrt{2}}\left(\chi^0 + i\chi'\right) \; , \quad \xi^0 \to v_\xi + \xi^0$$

$$\rho = \frac{v_\phi^2 + 4(v_\chi^2 + v_\xi^2)}{v_\phi^2 + 8v_\chi^2} \xrightarrow{v_\chi = v_\xi} \boxed{\rho = 1}$$

BACKUP SLIDES

$$F[m_1^2, m_2^2] = \begin{cases} 0 & \text{if } m_1^2 = m_2^2\,, \\ \dfrac{m_1^2 + m_2^2}{2} - \dfrac{m_1^2 m_2^2}{m_1^2 - m_2^2} \ln \dfrac{m_1^2}{m_2^2} & \text{otherwise}\,. \end{cases}$$

Collider Phenomenology

- Yukawa sector :

Only the doublet couples to fermions

triplet VEV (v_χ) : $v_\phi^2 + 8v_\chi^2 = v^2$ and $\tan\beta = \frac{v_\phi}{2\sqrt{2}v_\chi}$

$v_\chi \downarrow \qquad \tan\beta \uparrow$

……Similar phenomenology as in type-I 2HDM

Additional features :

The presence of a doubly charged scalar,
makes these models highly interesting for collider studies.